Erice School for Complexity, 23 Apr. 2024

Neural Networks, Quantum systems and Gravity

Koji Hashimoto (Kyoto U)

String Data 2024

Dec. 10-12, 2024 Kyoto Univ, Japan

Local organizers :
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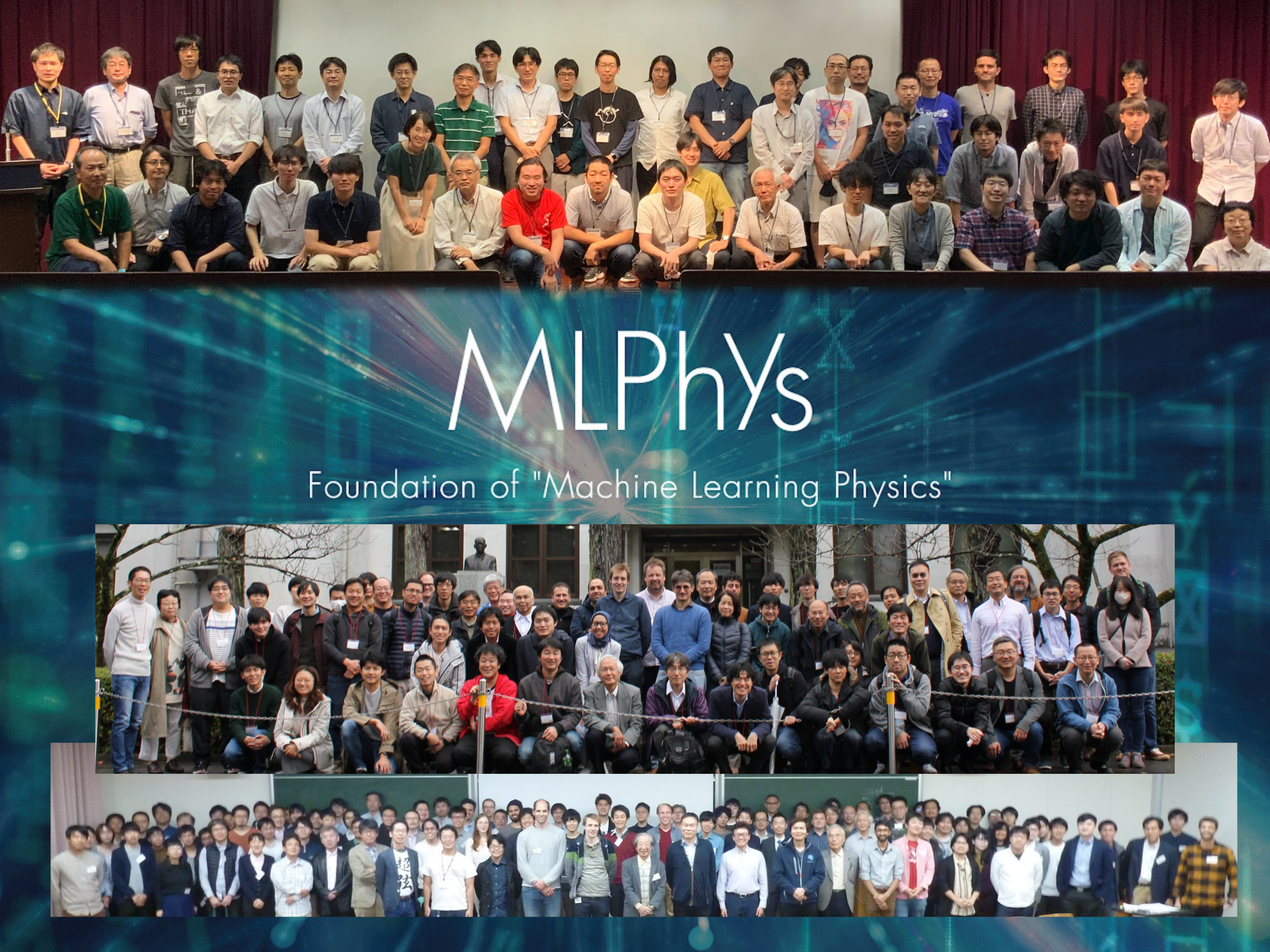
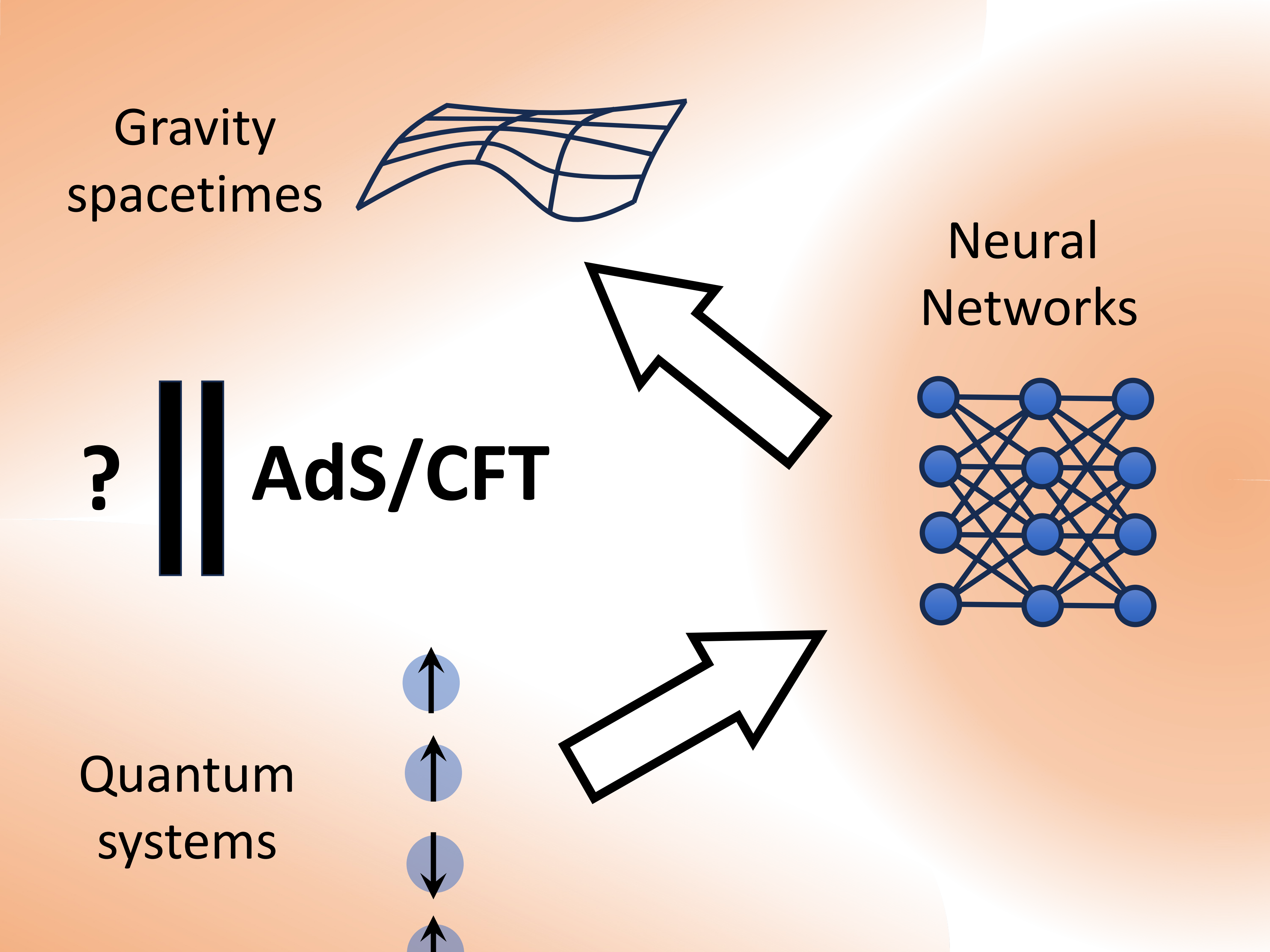

MLPhYs
Foundation of "Machine Learning Physics"


Gravity
spacetimes
Neural
Networks
? = AdS/CFT
Quantum
systems

References

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[Sugishita, Tanaka, Tomiya, KH 1802.08313]

“Deep Learning and Holographic QCD” [Sugishita, Tanaka, Tomiya, KH 1809.10536]

“Deep Boltzmann Machine and AdS/CFT” [KH 1903.04951]

“Deep Learning and AdS/QCD” [Akutagawa, Sumimoto, KH 2005.02636]

“Neural ODE and Holographic QCD” [Hu, You, KH 2006.00712]

“Deriving dilaton potential in improved holographic QCD from meson spectrum”
[Ohashi, Sumimoto, KH 2108.08091]

“Deriving dilaton potential in improved holographic QCD from chiral condensate”
[Ohashi, Sumimoto, KH 2209.04638]

“Machine Learning Spatial Geometry from Entanglement Features”
[You, Yang, Qi 1709.01223]

“Machine Learning Statistical Gravity from Multi-Region Entanglement Entropy”
[Lam, You 2110.01115]

[Vasseur, Potter, You, Ludwig 2019] [Tan, Chen 2019] [Yan, Wu, Ge, Tian 2020]
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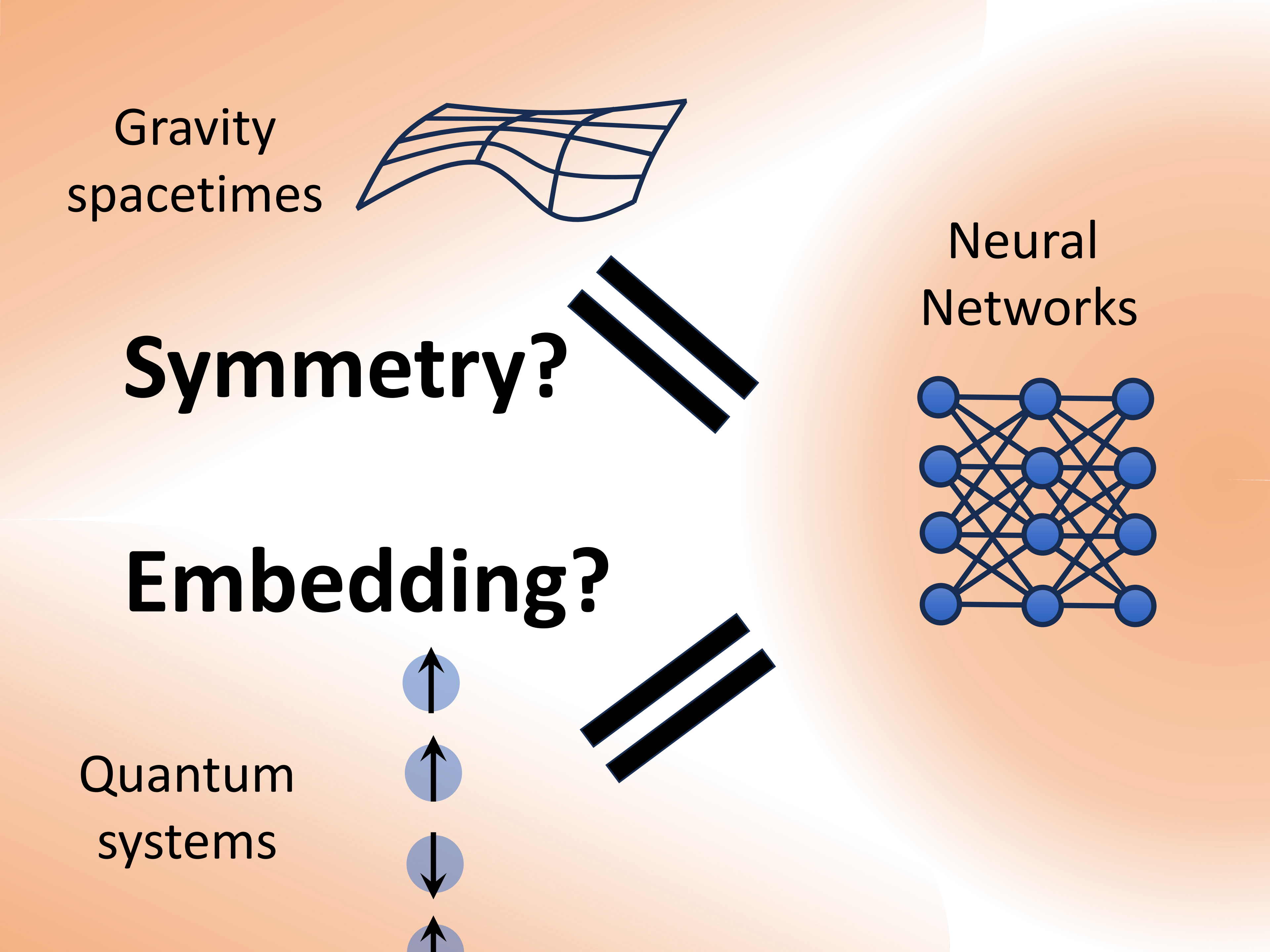
[Li, Ling, Liu, Wu 2023] ....


Gravity
spacetimes
Neural
Networks
Symmetry?
Embedding?
Quantum
systems

References

Symmetry?

"Unification of Symmetries Inside Neural Networks:
Transformer, Feedforward and Neural ODE"
[Hirono, Sannai, KH 2402.02362]

Embedding?

Neural Network Field Theory
[Halverson, Maiti, Stoner 2008.08601]
[Erbin, Lahoche, Samary 2018.01403] [Halverson 2112.04527]
[Demirtas, Halverson, Maiti, Schwartz, Stoner 2307.03223]
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"NN representation of quantum systems"
[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

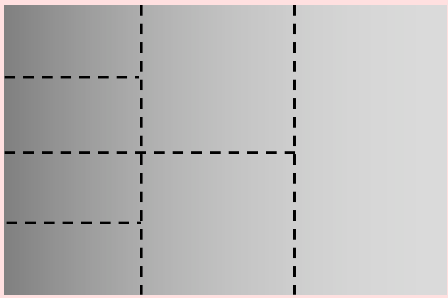
Random neural fields [Amari 1971] …

Roadmap

Quantum
gravity
in (d+1)-dim.

General
spacetime

Anti de Sitter
spacetime



‘tHooft `93
Susskind `94
Maldacena `97

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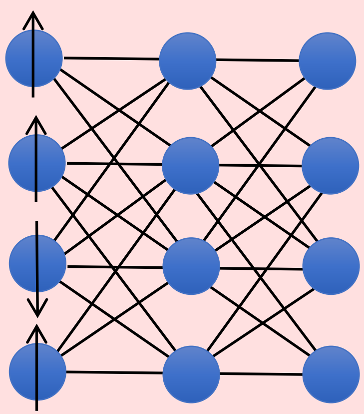
|| ?

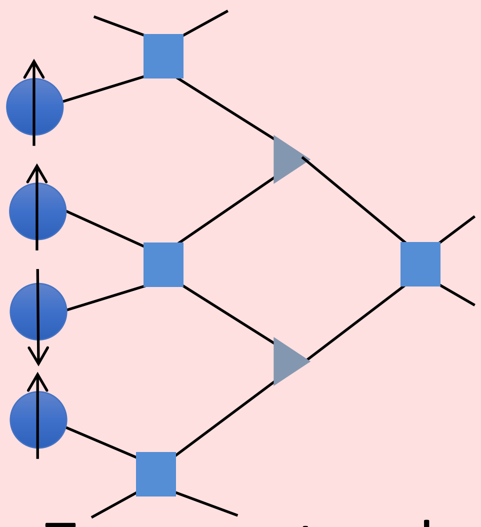
|| Swingle `10

Quantum
mechanics
in d-dim.



Neural network

Carleo,
Troyer `17



Tensor network

Roadmap

1.

Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

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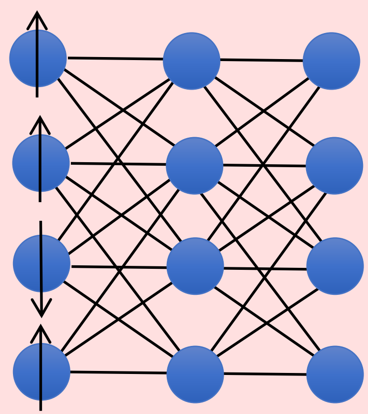
Quantum mechanics in d-dim.

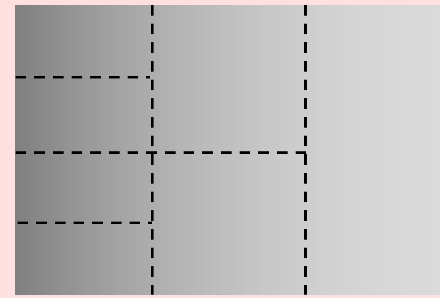
General spacetime

|| ?



Neural network

Anti de Sitter spacetime



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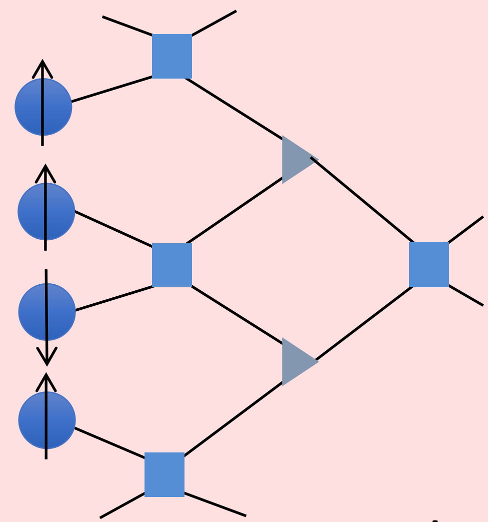

Tensor network

Carleo, Troyer `17

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Quantum gravity in (d+1)-dim.

General spacetime

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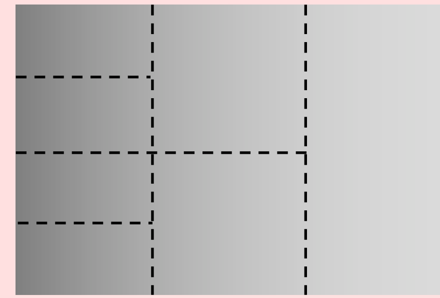

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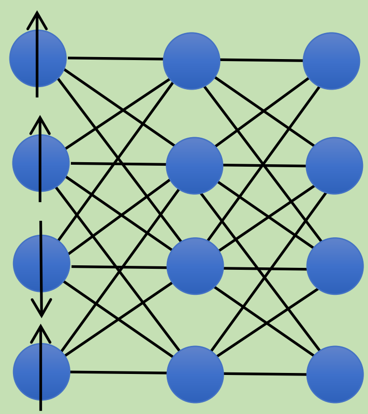
|| Swingle `10

Quantum mechanics in d-dim.

2.



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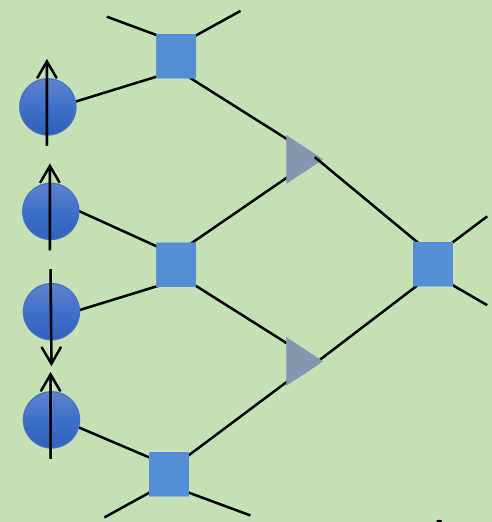

Tensor network

Roadmap

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Quantum mechanics in d-dim.

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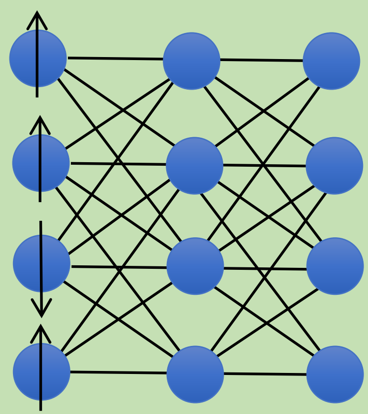

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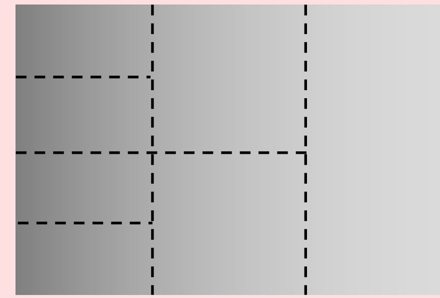
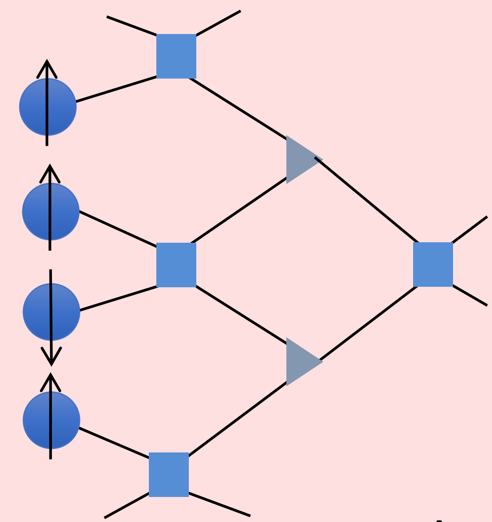


Swingle `10



Tensor network

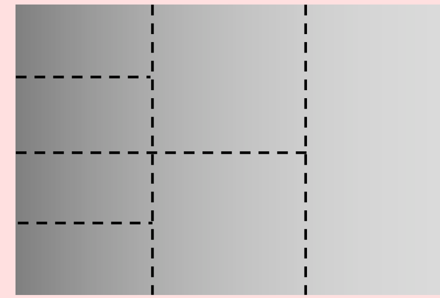
Roadmap

4.

