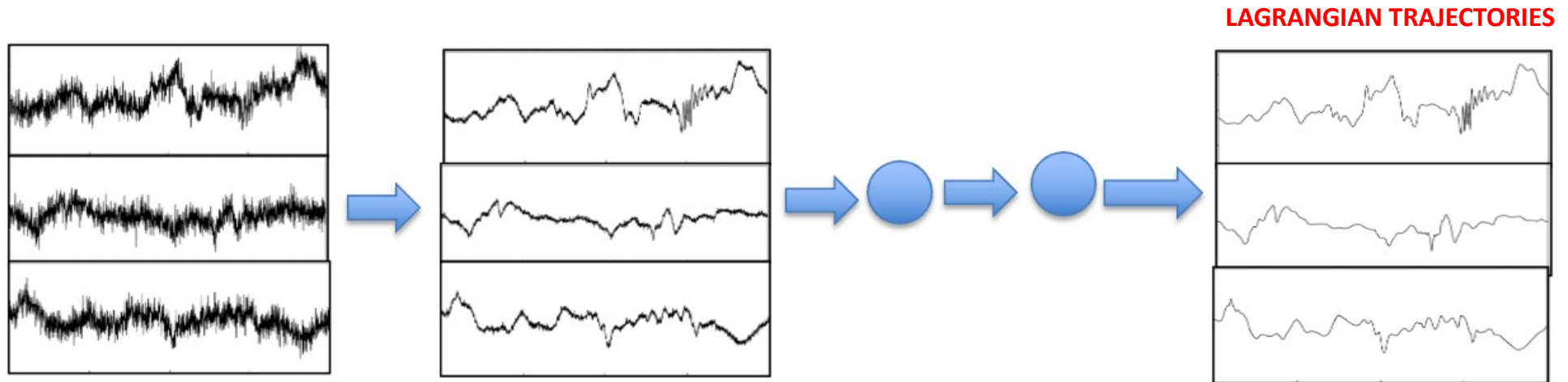


**ERICE – MACHINE LEARNING FOR COMPLEXITY  
2024 APRIL**

**Data driven tools for Eulerian and Lagrangian Turbulence**

CREDITS: M. CENCINI, C. CALASCIBETTA, L. PIRO, R. HEINONEN, T. LI, F. BONACCORSO, M. BUZZICOTTI, M. SCARPOLINI





Complex Fluids and Complex Flows Group  
Dept. Physics & INFN - University of Rome 'Tor Vergata'

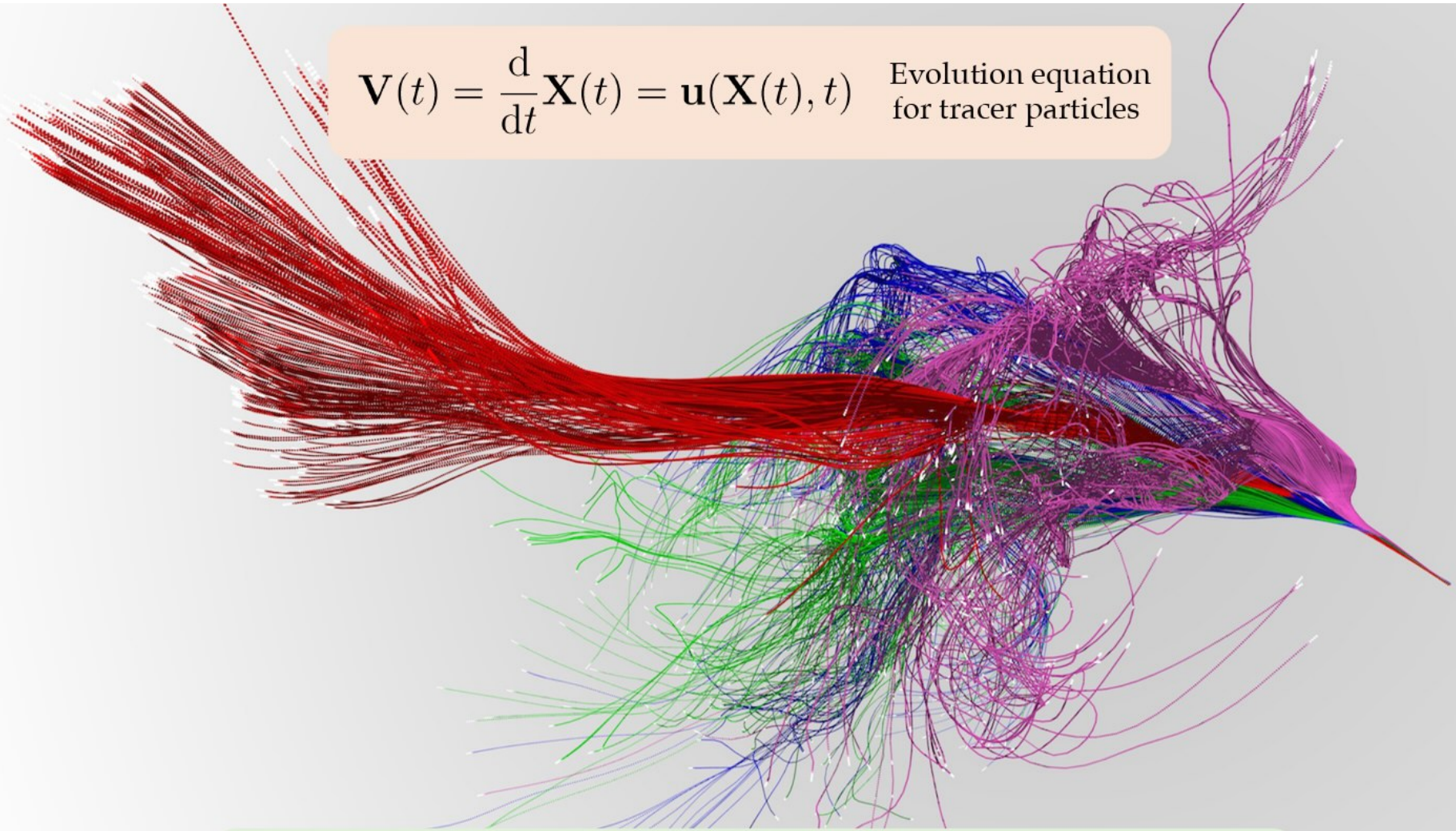
[biferale@roma2.infn.it](mailto:biferale@roma2.infn.it)

<https://biferale.web.roma2.infn.it/>

AQTIVATE



$$\mathbf{V}(t) = \frac{d}{dt}\mathbf{X}(t) = \mathbf{u}(\mathbf{X}(t), t) \quad \text{Evolution equation for tracer particles}$$



$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \Delta \mathbf{u} + \mathbf{F} \\ \nabla \cdot \mathbf{u} = 0 \end{cases} \quad \begin{array}{l} \text{Navier-Stokes} \\ \text{Eq.s} \end{array}$$

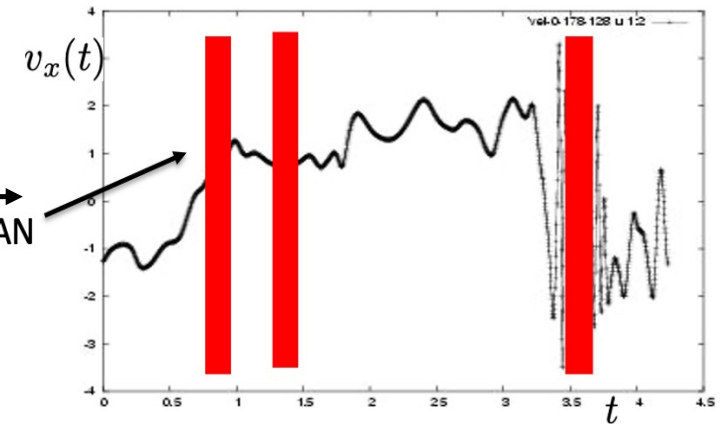
# STOCHASTIC MODELS FOR LAGRANGIAN TURBULENCE: WHY?

T. Li, LB, F. Bonaccorso, M. Scarpolini, M. Bucciotti.  
Synthetic Lagrangian Turbulence by Generative Diffusion Models.  
arXiv:2307.08529 (2023) – Nature Machine Intelligence APRIL 2024

**GENERATION OF LARGE SYNTHETIC DATA-BASE FOR**  
(I) RANKING OF PHYSICS FEATURES  
(II) TESTING DOWNSTREAM APPLICATIONS/MODELS

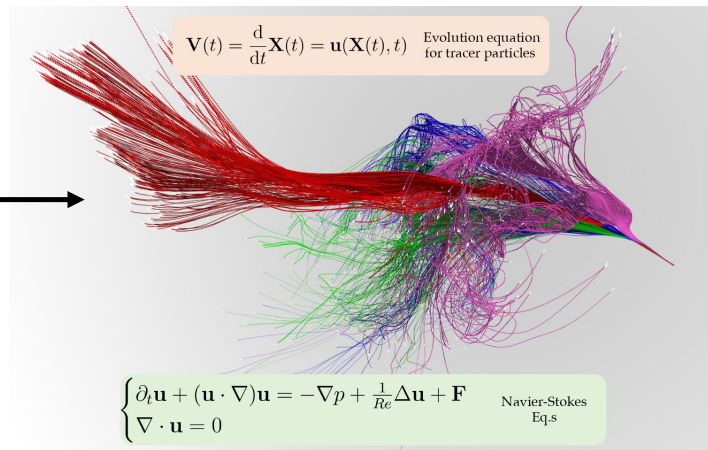
**DATA ASSIMILATION/INPAINTING FROM MISSING FIELD/EXPERIMENTAL OBERVATION**

LAGRANGIAN

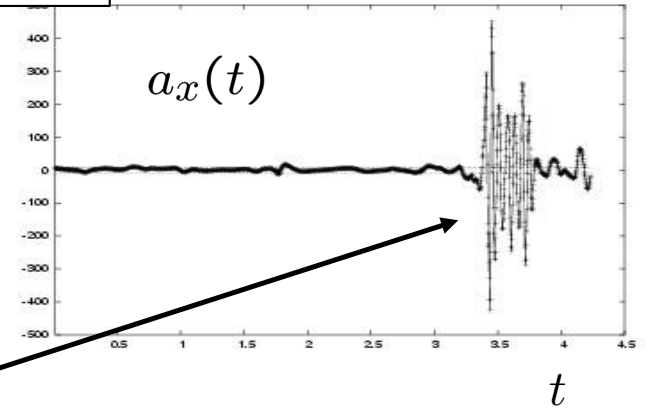
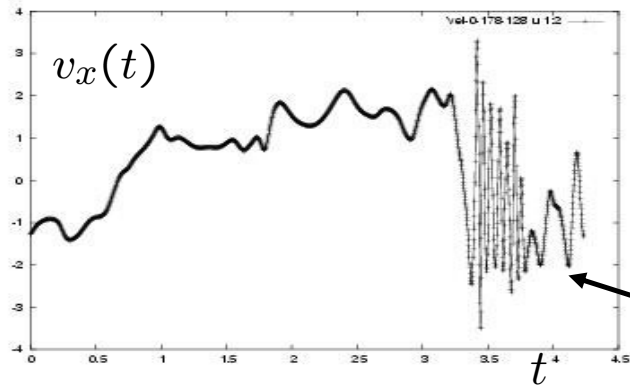


