Cosmological Phase Transitions Driven by Scalar Fields

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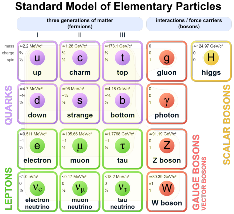
The Standard Model (SM):

- The SM describes matter and its interactions, except for gravity, at the most fundamental level:
  - Matter content: quarks and leptons
  - Force carriers: g, γ, $W^{\pm}$, Z^0
  - Higgs boson H:

    Masses of $W^{\pm}, Z^0$ but also fermions.

    The only fundamental scalar particle.



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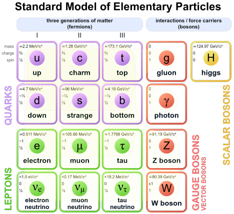
- Has been well tested and validated experimentally.
- However, the scalar sector is not so well understood (Higgs self-interaction).
- Also, various shortcomings motivating physics beyond the SM.

Shortcomings of the Standard Model (SM):

- The SM explains the electroweak (EW) and strong interactions, but does *not* explain gravity.

  Possible to achieve a unified description?
- "Aesthetic" issues:
  - Large number of parameters (25[1]).

    Are they free?
  - Fine-tuning/hierarchy problem:
    - Why is the Higgs boson so light?
    - Why is the EW interaction so much stronger than gravity?
- What is the nature of neutrino masses (as evidenced by neutrino oscillations)?



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[T. W. Kibble, 2015, J. M. Butterworth, 2016, S. Weinberg, 2018]

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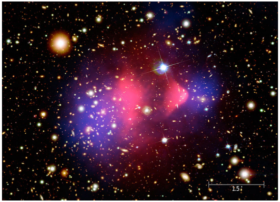
[1] If we include neutrino masses and neutrino oscillations.

Further shortcomings of the SM:

- The SM cannot explain why there is so much more matter than antimatter (baryon asymmetry).
- The SM cannot explain dark energy (severe mismatch between SM vacuum energy and dark energy density).
- The SM cannot explain the nature of dark matter.



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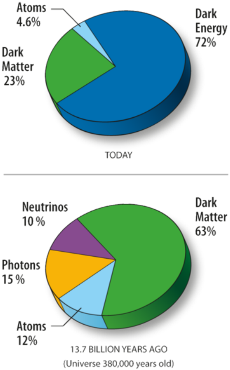

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[G. Bertone et al., 2005, A. Ahriche, 2007]

Baryon asymmetry continued:

- Why is there so much more matter than antimatter?
- Conditions for baryogenesis: [A. Sakharov, 1967]
  - Baryon number violation
  - Breaking of C- and CP-symmetry
  - Processes (strongly) out of thermal equilibiurm → phase transition (PT)

Credit: Wikipedia

- The SM cannot account for measured baryon asymmetry:
  - Too little CP-violation
  - Electroweak PT (EWPT) not strong enough

[G. Bertone et al., 2005, A. Ahriche, 2007]

Going beyond the SM

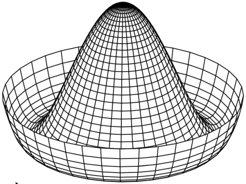
- There are many ways to go beyond the SM.
- For example, extend the scalar sector, by adding more Higgs-like fields:
  - Example: Two-Higgs doublet model (2HDM)

    [I. P. Ivanov, 2017, G. Branco et al., 2012, A. Ahriche, 2007]
- How to detect new physics?:
  - New particles: $h, H, A, H^{\pm}$ (2HDM), ...
  - Changes to EWPT → affects baryon asymmetry
  - Gravitational waves (GW) from cosmological PT:
    - Planned detectors: LISA, DECIGO, BBO

[C. Caprini et al., 2019, P. Amaro-Seoane *et al.* (LISA Collaboration), 2017, Kawamura Seiji *et al.*, 2011, Vincent Corbin and Neil J Cornish , 2006]

Higgs sector of the SM:

- $\mathcal{L}_{\text{Higgs}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V_{\text{Higgs}}(\Phi)$
  - Φ: $\mathrm{SU}(2)$ doublet
  - Covariant derivative:
    $$D_\mu = \partial_\mu - ig\frac{\tau_a}{2}W^a_\mu - ig'\frac{Y}{2}B_\mu$$
  - Higgs potential:
    $$V_{\text{Higgs}}(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \ (\lambda > 0)$$



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- Degenerate minimum given by
  $$\Phi^\dagger\Phi = -\mu^2/(2\lambda) \text{ (for } \mu^2 < 0)$$
- The system is going to be in *some* ground state $\Rightarrow$ spontaneous symmetry breaking

[P. W. Higgs, 1964]

Higgs sector of the SM (cont.):

- Expand around vacuum: $\Phi = (0, v + h)^T/\sqrt{2}$, where v is the vacuum expectation value (vev):

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{4}v^4\lambda + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 - \lambda v h^3 - \frac{1}{4}\lambda h^4 \\ + m_W^2 W_\mu^+ W^{\mu-} + m_W g W_\mu^+ W^{\mu-} h + \frac{1}{4}g^2 W_\mu^+ W^{\mu-} h^2 \\ + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{m_Z g}{2\cos\theta_W} Z_\mu Z^\mu h + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu h^2.$$

- The Higgs boson h appears as excitation w.r.t. the vev.
- Mass terms for W and Z bosons:
  - $m_W = gv/2$
  - $m_Z = \sqrt{g^2 + g'^2}v/2$
- Can also use Higgs doublet to accommodate fermion masses.