Quantum gravity in (d+1)-dim.

General spacetime

Anti de Sitter spacetime



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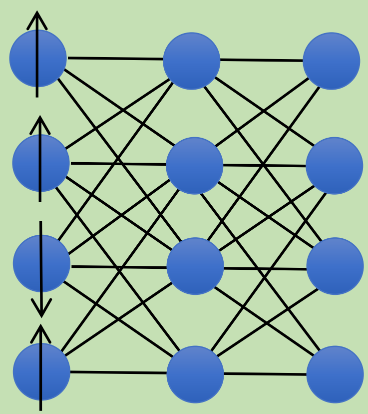
Swingle `10

Quantum mechanics in d-dim.



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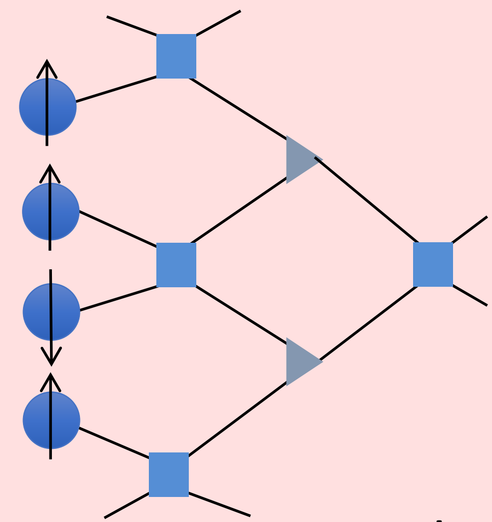

Tensor network

Deep Learning and Quantum Gravity

1. Quantum gravity — 4 pages
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Discussion: Quantum gravity $\subset$ ML ?

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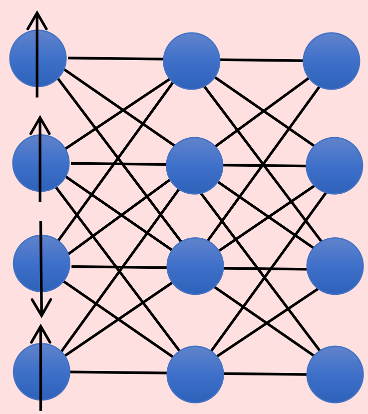
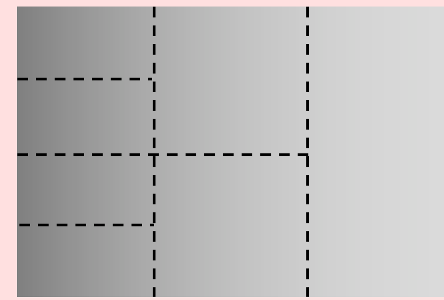


Neural network

Anti de Sitter spacetime



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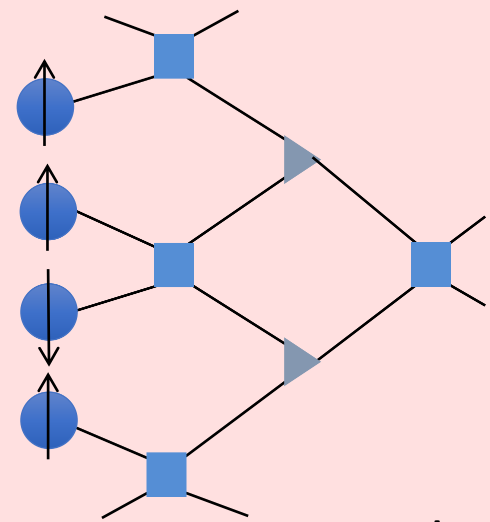


Tensor network

Carleo, Troyer `17

① Quantum Gravity

Brief History of quantum gravity

1974 ‘tHooft, Veltman:
Perturbation fails in Einstein gravity.

1970 Nambu, Susskind, Nielsen:
String theory of hadrons.

1974 Yoneya, Scherk, Schwarz:
String is quantum gravity.

1971 Bekenstein:
Black hole entropy.

1993 ‘tHooft, Susskind:
Holographic principle.

1997 Maldacena:
AdS/CFT correspondence.

① Quantum Gravity

AdS/CFT correspondence, no proof

[Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231]

"CFT"		"AdS"
"Large N" Quantum mechanics in d-dim. spacetime	=	Classical ~~Quantum~~ gravity in (d+1)-dim. spacetime

- Vast amount of examples known
- No proof! How does it work?
- Given Left, how can one get Right?

① Quantum Gravity

Dictionary : equating partition functions

[Gubser, Klebanov, Polyakov, Phys.Lett.B428(1998)105]
[Witten, Adv.Theor.Math.Phys. 2 (1998) 253]

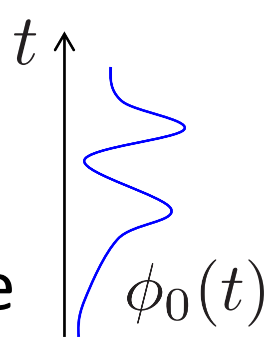
Partition function of Quantum mechanics $=$ Partition function of Classical gravity

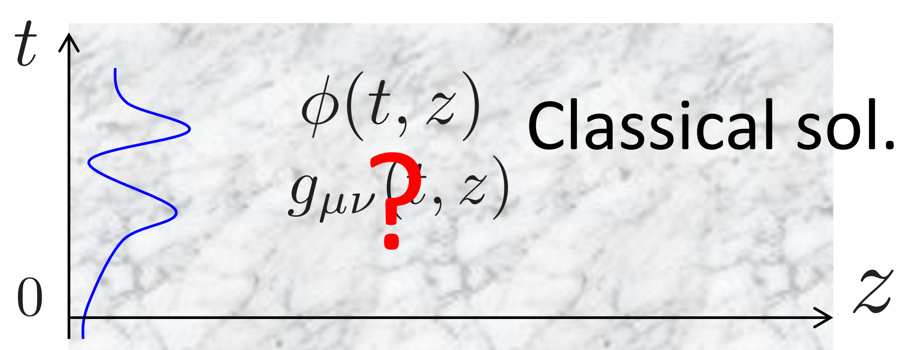
$$Z[\phi_0] = Z[\phi_0]$$

$$Z[\phi_0] = \int [\mathcal{D}q(t)] e^{-\int dt(\mathcal{L}[q,\dot{q}]+\phi_0(t)\mathcal{O}[q])}$$

$$Z[\phi_0] = e^{-\int dtdz\sqrt{-g}(R[g]+\mathcal{L}[\phi]+\cdots)}$$

source





$\phi_0(t) = \phi(t, z=0)$

① Quantum Gravity

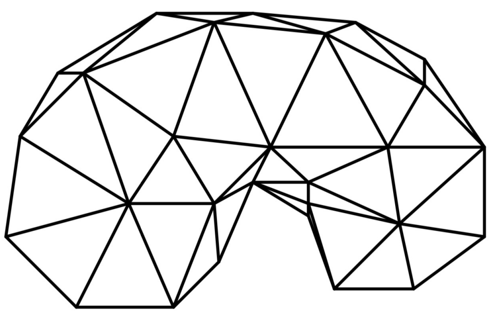
Quantum geometry is a network

Regge calculus

[Regge 1961]

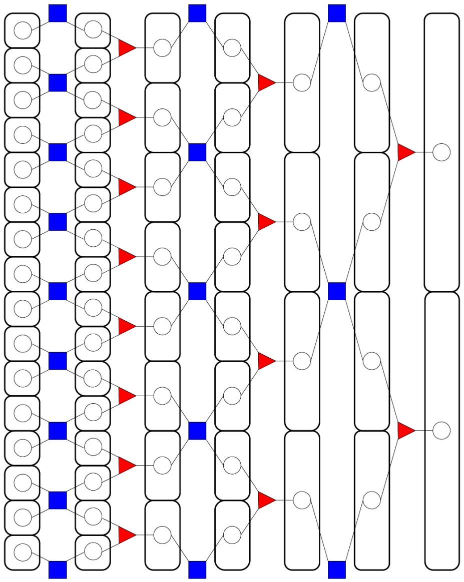
Causal dynamical triangulation

[Ambjorn, Loll 1998]



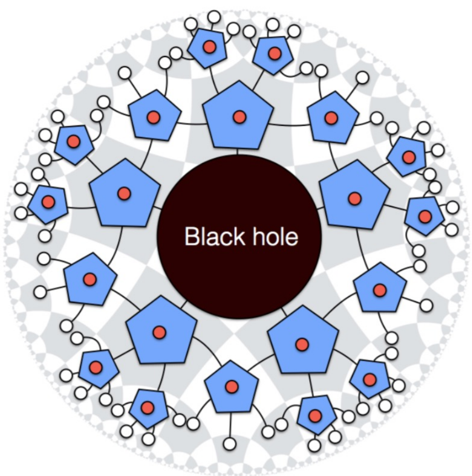
AdS/MERA (Tensor Network)

[Swingle `09]



Quantum codes for holography

[Pastawski, Yoshida, Harlow, Preskill `15]



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Roadmap

Quantum gravity in (d+1)-dim.

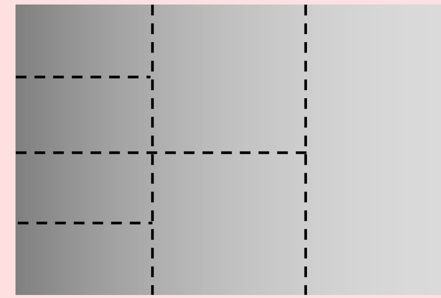
'tHooft `93
Susskind `94
Maldacena `97

Quantum mechanics in d-dim.

General spacetime

|| ?

Anti de Sitter spacetime



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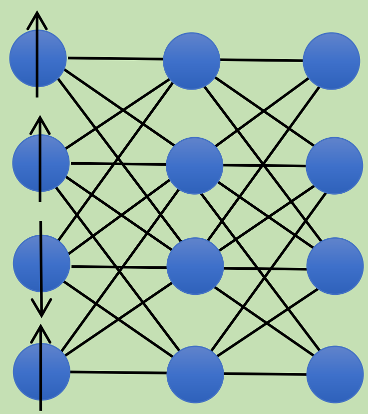
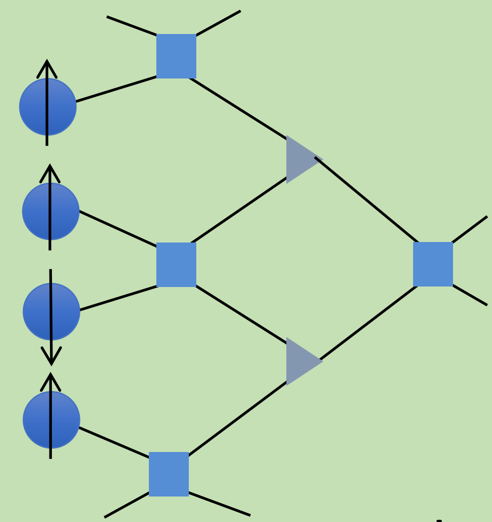
2.



Neural network

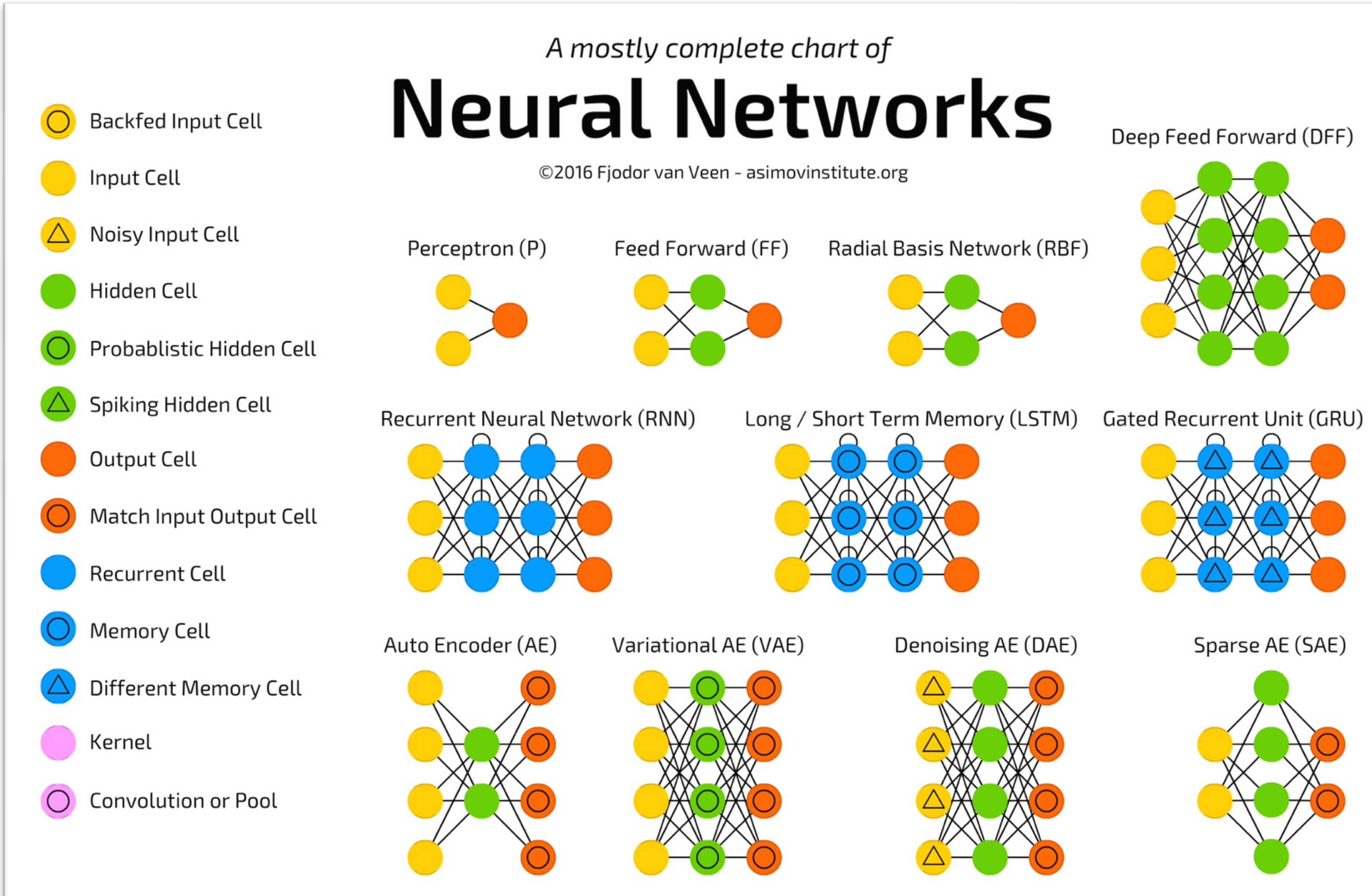
Carleo, Troyer `17



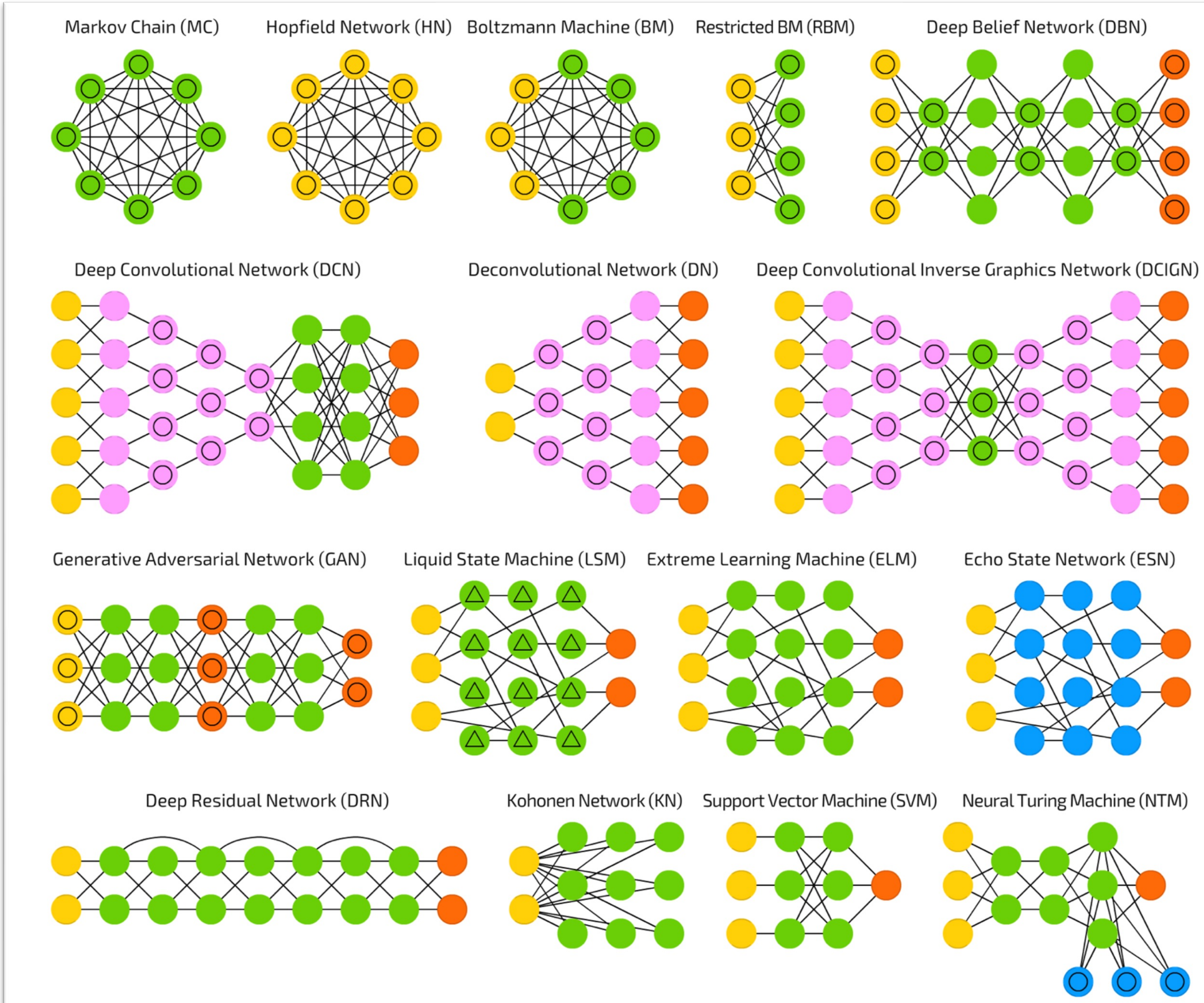
Tensor network

② Neural Network Quantum States 1/6

Various neural networks were invented



http://www.asimovinstitute.org/neural-network-zoo/

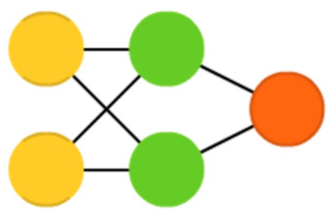

Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

② Neural Network Quantum States

Machine learning = function approximator

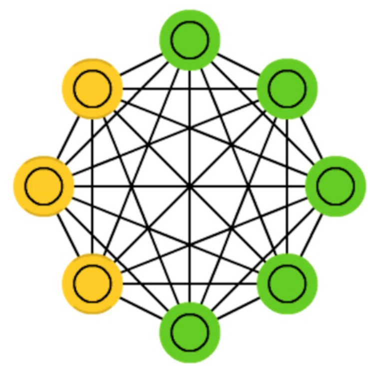
Input: a vector $(x_1, x_2, x_3, \cdots)$
Output: a value $f(x_1, x_2, x_3, \cdots)$

Network architecture is the function ansatz



Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

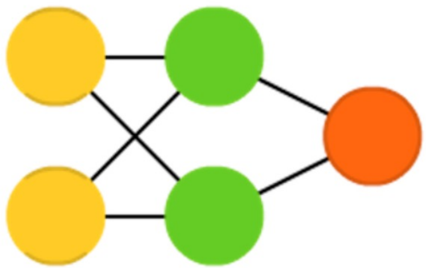


Boltzmann machine
[Ackley, Hinton, Sejnowski 1985]

② Neural Network Quantum States

Neural network for classification

Perceptron model



$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

"Unit" (circle) : Vector component

"Weight" (line) : Linear transformation to be optimized

"Activation function" (hidden line-end) : Nonlinear component-wise transf.

$$\varphi(x) \equiv \frac{1}{1 + e^{-x}}$$

- Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: (input, output)
2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right) \right|$

2. Neural Network Quantum States

Find ground state wave function $\psi(s_1, s_2, \cdots, s_N)$

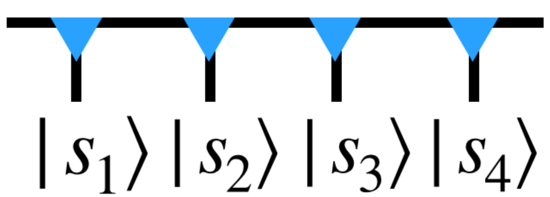
Q : Minimize its energy *E* for a given Hamiltonian *H*,

$$E = \frac{\sum_{s_1,\cdots s_N, s'_1,\cdots,s'_N} \psi^\dagger(s'_1,\cdots,s'_N)\hat{H}_{s'_1,\cdots s'_N,s_1,\cdots,s_N}\psi(s_1,\cdots,s_N)}{\sum_{s_1,\cdots s_N}\psi^\dagger(s_1,\cdots,s_N)\psi(s_1,\cdots,s_N)}$$

A : Use ansatz and optimize parameters!

