Complex Fluids and Complex Flows Group
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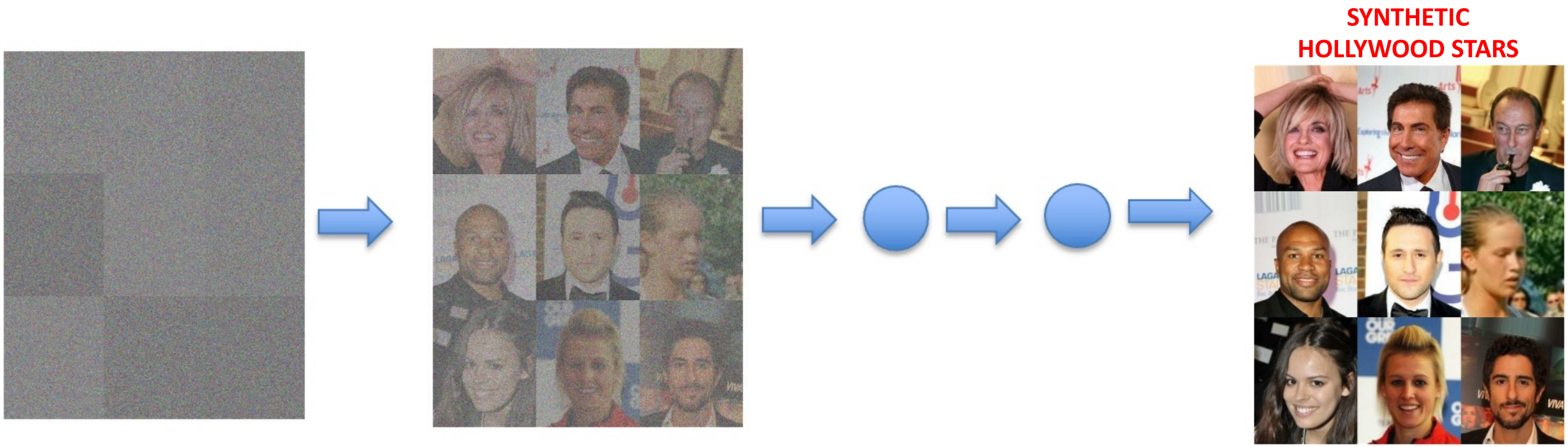


ERICE – MACHINE LEARNING FOR COMPLEXITY
2024 APRIL
Data driven tools for Eulerian and Lagrangian Turbulence

CREDITS: M. CENCINI, C. CALASCIBETTA, L. PIRO, R. HEINONEN, T. LI, F. BONACCORSO, M. BUZZICOTTI, M. SCARPOLINI

Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

1. Short introduction to Eulerian and Lagrangian Turbulence in 2D, 3D and in between
2. Data-driven and Equation-Informed tools for Eulerian Turbulence
3. Data-driven and Equation-Informed tools for Lagrangian Turbulence

COMPLEX FLUIDS & COMPLEX FLOWS

EULERIAN

$$\begin{cases} \partial_t v + v \cdot \partial_x v = -\partial_x p + \nu \partial^2 v + g\theta + b \cdot \partial_x b + f + \sum_i \delta(x - X_i)\mathcal{F} \\ \partial_x \cdot v = 0 \\ +b.c. \\ \partial_t \theta + v \cdot \partial_x \theta = \chi_\theta \partial^2 \theta \\ \partial_t b + v \cdot \partial_x b - b \cdot \partial_x v = \chi_b \partial^2 b \end{cases}$$

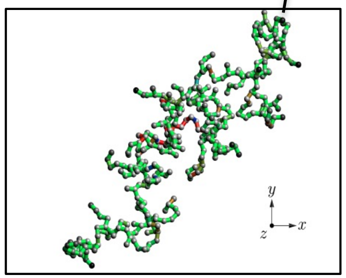
temperature

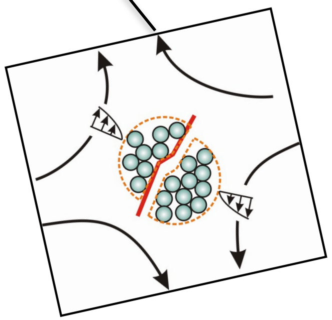
magnetic field

small active/passive particles/droplets/bubbles/colloidal aggregates

LAGRANGIAN

$$\begin{cases} \dot{X}_i = U_i \\ \dot{U}_i = \frac{v(X_i) - U_i}{\tau} + \beta D_t v(X_i) + U_i^C \end{cases}$$





COMPLEX FLUIDS & COMPLEX FLOWS

EULERIAN

$$\begin{cases} \partial_t v + v \cdot \partial_x v = -\partial_x p + \nu \partial^2 v \quad + f \\ \partial_x \cdot v = 0 \\ +b.c. \end{cases}$$

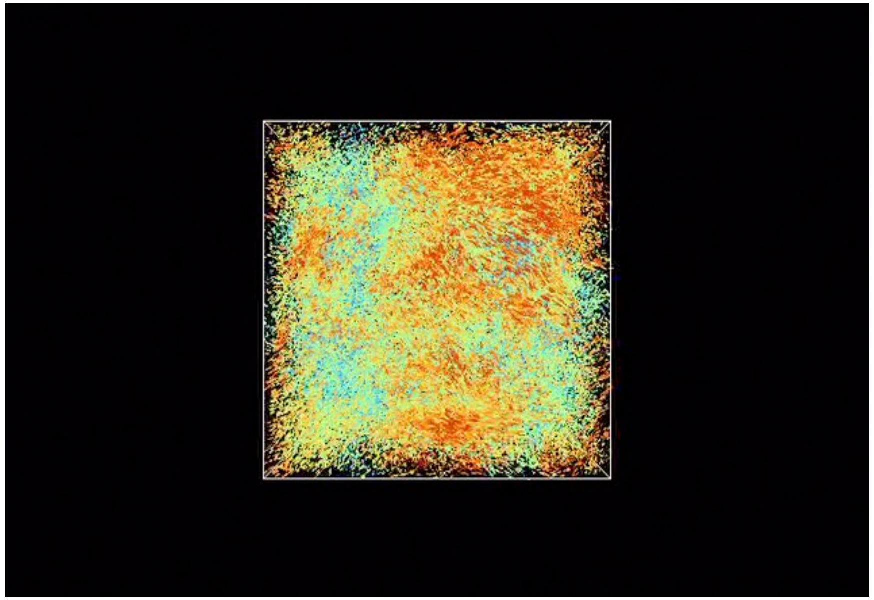
temperature

magnetic field

LAGRANGIAN

$$\left\{ \dot{X}_i = v(X_i(t), t) \right.$$

passive fluid tracers



CONTROL PARAMETER:

$$Re \sim \frac{v \partial_x v}{\nu \partial^2 v} \sim \frac{v_0 L_0}{\nu}$$

(NASA/Goddard Space Flight Center Scientific Visualization Studio)

NAVIER-STOKES 3D-2D:
DIMENSIONS MATTER!