[P. W. Higgs, 1964]

Higgs potential and symmetry breaking in 2HDM:

- Two $SU(2)$ Higgs doublets Φ_1, Φ_2
- Tree-level potential:

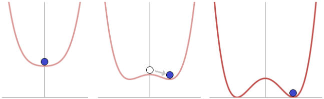
$$V_{\mathrm{2HDM}} = m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{ h.c.}] + \frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 \\ + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ + \left[\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + [\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2) + \text{ h.c.}\right]$$

- Minima of the potential (if not $[m_{ij}^2]$ is positively definite):
  - $\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}(0, v_1)^T$, $\langle\Phi_2\rangle = \frac{1}{\sqrt{2}}(0, v_2)^T$
  - Two vevs: v_1 and v_2
- Higgs spectrum: $h, H, A, H^\pm$

[I. P. Ivanov, 2017, G. Branco et al., 2012]

Overview

- Scalar fields: $\boldsymbol{\phi} = (\phi_1, ..., \phi_{n_S})$ Vacuum : $\boldsymbol{\omega} = (\omega_1, ..., \omega_{n_S})$
- Thermal effects modify the potential → Thermal effective potential $V_{\text{eff}}(\boldsymbol{\omega}, T)$:
  - One option: 4D potential + thermal corrections [R. R. Parwani, 1992]
  - Another option: Matching to 3D theory (dimensional reduction) [A. Ekstedt et al., 2023]
- Basic idea: As T changes, the depth of the minima change. When a lower minimum develops, a phase transition (PT) takes place.



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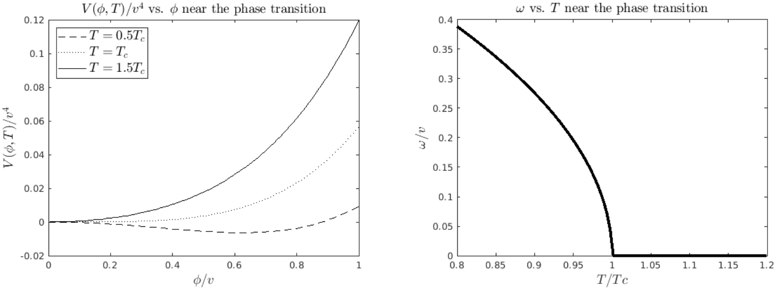
EWPT with SM-like potential

- Simplified effective potential at high T: [M. Quiros, 1999]

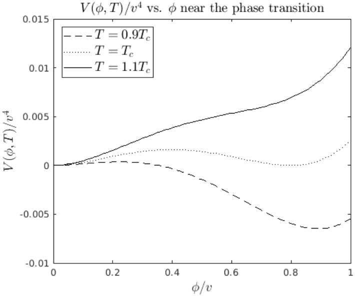
$$V_{\text{eff}}(\omega, T) = D(T^2 - T_0^2)\omega^2 - ET\omega^3 + \frac{1}{4}\lambda(T)\omega^4$$

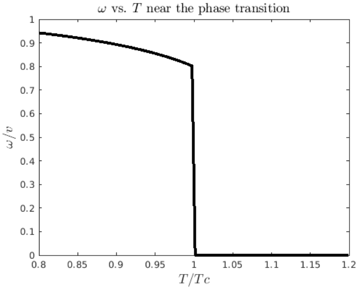
- PT occurs between degenerate minima at critical $T = T_c$.
- $E = 0$ gives 2:nd order PT: ω changes continuously at T_c.