**CLASSIFICATION/INFERRAL OF MISSING/INTERNAL PROPERTIES:**

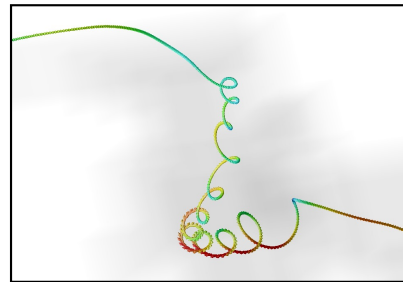
- (I) INERTIA
- (II) SHAPE
- (III) ACTIVE DEGREES OF FREEDOM
- (IV) ....



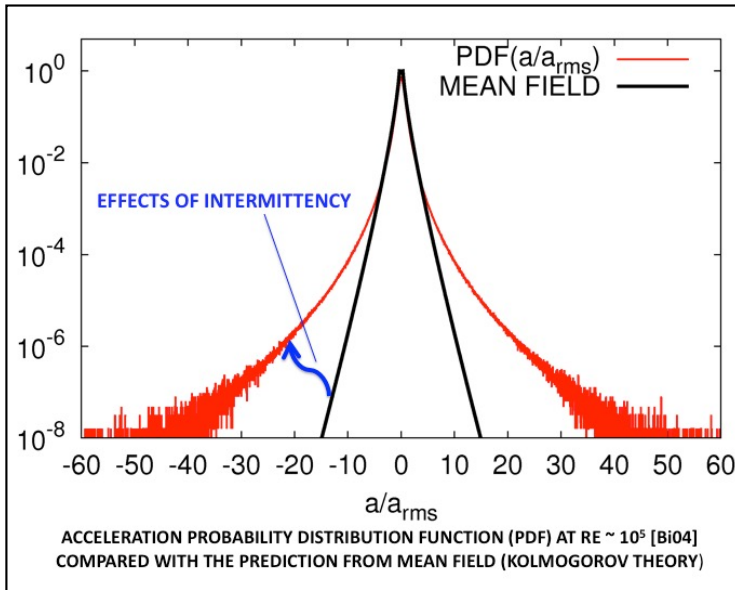
$$\begin{cases} \mathbf{a} = \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nu \Delta \mathbf{u} + \mathbf{f} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$



EXTREME EVENTS



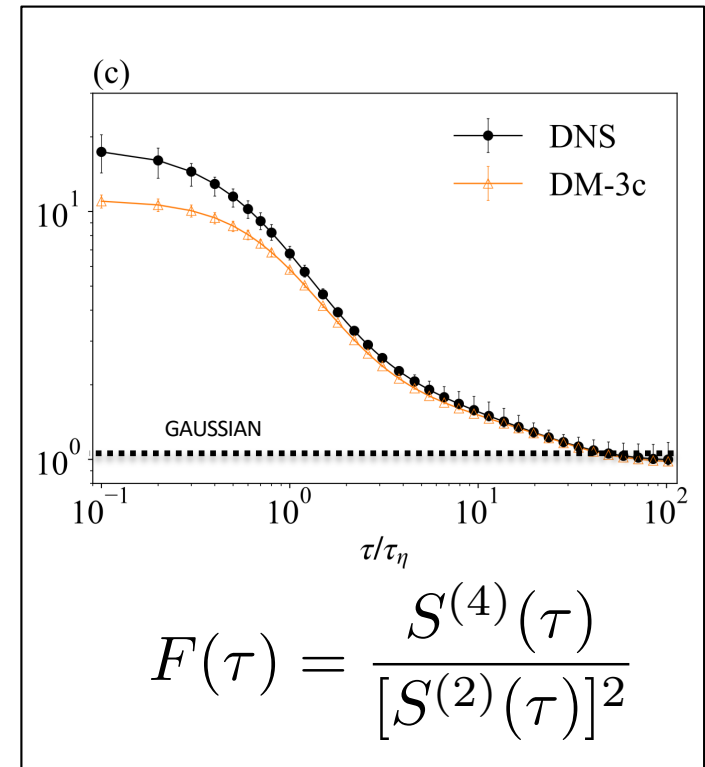
$$S_i^{(p)}(\tau) = \langle [v_i(t + \tau) - v_i(t)]^p \rangle$$



La Porta, G.A. Voth, A.M. Crawford, J. Alexander et al. Fluid particle accelerations in fully developed turbulence. *Nature*, 409(6823), 1017 (2001)

N. Mordant, P. Metz, O. Michel and J.F. Pinton. Measurement of Lagrangian velocity in fully developed turbulence. *Phys. Rev. Lett.* 87(21), 214501 (2001)

F. Toschi and E. Bodenschatz. Lagrangian Properties of Particles in Turbulence. *Annu. Rev. Fluid Mech.* 41, 375 (2009)



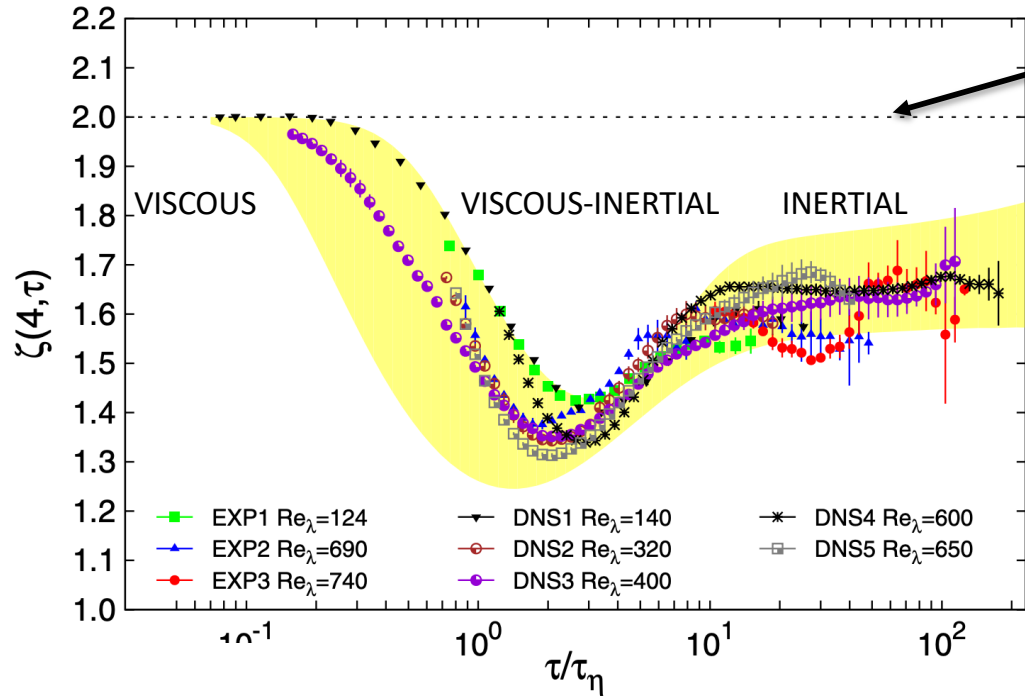
$$S_i^{(p)}(\tau) = \langle [v_i(t + \tau) - v_i(t)]^p \rangle$$

$$\zeta(4, \tau) = \frac{d \log S^{(4)}(\tau)}{d \log S^{(2)}(\tau)}$$

Universal Intermittent Properties of Particle Trajectories in Highly Turbulent Flows

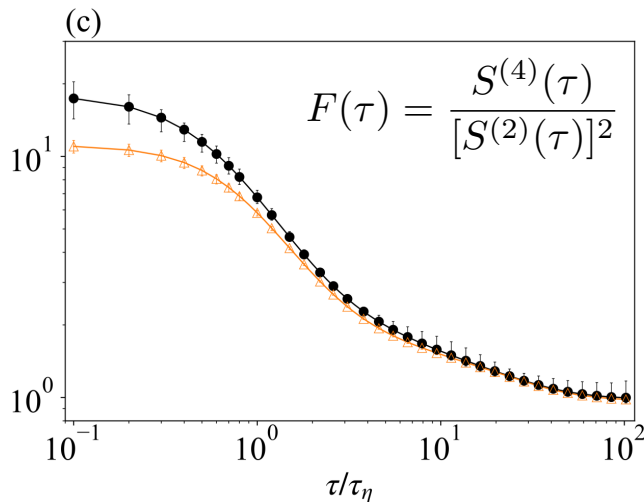
A. Arnéodo,<sup>1</sup> R. Benzi,<sup>2</sup> J. Berg,<sup>3</sup> L. Biferale,<sup>4,\*</sup> E. Bodenschatz,<sup>5</sup> A. Busse,<sup>6</sup> E. Calzavarini,<sup>7</sup> B. Castaing,<sup>1</sup> M. Cencini,<sup>8,\*</sup> L. Chevillard,<sup>1</sup> R. T. Fisher,<sup>9</sup> R. Grauer,<sup>10</sup> H. Homann,<sup>10</sup> D. Lamb,<sup>9</sup> A. S. Lanotte,<sup>11,\*</sup> E. Lévêque,<sup>1</sup> B. Lüthi,<sup>12</sup> J. Mann,<sup>3</sup> N. Mordant,<sup>13</sup> W.-C. Müller,<sup>6</sup> S. Ott,<sup>3</sup> N. T. Ouellette,<sup>14</sup> J.-F. Pinton,<sup>1</sup> S. B. Pope,<sup>15</sup> S. G. Roux,<sup>1</sup> F. Toschi,<sup>16,17,\*</sup> H. Xu,<sup>5</sup> and P. K. Yeung<sup>18</sup>

(International Collaboration for Turbulence Research)



GAUSSIAN – NO INTERMITTENCY

- UNIVERSALITY
- SCALE-BY-SCALE INTERMITTENCY
- VISCOUS-SCALE FLUCTUATIONS
- MF-PREDICTION



M. Borgas “The multifractal Lagrangian nature of turbulence”, PTRSA 342, 379 (1993)  
 G. K. Batchelor. “Pressure fluctuations in isotropic turbulence” Proc. Camb. Philos. Soc. 47, 359 (1951)  
 G. Paladin and A. Vulpiani, “Degrees of freedom of turbulence,” Phys. Rev. A 35, 1971 (1987)  
 C. Meneveau, “Transition between viscous and inertial-range scaling of turbulence structure functions” Phys. Rev. E 54, 3657 (1996)

Sawford, B. L. Reynolds number effects in Lagrangian **stochastic models** of turbulent dispersion. Phys. Fluids A: Fluid Dyn. 3, 1577–1586 (1991).