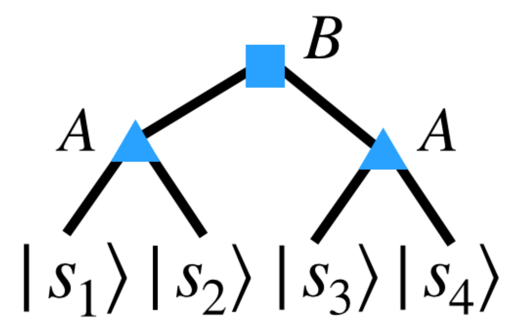
- Matrix product states

$$\psi(s_1, s_2, \cdots) = \mathrm{tr}[A^{(s_1)}A^{(s_2)}\cdots]$$



- Tensor network states

$$\psi(s_1, s_2, \cdots) = \sum_{m,n} B_{mn}A_{ms_1s_2}A_{ns_3s_4}$$



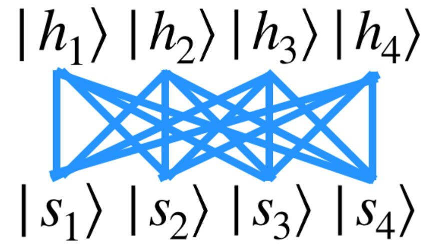
② Neural Network Quantum States

Neural network can be wave functions

- Boltzmann machine states

  [Carleo, Troyer `17],

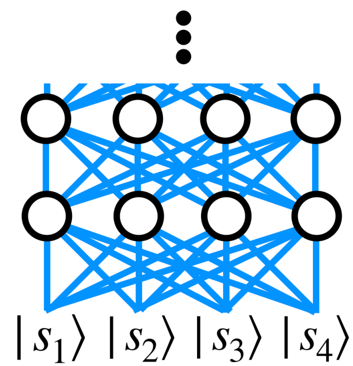
  [Nomura, Darmawan, Yamaji, Imada `17], ..



$$\psi(s_1, \cdots, s_N) = \sum_{h_A} \exp\left[\sum_a a_a s_a + \sum_A b_A h_A + \sum_{a,A} J_{aA} s_a h_A\right]$$

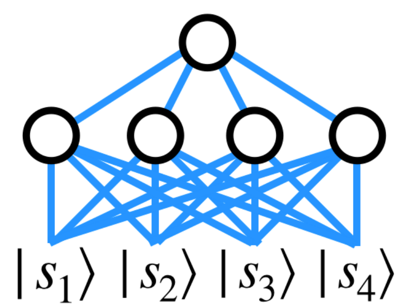
- Deep Boltzmann machine states

  [Carleo, Nomura, Imada `18], ..



- Feedfoward network states [Saito `18], ..

$$\psi(s_1, \cdots, s_N) = \sum_i f_i\, \sigma\left(\sum_j W_{ij} s_j + b_i\right)$$



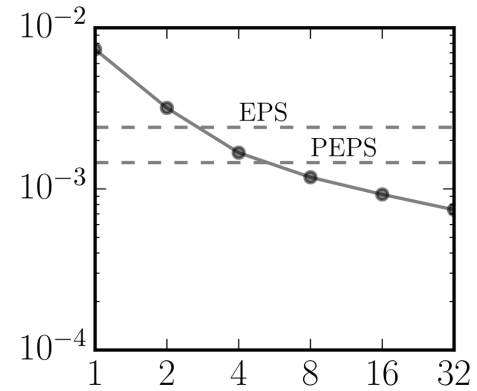
② Neural Network Quantum States

Better? and Why?

Neural states may beat conventional ones.

Ex) 2-dimensional antiferromagnetic Heisenberg model [Carleo, Troyer `17]

Energy with RBM states



# of hidden units

Discovered intimate relations are there.

1) Boltzmann machine states are tensor network states [Chen, Cheng, Xie, Wang, Xiang `18]

2) Tensor states are deep Boltzmann [Gao, Duan `17] [Huang, Moore `17]

3) Tensor states are feedforward with "product pooling" [Cohen, Shashua `18]

Ex) Unified approach: MPO-Net [Gao, Cheng, He, Xie, Zhao, Lu, Xiang `19]

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gravity
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'tHooft `93
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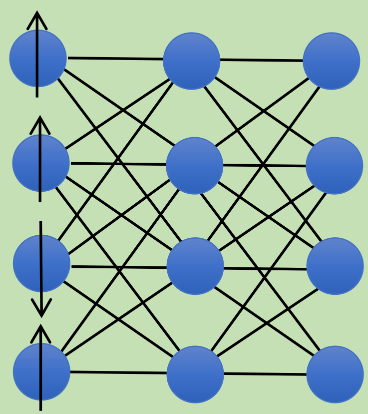
Quantum
mechanics
in d-dim.

3.

General
spacetime

|| ?



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Carleo,
Troyer `17

Anti de Sitter
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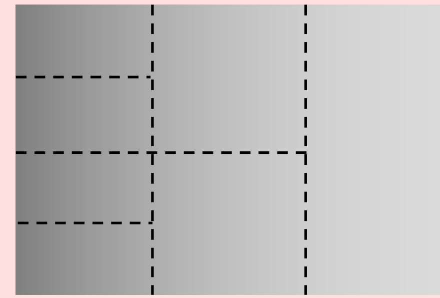
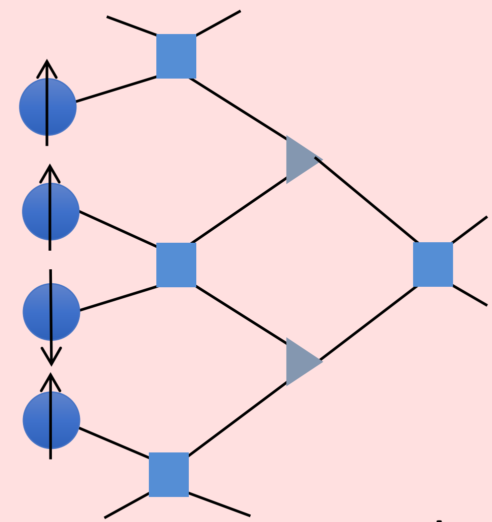
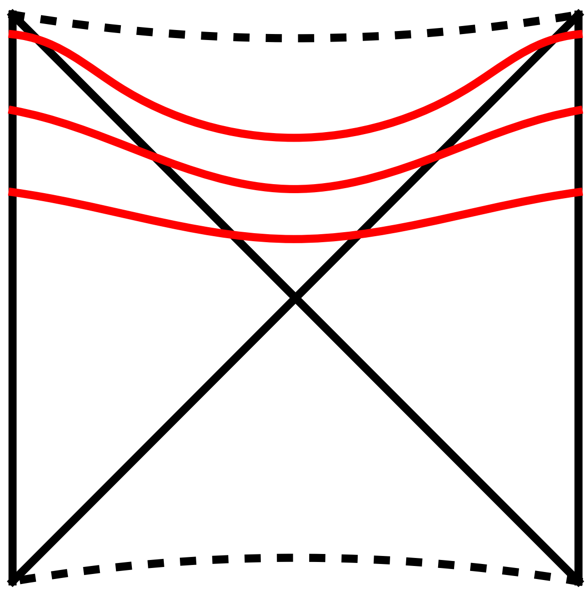


|| Swingle `10

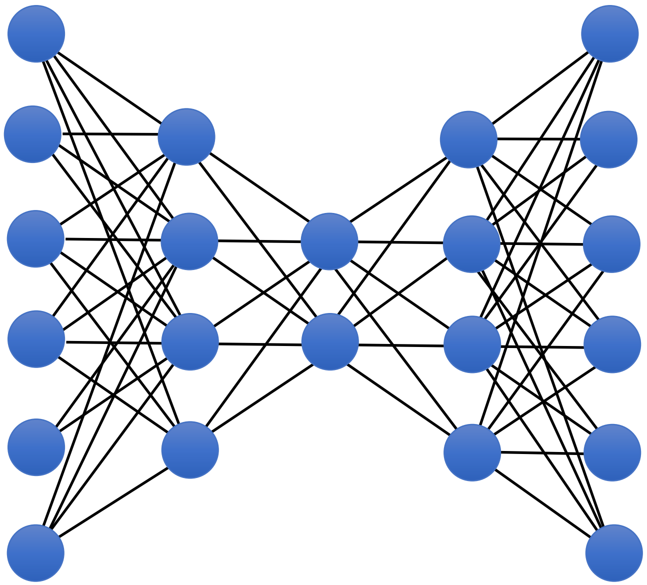


Tensor network

Similarity!?



Wormholes in Penrose diagram of maximally extended eternal AdS Schwarzschild black hole [Iizuka, Sugishita, KH `17]

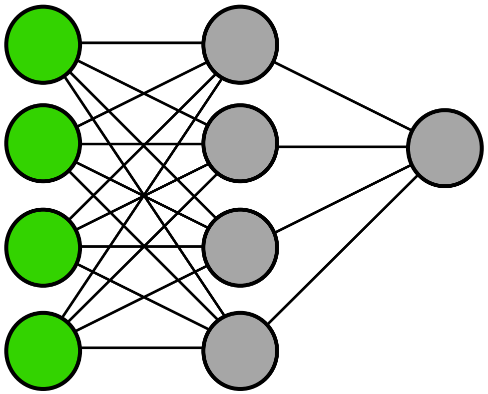


Deep Autoencoder

3. When is NN a spacetime?

General NN is not a space

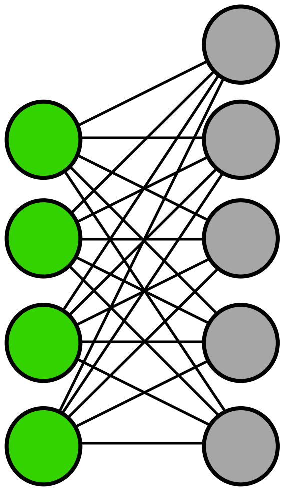
No notion of which unit is close to which



Perceptron model

[Rosenblatt 1958]

[Rumelhart, McClelland 1986]



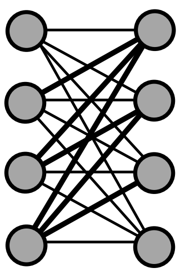
Boltzmann machine

[Ackley, Hinton, Sejnowski 1985]

3. When is NN a spacetime?

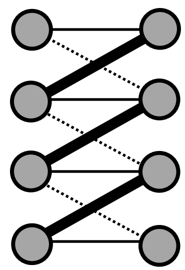
Sparsity + weight sharing, for NN to be a space

No locality



Fully connected

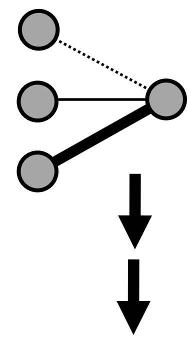
Locality imposed



Convolutional layer

[Fukushima `80]

=



Parallelly translated

Input: $\phi(n\Delta x)$

Output:

$$a\phi(n\Delta x) + b\partial_x\phi(n\Delta x) + c\partial_x^2\phi(n\Delta x) + \cdots$$

③ When is NN a spacetime?

NN depth as time

Dynamical system

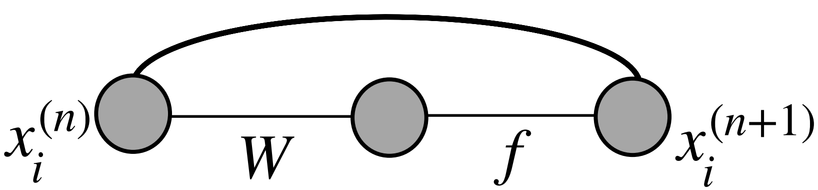
$$\dot{x}_i = f_i(x(t)) \quad \Longrightarrow \quad x_i(t_{n+1}) = \underline{x_i(t_n) +} \, \Delta t \cdot f_i(x(t_n))$$

$$t_{n+1} = t_n + \Delta t$$

Discretized time

ResNET (Residual network) : easily trained deep model

[K.He et al.,1512.03385]



$$x_i^{(n+1)} = f(W_{ij} x_j^{(n)}) \underline{+ x_i^{(n)}}$$

Skip connection

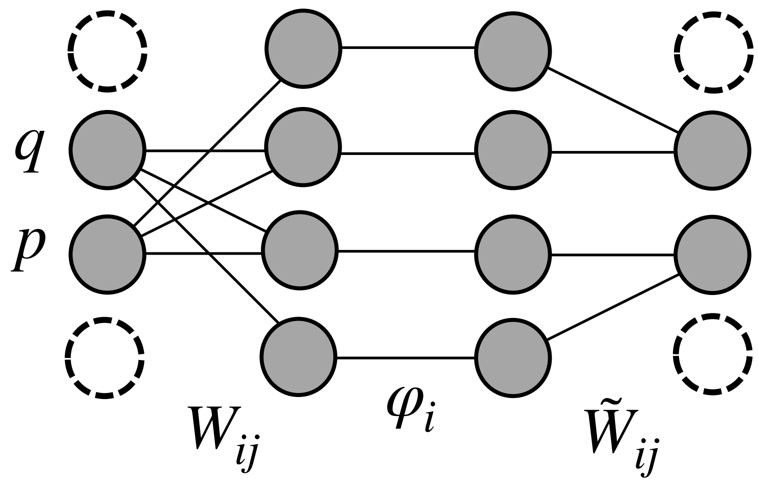
③ When is NN a spacetime?

Hamilton dynamics is a NN

1802.08313

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

Time-dependent Hamiltonian = weights/activation



$$W = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 1+\Delta t w_{11} & \Delta t w_{12} & 0 \\ 0 & \Delta t w_{21} & 1+\Delta t w_{12} & 0 \\ 0 & u & 0 & 0 \end{pmatrix}$$

$$\varphi_i = \begin{pmatrix} \Delta t f(x) \\ 1 \\ 1 \\ \Delta t g(x) \end{pmatrix} \qquad \tilde{W} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \lambda_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H = w_{11} pq + \frac{1}{2} w_{12} p^2 - \frac{1}{2} w_{21} q^2 + \frac{\lambda_1}{v} F(vp) - \frac{\lambda_2}{u} G(uq)$$

$$(F' = f, \quad G' = g)$$

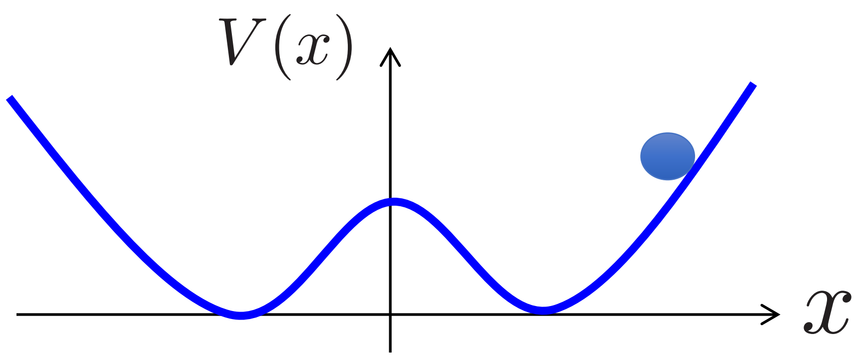
③ When is NN a spacetime?

Q. Find a Hamiltonian

Consider a particle motion $x(t)$ in a given potential $V(x)$ in 1 dimension, with unknown time-dependent friction force $h(t)\dot{x}$.
One tried many initial conditions $(x(t=0), \dot{x}(t=0))$ and collected those which stop at $t=10$.

Q. From given data of the initial conditions, find $h(t)$.

$$m\ddot{x} = h(t)\dot{x} + \frac{\partial V(x)}{\partial x}$$



Deep Learning and Quantum Gravity

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 7 pages

Discussion: Quantum gravity $\subset$ ML ?

Roadmap

4.

General spacetime

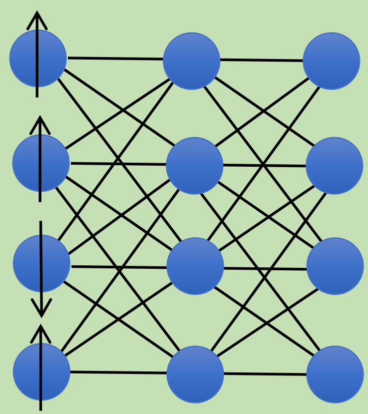
Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

||

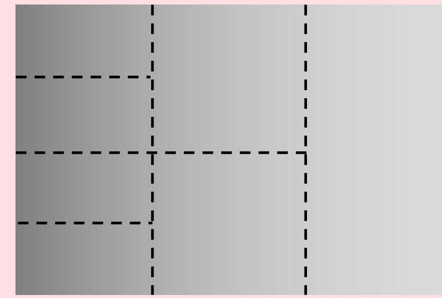
|| ?

Quantum mechanics in d-dim.