Entry #: 84174

Vortices within vortices: hierarchical nature of vortex tubes in turbulence

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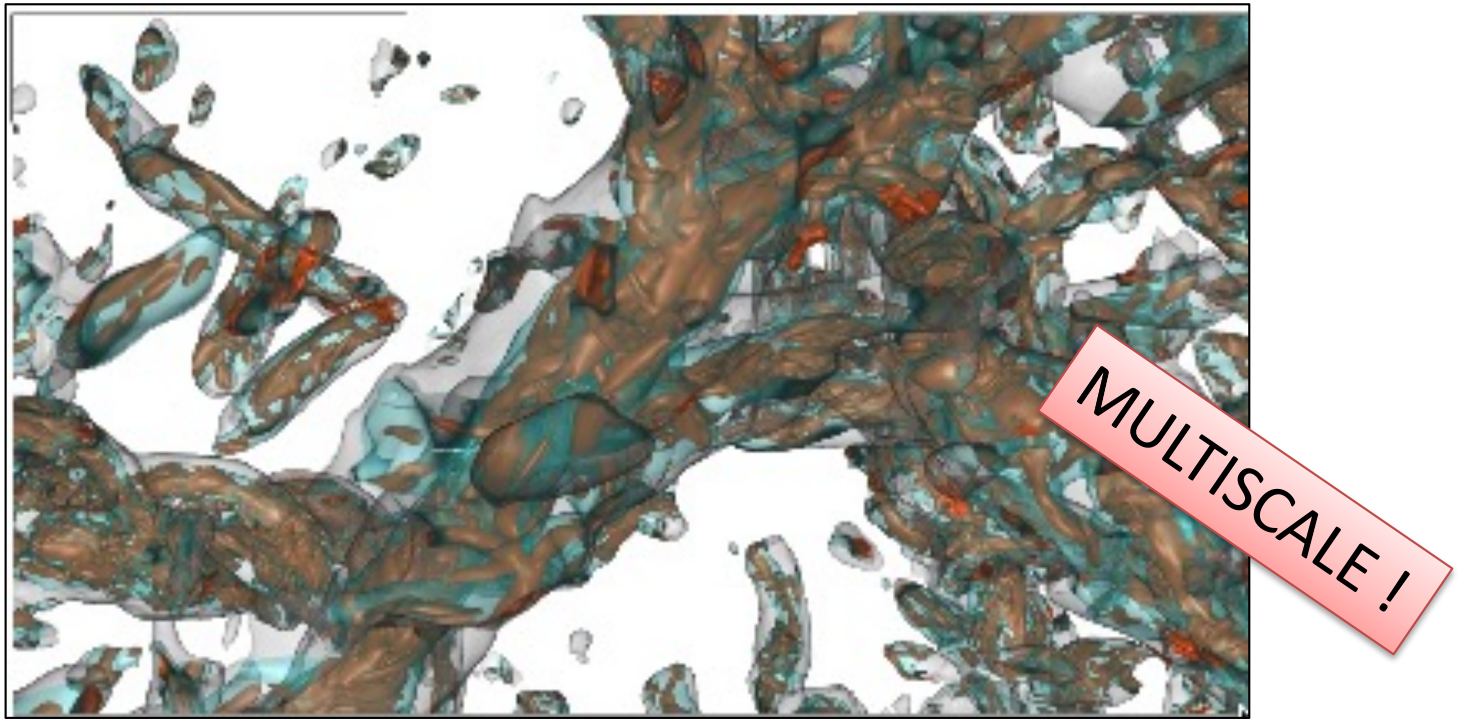
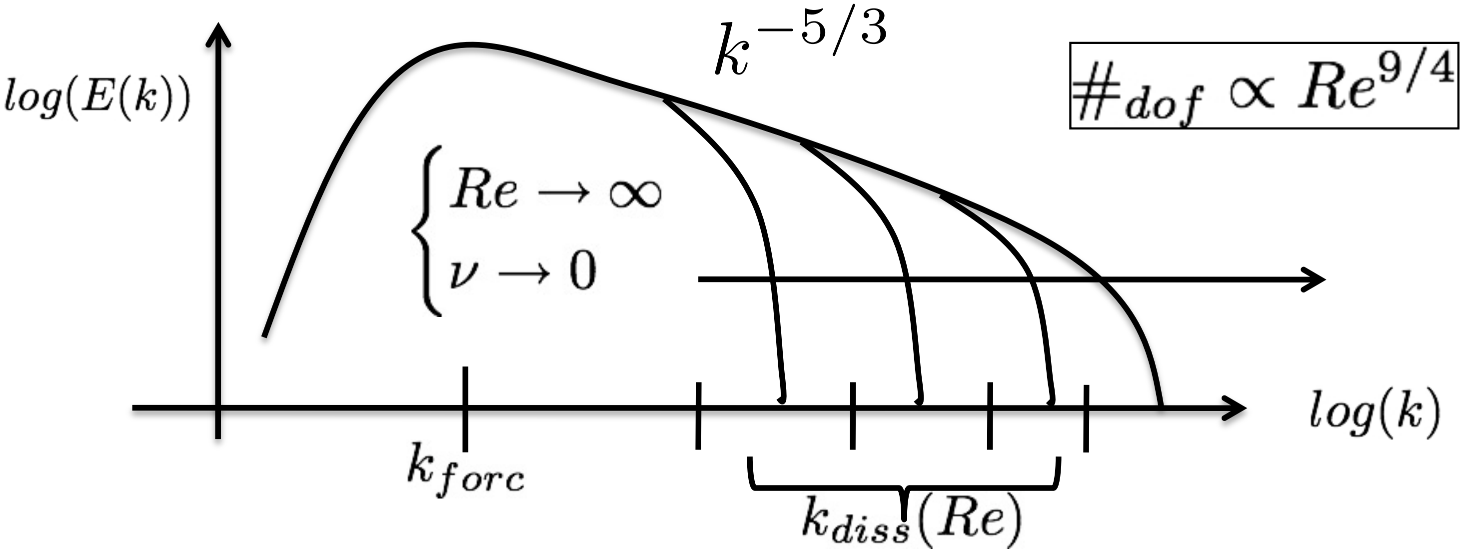
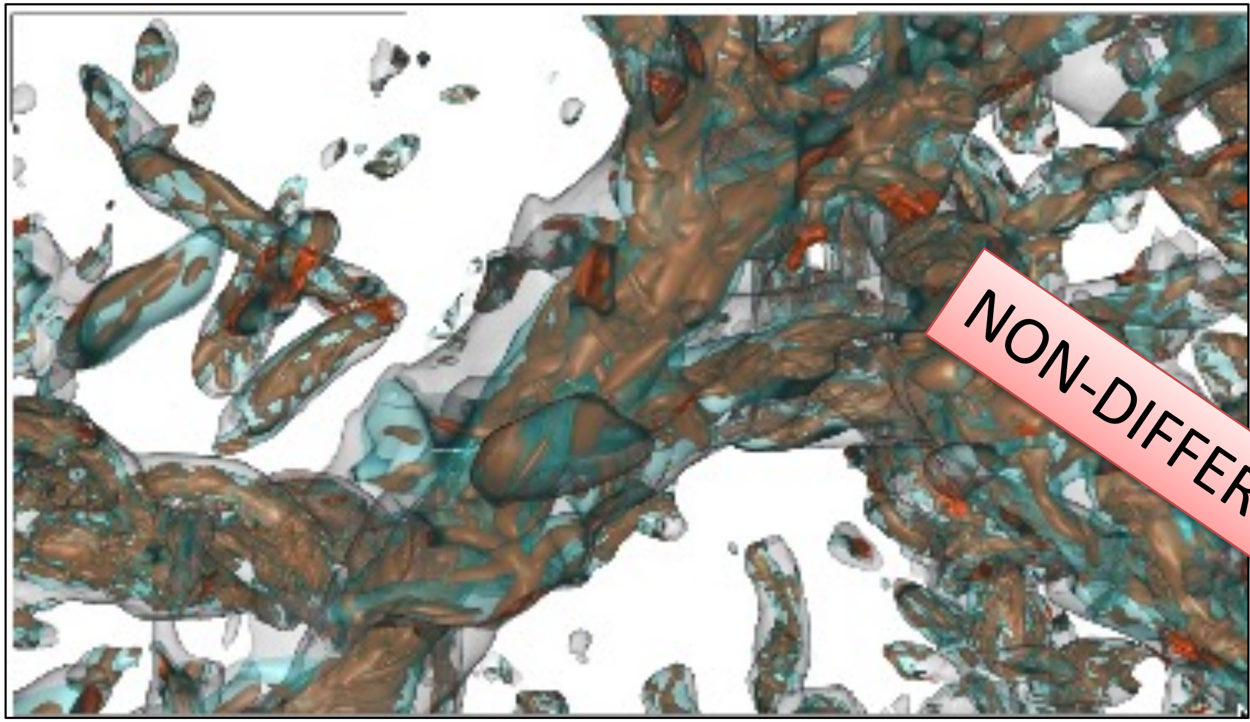
NAVIER-STOKES 3D