Wilson, J. D. & Sawford, B. L. Review of lagrangian stochastic models for trajectories in the turbulent atmosphere. Boundary-layer meteorology 78, 191–210 (1996).

Biferale, L., Boffetta, G., Celani, A., Crisanti, A. & Vulpiani, A. Mimicking a turbulent signal: **Sequential multiaffine processes**. Physical Review E 57, R6261 (1998).

Arneodo, A., Bacry, E. & Muzy, J.-F. **Random cascades on wavelet** dyadic trees. Journal of Mathematical Physics 39, 4142–4164 (1998).

Lamorgese, A., Pope, S. B., Yeung, P. & Sawford, B. L. A **conditionally cubic-gaussian stochastic** lagrangian model for acceleration in isotropic turbulence. Journal of Fluid Mechanics 582, 423–448 (2007).

Arnéodo, A. et al. Universal intermittent properties of particle trajectories in highly turbulent flows. Physical Review Letters 100, 254504 (2008)

Pope, S. B. Simple models of turbulent flows. Physics of Fluids 23, 011301 (2011).

Minier, J.-P., Chibbaro, S. & Pope, S. B. Guidelines for the formulation of lagrangian stochastic models for particle simulations of single-phase and dispersed two-phase turbulent flows. Physics of Fluids 26, 113303 (2014).

Chevillard, L., Garban, C., Rhodes, R. & Vargas, V. On a skewed **and multifractal unidimensional random field**, as a probabilistic representation of kolmogorov's views on turbulence. In Annales Henri Poincaré, vol. 20, 3693–3741 (Springer, 2019).

Viggiano, B. et al. Modelling lagrangian velocity and acceleration in turbulent flows **as infinitely differentiable stochastic processes**. Journal of Fluid Mechanics 900, A27 (2020).

Sinhuber, M., Friedrich, J., Grauer, R. & Wilczek, M. **Multi-level stochastic refinement** for complex time series and fields: a data-driven approach. New Journal of Physics 23, 063063 (2021).

Zamansky, R. **Acceleration scaling and stochastic dynamics** of a fluid particle in turbulence. Physical Review Fluids 7, 084608 (2022).

Lubcke, J, Friedrich, J., Grauer, R. **Stochastic interpolation** of sparsely sampled time series by a superstatistical random process and its synthesis in Fourier and wavelet space. J. Phys. Complex. 4 015005 (2023)



# Diffusion Models

**Training set:** a set of images  $\vec{a}^\mu \in \mathbb{R}^N$   $\mu = 1, \dots, P$   
N is the dimension of the data, P their number

**Langevin equation** for an Ornstein-Uhlenbeck process

$$\frac{d\vec{x}}{dt} = -\vec{x} + \vec{\eta}(t) \quad \langle \eta_i(t) \eta_j(t') \rangle = 2T \delta_{ij} \delta(t - t')$$

$\vec{x}^\mu(t = 0) = \vec{a}^\mu$  It transforms the data in iid Gaussian  $\mathcal{N}(0, 1)$  at  $t \gg 1$

$$P_t(\vec{x}) = \int d\vec{a} P_0(\vec{a}) \frac{1}{\sqrt{2\pi\Delta_t}^N} \exp\left(-\frac{1}{2} \frac{(\vec{x} - \vec{a}e^{-t})^2}{\Delta_t}\right) = \int d\vec{a} P_t(\vec{a}, \vec{x})$$

$\Delta_t = T(1 - e^{-2t})$

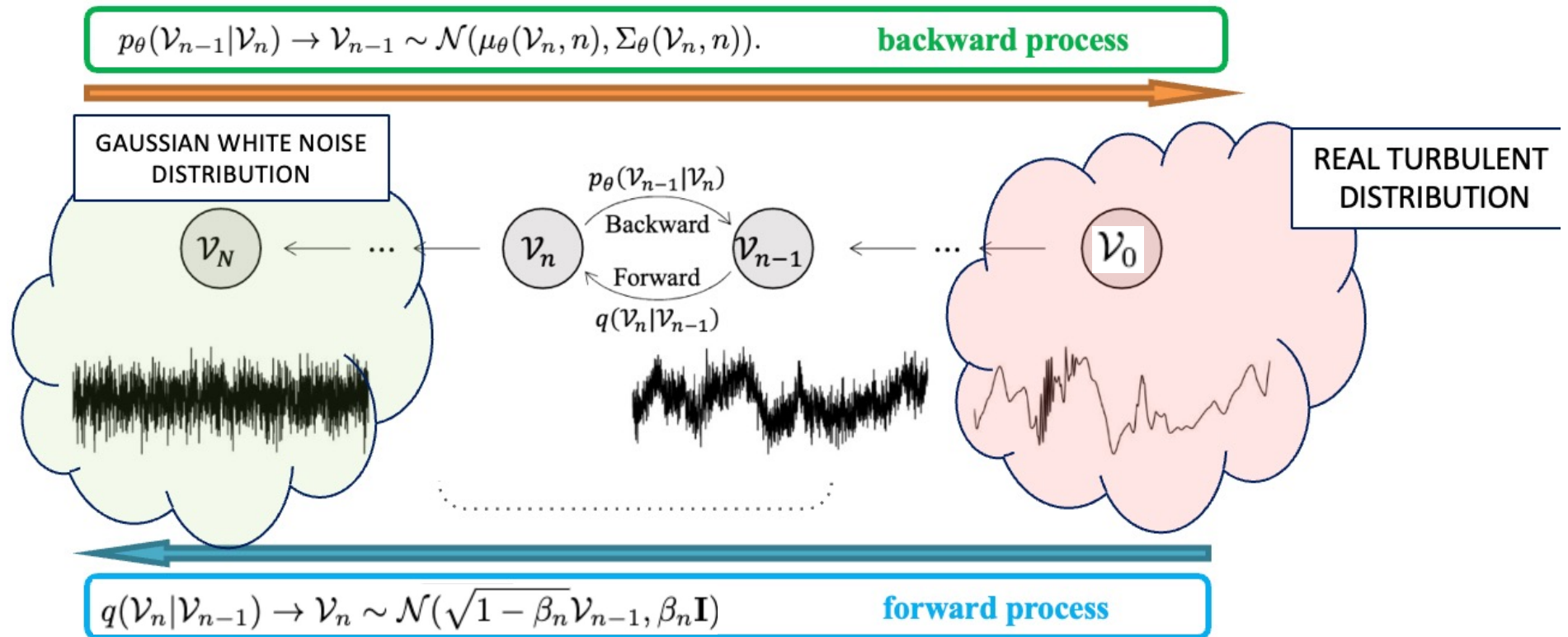


**Score function** provides the force field to go back in time

$$\mathcal{F}_i(\vec{x}, t) = \frac{\partial \log P_t(\vec{x})}{\partial x_i} \quad -\frac{dy_i}{dt} = y_i + 2T \mathcal{F}_i(y, t) + \eta_i(t)$$

# Diffusion Models

‘Synthetica Lagrangian Turbulence: all you need is Diffusion Models’ T. Li, L.B, F. Bonaccorso, M. Scarpolini and M. Buzzicotti (arXiv:2307.08529 2023, submitted Nature Machine Intelligence)