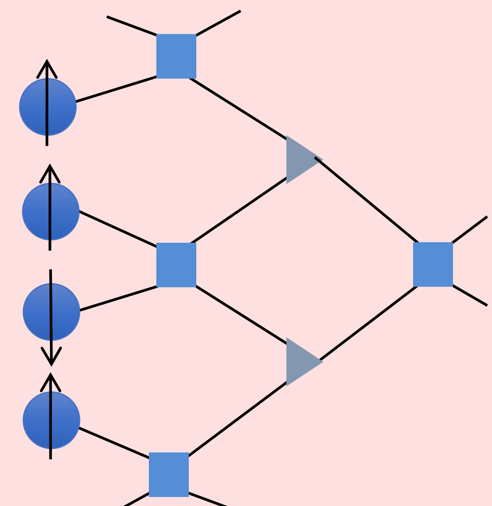
Neural network

Anti de Sitter spacetime

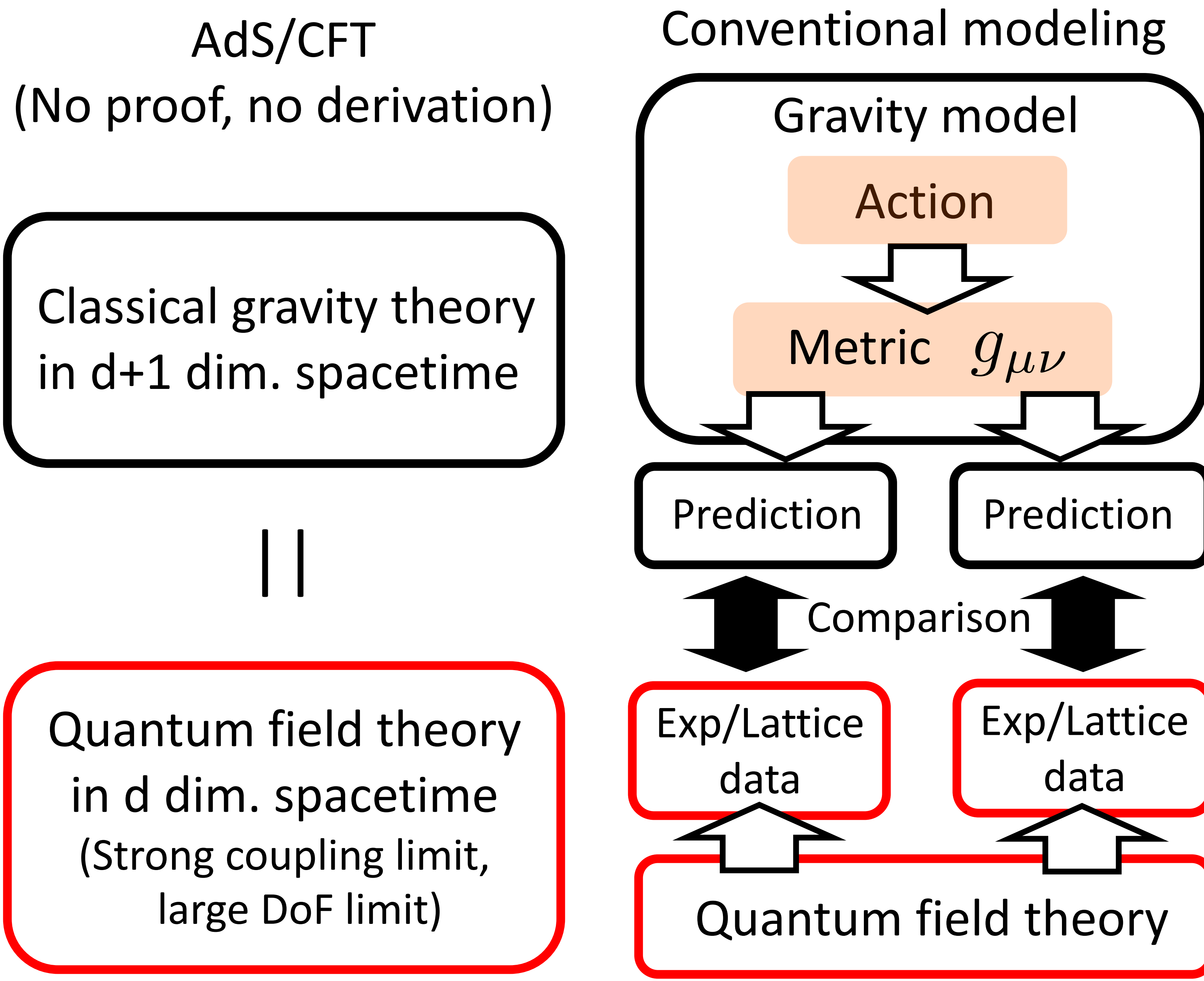


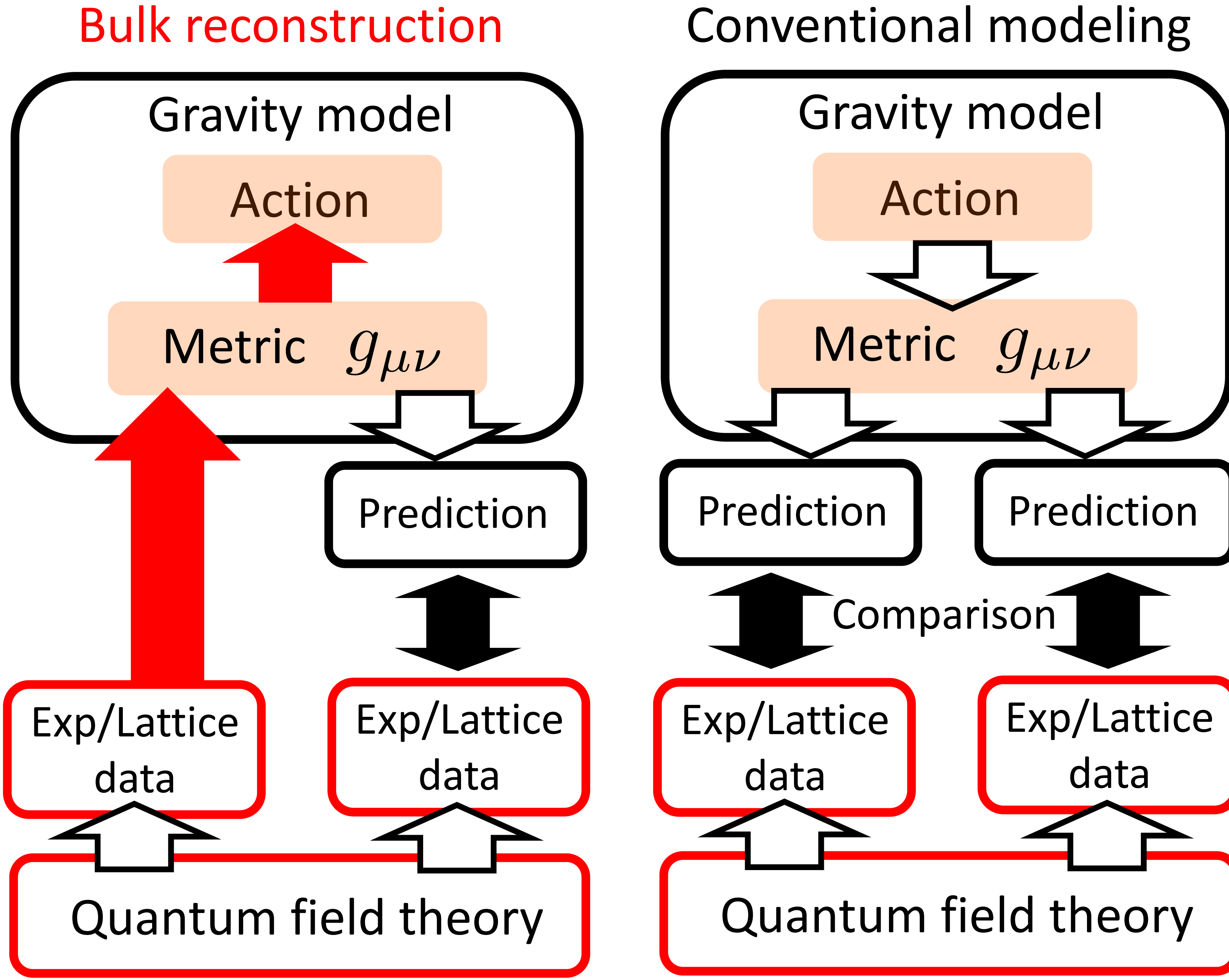
|| Swingle `10

Carleo, Troyer `17



Tensor network


AdS/CFT
(No proof, no derivation)
Classical gravity theory in d+1 dim. spacetime
=
Quantum field theory in d dim. spacetime
(Strong coupling limit, large DoF limit)
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory



Bulk reconstruction
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Exp/Lattice data
Quantum field theory
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory

④ Spacetime emergent from data

Gravity side

Classical scalar field theory in unknown 5-dim. spacetime

$$S = \int d\eta d^4x \sqrt{\det g} \left[(\partial_\eta \phi)^2 - V(\phi) \right]$$

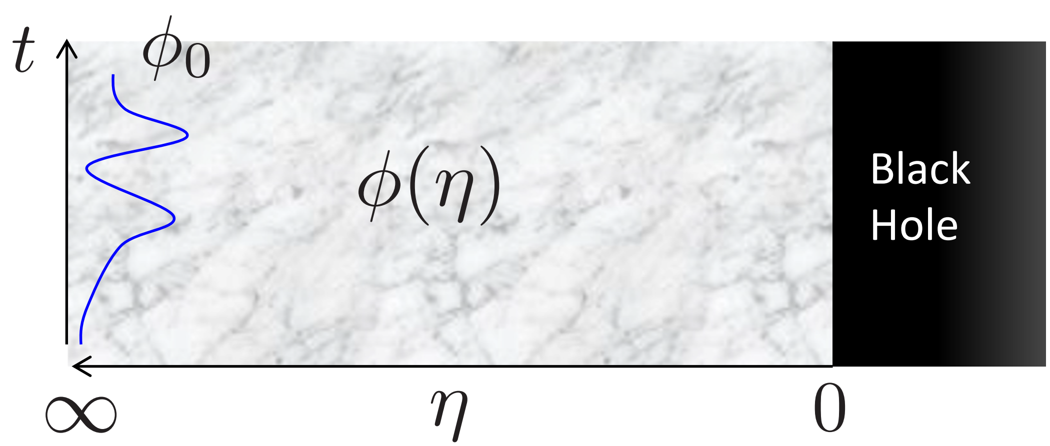
1802.08313
1809.10536

$$\begin{cases} ds^2 = -f(\eta) dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \\ V[\phi] = -\dfrac{3}{L^2}\phi^2 + \dfrac{\lambda}{4}\phi^4 \end{cases}$$

Data: $(\phi_0, Z[\phi_0])$

$\Updownarrow$

$(\phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=0})$



4. Spacetime emergent from data

Equation of motion as a feedforward NN

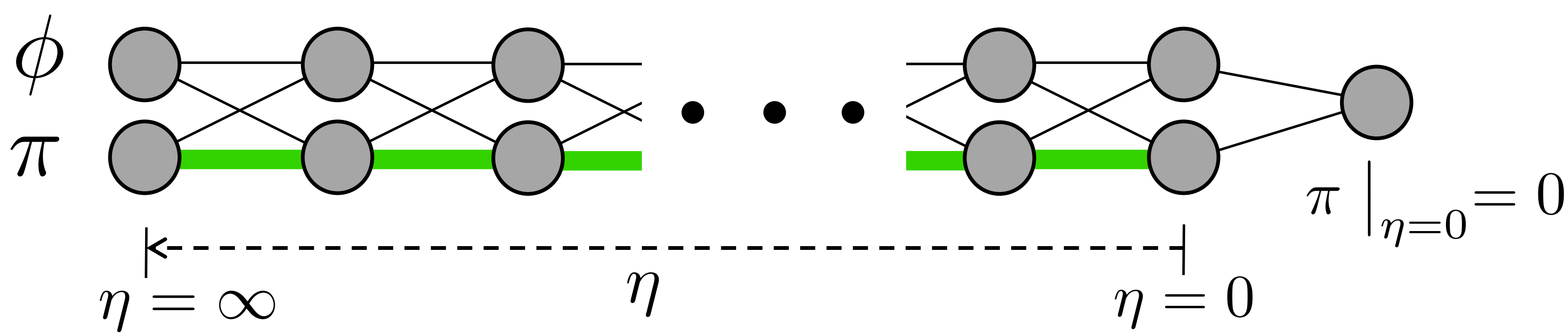
Eq. of motion $\partial_\eta^2 \phi + h(\eta)\partial_\eta \phi - \dfrac{\delta V[\phi]}{\delta \phi} = 0$

metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

Discretization
Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta\phi(\eta)} \right) \end{cases}$$

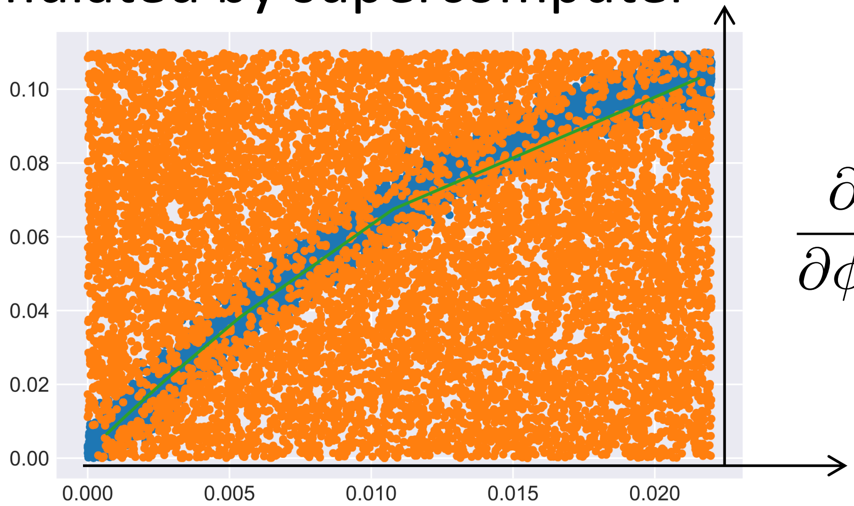
Feedfoward neural network for classification



④ Spacetime emergent from data

Training with data of quark condensate

Data of quantum chromodynamics
simulated by supercomputer

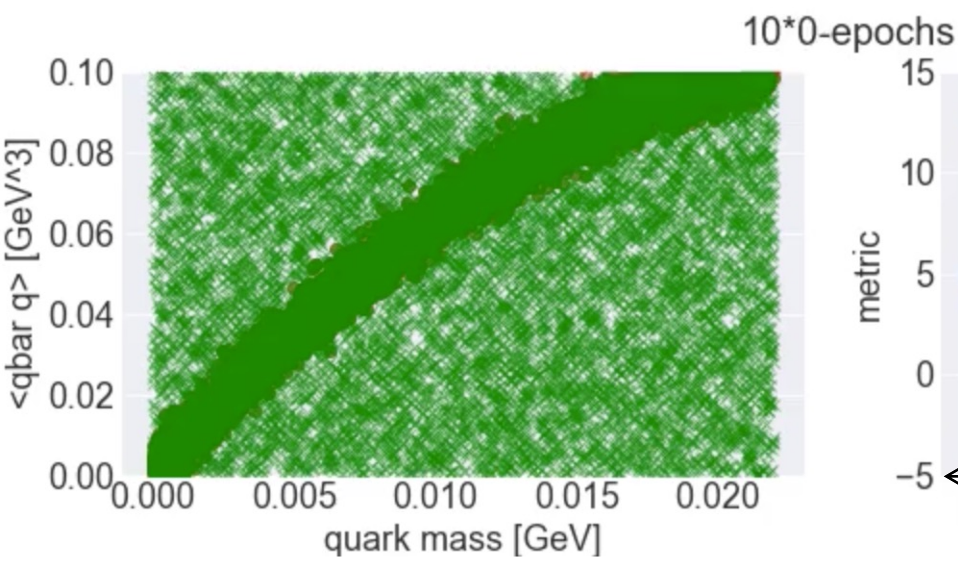


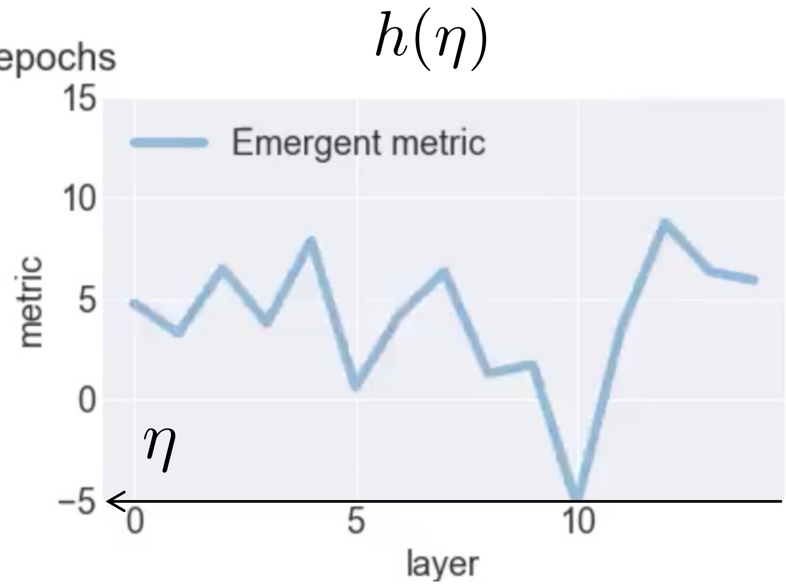
$\dfrac{\partial}{\partial \phi_0} Z[\phi_0]$: Quark condensate

ϕ_0 : Quark mass

④ Spacetime emergent from data

Spacetime metric emergent as NN



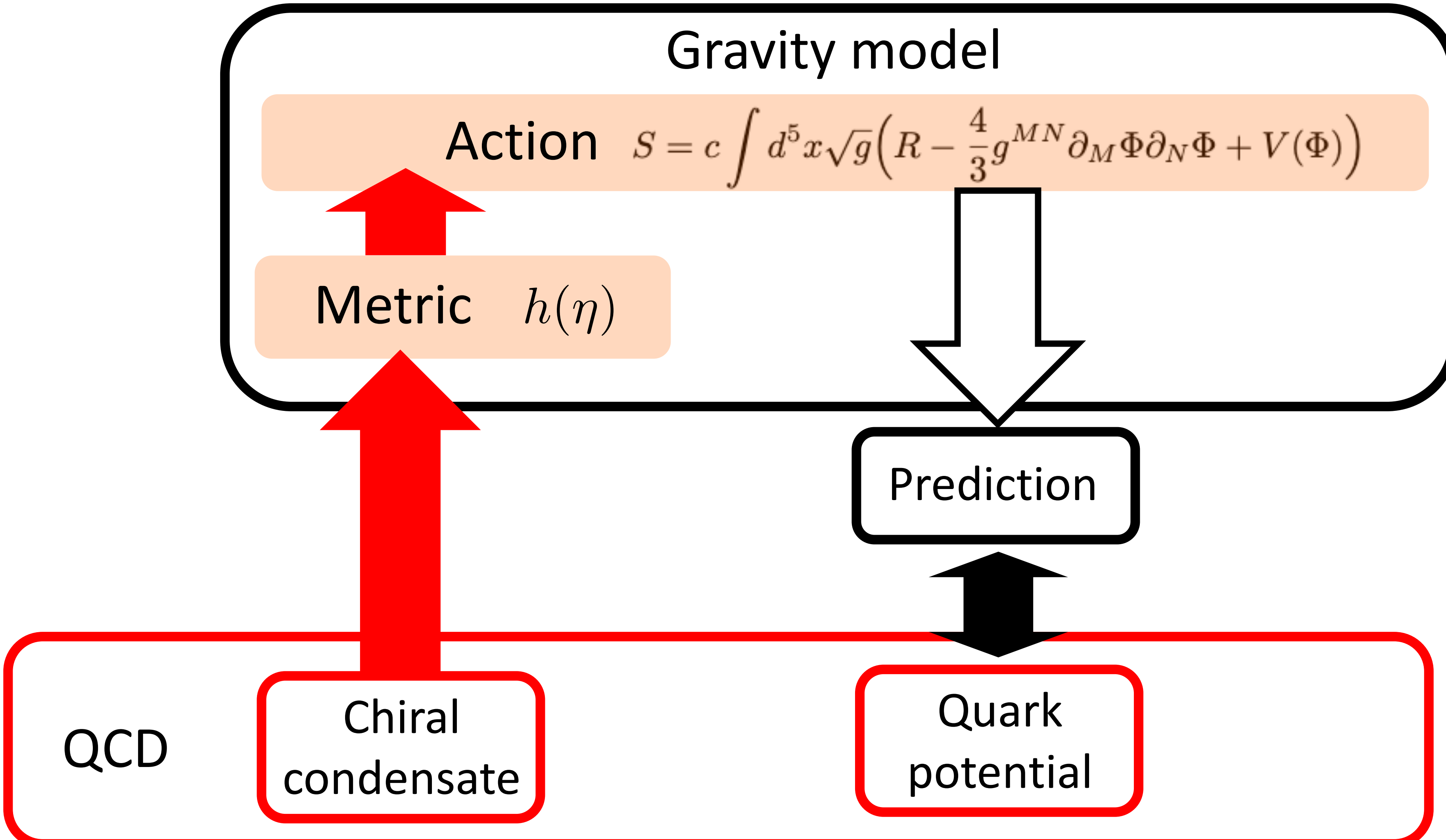


Trained values of potential :

1/L = 237(3)[MeV] ,   λ/L = 0.0127(6)

4. Spacetime emergent from data

Reconstructing gravity model dual to QCD



4. Spacetime emergent from data

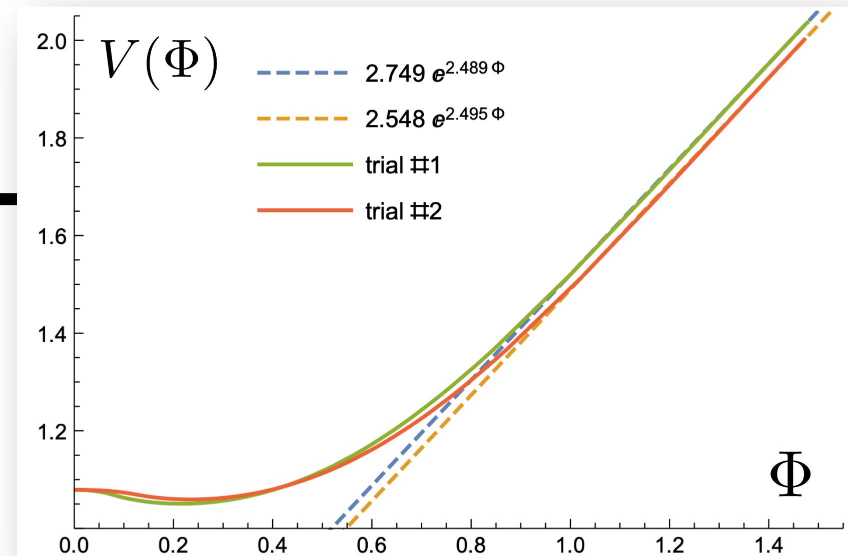
Deriving the dilaton potential

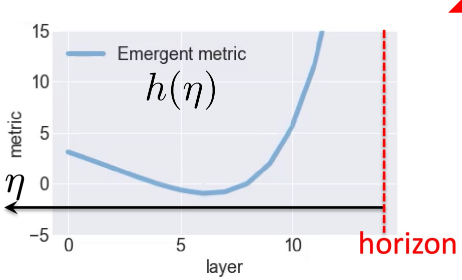
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

2209.04638

Metric $h(\eta)$





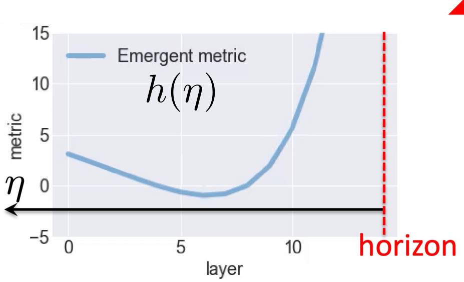
Chiral condensate

Prediction of QCD string breaking

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

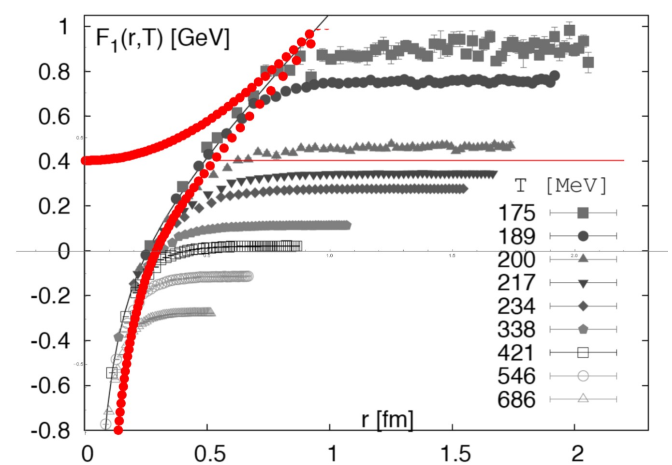
Metric $h(\eta)$



Chiral condensate

Prediction

Quark potential



Deep Learning and Quantum Gravity

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Discussion: Quantum gravity $\subset$ ML ?