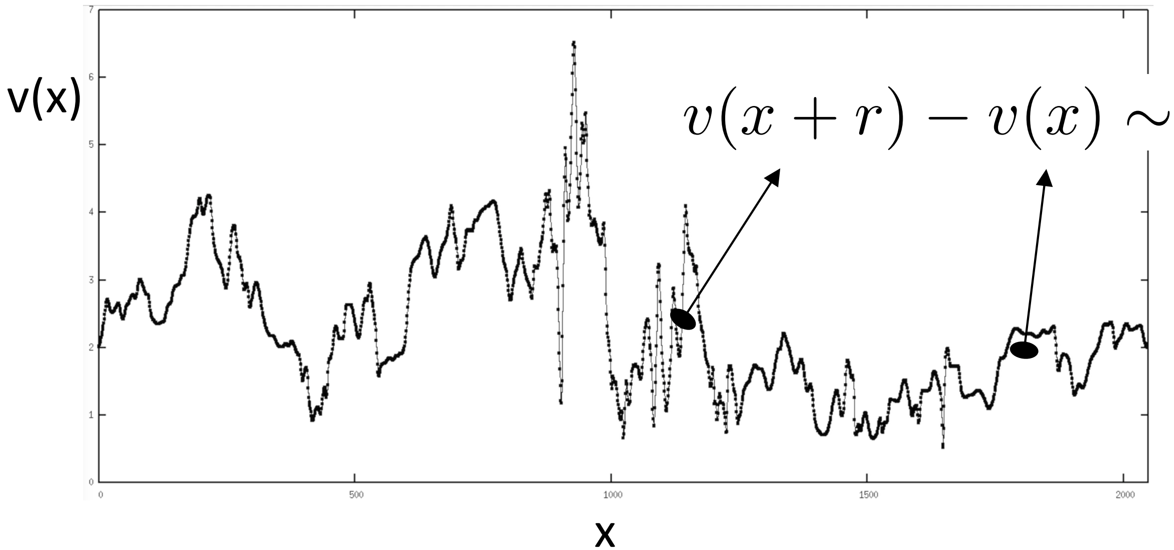




NAVIER-STOKES 3D



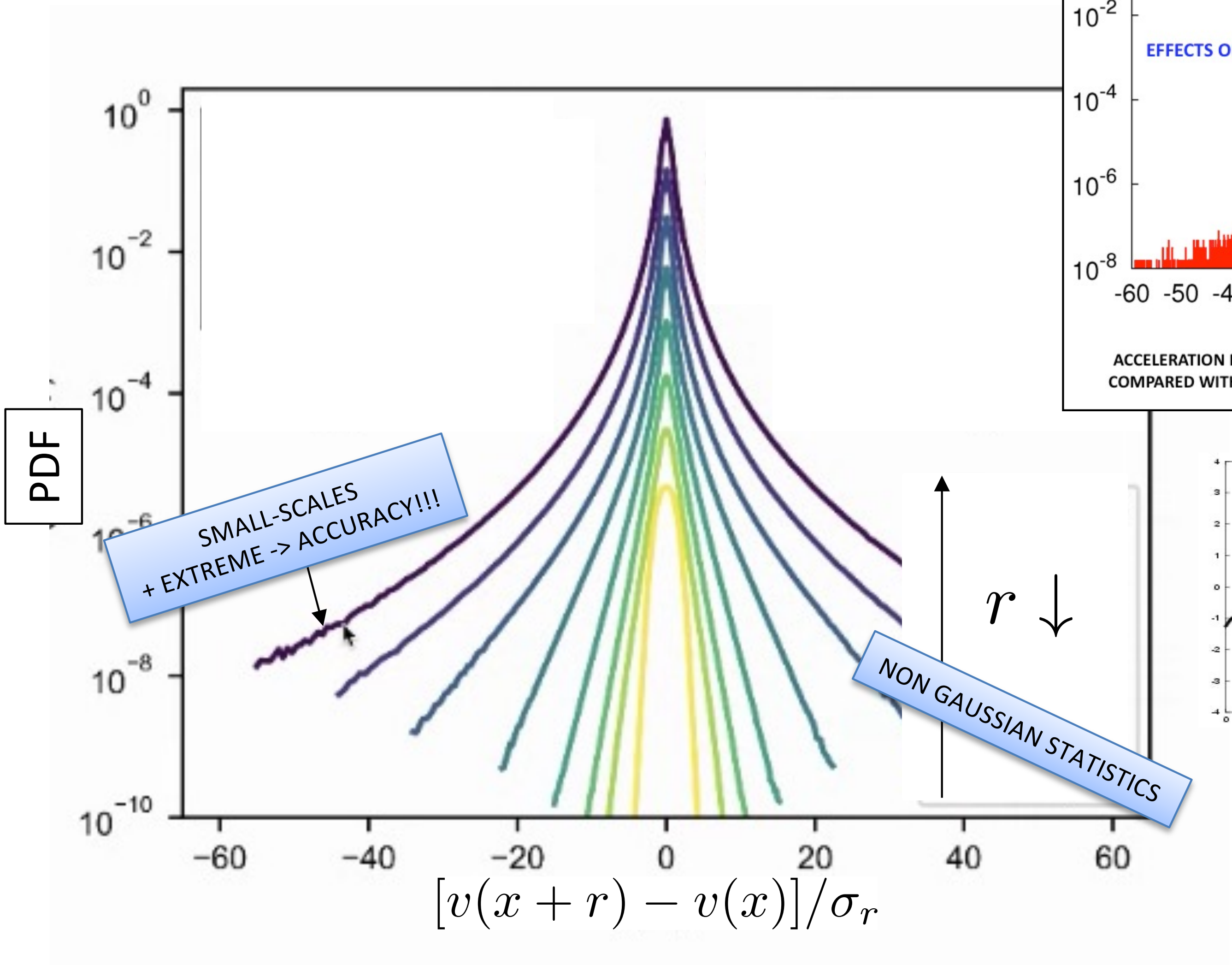
NON-DIFFERENTIABLE !

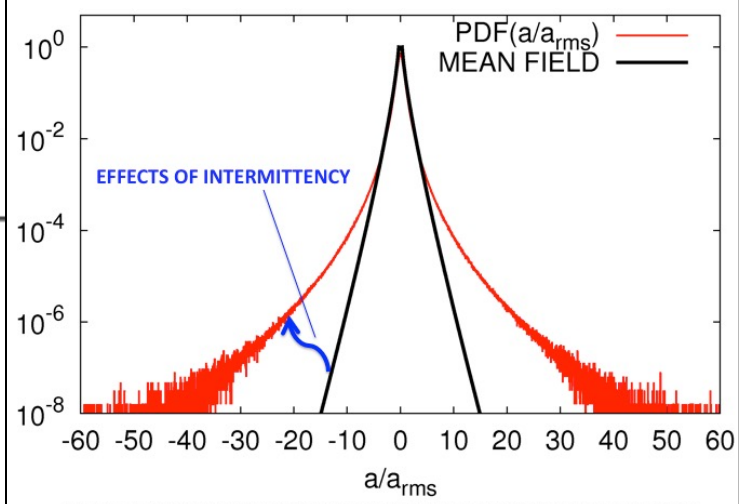


$$v(x+r) - v(x) \sim r^{1/3}$$

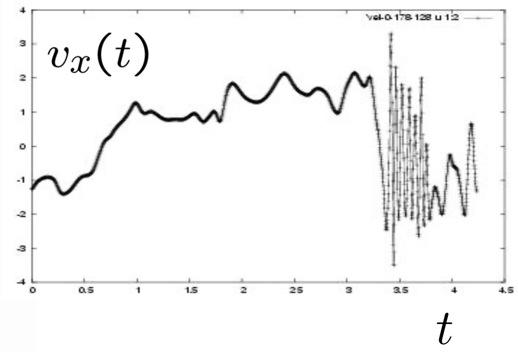
Holder continuos 1/3
non-differentiable!