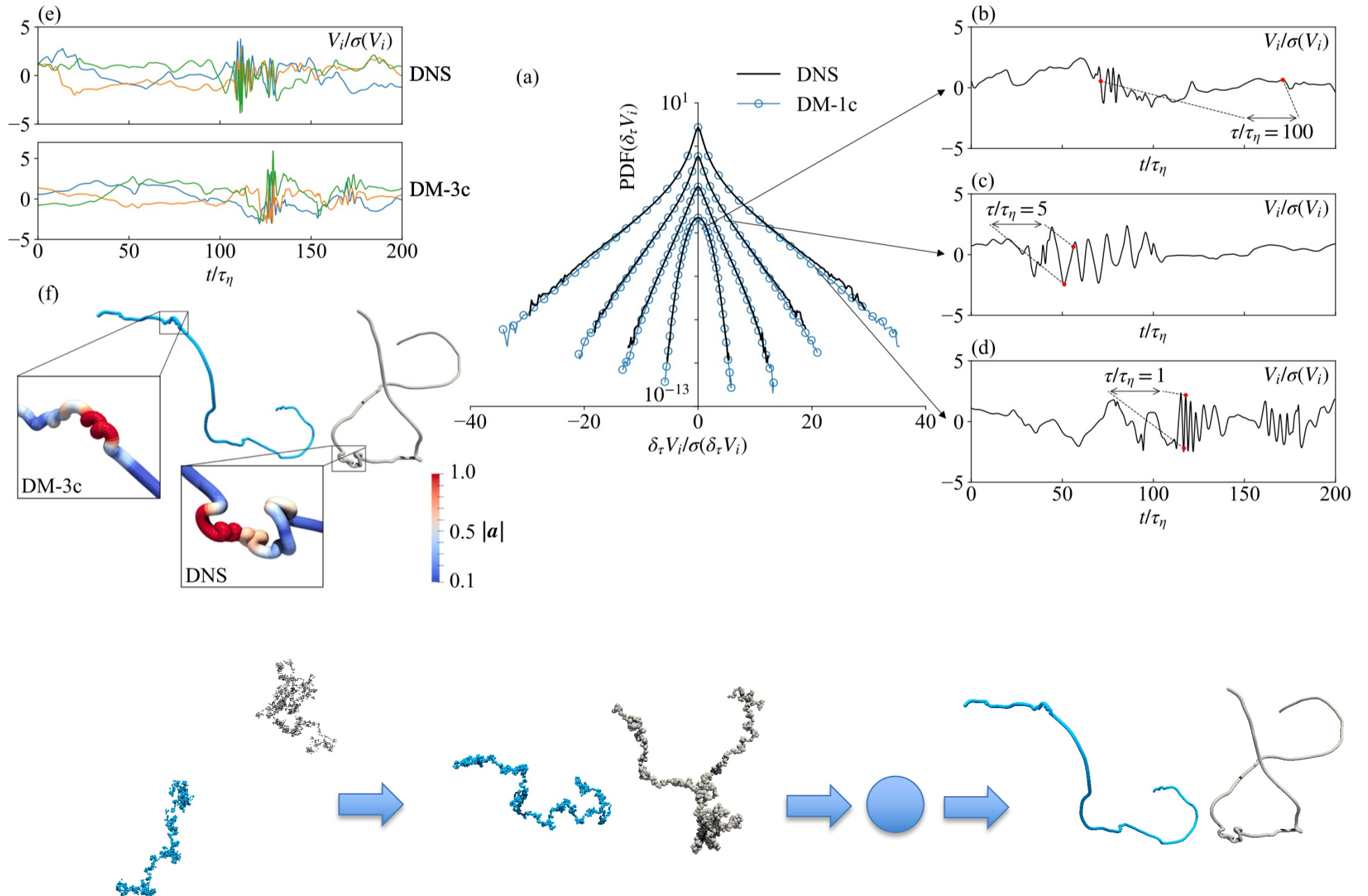


[Sohl-Dickstein et al., Deep Unsupervised Learning using Nonequilibrium Thermodynamics, ICML 2015](#)

[Ho et al., Denoising Diffusion Probabilistic Models, NeurIPS 2020](#)

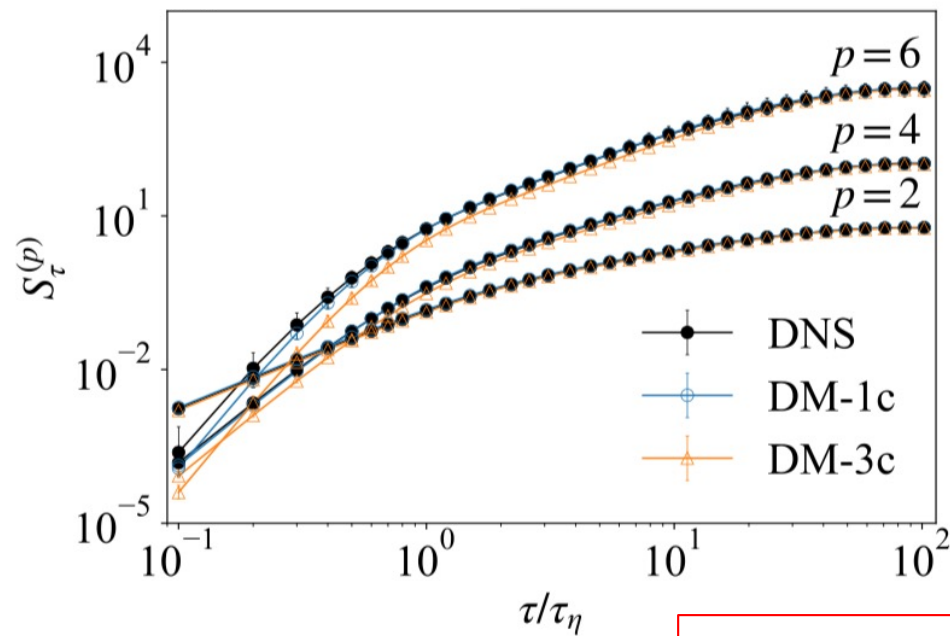
[Song et al., Score-Based Generative Modeling through Stochastic Differential Equations, ICLR 2021](#)

$$\delta_\tau V_i(t) = V_i(t + \tau) - V_i(t),$$



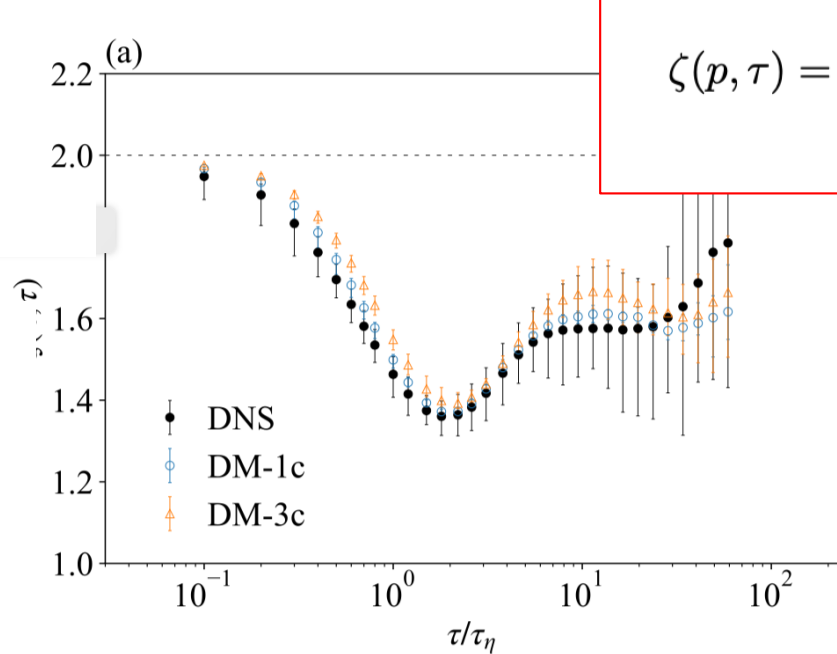
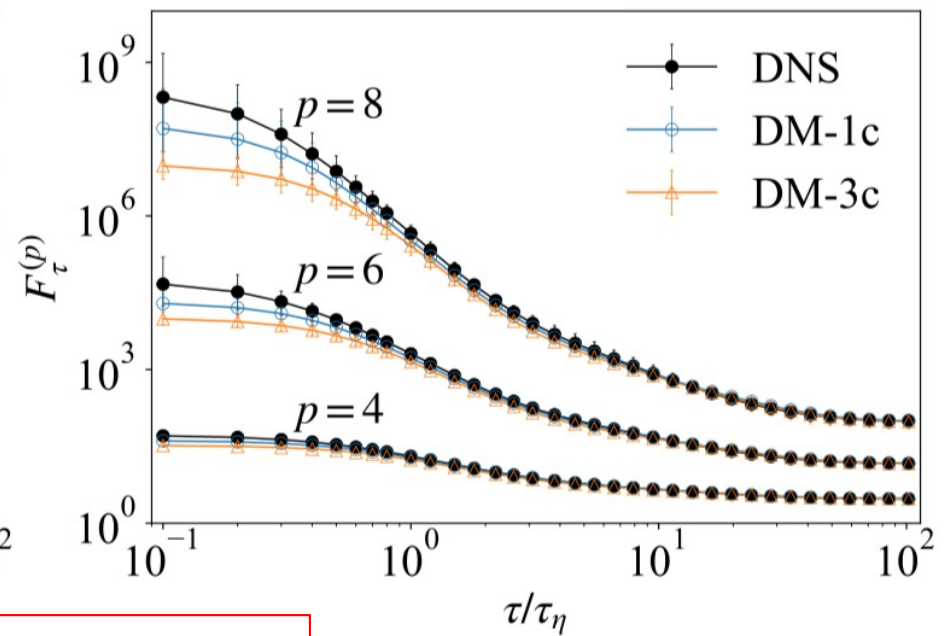
# LAGRANGIAN STRUCTURE FUNCTIONS