Discussion: Quantum gravity ⊂ ML ?

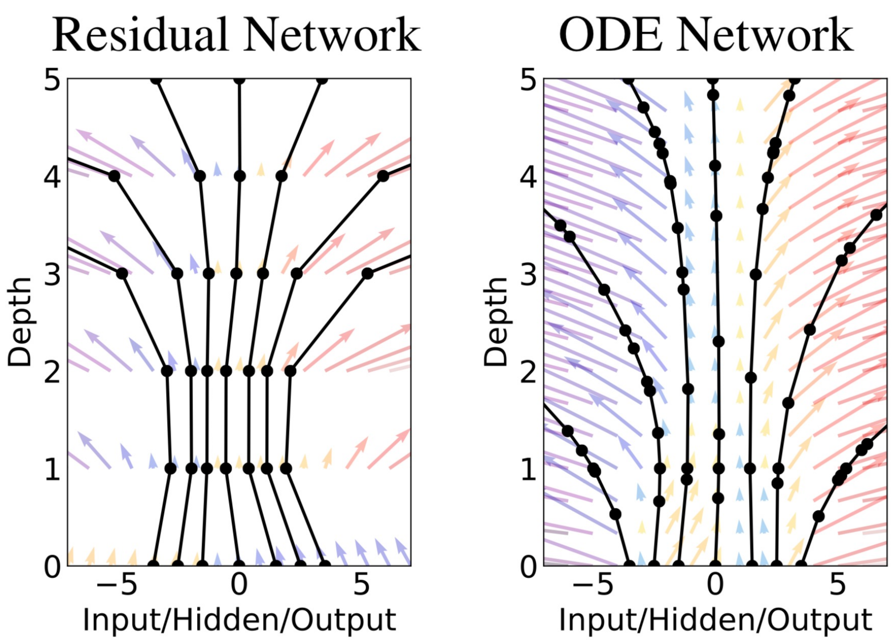
3 steps for quantum gravity

Quantum Mechanics side	Gravity side		NN architecture
	metric $g_{\mu\nu}$	field ϕ	
Large DoF limit	Classical	Classical	Feedforward NN
Large DoF expansion	Classical	Quantum	Deep Boltzmann
Finite DoF	Quantum	Quantum	?

- Neural ODE : free from discretization
- Quantum AdS/CFT ⊂ Deep Boltzmann machine
- Which part of geometry is the neurons?

Discussion: Quantum gravity ⊂ ML ?

Neural ODE : free from discretization



Neural ODE [R.T.Q.Chen, Y.Rubanova, J.Bettencourt, D.Duvenaud 1806.07366]

2006.00712

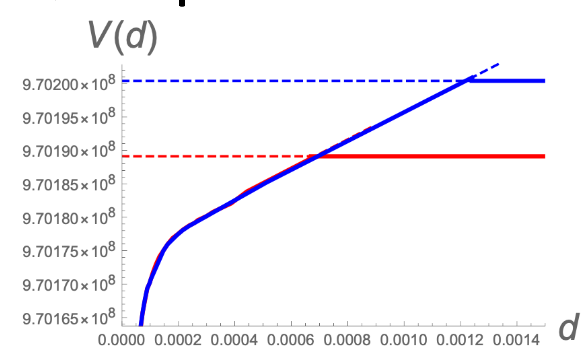
$$\frac{d\phi(\eta)}{d\eta} = f\left(\phi(\eta), \eta, h(\eta)\right)$$

Emergent metric

$$\begin{aligned} h(\eta) = & \, 8.2351\tilde{\eta}^8 + 8.0108\tilde{\eta}^7 + 7.6071\tilde{\eta}^6 \\ & + 6.9468\tilde{\eta}^5 + 150.8853\tilde{\eta}^4 - 130.8117\tilde{\eta}^3 \\ & + 55.5384\tilde{\eta}^2 - 2.22235\tilde{\eta}^1 + 3.7719. \end{aligned}$$

$$\tilde{\eta} = 1 - \eta$$

Q Qbar potential

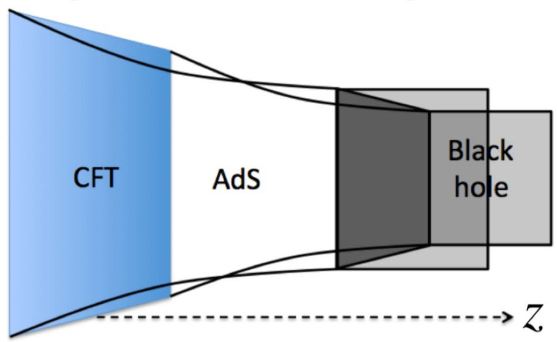


Discussion: Quantum gravity ⊂ ML ?

Quantum AdS/CFT ⊂ Deep Boltzmann

AdS/CFT

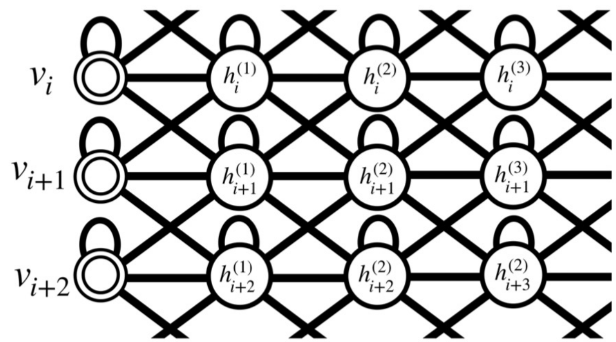
[Maldacena 1997]



$$Z_{\mathrm{QFT}}[J] = \int_{\phi(z=0)=J} \mathcal{D}\phi \, \exp\left(-S_{\mathrm{gravity}}[\phi]\right)$$

Deep Boltzman machine

[Salakhutdinov, Hinton 2009]



$$P(v_i) = \sum_{h_i \in \{0,1\}} \exp\left[-\mathcal{E}(v_i, h_i)\right]$$

[KH `19] [You,Yang,Qi `18] (See also [Gan,Shu `17][Howard `18])

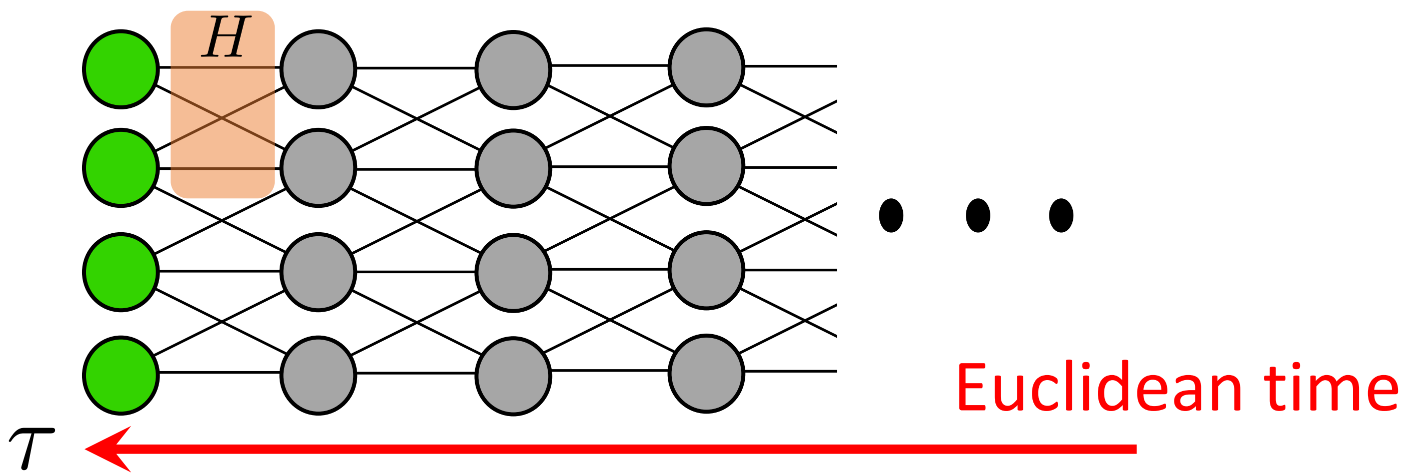
Discussion: Quantum gravity ⊂ ML ?

Physical picture of Deep Boltzmann

Ground state wave function for given Hamiltonian is identified as a deep Boltzmann machine

[Carleo, Nomura, Imada `18], ..

$$|\psi\rangle = \lim_{\tau\to\infty} e^{-\tau H}|\text{any}\rangle = e^{-\Delta\tau H}e^{-\Delta\tau H}\cdots|\text{any}\rangle$$



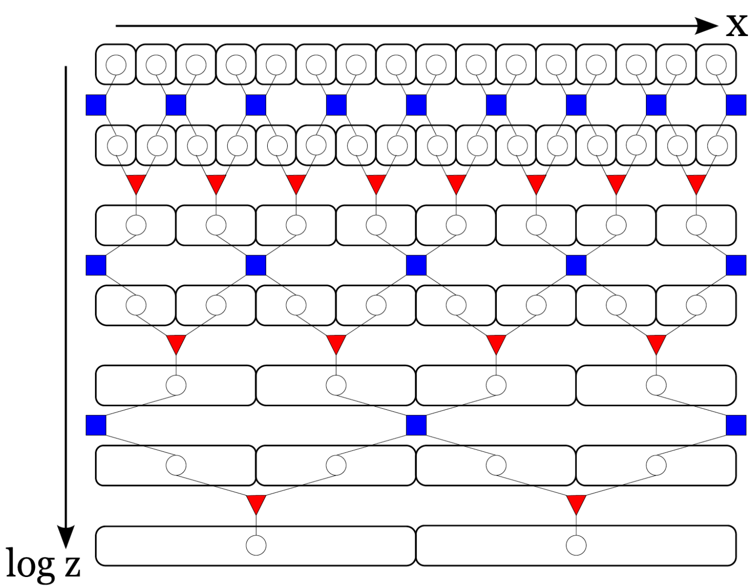
$$\psi(x_i) = \sum_{h_j^{(n)}\in\{0,1\}} \exp\left[-\sum_{ij} w_{ij}^{(0)} x_i h_j - \sum_n \sum_{ij} w_{ij}^{(n)} h_i^{(n)} h_j^{(n+1)}\right]$$

Discussion: Quantum gravity ⊂ ML ?

AdS/CFT discretized the bulk, but fixed

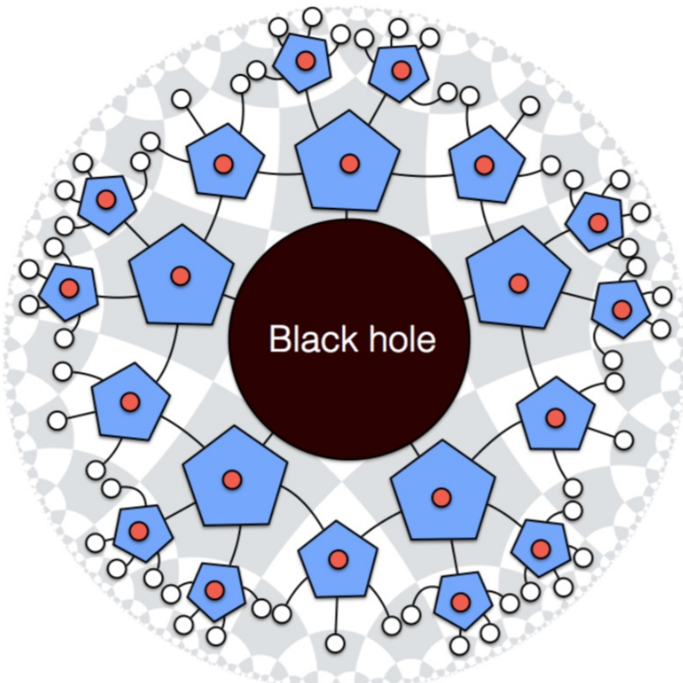
AdS/MERA

[Swingle `09]



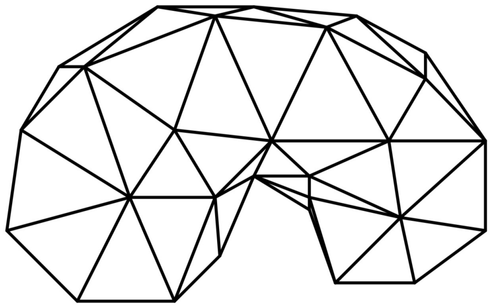
Quantum codes for holography

[Pastawski, Yoshida, Harlow, Preskill `15]



Discussion: Quantum gravity ⊂ ML ?

Quantum spacetime? Regge vs Matrix



Regge calculus
[Regge `61]

Fixed lattice architecture, variable lengths

⇩

Suits conventional NN

Dynamical triangulation
[Ambjorn, Loll `98]

Randomly generated lattice architecture, fixed lengths

⇩

Novel “QG NN”

Roadmap

Quantum gravity in (d+1)-dim.

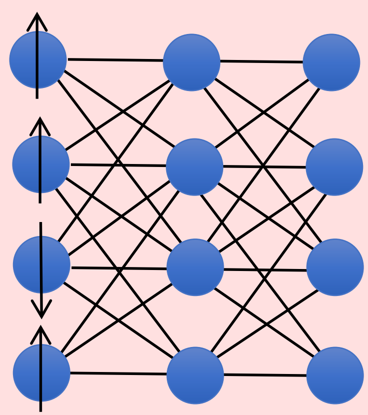
‘tHooft `93
Susskind `94
Maldacena `97

||

Quantum mechanics in d-dim.

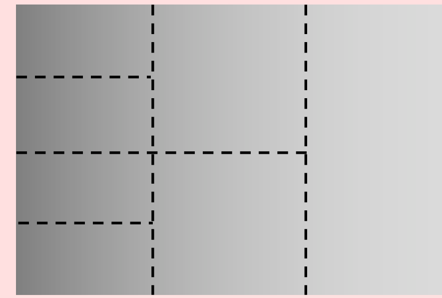
General spacetime

|| ?

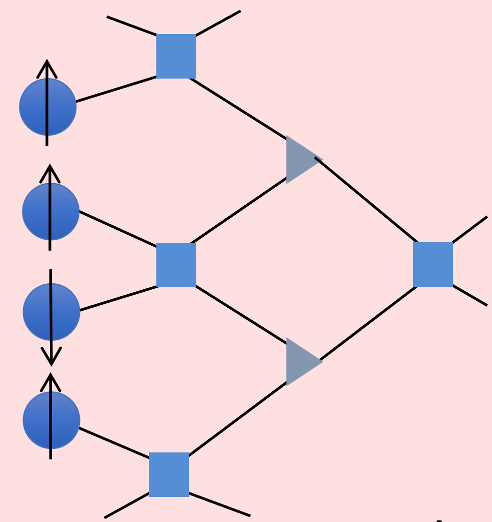


Neural network

Anti de Sitter spacetime



|| Swingle `10



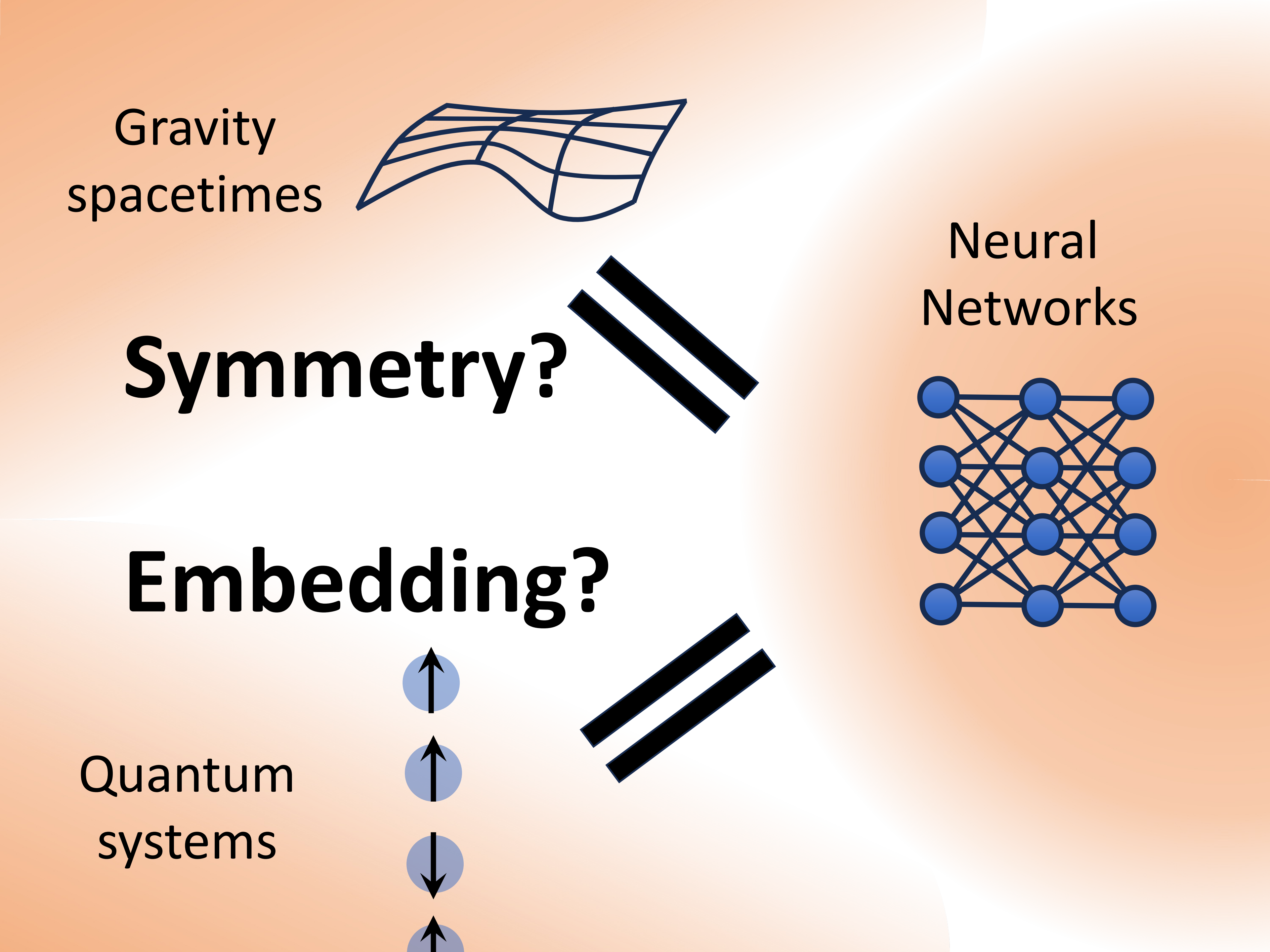
Tensor network

Carleo, Troyer `17

Erice School for Complexity, 23 Apr. 2024

Neural Networks, Quantum systems and Gravity

Koji Hashimoto (Kyoto U)


Gravity
spacetimes
Symmetry?
Embedding?
Neural
Networks
Quantum
systems

References

Symmetry?

“Unification of Symmetries Inside Neural Networks:
Transformer, Feedforward and Neural ODE”
[Hirono, Sannai, KH 2402.02362]

Embedding?