NAVIER-STOKES 3D





ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04]
COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)



Bentkamp, L, Cr C. Lalescu, and M. Wilczek. "*Nature communications* 10.1 (2019): 1-8.

Turbulent luminance in impassioned van Gogh paintings

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Road with Cypress and Star

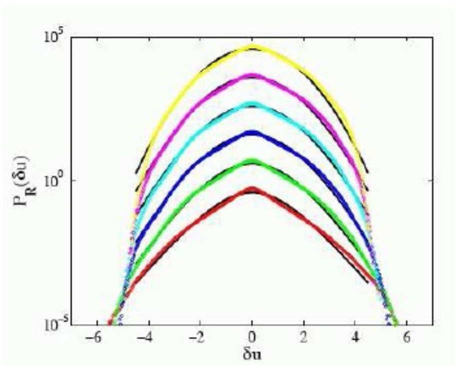


FIG. 2: Semilog plot of the probability density $P_R(\delta u)$ of luminance changes δu for pixel separations R = 60, 240, 400, 600, 800, 1200 (from bottom to top). Curves have been vertically shifted for better visibility. Data points were fitted, according to Ref. [13], and the results are shown in full lines; parameter values are λ = 0.2, 0.15, 0.12, 0.11, 0.09, 0.0009 (from bottom to top).

Starry night

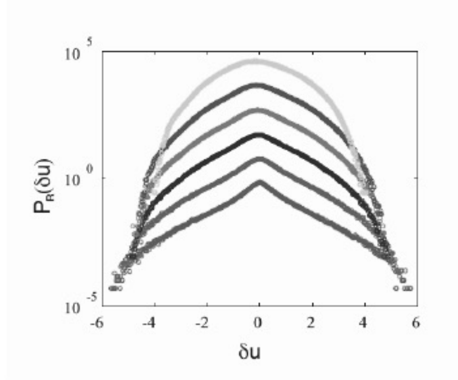
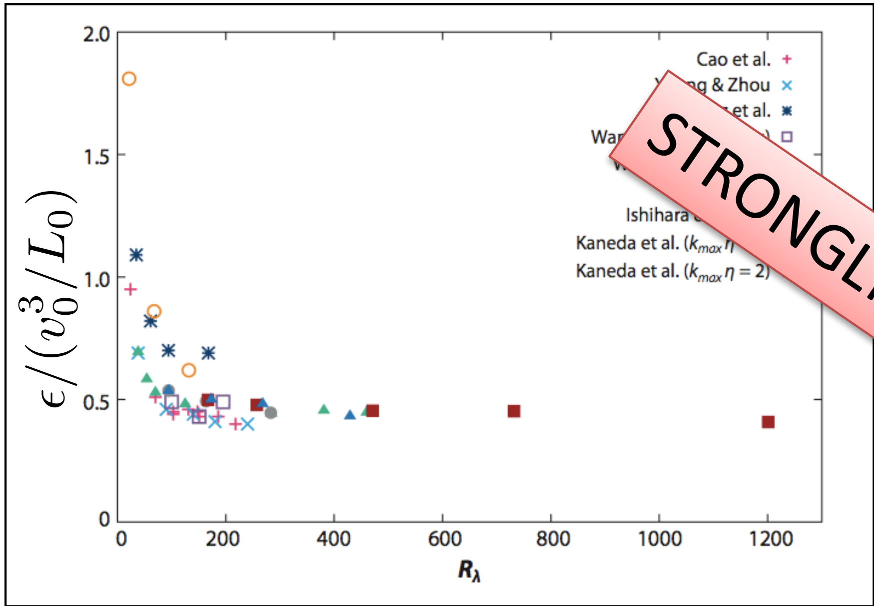


FIG. 5: Left: *Road with Cypress and Star* (Rijksmuseum Kröller-Müller, Otterlo). Right: PDF for pixel separations R = 2, 5, 15, 20, 30, 60 (from bottom to top). The studied image was taken from the WebMuseum-Paris, webpage.

NAVIER-STOKES 3D

NO ROOM FOR QUASI-EQUILIBRIUM APPROACHES



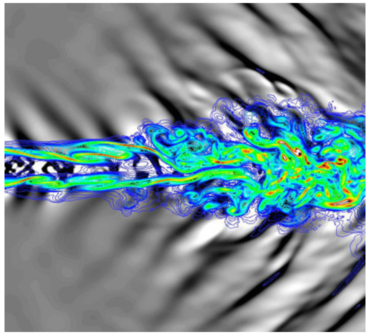
STRONGLY OUT-OF-EQUILIBRIUM!

$$\partial_t v + v\partial v = -\partial p + \nu \Delta v$$

$$\lim_{Re\to\infty} \epsilon = \lim_{\nu\to 0} \nu\langle(\partial v)^2\rangle \to const.$$

DISSIPATIVE ANOMALY

$$\#_{dof} = (\frac{k_{diss}}{k_{forc}})^3 \sim Re^{9/4}$$



laboratory flow

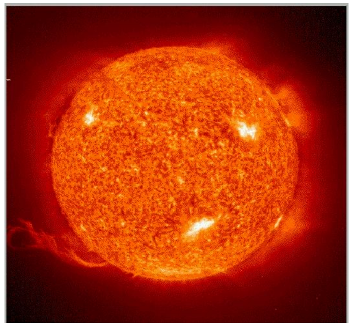
$$Re \sim 10^5 - 10^9$$

$$\#_{dof} \sim 10^{11} - 10^{20}$$

atmosph. flow

$$Re \sim 10^8 - 10^{12}$$

$$\#_{dof} \sim 10^{18} - 10^{30}$$



astrophys. flow

$$Re > 10^{15}$$

$$\#_{dof} \sim \infty$$

FIRST EXASCALE COMPUTATION WORLD-RECORD

76th Annual Meeting of the Division of Fluid Dynamics
Sunday–Tuesday, November 19–21, 2023; Washington, DC

Session T02: Turbulence: DNS
4:25 PM–5:56 PM, Monday, November 20, 2023
Room: Ballroom B

Chair: Robert Moser, University of Texas at Austin

Abstract: T02.00005 : Turbulence simulations at grid resolution up to 32768^3 enabled by Exascale computing*
5:17 PM–5:30 PM

Presenter:
Pui-Kuen (P.K) Yeung
(Georgia Institute of Technology)

32768^3 ~ 35 TRILLION GRID POINT
1 CONF ~ 1 PETABYTE
FEATURES RANKING!!!