EWPT with SM-like potential, continued

- $E \neq 0$ gives 1:st order PT: ω discontinuous at T_c,



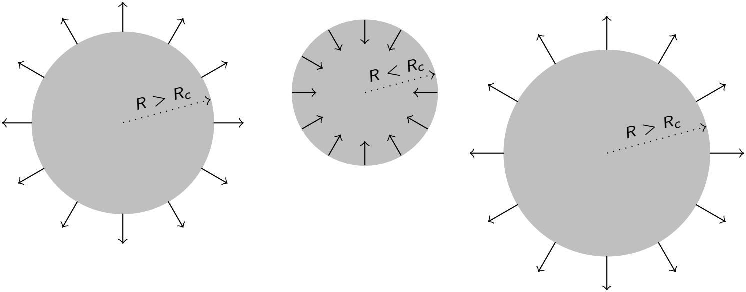


- Order parameter:
  $\Delta v_c/T_c = [\lim_{T\to T_c^-}\omega(T) - \lim_{T\to T_c^+}\omega(T)]/T_c$
- Strongly first order: $\Delta v_c/T_c \gtrsim 1$.
- Note: Not this strong with SM-parameters (actually a crossover) $\Rightarrow$ insufficient baryogenesis. [A. Ahriche, 2007]

Dynamics of cosmological FOPT

- The phase transition is not instantaneous due to potential barrier.
- Thermal jumps and tunnelling across barrier drive transition.

  [S. Coleman, 1977, A. Linde, 1983, M. E. Carrington et al., 1993]
- Transition is not homogeneous in space → bubble nucleation



- Free energy volume term $\propto R^3$ and surface term $\propto R^2$ → critical bubble radius R_c.

Dynamics of cosmological FOPT, continued

- Extremize the Euclidean action at finite temperature:

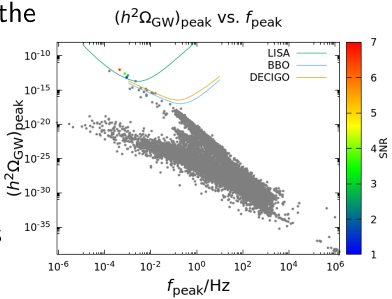
$$S_3[\phi] = 4\pi \int r^2 dr \left[(1/2)(d\phi/dr)^2 + V_{\mathrm{eff}}(\phi, T) \right]$$

- Key parameters to extract: [C. Caprini et al., 2016]
  - Nucleation rate: $\Gamma/V = Ae^{-S_3/T}$
  - Nucleation temperature: T_n
  - Energy release/radiation energy in plasma: α
  - Inverse duration: β/H
- Gravitational waves (GW) generated by bubble dynamics:
  - Collisions of bubble walls.
  - Sound waves in the plasma.
  - Turbulence due to magnetohydrodynamic effects.
- Power spectrum of GWs can be calculated from PT params:

$$h^2\Omega_{\mathrm{GW}}(f) = h^2 \frac{d(\rho_{\mathrm{GW}}/\rho_c)}{d \ln f}$$

Phenomenology of multiscalar models

- A model like 2HDM is a whole class of models: the phenomenology depends on the parameter values (model realization)
- Overall approach:
  - Scan parameter space.
  - Exclude model realizations ruled out by collider data.
  - Study PT and calculate GW spectrum of surviving models.
- Current focus: Develop tools and methods to reliably calculate PT parameters.



Calculating the bubble action

- Recall: PT parameters are governed by the bubble action S_3
- The S_3 action is found by extremizing the action functional

$$S_3[\phi] = 4\pi \int r^2 dr \left[(1/2)(d\phi/dr)^2 + V_{\text{eff}}(\phi, T)\right]$$

  which has to be done numerically.
- Two directions of research:
  - Generic tool for calculating V_{eff}
    - 4D approach ("standard") has many issues
    - Matching to 3D EFT (dimensional reduction) more sound
  - Numerical tool to robustly calculate S_3
- Many other problems to solve:
  - Bubble wall speed
  - Nucleation "prefactor" A in $\Gamma/V = Ae^{-S_3/T}$

Calculating V_{eff} through Dimensional Reduction

- Imaginary time formalism: $t \to -i\tau$, where τ is periodic (antiperiodic) for bosons (fermions) with period $\beta = 1/T$
- The time dimension disappears and is replaced by a sum over discrete frequencies $\omega_n = n\pi T, n \in \mathbb{Z}$ (Matsubara modes) $\Rightarrow$

  Tower of particle masses: $m^2 \to m^2 + (n\pi T)^2, n \in \mathbb{Z}$
- Fermionic modes have n odd $\Rightarrow n \neq 0$, so they are *heavy*.

  Bosonic modes have n even. Those with $n \neq 0$ are also *heavy*.
- Integrating out (averaging over) the heavy modes leaves us with a 3D effective field theory (EFT) matched to the 4D theory: DRalgo package [A. Ekstedt et al., 2023]
- My current research: Creating a pipeline from DRalgo to the calculation of PT parameters for generic models

Conclusion:

- There are several reasons to go beyond the SM:
  - Dark matter, dark energy, baryogenesis, . . .
  - Unification, hierarchy problem, . . .
- Signals from BSM models can show up in many places:
  - Collider experiments
  - Dark matter properties
  - Gravitational waves due to phase transition

    ⋮
- Extracting phase transition parameters in a robust and fast manner is a challenging problem.
- Solving this problem is crucial for the understanding of the cosmological implications of BSM models.

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