Neural Network Field Theory
[Halverson, Maiti, Stoner 2008.08601]
[Erbin, Lahoche, Samary 2018.01403] [Halverson 2112.04527]
[Demirtas, Halverson, Maiti, Schwartz, Stoner 2307.03223]
Ref. [Grosvenor, Jefferson] [Bachtis, Aarts, Lucini] [He]…

“NN representation of quantum systems”
[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

Random neural fields [Amari 1971] …

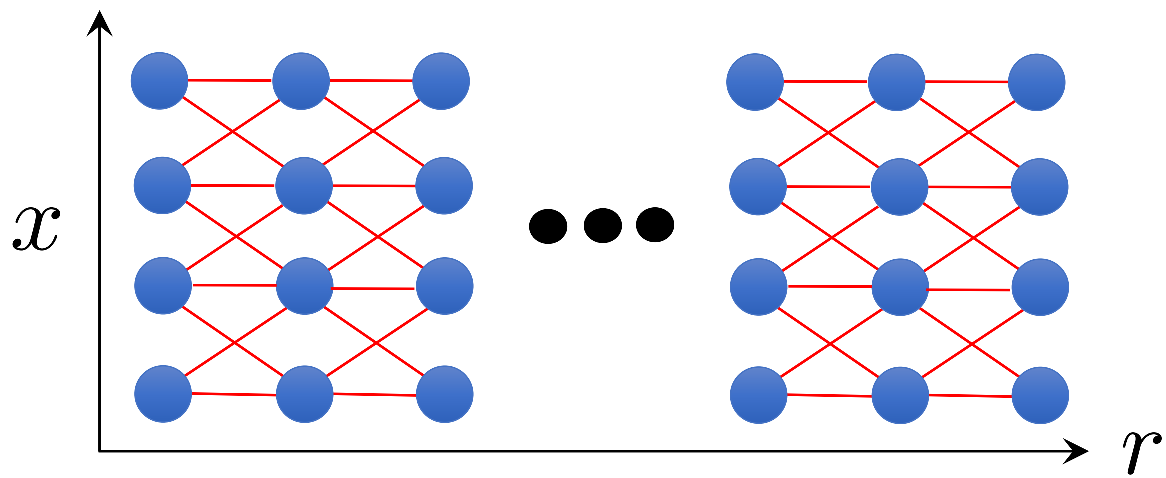
Symmetry? Reparametrization inside NN

Mainly on [Y.Hirono, A.Sannai, KH 2402.02362]

1. Motivation: gauge sym in NN

There should be diffeo, a la AdS/DL

Deep feedforward NN = AdS spacetime



Bulk reconstruction by entanglement [Lam You] [You Yang Qi] …

Bulk reconstruction by QFT correlators

[Tanaka Tomiya Sughishita KH] [Akutagawa Sumimoto KH] [Hu You KH]
[Tan Chen] [Song Oh Ahn Kim] [Yan Wu Ge Tian] …

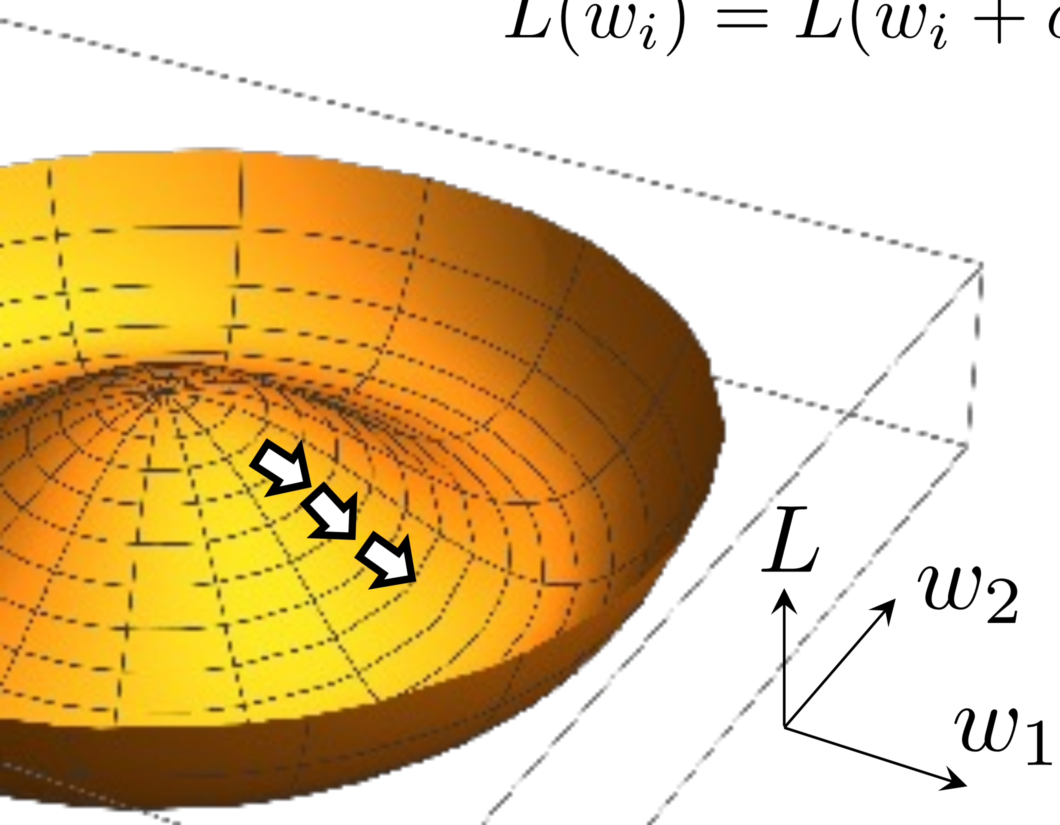
Deep Boltzmann machine = AdS/CFT [KH]

1. Motivation: gauge sym in NN

Symmetries = redundancy → trainability

NN symmetry : Invariance of loss function $L(w_i)$ under a transformation on weights w_i

$$L(w_i) = L(w_i + \delta_i(w))$$



Learning dynamics may depend on symmetries.

[Amari] [Amari Ozeki Karakida Yoshida Okada] [Badrinarayanan Mishra Cipolla] [Neyshabur Salakhutdinov Srebro] [Ziyin 2023] …

hep-th/0403001
CITA-04-3
SLAC-PUB-10343
SU-ITP-04/05

Beauty is Attractive: Moduli Trapping at Enhanced Symmetry Points

Lev Kofman,[a] Andrei Linde,[b] Xiao Liu,[b,c]
Alexander Maloney,[b,c] Liam McAllister,[b,c] and Eva Silverstein[b,c]

[a] CITA, University of Toronto, Toronto, ON M5S 3H8, Canada
[b] Department of Physics, Stanford University, Stanford, CA 94305, USA
[c] SLAC, Stanford University, Stanford, CA 94309, USA

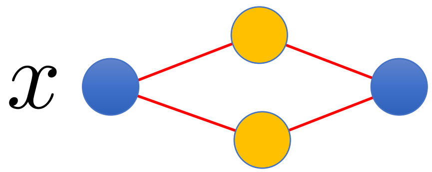
We study quantum effects on moduli dynamics arising from the production of particles which are light at special points in moduli space. The resulting forces trap the moduli at these points, which often exhibit enhanced symmetry. Moduli trapping occurs in time-dependent quantum field theory, as well as in systems of moving D-branes, where it leads

Symmetry? Reparametrization inside NN

Mainly on [Y.Hirono, A.Sannai, KH 2402.02362]

2. Candidates for diffeo in NN

1. Swapping of neurons



$$f(x) = w_1\varphi(\tilde{w}_1 x + \tilde{b}_1) + w_2\varphi(\tilde{w}_2 + \tilde{b}_2) + b$$

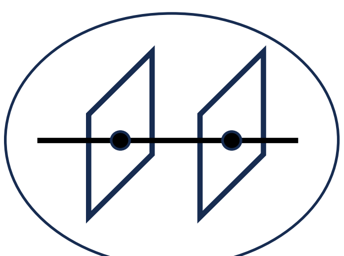
Multi-Layer Perceptron

<u>NN symmetry</u> : swapping of neurons

$$w_1 \leftrightarrow w_2, \quad \tilde{w}_1 \leftrightarrow \tilde{w}_2, \quad \tilde{b}_1 \leftrightarrow \tilde{b}_2$$

Note : this Z_2 sym enhances at singularity

[Amari Ozeki Karakida Yoshida Okada 2016]

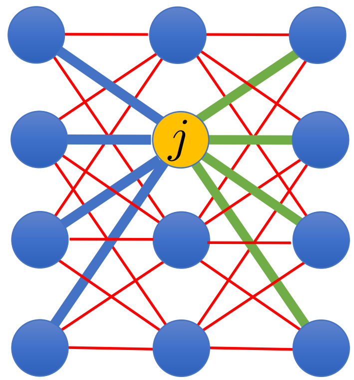


2. Candidates for diffeo in NN

2. Rescaling symmetry

Ex. [Neyshabur, Salakhutdinov, Srebro 2015]

x_k

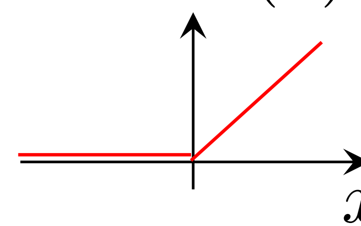


$$f_i = w_{ij}\varphi(\tilde{w}_{jk}x_k + \tilde{b}_j) + b_i$$

ReLU activation

$\varphi(x) = \mathrm{ReLU}(x)$

$\mathrm{ReLU}(x)$



<u>NN symmetry</u> : For any fixed j, rescale

$$w_{ij} \mapsto \alpha w_{ij}, \quad \tilde{w}_{jk} \mapsto \alpha^{-1}\tilde{w}_{jk}, \quad \tilde{b}_j \mapsto \alpha^{-1}\tilde{b}_j$$

due to ReLU scaling property

$$\mathrm{ReLU}(\alpha^{-1}x) = \alpha^{-1}\mathrm{ReLU}(x)$$

2. Candidates for diffeo in NN

3. Self-attention in transformers

Transformer: [Vaswani et al. 2017]

$$h_i = \sum_{j=1}^{n} \mathrm{ReLU}\left((xw^{(q)})_i (xw^{(k)})_j^{\mathrm{T}} \right) (xw^{(v)})_j$$

$x \in \mathbf{R}^{n\times d}$: set of data $x_i \in \mathbf{R}^d (i = 1, 2, \cdots, n)$

$w^{(q),(k),(v)} \in \mathbf{R}^{d\times d}$: query, key and value weights

<u>NN sym 1</u> : rescaling $w^{(a)} \mapsto \alpha^{(a)} w^{(a)}, \quad w^{(q)} w^{(k)} w^{(v)} = 1$

<u>NN sym 2</u> : Internal sym

$$w^{(q)} \mapsto w^{(q)} A, \quad w^{(k)} \mapsto w^{(k)} (A^{-1})^T, \quad A \in SL(d, \mathbf{R})$$

Symmetry? Reparametrization inside NN

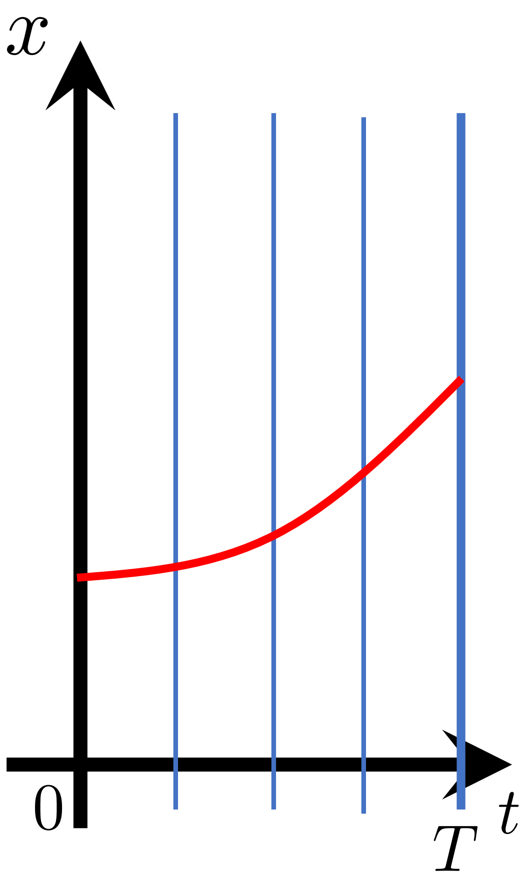
Mainly on [Y.Hirono, A.Sannai, KH 2402.02362]

1. Motivation: gauge sym in NN — 2 pages
2. Candidates for diffeo in NN — 3 pages
3. Diffeo in neural ODEs — 4 pages
4. Physics of NN symmetries? — 2 pages

3. Diffeos in neural ODEs

Neural ODEs = continuous ver. of NN

[Chen, Rubanova, Bettencourt, Duvenaud 2018]



$$\dot{x}(t) = F(t, x(t))$$

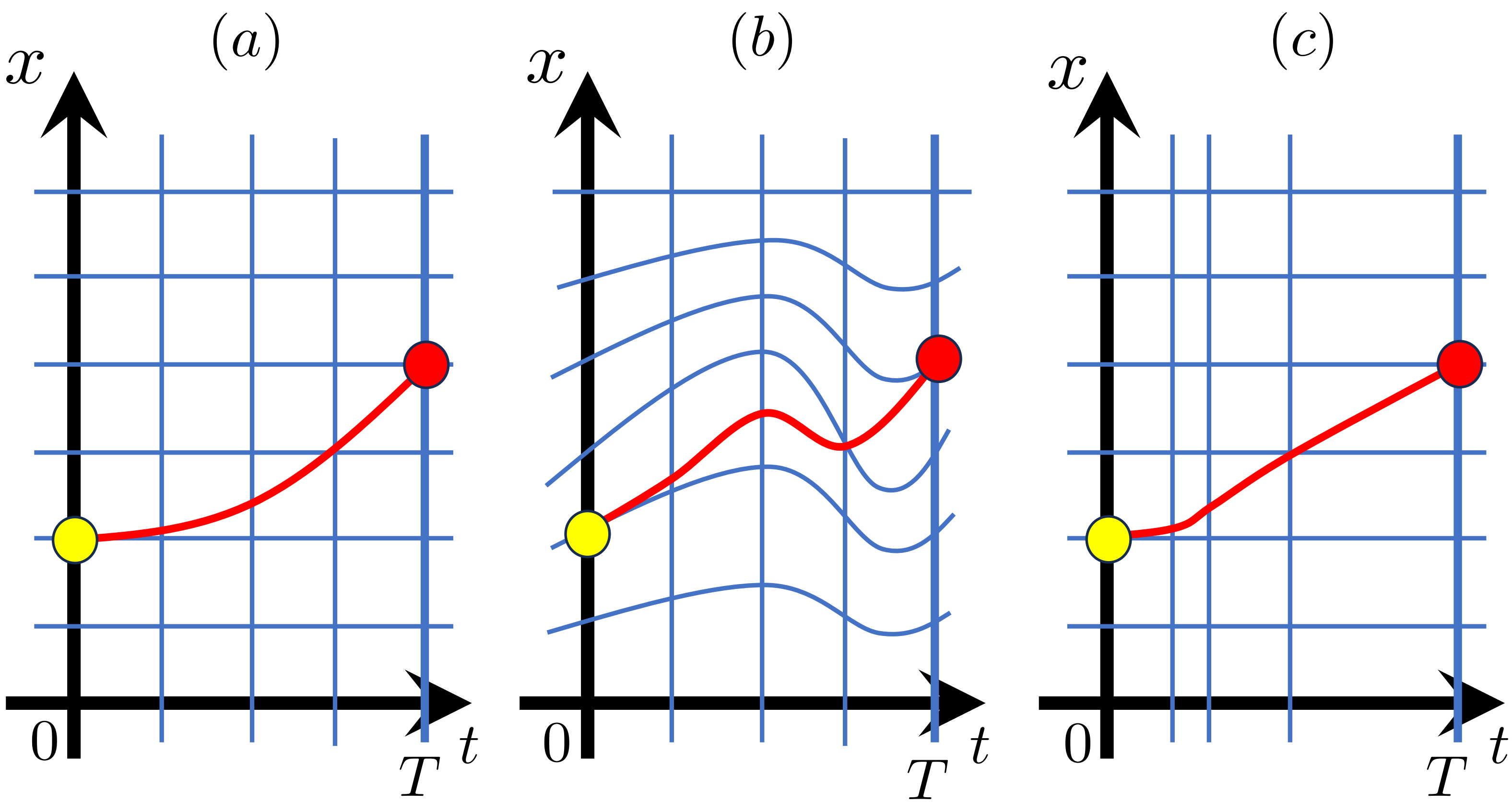
Training : train $F(t, x)$ such that the relation $(x(0), x(T))$ reproduces given set of data $\{(x_i, x_f)\}$.

Discretizing it gives a residual NN :

$$x(t + \delta t) = x(t) + F(t, x(t))\delta t$$

3. Diffeos in neural ODEs

Diffeos in ADM-like decomposition in neural ODE



Original frame

Spatial diffeo

$$x(t) \mapsto x(t) + \epsilon(t, x(t))$$

Time reparam.

$$t \mapsto t + f(t)$$

3. Diffeos in neural ODEs

Linear neural ODE can be solved

Linear neural ODE : $\dot{x}(t) = w(t)x(t) + b(t)$

Explicit solution of the ODE

$$x(T) = \underbrace{e^{\int_0^T w(t')dt'}}_{\text{“Wilson loop”}} x(0) + \left(\int_0^T e^{-\int_0^{t'} w(t'')dt''} b(t')dt' \right) e^{\int_0^T w(t'')dt''}$$

<u>Spatial diffeo</u> $x(t) \mapsto x(t) - g(t)x(t)$ is weight transf.

$$w(t) \mapsto w(t) + \dot{g}(t), \quad b(t) \mapsto e^{g(t)-g(t=0)}\, b(t)$$

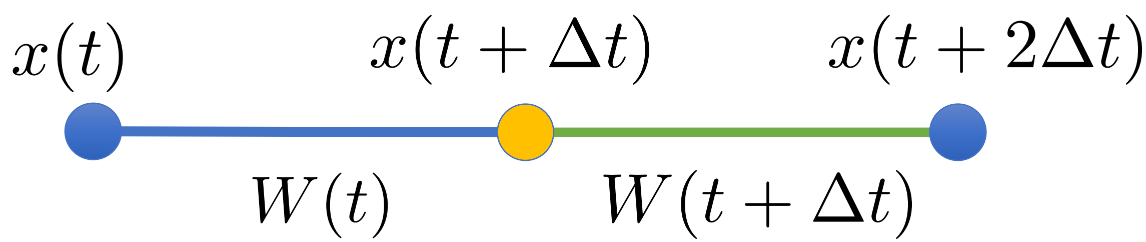
“Gauge field” “Higgs field”

<u>Time reparam.</u> $t \mapsto t + f(t)$ is weight transf.

$$w(t) \mapsto w(t) - \frac{d}{dt}(w(t)f(t)), \quad b(t) \mapsto b(t) - \frac{d}{dt}(b(t)f(t))$$

3. Diffeos in neural ODEs

Rescaling = a spatial diffeo



Spatial diffeo : Integrated weight transforms as Wilson line

$$W(t) \mapsto e^{-g(t)} W(t) e^{g(t+\Delta t)}$$

$$W(t+\Delta t) \mapsto e^{-g(t+\Delta t)} W(t) e^{g(t+2\Delta t)}$$

which reproduces the rescaling symmetry

$$w_{ij} \mapsto \alpha w_{ij}, \quad \tilde{w}_{jk} \mapsto \alpha^{-1} \tilde{w}_{jk}$$

Symmetry? Reparametrization inside NN

Mainly on [Y.Hirono, A.Sannai, KH 2402.02362]

4. Physics of NN symmetries?

NN gauge symmetries are broken in general

- Discretization breaks spacetime sym, as in lattice QFT.

- Limited architecture has NN symmetries :
  Other than (leaky) ReLU, no rescaling symmetry.
  [Godfrey Brown Emerson Kvinge 2205.14258(cs.LG)]

- Special neural ODE allows a metric interpretation.

"Neural geodesic equation" $$\begin{cases} \dot{x}^i = v^i \\ \dot{v}^i = -\Gamma^i_{jk}(x) v^j v^k \end{cases}$$

4. Physics of NN symmetries?

Training and symmetry breaking

Nevertheless, amusing to see similarities to gauge theory!