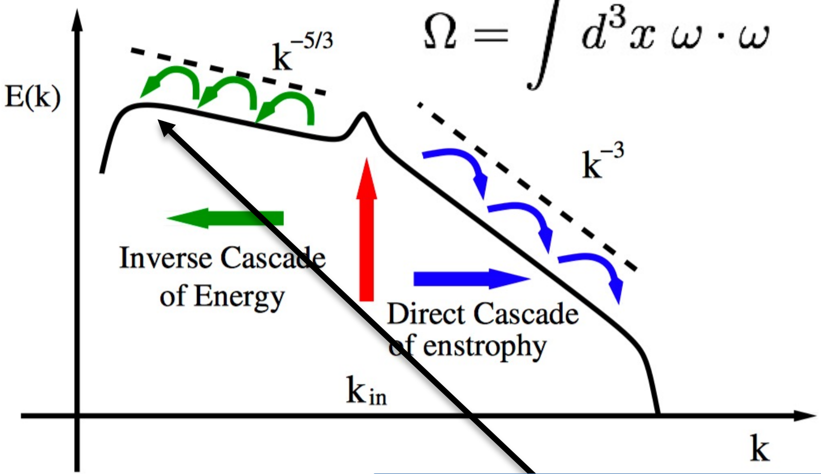
e

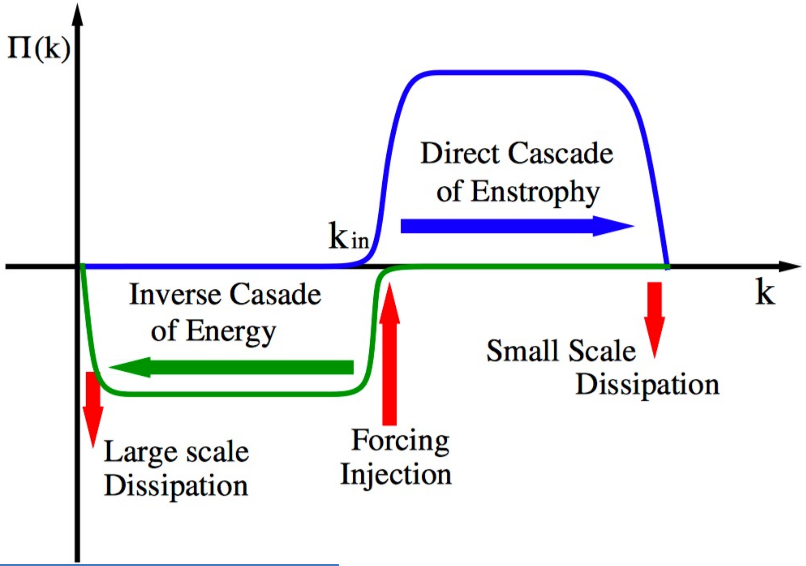
Moral: brute force Direct Numerical Simulations able to saturate any computing power (present and/or future): Computo ergo sum?

2D –TURBULENCE: 2 POSITIVE DEFINITE INVARIANTS

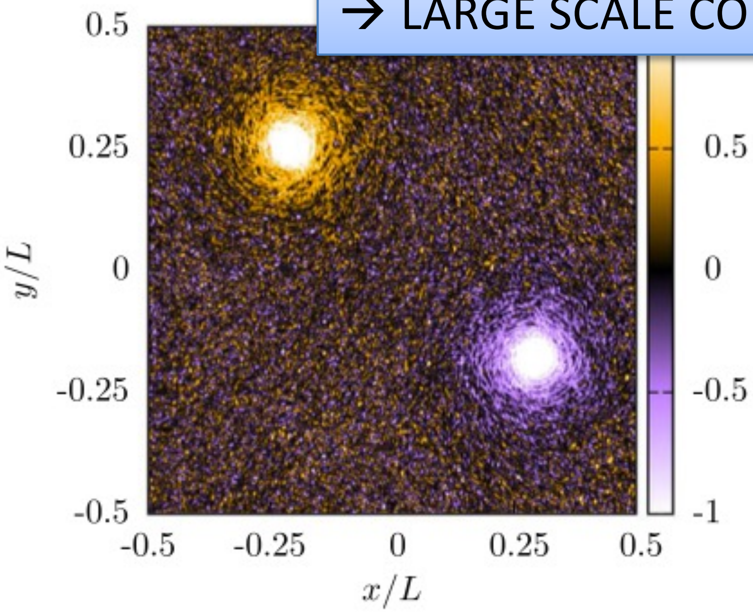
$$E = \int d^3x \, \mathbf{v} \cdot \mathbf{v}$$

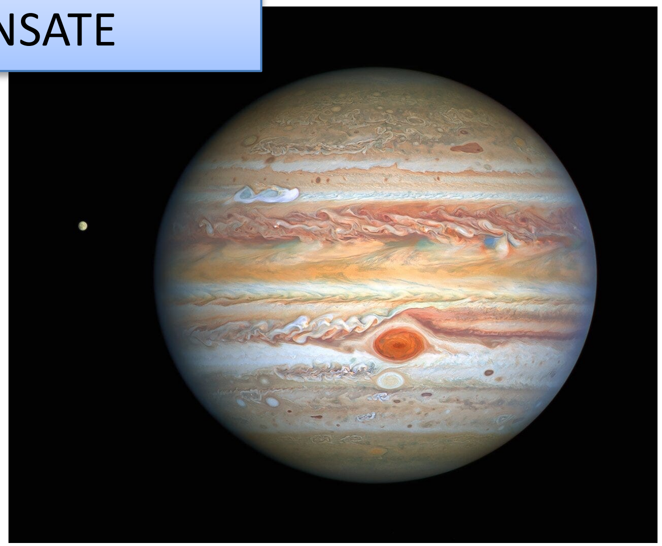
$$\Omega = \int d^3x \, \omega \cdot \omega$$





INVERSE CASCADE + IR CUT-OFF →
→ LARGE SCALE CONDENSATE





J. Laurie et al. PRL 113, 254503 (2014); H. Xia et al. PRL 101, 194504 (2008)

2D/3D NOT THE END OF THE HISTORY: SPLIT ENERGY CASCADE

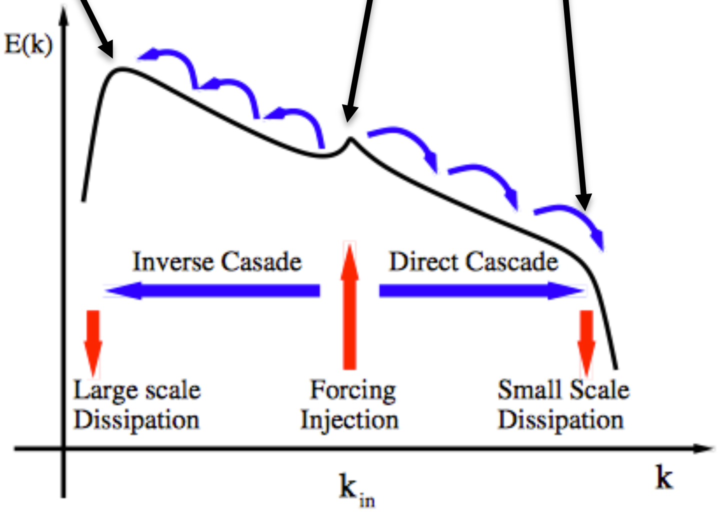
$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P + \nu\Delta\mathbf{u} - \alpha\mathbf{u} + \mathbf{f}.$$

$$\partial_t \mathcal{E} = \epsilon_{in}(t) - \epsilon_{\nu}(t) - \epsilon_{\alpha}(t)$$

$$\epsilon_\alpha(t) = \alpha\langle|\mathbf{u}|^2\rangle$$

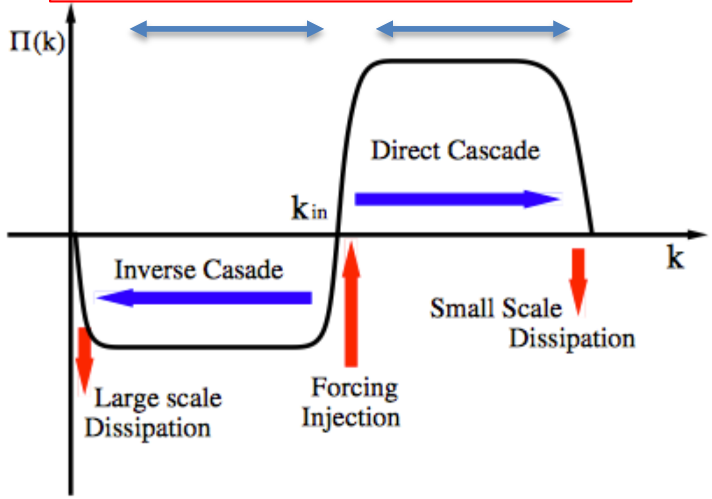
$$\epsilon_{in}(t) = \langle\mathbf{u}\cdot\mathbf{f}\rangle$$

