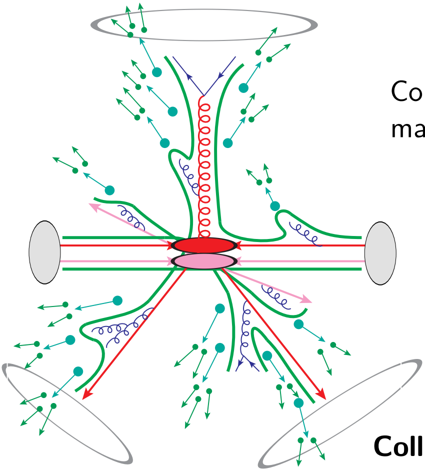
Perturbative QCD & Jets

COST school on hard & soft QCD probes, Lund
February 25, 2019 Stefan Prestel (LU)



Colliding composite objects kick-starts many processes:

hard scattering

radiation cascade

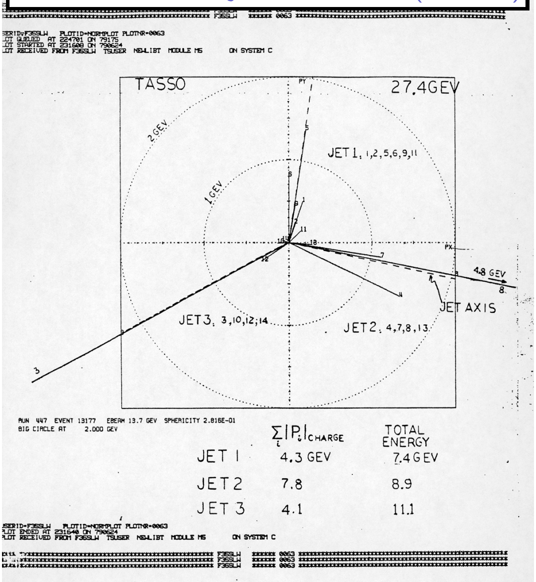
multiparton interactions

hadronization and decay

Colliders provide rich phenomena & fun things to measure + calculate!

High-energy scatterings are not isotropic!

1st hints of three jets in TASSO (25 GeV)



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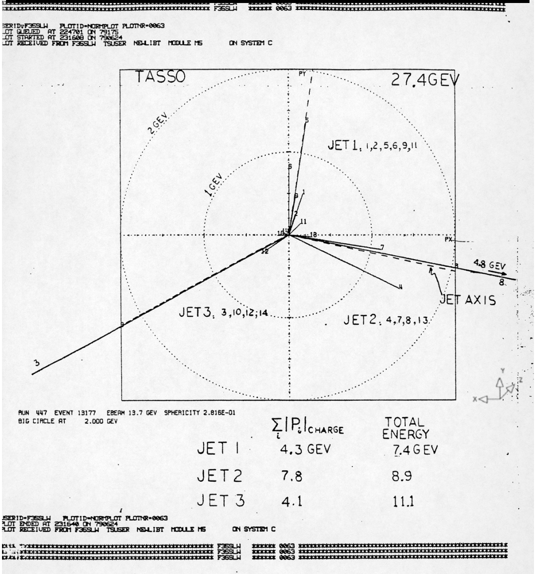
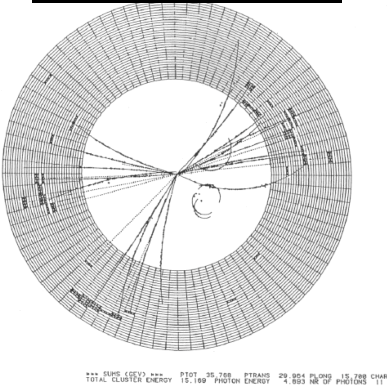