$$S_\tau^{(p)} = \langle (\delta_\tau V_i)^p \rangle$$

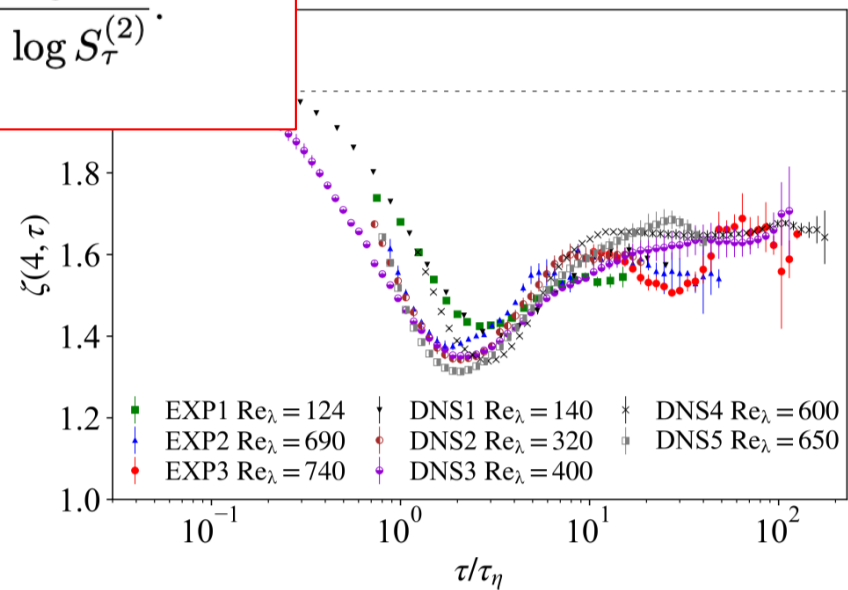


# GENERALIZED FLATNESS

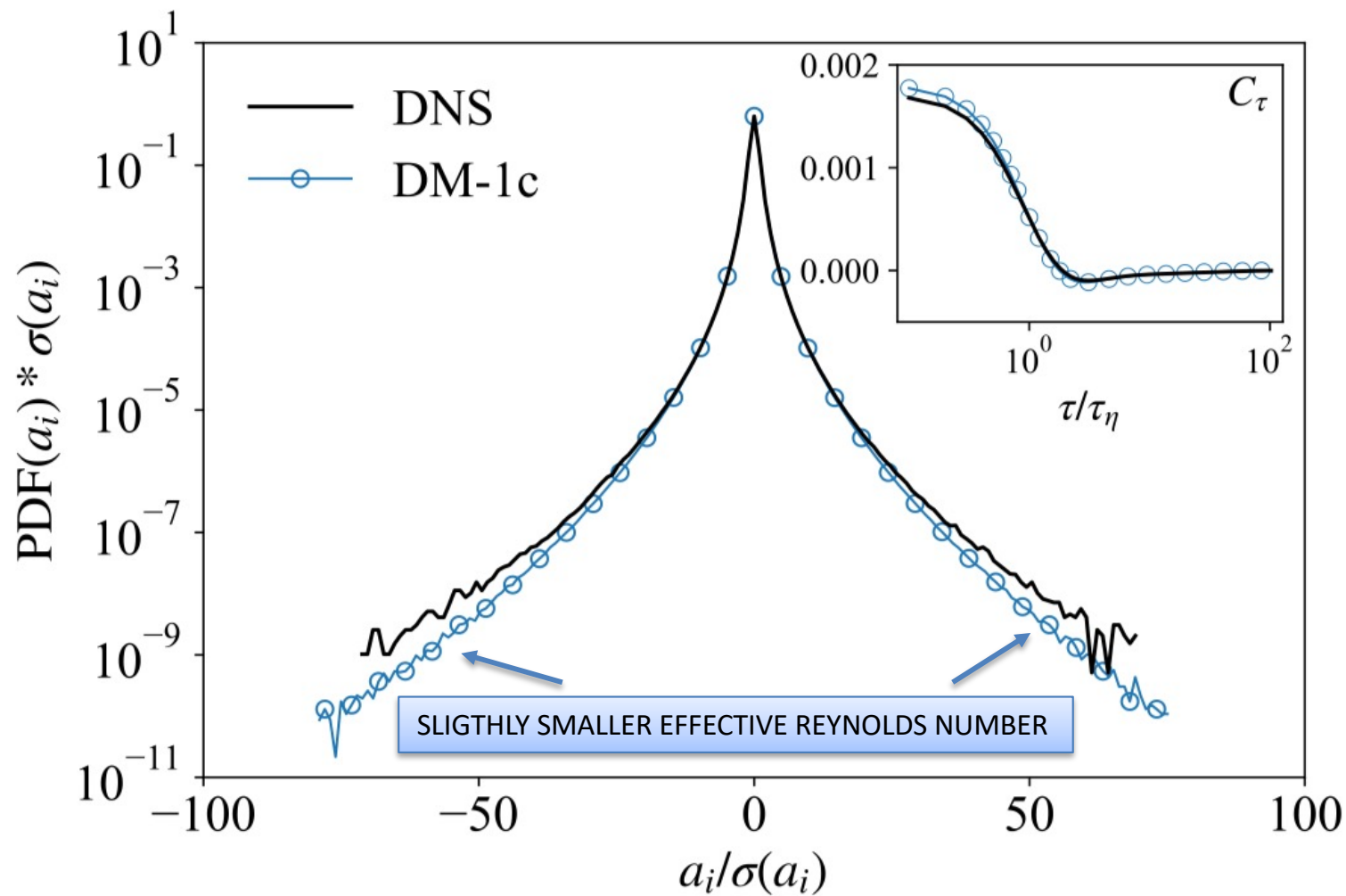
$$F_\tau^{(p)} = S_\tau^{(p)} / [S_\tau^{(2)}]^{p/2}$$

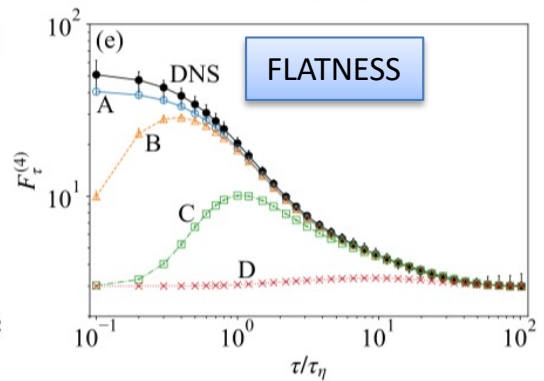
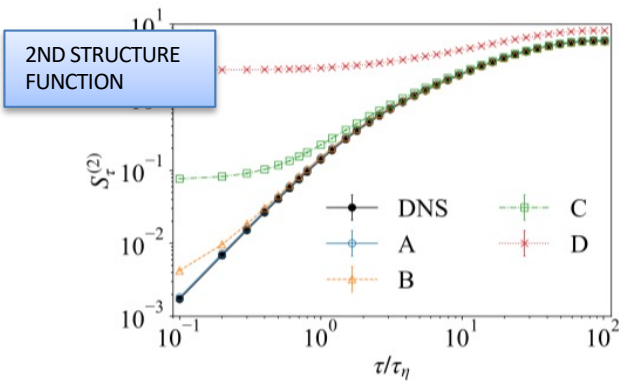
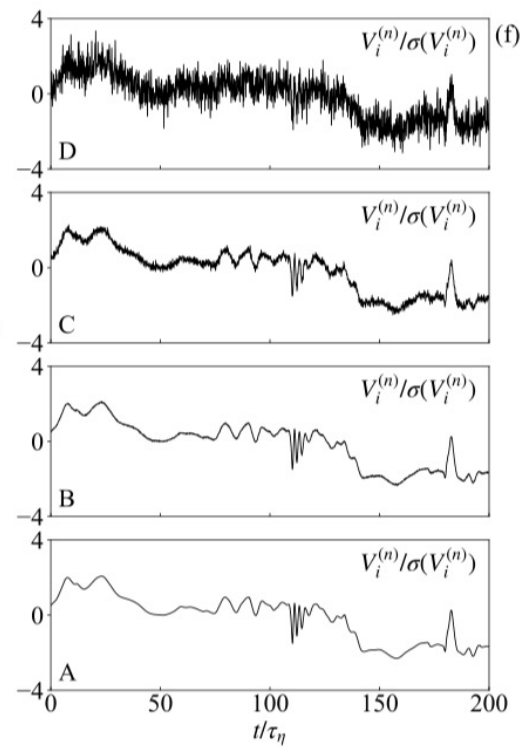
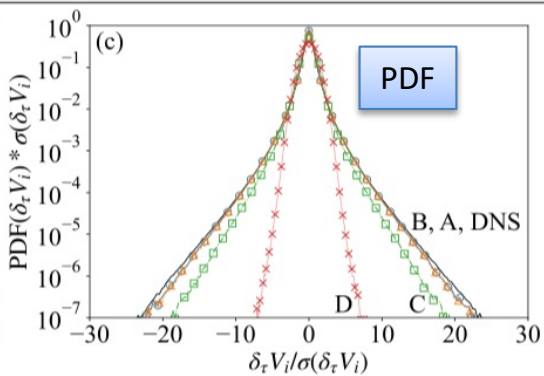
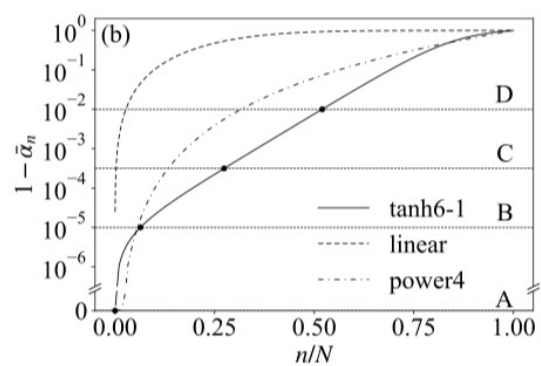
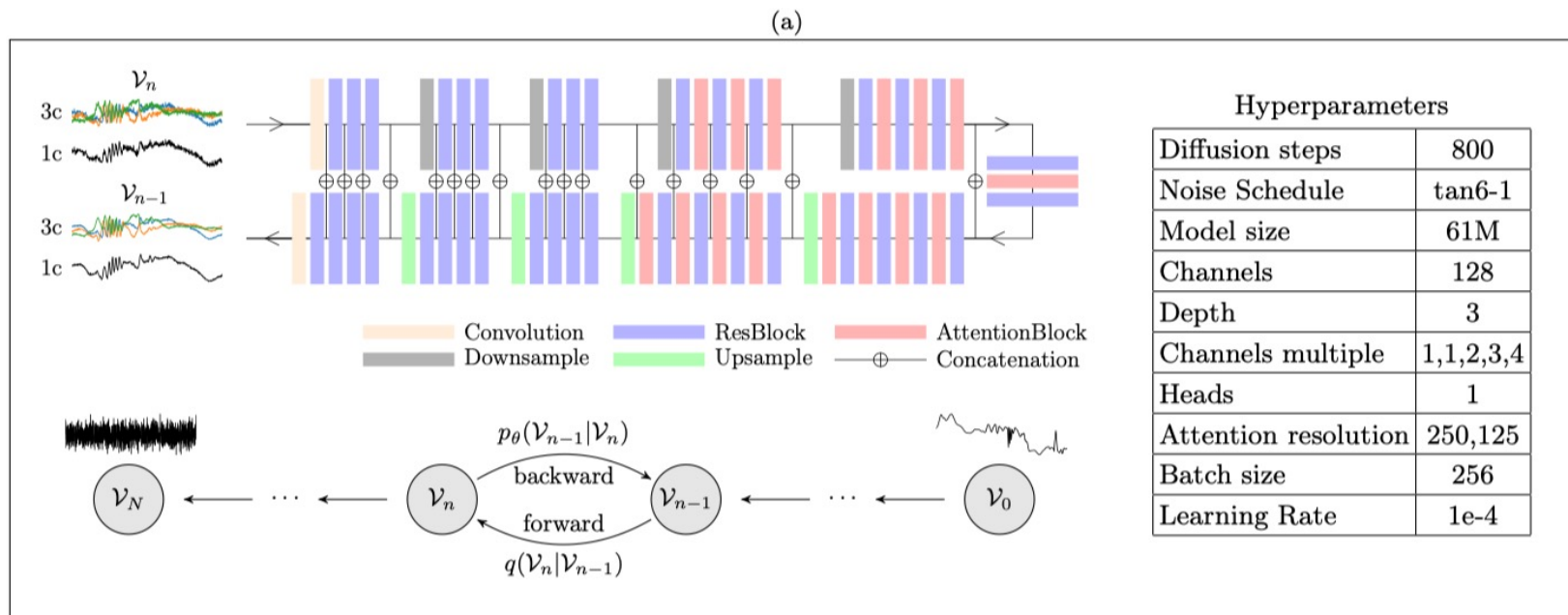