$$\epsilon_\nu(t) = \nu\langle|\nabla\mathbf{u}|^2\rangle$$



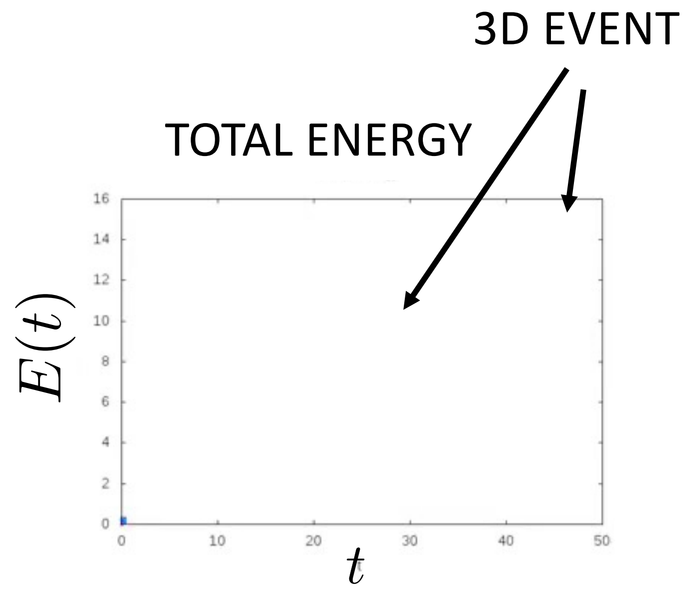
CONTROL PARAMETERS

$$Kr = \frac{u_f k_{in}}{\alpha} \qquad Re = \frac{u_f}{\nu k_{in}}$$

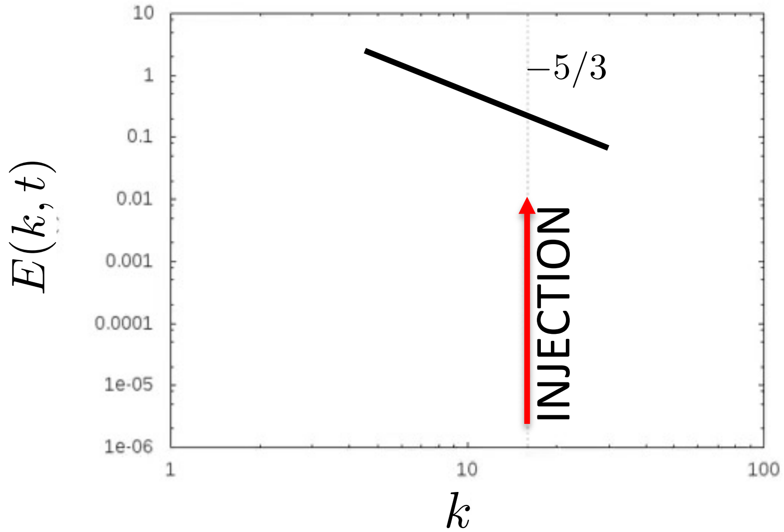


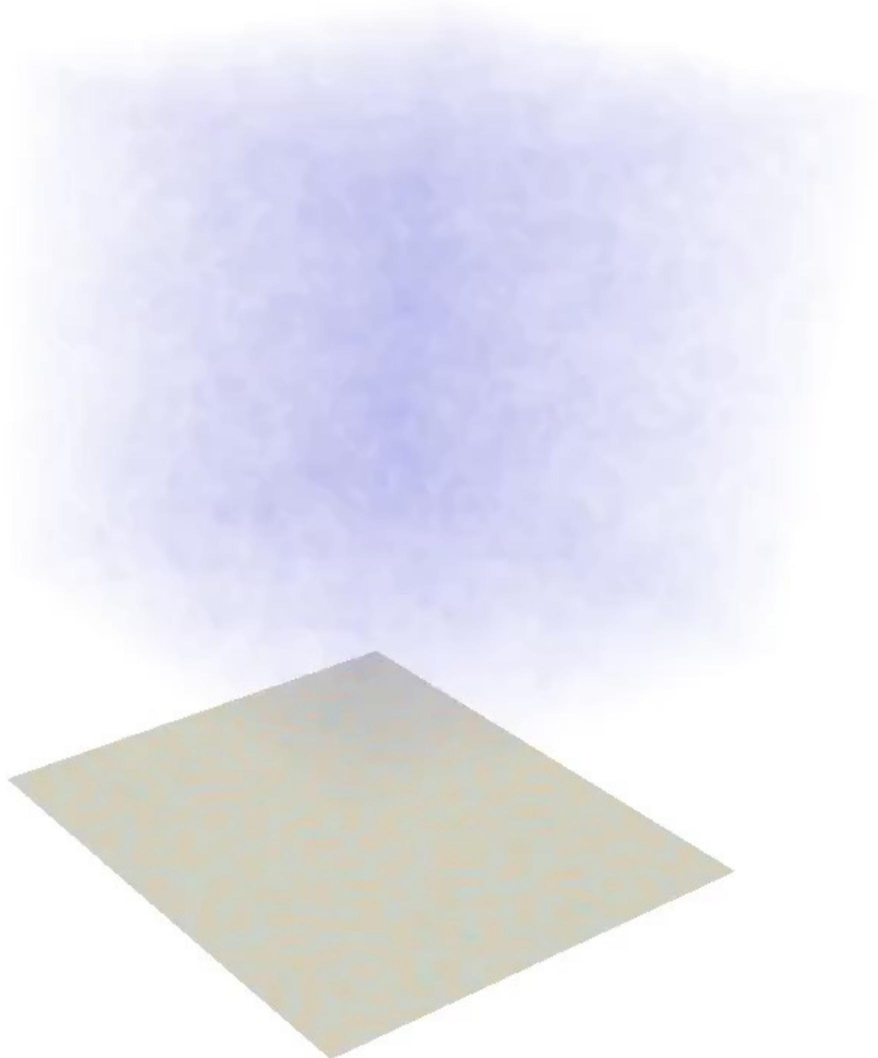
A. Alexakis and L.B "Transitions and cascades in turbulent flows" Phys. Rep 2018