More complex at 31 GeV



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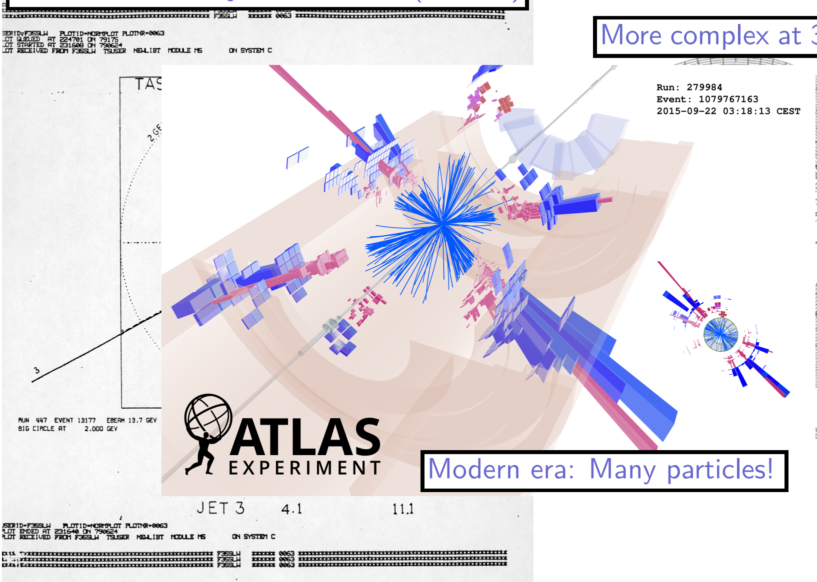
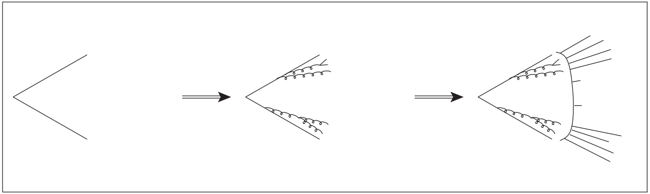


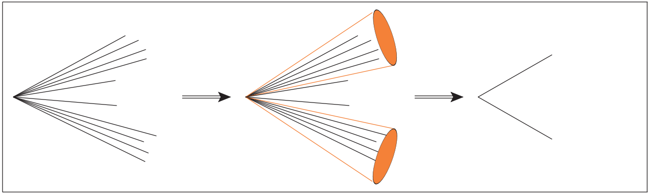
More complex at 31 GeV

Modern era: Many particles!

Part 1: What is a jet, and how do the jets form?
How do simple laws lead to complex consequences?



Part 2: How to reconstruct jets?
How should we analyze complex data to extract simple physics?



EXPERIMENTALLY...

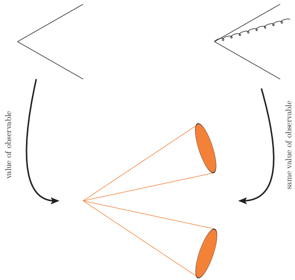
...we see collimated bunches of energy deposition or particles.
...every particle will come bunched together with other particles.
Jets ≈ *energetic* bunches with particles above some $E_{\mathsf{min}}/p_{\perp\mathsf{min}}$

The definition of a jet (bunches of many → jets) is a way of "coarse-graining" the information in one scattering event.
⇒ Makes handling information more manageable.

Jet definitions are a contract between experimentalists & theorists about presenting information.

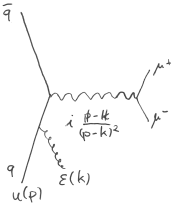
Why do we see jets?

Jet formation intimately linked to the infra-red structure of QFT:
Fixed-particle (number) S-matrix $= 0$.
n-hard-particle x-section $= \infty$.
Sum of all n-hard+soft particle x-sections $=$ finite.



Detector resolution means we measure partially inclusive states:
n observed particles come with any number of unobservable ones.
Result not fully inclusive. Hit different detector cells $\Rightarrow$ change result.

$\Rightarrow$ Non-inclusive result: IR divergences do not completely cancel. New particle emission rate logarithmically enhanced.