$$\zeta(p, \tau) = \frac{d \log S_\tau^{(p)}}{d \log S_\tau^{(2)}}$$



ACCELERATION PDF

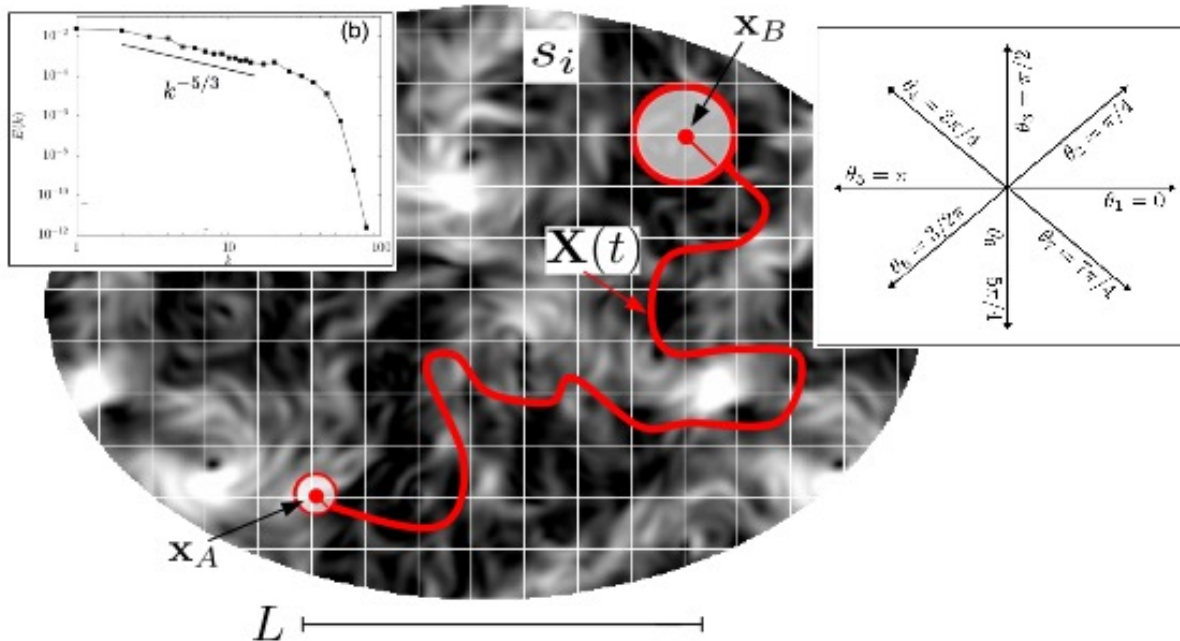




# Zermelo's problem: Optimal point-to-point navigation in 2D turbulent flows using Reinforcement Learning

L. Biferale,<sup>1</sup> F. Bonaccorso,<sup>1,2</sup> M. Bucciotti,<sup>1</sup> P. Clark Di Leoni,<sup>1,3</sup> and K. Gustavsson<sup>4</sup>

Chaos: An Interdisciplinary Journal of Nonlinear Science  
29.10 (2019): 103138.  
arXiv preprint:1907.08591



$$\begin{cases} \dot{\mathbf{X}}_t = \mathbf{u}(\mathbf{X}_t) + \mathbf{U}^{ctrl}(\mathbf{X}_t) \\ \mathbf{U}^{ctrl}(\mathbf{X}_t) = V_s \mathbf{n}(\mathbf{X}_t) \end{cases}$$

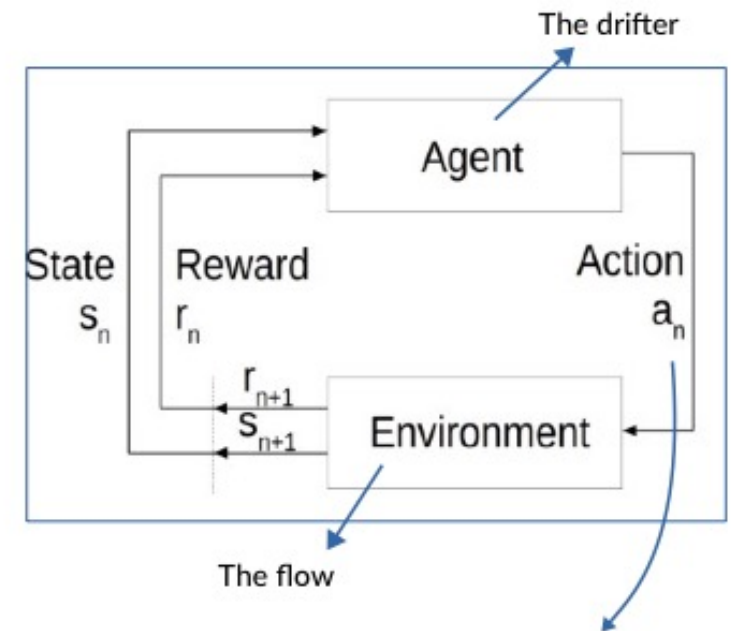
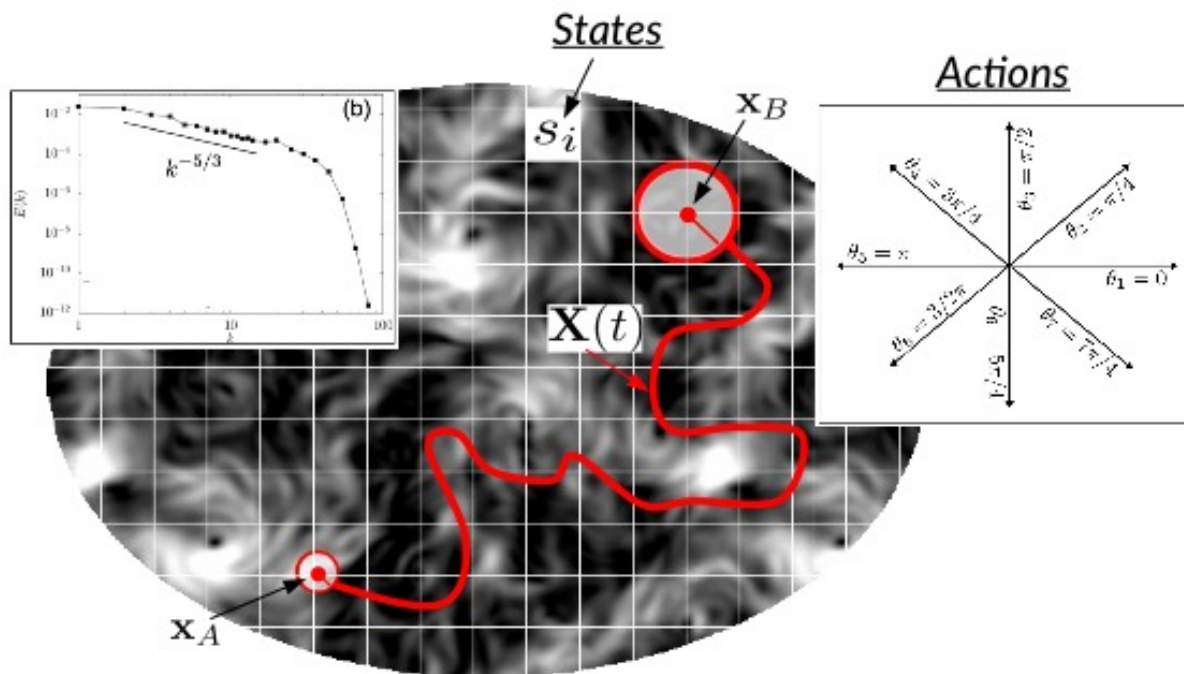
$$\mathbf{n}(\mathbf{X}_t) = (\cos[\theta_t], \sin[\theta_t]),$$

$V_s \rightarrow$  Navigation speed is small compared to the velocity of the underling flow!

E. Zermelo, "Über das navigationsproblem bei ruhender oder veränderlicher windverteilung," ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik **11**, 114-124 (1931).

A. E. Bryson and Y. Ho, *Applied optimal control: optimization, estimation and control* (New York: Routledge, 1975).

# Reinforcement Learning; Policy Gradient Methods



Parameterized policy:

$$\pi(a_j | s_i, \mathbf{q}) = \frac{\exp h(s_i, a_j, \mathbf{q})}{\sum_{k=1}^{N_a} \exp h(s_i, a_k, \mathbf{q})}$$

Parameterized state value function:

$$\hat{v}(s_i, \mathbf{w}) = \sum_{j=1}^{N_s} w_j \delta_{j,i}$$

Reward

$$r_t = -\Delta t$$

$$r_{tot} = -T_{A \rightarrow B}$$

Actor-Critic algorithm

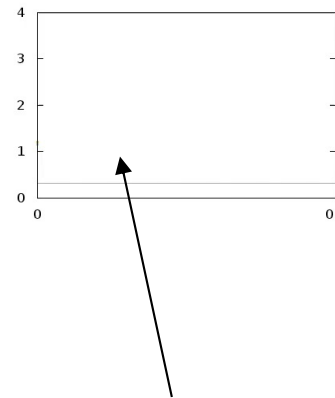
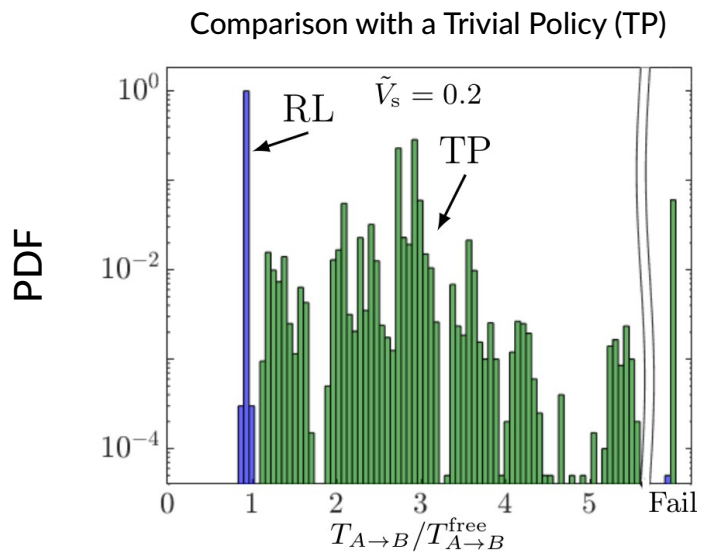
$$\begin{cases} \mathbf{q}_{t+\Delta t} = \mathbf{q}_t + \alpha_t \beta_t \nabla_{\mathbf{q}} \ln(\pi(a_t | s_t, \mathbf{q}_t)) \\ \mathbf{w}_{t+\Delta t} = \mathbf{w}_t + \alpha'_t \beta_t \nabla_{\mathbf{w}} \hat{v}(s_t, \mathbf{w}_t) \end{cases}$$

$$\beta_t = [\hat{r}_{t+\Delta t} - \hat{v}(s_t, \mathbf{w}_t)] \rightarrow \text{baseline}$$