TURBULENCE UNDER ROTATION

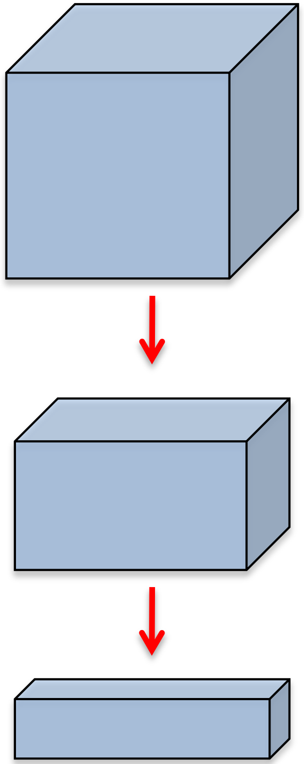


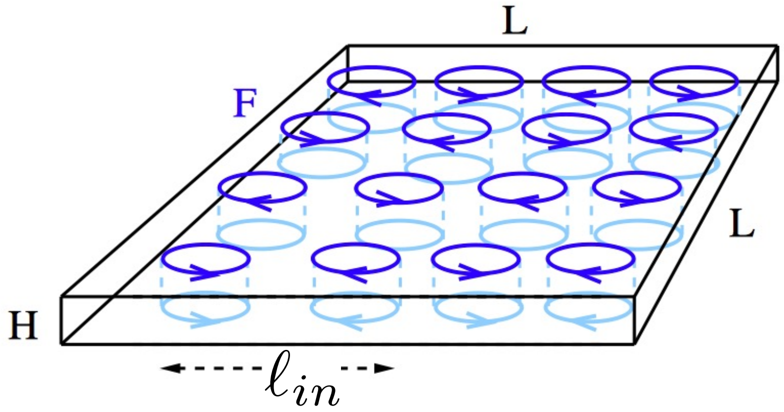
ENERGY SPECTRUM





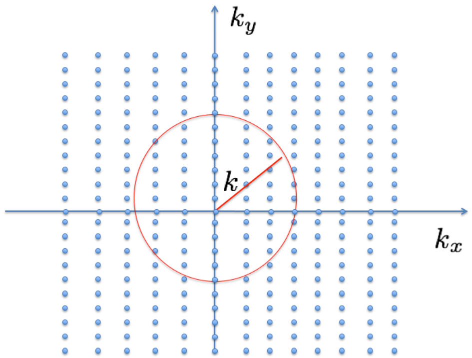
3D ← → 2D: THIN LAYERS

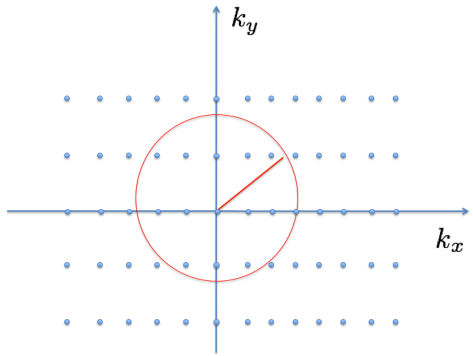




CONTROL PARAMETER

$$\frac{H}{\ell_{in}}$$





$$\partial_t \mathbf{u}_{2D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{2D} = -\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \overline{\nabla P} + \nu \Delta \mathbf{u}_{2D} - \alpha \mathbf{u}_{2D} + \mathbf{f}_{2D},$$

$$\partial_t \mathbf{u}_{3D} + \mathbf{u}_{2D} \cdot \nabla \mathbf{u}_{3D} = -\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{2D} + (\overline{\mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}} - \mathbf{u}_{3D} \cdot \nabla \mathbf{u}_{3D}) - \nabla P + \nu \Delta \mathbf{u}_{3D} + \mathbf{f}_{3D},$$

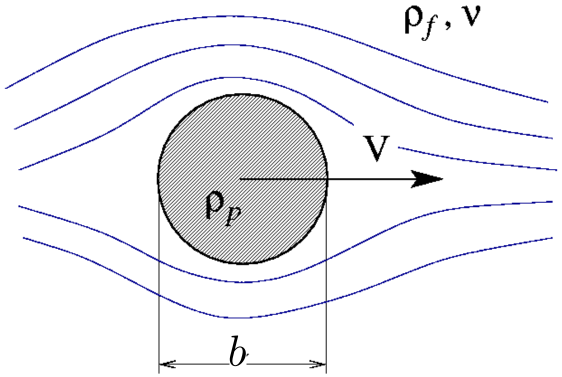
ENERGY BALANCE

$$\partial_t \mathcal{E}_{2D}(t) = \epsilon_{in}^{2D} - \mathcal{T} - \epsilon_{\nu}^{2D} - \epsilon_{\alpha}^{2D}$$

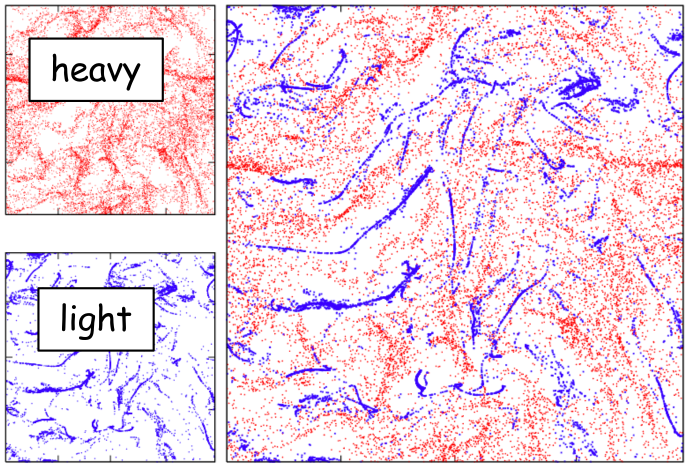
$$\partial_t \mathcal{E}_{3D}(t) = \epsilon_{in}^{3D} + \mathcal{T} - \epsilon_{\nu}^{3D} - \epsilon_{\alpha}^{3D}$$

L.M. Smith et al PRL 77, 2467 (1996); A. Celani et al PRL 104, 184605 (2010); H. Xia et al Nature Physics (2011); S. Musacchio and G. Boffetta PoF 29, 111106 (2017)

COMPLEX PARTICLES IN COMPLEX FLOWS



$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v}\cdot\partial_{\mathbf{x}}\mathbf{v} + \partial_{\mathbf{x}} P = \nu \Delta \mathbf{v} \\ \dot{\mathbf{X}}_i = \mathbf{U}_i \\ \dot{\mathbf{U}}_i = -\frac{\mathbf{U}_i - \mathbf{v}}{\tau} + \beta D_t \mathbf{v} - g(1-\beta)\hat{\mathbf{z}} \end{cases}$$



$$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$$

$\beta<1$ heavy particles

$\beta>1$ light particles

$$\tau = \frac{b^2}{3\nu\beta}$$

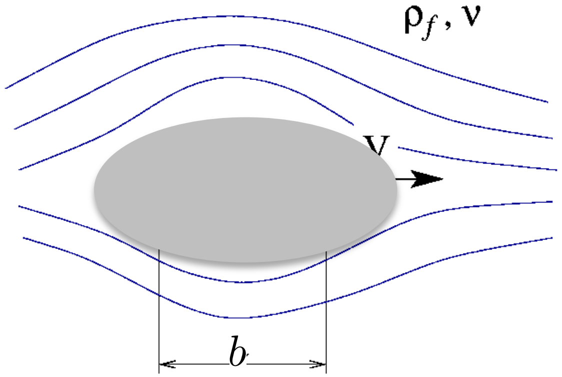
Drag: **Stokes Time**

Preferential concentration

Naive light(heavy) particles accumulate inside(outside) highly vortical regions

M.R. Maxey, J. Fluid Mech. **174**, 441 (1987); G. Falkovich et al, Phys. Rev. Lett. **86**, 2790 (2001)

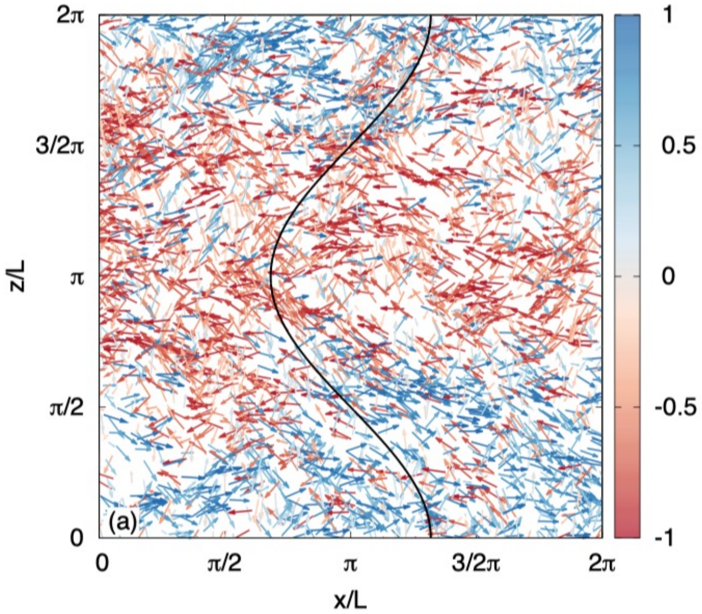
COMPLEX PARTICLES IN COMPLEX FLOWS



$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F} ,$$

$$\dot{\boldsymbol{x}} = \boldsymbol{u}(\boldsymbol{x}, t) + v_s \boldsymbol{n} ,$$