- Weights = gauge field, Biases = Higgs field
- Weight decay = Gauge mass term

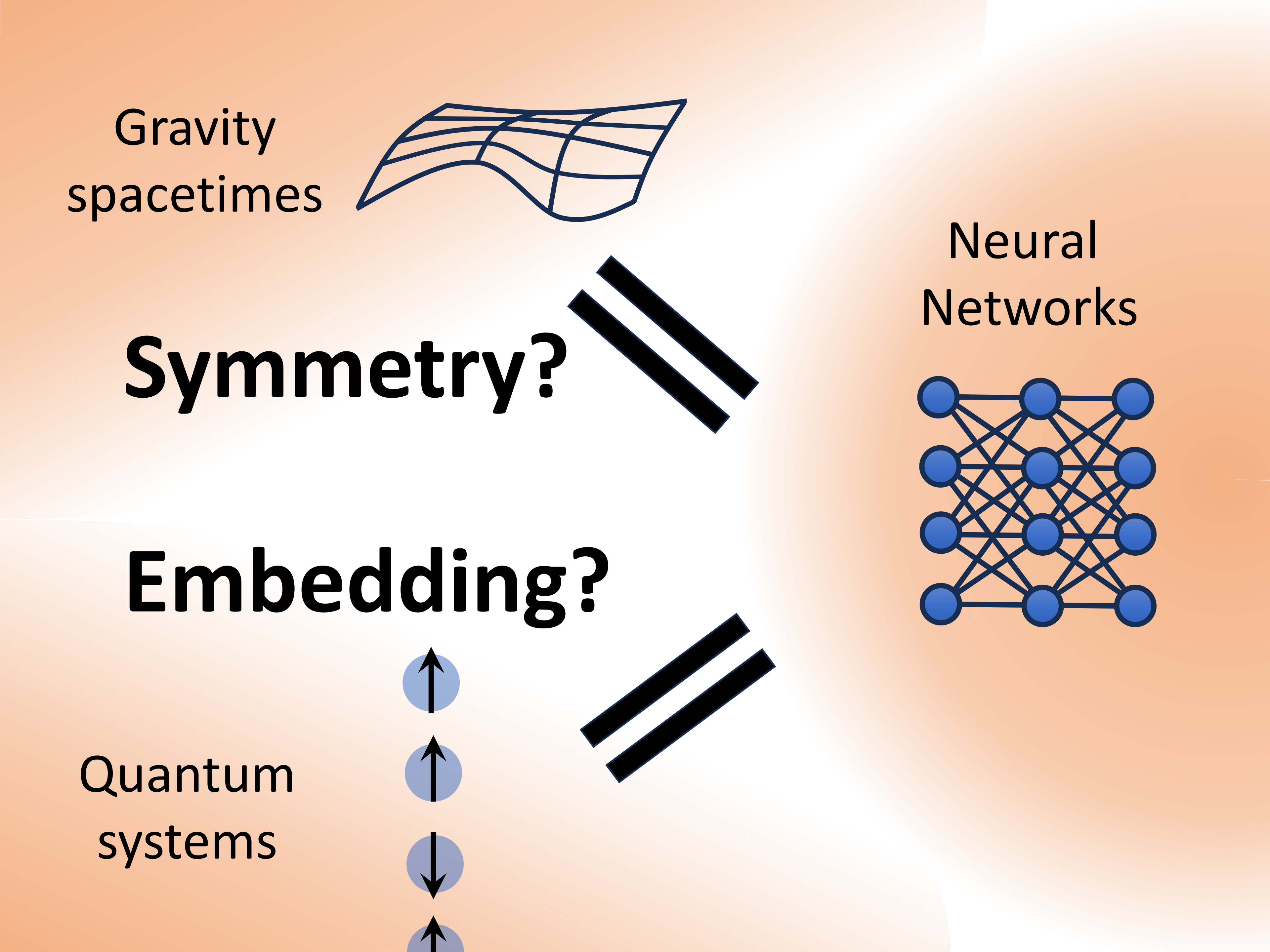
$$\sum_{i,j} w_{ij}^2 \sim \int dt\, w(t)^2$$

- Linear transport condition = Lorentz gauge fixing term

$$L_{\mathrm{R}} = \lambda \int_0^T dt\, \left[(\dot{w} + w^2)^2 + \left[\, (\partial_t + w) b \,\right]^2 \right]$$

Symmetry? Reparametrization inside NN

Mainly on [Y.Hirono, A.Sannai, KH 2402.02362]


Gravity
spacetimes
Symmetry?
Embedding?
Neural
Networks
Quantum
systems

References

Symmetry?

“Unification of Symmetries Inside Neural Networks:
Transformer, Feedforward and Neural ODE”
[Hirono, Sannai, KH 2402.02362]

Embedding?

Neural Network Field Theory
[Halverson, Maiti, Stoner 2008.08601]
[Erbin, Lahoche, Samary 2018.01403] [Halverson 2112.04527]
[Demirtas, Halverson, Maiti, Schwartz, Stoner 2307.03223]
Ref. [Grosvenor, Jefferson] [Bachtis, Aarts, Lucini] [He]…

“NN representation of quantum systems”
[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

Random neural fields [Amari 1971] …

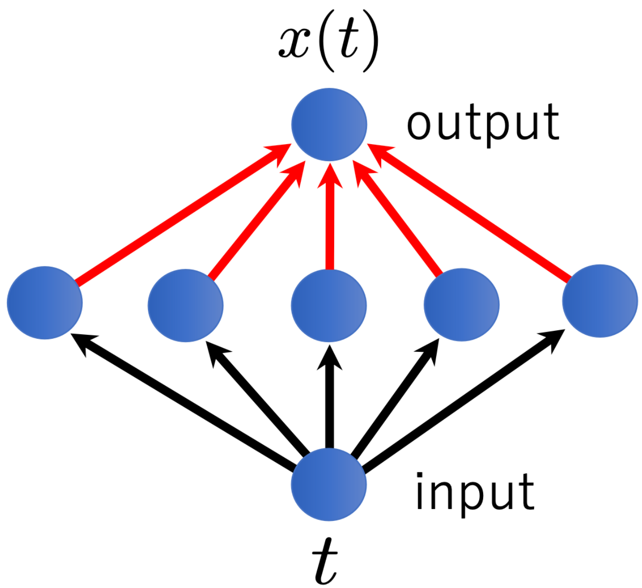
NN Representation of Quantum Systems

[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

1. Neural network field theory

Wide limit of NN gives Gaussian process

3-layer perceptron: [Niel 1994], [Williams 1997]
Deep NN: [Lee, Bahri, Novak, Schoenholz, Pennington, Sohl-Dickstein 2018]



Suppose:
- Weights are random i.i.d
- Infinite width of NN

➔ The central limit theorem applies.

➔ Output correlators are Gaussian.

$$\langle x(t_1)x(t_2)x(t_3)x(t_4)\rangle$$
$$= \langle x(t_1)x(t_2)\rangle\langle x(t_3)x(t_4)\rangle + \langle x(t_1)x(t_3)\rangle\langle x(t_2)x(t_4)\rangle + \langle x(t_1)x(t_4)\rangle\langle x(t_2)x(t_3)\rangle$$

1. Neural network field theory

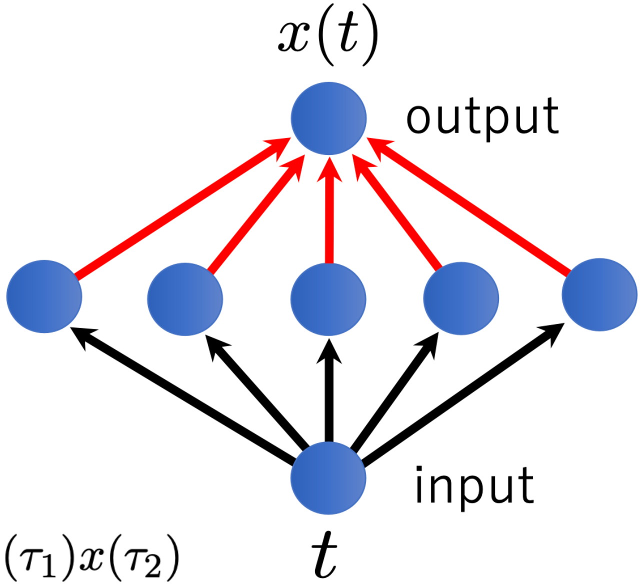
Neural Network Field Theory (NNFT)

[Halverson, Maiti, Stoner 2008.08601 [cs.LG]]
[Halverson 2112.04527 [hep-th]]
[Demirtas, Halverson, Maiti, Schwartz, Stoner 2307.03223 [hep-th]]

Outputs of a NN can reproduce free quantum field theory!

$$x(\tau) = \sum_{n=1}^{N} a_n \frac{1}{\sqrt{b_n^2 + k/m}} \cos\left(b_n \tau + c_n\right)$$

$$\Downarrow \quad \begin{aligned} a_n &\sim P(a) = \mathcal{N}(0, 1/N)\,, \\ b_n &\sim P(b) = \mathcal{U}(-\Lambda, \Lambda)\,, \\ c_n &\sim P(c) = \mathcal{U}(-\pi, \pi)\,. \end{aligned}$$



$$\begin{aligned} \langle x(\tau_1) x(\tau_2) \rangle &= \int \prod_n \left(P(a_n) da_n P(b_n) db_n P(c_n) dc_n\right) x(\tau_1) x(\tau_2) \\ &= \int_{-\Lambda}^{\Lambda} db\, \frac{\cos(b(\tau_1 - \tau_2))}{b^2 + k/m}\,. \end{aligned}$$

Harmonic Oscillator.

1. Neural network field theory

Neural Network Field Theory (NNFT)

[Halverson, Maiti, Stoner 2008.08601 [cs.LG]]
[Halverson 2112.04527 [hep-th]]
[Demirtas, Halverson, Maiti, Schwartz, Stoner 2307.03223 [hep-th]]

Outputs of a NN can reproduce free quantum field theory!

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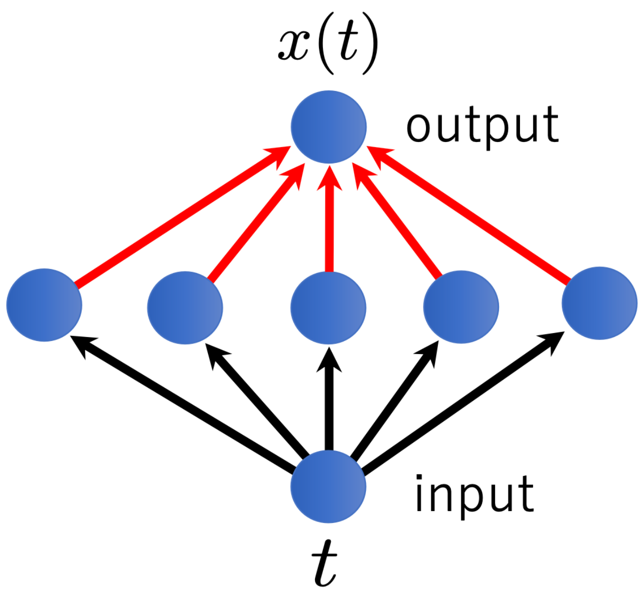


<u>NN Characteristics</u>:

- Cosine activation w/ a normalization
- Large N units at the second layer
- i.i.d. parameter samples

➔ Central limit theorem is applied to have free QFT

NN Representation of Quantum Systems

[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

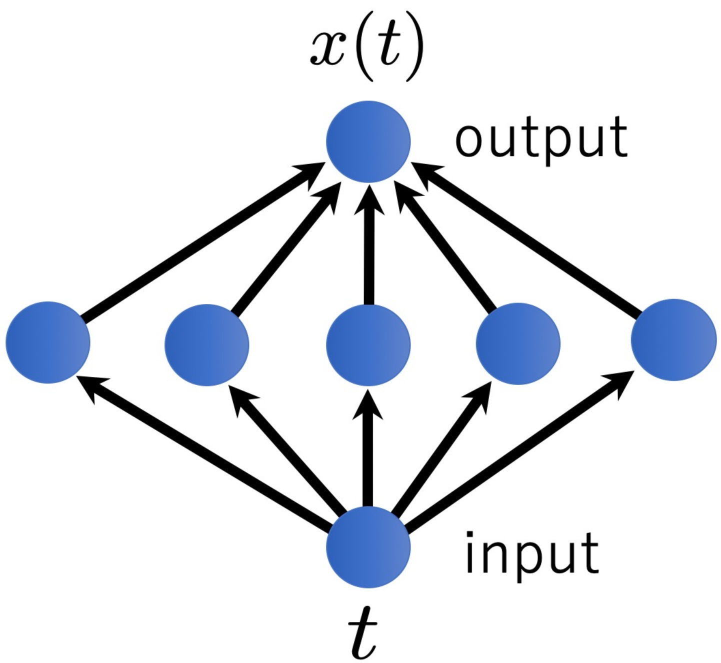
1. Neural network field theory — 2 pages
2. Universal approximation of path integral — 5 pages
3. Various activations — 4 pages
4. Generality: NN=QM? — 4 pages

2. Universal Approx. of Path Integral

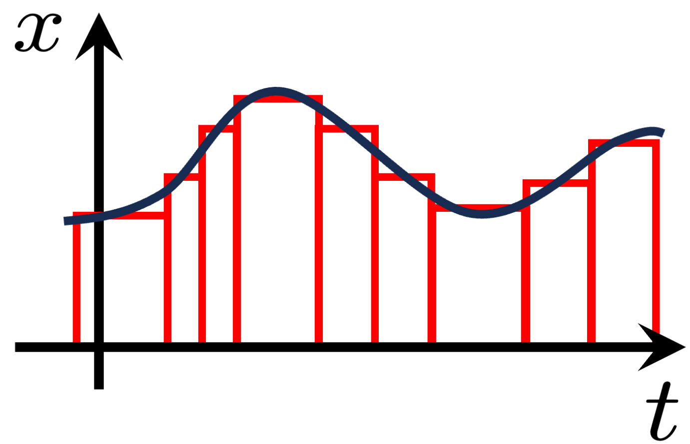
Universal approximation of NN

[Cybenko 1989],[Hornik, Stinchcombe, White 1989]
[Hornik 1991],[Leshno, Lin, Pinkus, Schocken 1993]

Theorem: Infinitely wide NN can approximate any function



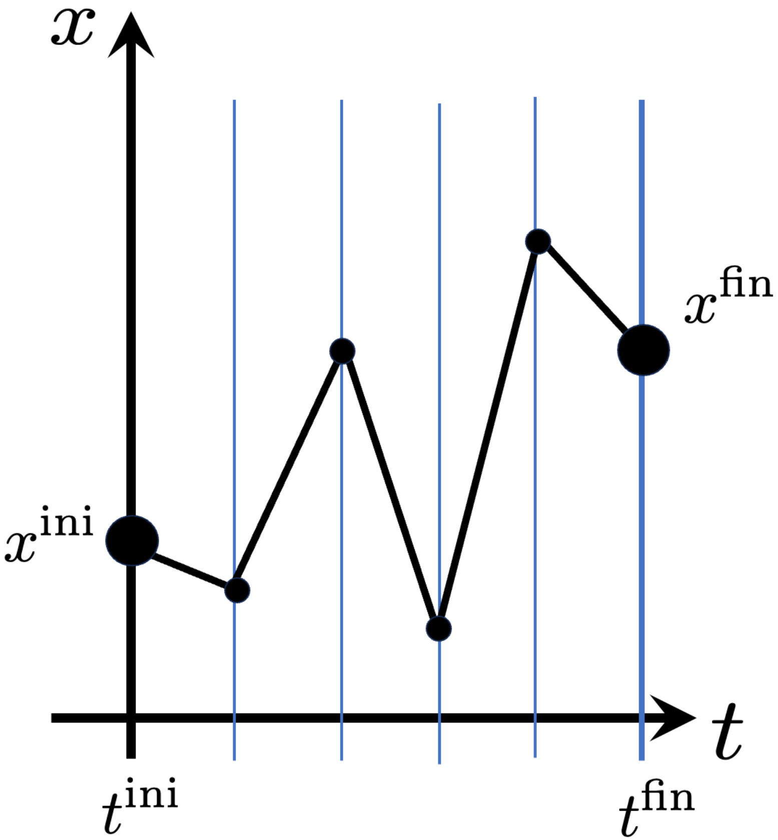
$$x(t) = \sum_n a_n \theta(t + c_n)$$



2. Universal Approx. of Path Integral

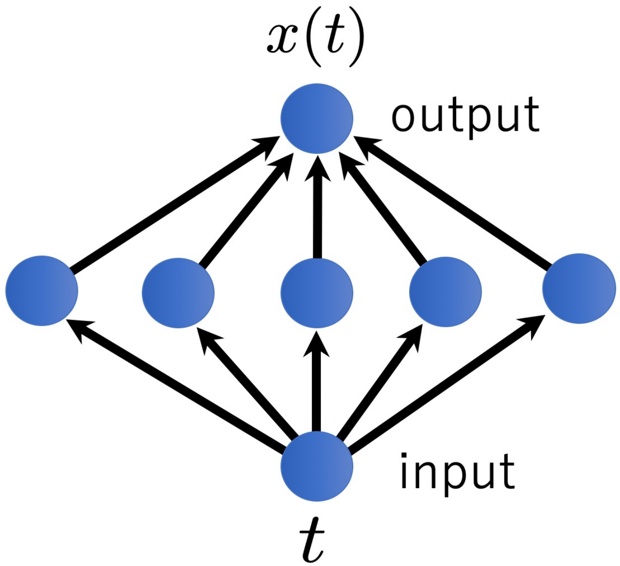
Step 1) Universally approximate the paths

Arbitrary path in path integral



Universal approximation theorem

$$x(t) = \sum_{n=1}^{N} w_n \mathrm{ReLU}(t - t_{n-1}) + b$$



2. Universal Approx. of Path Integral

Step 2) Statistical sum over NN parameters

Path integral measure $\prod_{n=1}^{N-1} dx_n = (\Delta t)^{N-1} \prod_{n=1}^{N-1} dw_n$

Path integral weight $\exp\left[\frac{i}{\hbar} S\right]$

$$S = \int dt\, \mathcal{L}[x, \dot{x}] = \Delta t \sum_{n=1}^{N} \mathcal{L}\left[\frac{x_n + x_{n-1}}{2}, \frac{x_n - x_{n-1}}{\Delta t}\right]$$

$$= \Delta t \sum_{n=1}^{N} \mathcal{L}\left[x^{\text{ini}} + \Delta t \sum_{k=1}^{n}\left(k - \frac{1}{2}\right) w_{n-k+1}\, , \sum_{k=1}^{n} w_k\right]$$

<u>NN Characteristics</u>:

- ReLU activation w/ time division bias
- Large N units at the second layer
- No i.i.d. parameter samples

2. Universal Approx. of Path Integral

Ex. Free particle, Harmonic oscillator

Path integral $\langle x^{\text{fin}} | x^{\text{ini}} \rangle = (\Delta t)^{N-1} \int \exp\left[\frac{i}{\hbar} S\right] \prod_{n=1}^{N-1} dW_n \qquad W_n \equiv \sum_{k=1}^{n} w_k$

Path integral weight $\exp\left[\frac{i}{\hbar} S\right]$

Free particle : $S = \int dt \, \frac{m}{2} \dot{x}^2 = \frac{m \Delta t}{2} \left(\sum_{n=1}^{N-1} (W_n)^2 + \left(\frac{x^{\text{fin}} - x^{\text{ini}}}{\Delta t} - \sum_{k=1}^{N-1} W_k \right)^2 \right)$

Harmonic oscillator:

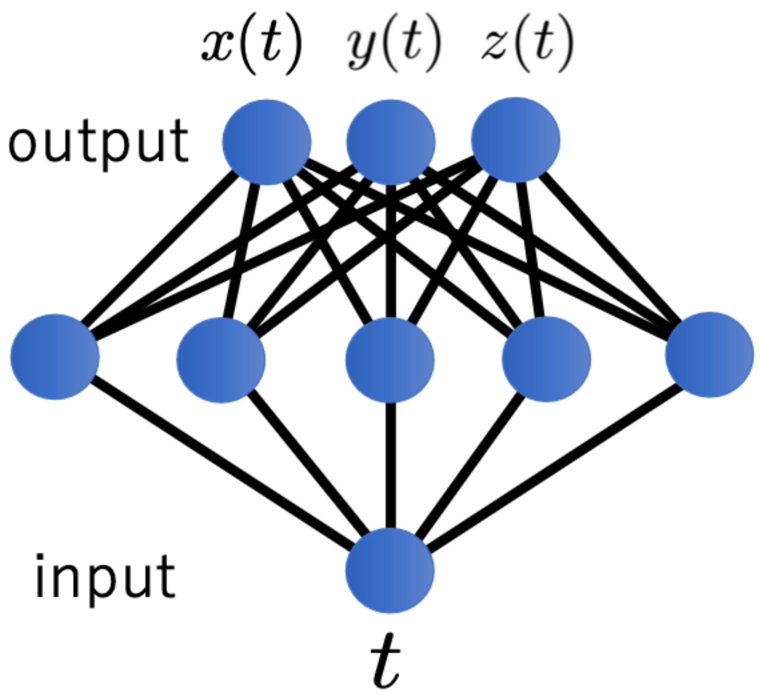
$$S = \int dt \left(\frac{m}{2} \dot{x}^2 - \frac{k}{2} x^2 \right) = \frac{m \Delta t}{2} \left(\sum_{n=1}^{N-1} (W_n)^2 + \left(\frac{x^{\text{fin}} - x^{\text{ini}}}{\Delta t} - \sum_{k=1}^{N-1} W_k \right)^2 \right)$$

$$- \frac{k \Delta t}{2} \sum_{n=1}^{N-1} \left(x^{\text{ini}} - \frac{1}{2} \Delta t \, W_n + \Delta t \sum_{k=1}^{n} W_k \right)^2$$