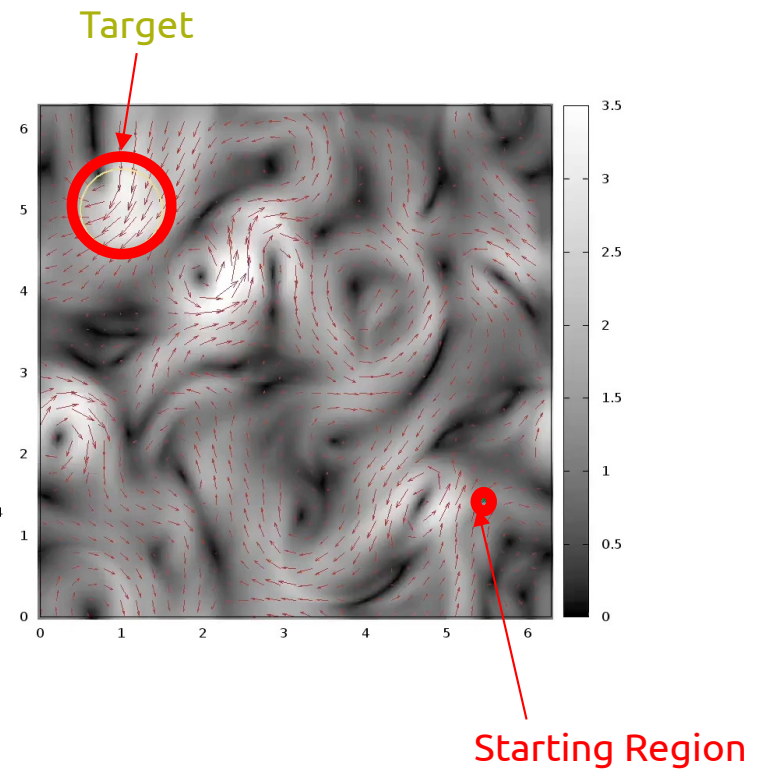


# TIME-DEPENDENT 2D TURBULENT FLOWS

REINFORCEMENT LEARNING (BLUE) VS TRIVIAL POLICY (GREEN)  $\tilde{V}_s = 0.2$

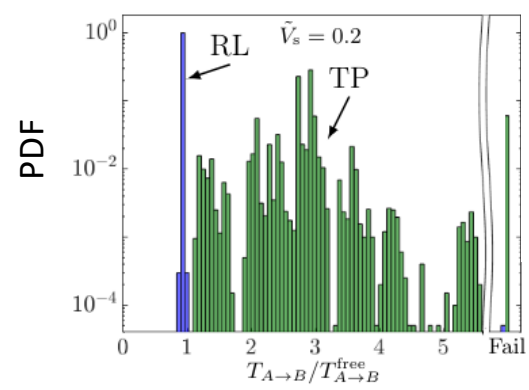
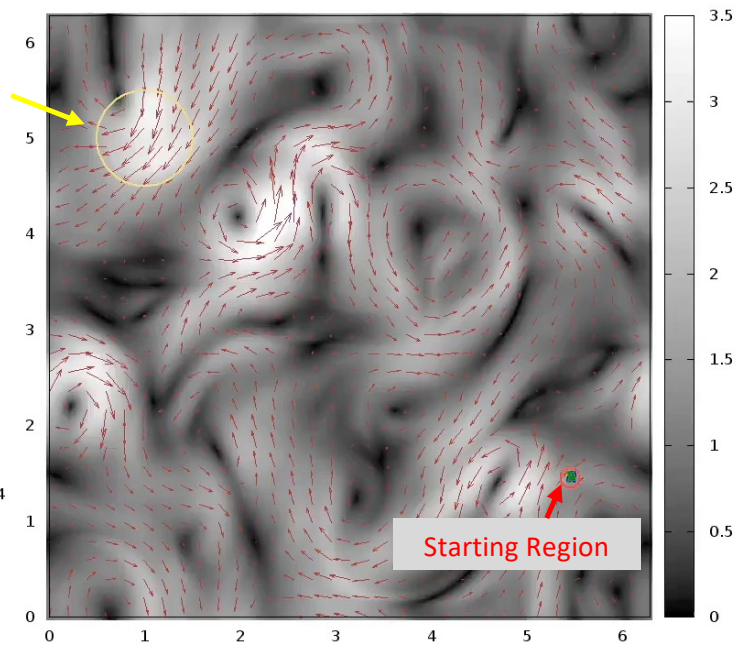
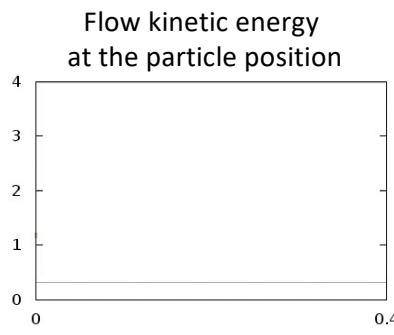
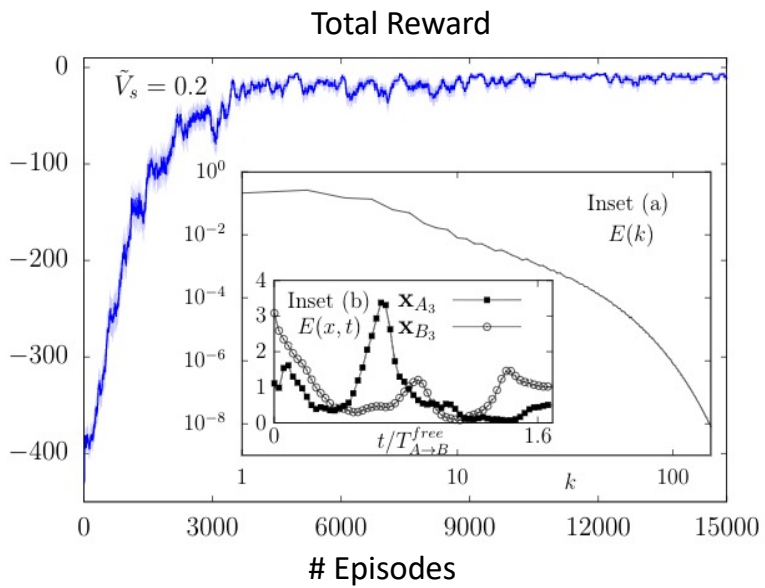


Flow kinetic energy  
at the particle position



Time-Dependent 2D Turbulent Flow

$$\frac{V_s}{\max(u)} \sim 0.2$$



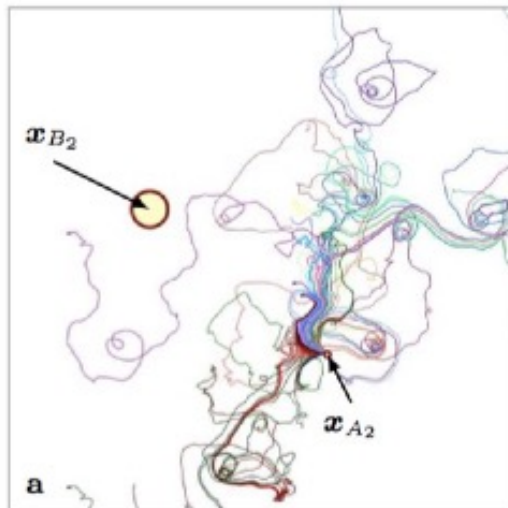
Reinforcement Learning (blue)  
vs  
Trivial Policy (green)

## COMPARISON RL VS OPTIMAL NAVIGATION

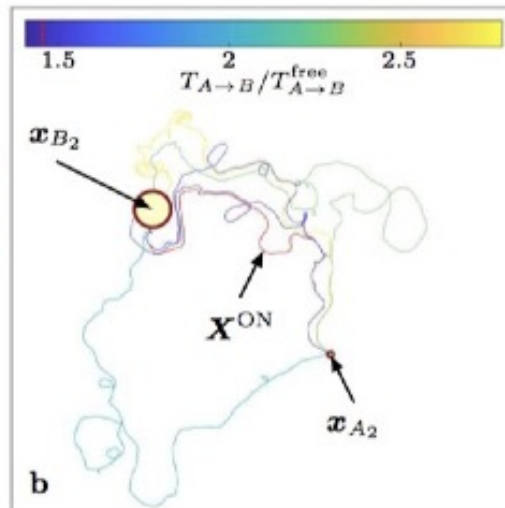
A. E. Bryson and Y. Ho, Applied optimal control: optimization, estimation and control (New York: Routledge, 1975).