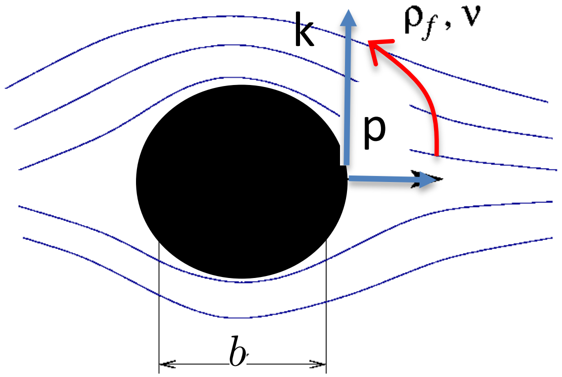
$$\dot{\boldsymbol{n}} = [\mathbb{O}(\boldsymbol{x}, t) + \Lambda \mathbb{S}(\boldsymbol{x}, t)]\boldsymbol{n} - \Lambda[\boldsymbol{n} \cdot \mathbb{S}(\boldsymbol{x}, t)\boldsymbol{n}]\boldsymbol{n} ,$$



Preferential Alignment/Concentration

M. Borgnino et al, arXive 2203.04833 (2002)

COMPLEX PARTICLES IN COMPLEX FLOWS

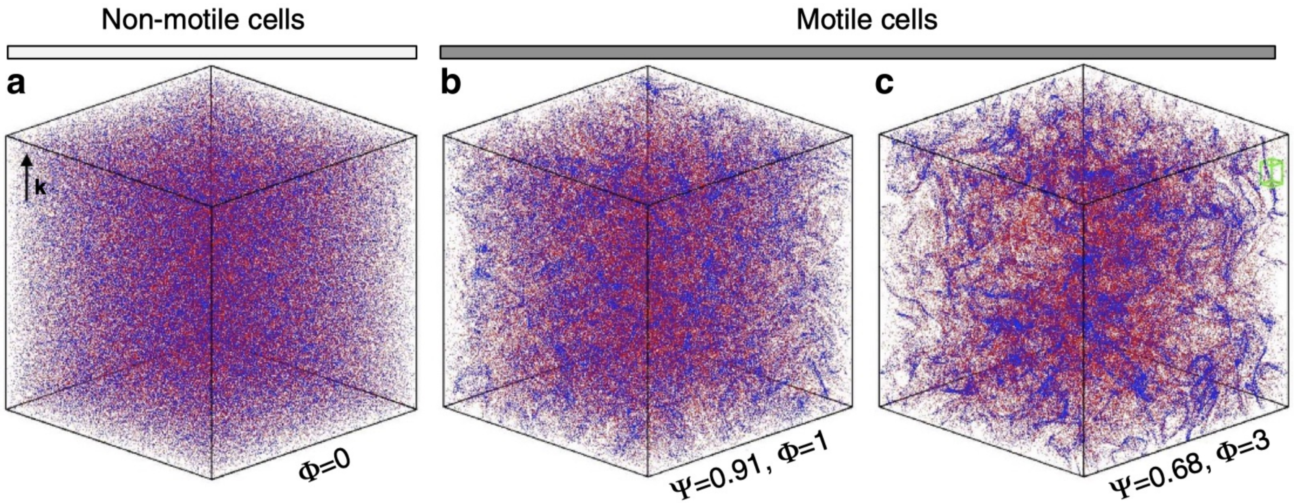


$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \Delta \boldsymbol{u} + \boldsymbol{F},$$

$$\frac{\mathrm{d}\mathbf{X}^*}{\mathrm{d}t^*} = V_{\mathrm{C}}\mathbf{p} + \mathbf{u}^*(\mathbf{X}^*).$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t^*} = \frac{1}{2B}[\mathbf{k} - (\mathbf{k}\cdot\mathbf{p})\mathbf{p}] + \frac{1}{2}(\boldsymbol{\omega}^* \times \mathbf{p})$$

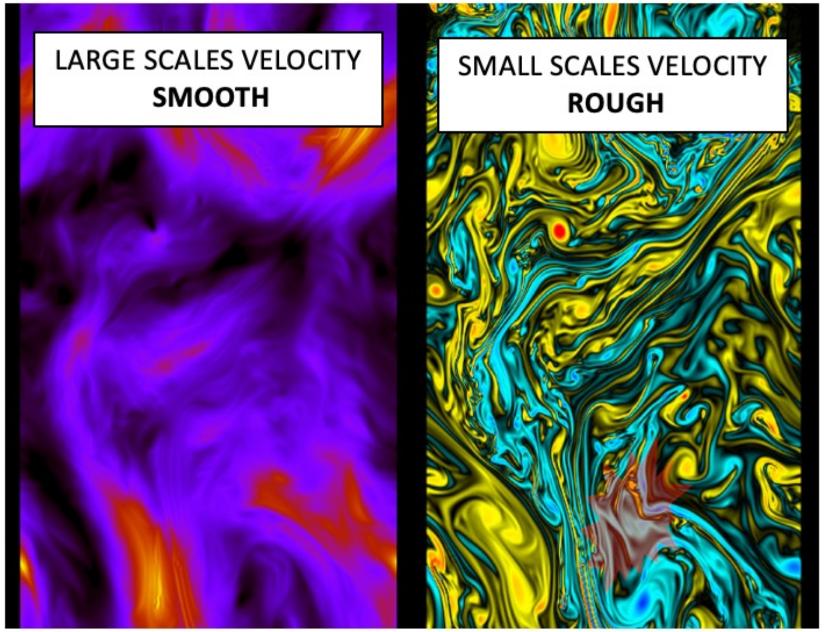
Preferential Concentration Gyrotactit Plankton



M. Durham et al. Nature Comm. (2013)

TURBULENCE IS ….

1) STRONGLY OUT-OF-EQUILIBRIUM SYSTEMS (YOU NEED TO STIRR TO KEEP IT IN MOTION)
2) MULTI-SCALE (MANY DEGREES-OF-FREEDOM)
3) STRONGLY NON-GAUSSIAN
4) STRONGLY NON-LINEAR



…. OUT OF CONTROL FROM

1) MATHEMATICAL ASPECTS (NOT SURE THERE EXISTS A SOLUTION TO NSE FOR ALL TIMES!)
2) NUMERICAL ASPECTS (TOO MANY DEGREES-OF-FREEDOM (COMPUTO ERGO SUM?)
3) MOST OF THE TIME UNACCESSIBLE FROM EXPERIMENTAL MEASUREMENTS (TOO INVASIVE OR TOO FAR, I.E. ATMOSPHERE, PLASMAS, SOLAR WINDS…)

3D HIT: DIRECT ENERGY CASCADE + DIRECT HELICITY CASCADE - CO-DIRECTIONAL CASCADES
2D HIT: INVERSE ENERGY CASCADE AND DIRECT ENSTROPHY CASCADE- COUNTER-DIRECTIONAL CASCADES
3D HAT + ROTATION: SPLIT ENERGY CASCADE

3D HIT: + BIDIRECTIONAL CASCADE (HOMO-CHIRAL AND HETERO-CHIRAL CHANNELS)
3D HAT + ROTATION: FLUX-LOOP CASCADE (ZERO FLUX BUT OUT OF EQUILIBRIUM)

2D STRATIFIED TURBULENCE: FLUX LOOP
2D-3D THICK LAYER: FLUX LOOP
2D-3D THICK LAYER + ROTATION ….
PASSIVE SCALARS IN COMPRESSIBLE/INCOMPRESSIBLE FLOWS: DIRECT OR INVERSE CASCADE
MHD IN 2D OR 3D (+ MEAN MAGNETIC FIELD)
STRATIFIED AND ROTATING FLOWS

Physics Reports 767–769 (2018) 1–101

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Cascades and transitions in turbulent flows

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[b] Department of Physics and INFN University of Rome 'Tor Vergata', Via della Ricerca Scientifica 1, 00133 Rome, Italy