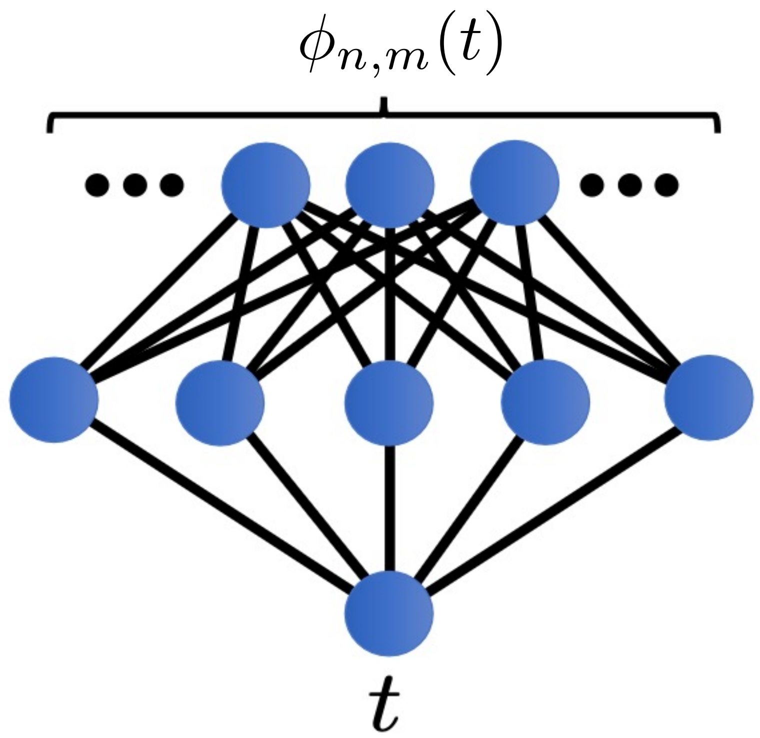
2. Universal Approx. of Path Integral

Ex. QFT

Quantum mechanics
In multi dimensions



Quantum field theory
on a lattice



NN Representation of Quantum Systems

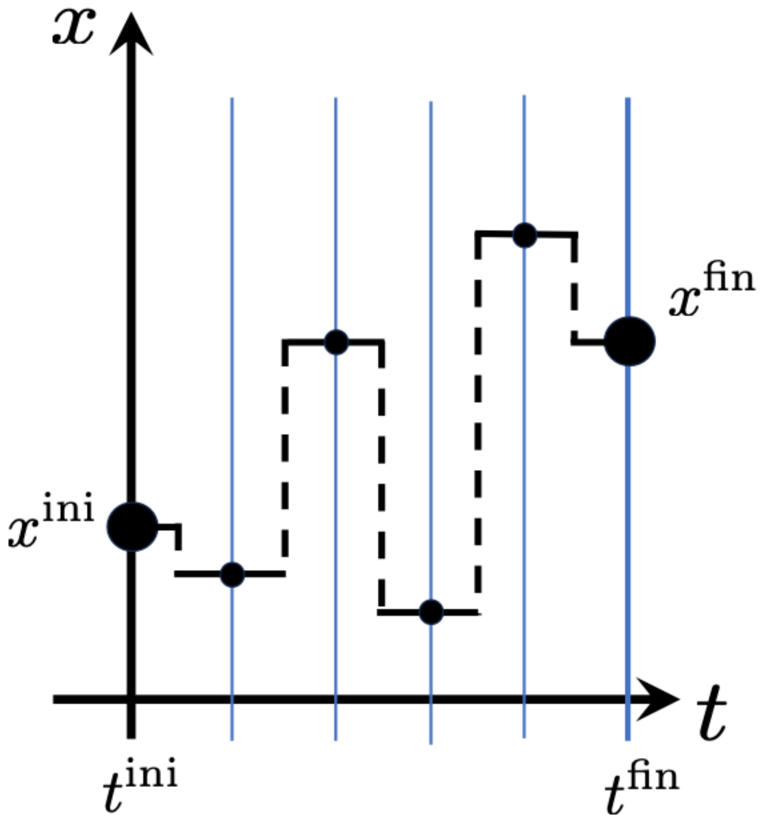
[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

1. Neural network field theory — 2 pages
2. Universal approximation of path integral — 5 pages
3. Various activations — 4 pages
4. Generality: NN=QM? — 4 pages

3. Various activations

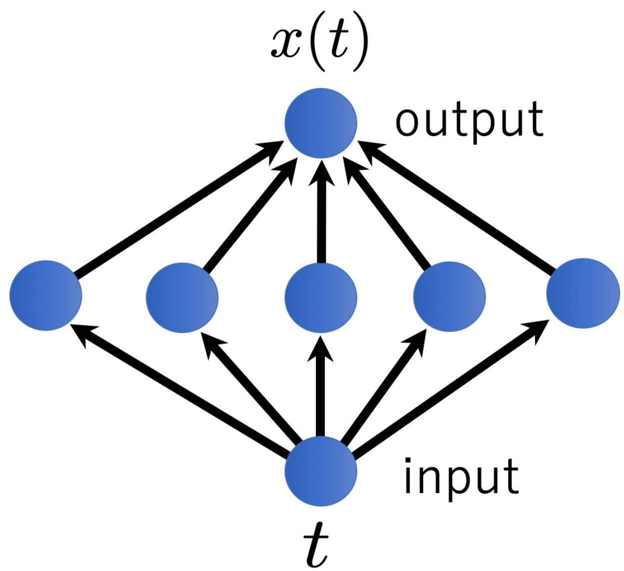
Can use step activation

Arbitrary path in path integral



Universal approximation theorem

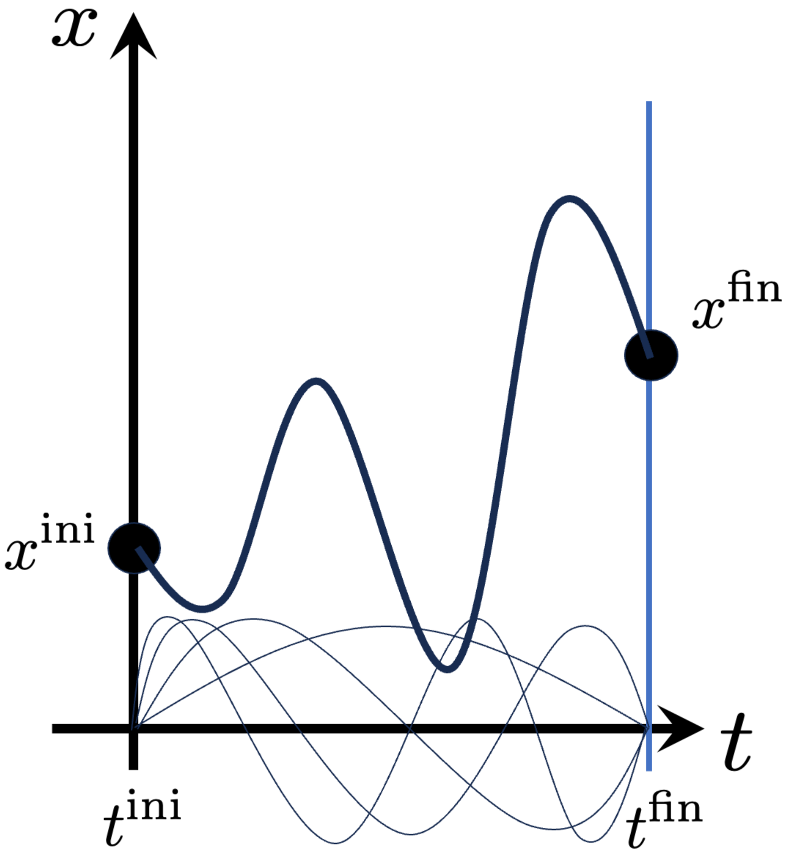
$$x(t) = x^{\mathrm{ini}} + \Delta t \sum_{n=1}^{N} W_n \, \theta\left(t - t_{n-1} - \frac{1}{2}\right)$$



3. Various activations

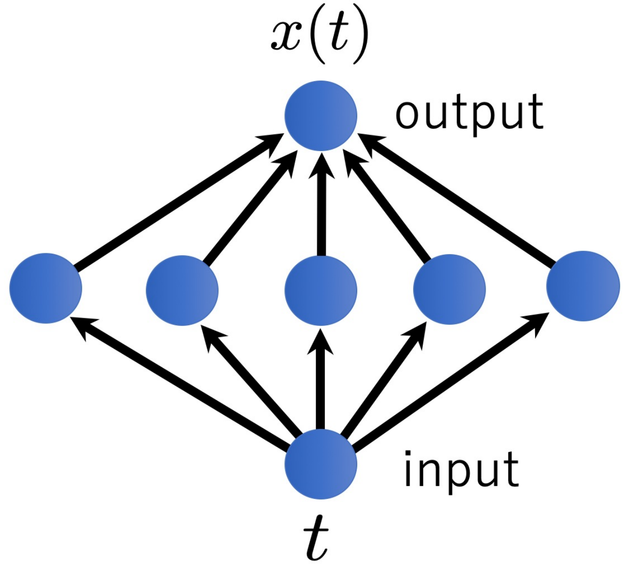
Can use cosine activation

Arbitrary path in path integral



Fourier decomposition

$$x(t) = \sum_n a_n \cos\left(\frac{n\pi}{T} t\right)$$

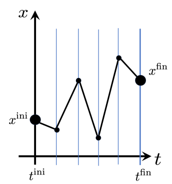


3. Various activations

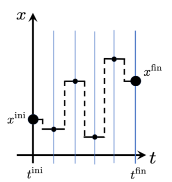
Physical meaning of NN parameters

$$x(t) = \sum_n a_n \sigma(b_n t + c_n)$$

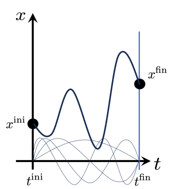
NN Repr.



ReLU : $x(t) = \sum_{n=1}^{N} w_n \mathrm{ReLU}(t - t_{n-1})$



Step : $x(t) = \Delta t \sum_{n=1}^{N} W_n \, \theta \left(t - t_{n-1} - \frac{1}{2} \right)$

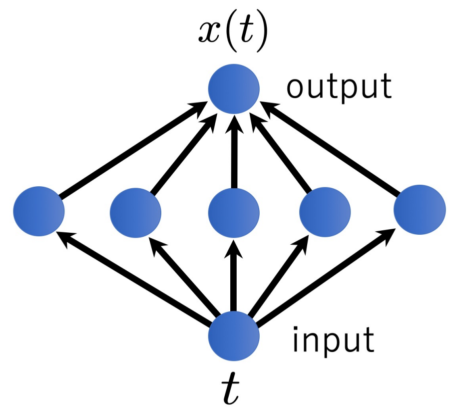


Cosine : $x(t) = \sum_{n} a_n \cos\left(\frac{n\pi}{T} t \right)$

Activation	Meaning of network parameters a	b	c
ReLU	Acceleration	Fixed	Time divisions (fixed)
Step	Velocity	Fixed	Time divisions (fixed)
Cosine	Fourier Coefficient	Frequency (fixed)	Fixed

3. Various activations

NNFT is Fourier transf. of our formulation



Central limit theorem

Wide limit

Universal approximation theorem

Wide limit

NNFT

Our formulation

||

||

Path integral over momenta

Fourier transformation

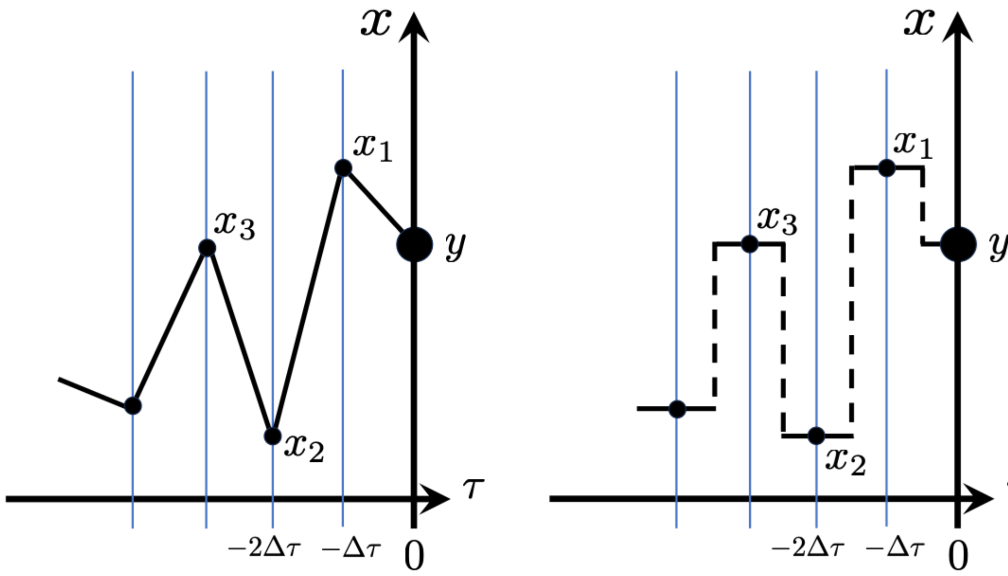
Path integral over coordinates

NN Representation of Quantum Systems

[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

4. Generality : NN=QM?

1) Ground state is statistical sum of NNs



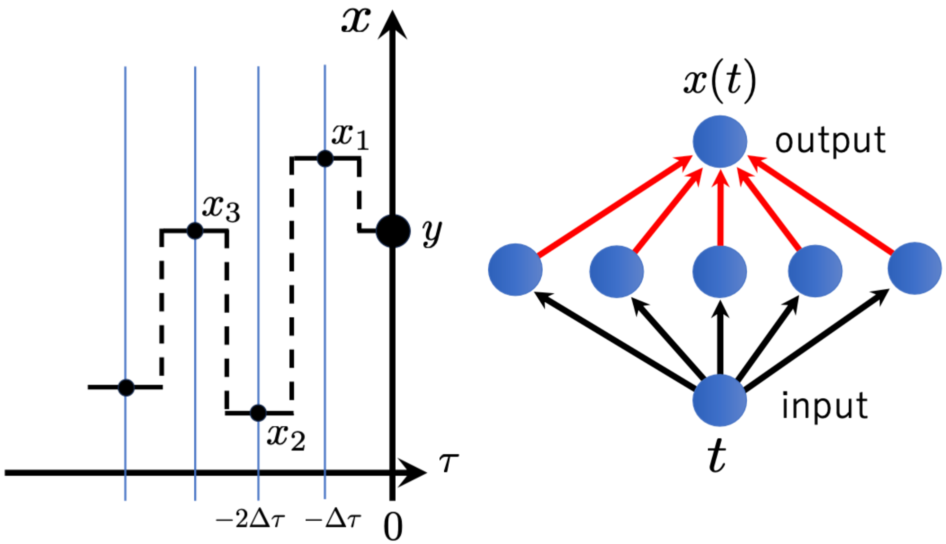
Ground state: $$\psi(y) \propto \int_{x(\tau=-\infty)=\text{any}}^{x(\tau=0)=y} \prod dx_n \exp\left[-\frac{1}{\hbar}S_{\mathrm{E}}\right]$$

$$\propto \int \prod_{n=1}^{\infty} dw_n \exp\left[-\frac{1}{\hbar}S_{\mathrm{E}}(\{w_n\}, y)\right]$$

4. Generality : NN=QM?

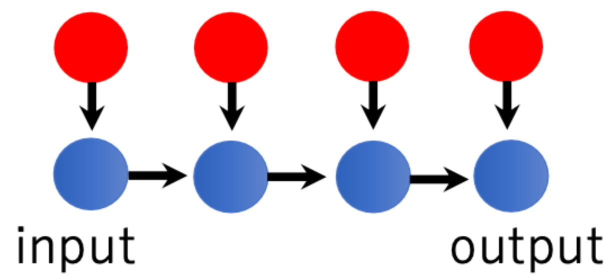
2) Step activation is stochastic quantization

Path integral



Stochastic quantization

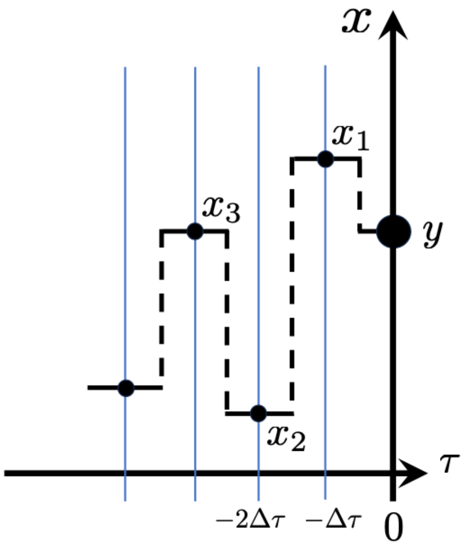
$$\frac{\partial x(\tau)}{\partial \tau} = F(x(\tau), \tau) + \sqrt{2D}\,\eta(\tau)$$



$$x_n = y + \Delta\tau \sum_{k=1}^{n} W_k \quad \Leftrightarrow \quad x_{n+1} = x_n + \Delta\tau\, W_{n+1}$$

4. Generality : NN=QM?

3) Gaussian random NN = Free particle



NN repr. of a free particle
w/ step activation :

$$\int \prod_{n=1}^{\infty} dW_n \exp\left[-\frac{1}{\hbar}\frac{m\Delta t}{2}\sum_{n=1}^{N-1}(W_n)^2\right]$$

i.i.d. samples, Gaussian.

➔ Whole knowledge of Random NN [Amari `69] [Rozonoer `69] can allow Quantum Interpretation?

4. Generality : NN=QM?

4) In fact: no need of parameter fixing

Non-fixed NN $$x(t) = \sum_{n=1}^{N} w_n \mathrm{ReLU}(\tilde{w}_n t + \tilde{b}_n) + b$$

⇩ Use rescaling symmetry $\mathrm{ReLU}(\alpha x) = \alpha \mathrm{ReLU}(x)$

Equivalent NN $$x(t) = \sum_{n=0}^{N-1} w_n \tilde{w}_n \mathrm{ReLU}(t + \tilde{b}_n / \tilde{w}_n) + b$$

⇩ Redefine parameters

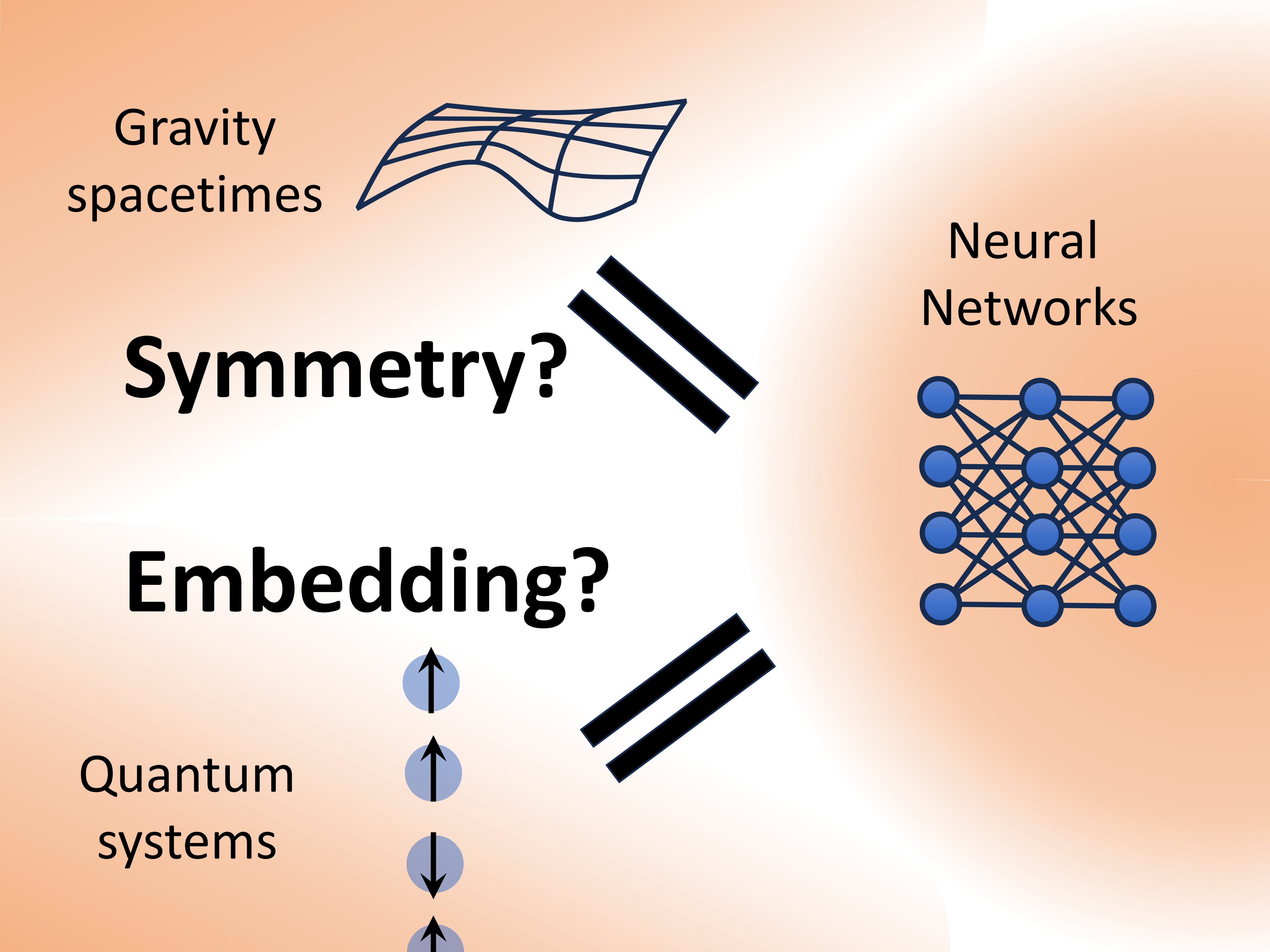
Partially-fixed NN $$x(t) = \sum_{n=0}^{N-1} w_n \mathrm{ReLU}(t + \tilde{b}_n) + b$$

Arbitrary time-division formulation of QM

NN Representation of Quantum Systems

[Hirono, Maeda, Totsuka-Yoshinaka, KH 2403.11420]

To what extent
neural networks
are similar to
quantum mechanics?


Gravity spacetimes
Neural Networks
Symmetry?
Embedding?
Quantum systems