Time independent flow

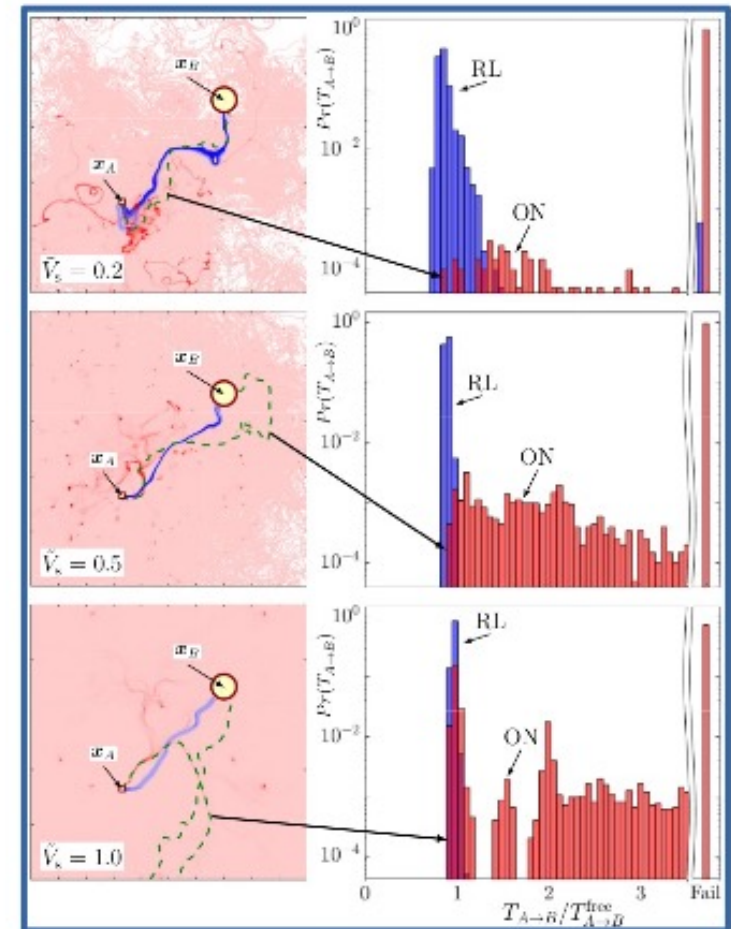
$$\begin{cases} \dot{\mathbf{X}}_t = \mathbf{u}(\mathbf{X}_t) + \mathbf{U}^{ctrl}(\mathbf{X}_t) & \mathbf{n}(\mathbf{X}_t) = (\cos[\theta_t], \sin[\theta_t]), \\ \mathbf{U}^{ctrl}(\mathbf{X}_t) = V_s \mathbf{n}(\mathbf{X}_t) & A_{ij} = \partial_i u_j \\ \dot{\theta}_t = A_{21} \sin^2 \theta_t - A_{12} \cos^2 \theta_t + (A_{11} - A_{22}) \cos \theta_t \sin \theta_t, \end{cases}$$

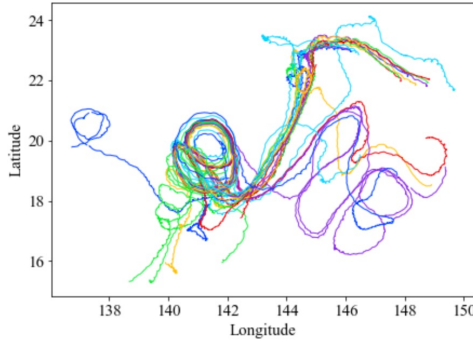
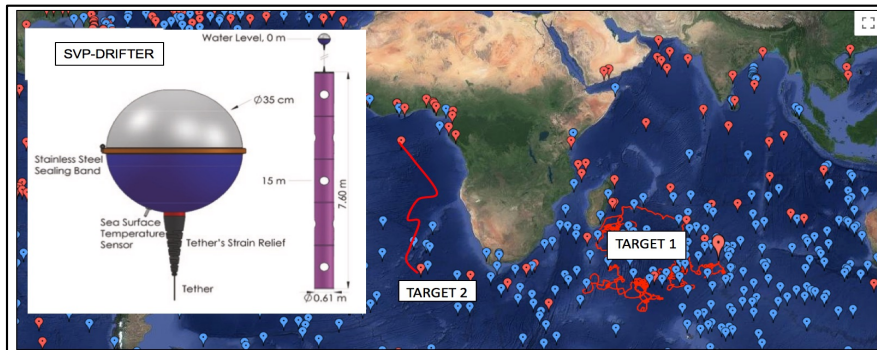


1000 trials (all failures)



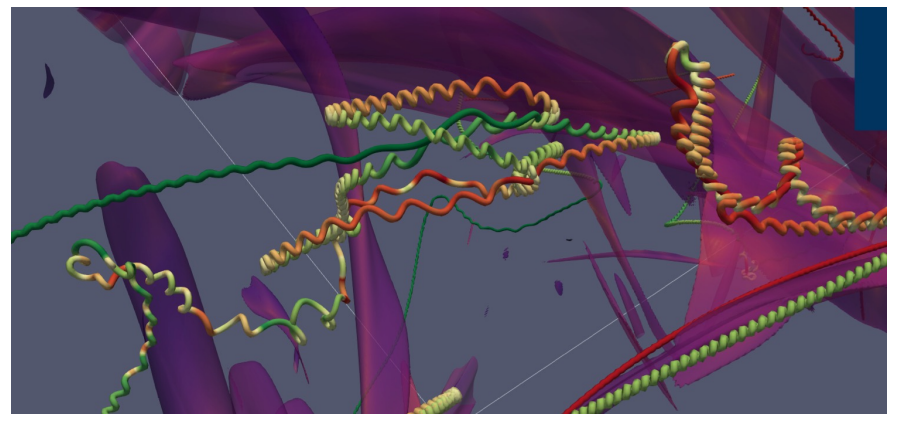
100k trials (10 successes)





DRIFTERS FROM THE GLOBAL DRIFTER MAP  
 PARTIAL TARGETS: (1) KEEP THE PROBES INSIDE A  
 AMONG TWO END-POINTS (ZERMELO PROBLEM).  
 BE AT 15M DEPTH.

CHARGED-PARTICLES IN  
 MHD  
 with RL. Centurioni  
 (UCSD, USA)



CHARGED-PARTICLES IN MHD  
 with R. Grauer & J. Lubcke (Bochum U., GER)

**-WHAT-IF QUESTIONS: EXPLICABILITY OF THE GENERATED DATA, FEATURES RANKINGS,  
 PHYSICS DISCOVERY**

**Wavelet Score-Based Generative Modeling**

**Florentin Guth**  
 Computer Science Department,  
 ENS, CNRS, PSL University

**Simon Coste**  
 Computer Science Department,  
 ENS, CNRS, PSL University

**Valentin De Bortoli**  
 Computer Science Department,  
 ENS, CNRS, PSL University

**Stéphane Mallat**  
 Collège de France, Paris, France  
 Flatiron Institute, New York, USA

[arxiv 2208.05003](https://arxiv.org/abs/2208.05003)

## WE HAVE NEW TOOLS IN THE BOX!

### OPEN ISSUES:

0. **EXPLENABILITY.** VERY FAR, AT THE MOMENT. COMPUTO ERGO SUM?
1. **PROOF-OF-CONCEPTS STAGE.** VERY FEW NEW DISCOVERIES/FAR FROM FRONTIER RESEARCH
2. **GENERALIZABILITY/ROBUSTNESS.** CHANGE ANYTHING, CHANGES EVERYTHING?
3. **SCALABILITY.** VS NETWORK ARCHITECTURE AND VS CONTROL PHYSICAL PARAMETERS
4. **UNCERTAINTY QUANTIFICATION/TRUSTABILITY**

### WHAT WE MISS:

1. COMMUNITY EFFORT TO:
  - (A) DEPLOY AND MAINTAIN HIGH QUALITY AND HIGH QUANTITY DATA (OPEN)
  - (B) IDENTIFY BENCHMARKS, VALIDATION STEPS, BASELINES
  - (C) IDENTIFY GRAND CHALLENGES



## Guide for users

What is **Smart-TURB**? It is a brand new software infrastructure (born June 2020) for the research community working on turbulence and complex flows with particular emphasis to collect/standardize and preserve huge datasets of big-data and Machine Learning approaches to fluid mechanics in general. It is an easily accessible web platform for high quality data. It is to host, standardize and manage a large collection of experimental and numerical data sets from high-end fluid dynamics studies and High Performance Computational centers. Smart-TURB offers excellent performances when accessing/uploading/searching data. The research community is asked to contribute, by deploying freely downloadable, accurate and validated dataset for the sake of "reproducibility": The process of documenting procedures and archiving data so that others can fully reproduce scientific results. Please contact the administrator for infos about how to upload your dataset. We started by deploying a first dataset made of 2d and 3d turbulent configurations under the name of TURB-Rot. More will come.

<https://smart-turb.roma2.infn.it/>

### TURB-ROT. A LARGE DATABASE OF 3D AND 2D SNAPSHOTS FROM TURBULENT ROTATING FLOWS

A PREPRINT

**L. Biferale**

Dept. Physics and INFN  
University of Rome Tor Vergata, Italy, and IIC-Paris, France  
biferale@roma2.infn.it

**F. Bonaccorso**

Center for Life Nano Science@La Sapienza  
Istituto Italiano di Tecnologia and INFN  
University of Rome Tor Vergata, Italy.  
fabio.bonaccorso@roma2.infn.it

**M. Buzzicotti**

Dept. Physics and INFN  
University of Rome Tor Vergata, Italy.  
michele.buzzicotti@roma2.infn.it

**P. Clark Di Leoni**

Department of Mechanical Engineering,  
Johns Hopkins University, Baltimore, USA.  
patc@jhu.edu

Search for datasets

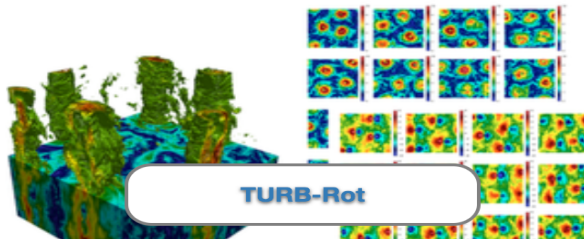


1

Datasets

**TURB-Rot**

A large database of 3d and 2d snapshots from turbulent rotating



**TURB-Rot**



2

Organizations

web\_admin  
web\_admin group

1  
member

# THANK YOU !

If you are interested to /working in AI  
applications to fluid dynamics  
participate to the questionnaire-->

The idea is to survey some information on the subject from  
the community and present / discuss the results in a  
JFM Perspective paper

with L. Biferale & M. Buzzicotti

LINK TO THE QUESTIONNAIRE

<https://forms.gle/j1ubGQQ8V2S9UwjP8>



QR CODE TO REACH THE QUESTIONNAIRE