For $E_{(p-k)} \approx z E_p$ and small gluon $k_\perp^2$:

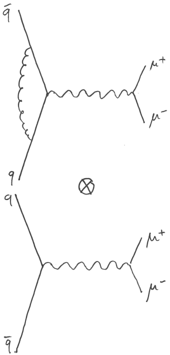
$$\frac{\not p - \not k}{(p-k)^2} \approx \frac{u(p_a)\bar u(p_a)}{p_a^2}$$

$$V_{QCD} \propto \bar u \not\varepsilon u \propto p_a^2$$

$$\Rightarrow |\mathcal{M}_{+g}|^2 \propto \frac{1}{p_a^2} |\mathcal{M}|^2$$

$$\frac{d^3k}{2E_k} \propto \frac{dz\, d\phi\, dk_\perp^2}{(1-z)}$$

$$\Rightarrow d\sigma_{+g} \propto d\sigma \cdot \frac{\alpha_s}{2\pi} \frac{dk_\perp^2}{k_\perp^2} dz \frac{1+z^2}{1-z}$$



contains an integral over all (experimentally indistinguishable) internal gluon momenta.

Both real emission and loop integrals give divergent results. Adding both yields a finite result.

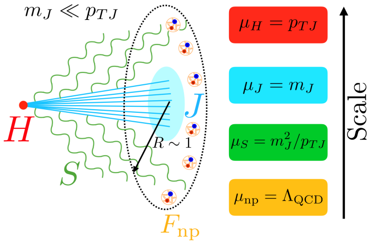
However, loops give indistinguishable kinematics, whereas detectors can "cut" the real emission into pieces!

$\Rightarrow$ Miscancellation.
$\Rightarrow$ Log. enhanced emission rate (cut off by resolution Λ)
$\Rightarrow$ Jets of partons

A "natural" resolution for partons in QCD is the hadronic scale $\Lambda_{\text{QCD}} \approx 1$ GeV.

Figures taken from arXiv:1709.04464

Partonic jets are a direct consequence of the IR behavior of the cross section. Thus
jets will have a "hard core" of high-energy particles,
surrounded by a "soft" low-energy particle cloud



The transition of partonic jets to jets of hadrons can be sensitive to both effects $\hookrightarrow$ Torbjörn

Emission x-section factorizes into simple low-multiplicity x-sections and *universal radiation functions*.

We may construct jet x-sections iteratively: Add one emission, then next emission on top, then next emission... $\Rightarrow$ Parton Shower

To calculate partonic jets, we rely on factorization of long-distance (hadronic) effects from short-distance (partonic) physics:

$$\begin{aligned}\sigma \quad = \quad & \int d\sigma_{(ab\to X+N\ \text{partons})}(\text{high energy}) \\ & \otimes\ f_{a\in \mathrm{A}}(\{x\}_a, \text{high energy}) \otimes\ f_{b\in \mathrm{B}}(\{x\}_b, \text{high energy}) \\ & \otimes\ \mathcal{D}(p_A, p_B, p_1, \ldots, p_N)\end{aligned}$$

$f(\{x\}, \text{energy}) \mathrel{\widehat{=}}$ Parton density in hadron at "resolution" $1/\text{energy}$
$\mathcal{D} \mathrel{\widehat{=}}$ Fragmentation mechanism.

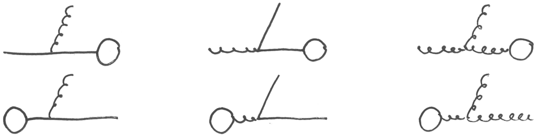
We then measure/extract f and $\mathcal{D}$ where radiative corrections are small (small characteristic momentum transfers).

Better calculations @ short-distance $\rightarrow$ better momentum distribution inputs to fragmentation $\rightarrow$ more universal parameter extractions.

Every x-section containing an additional collinear parton can be factorized as

$$d\sigma(pp \to X + g) \approx d\sigma(pp \to X) \int \frac{dt}{t} \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{f(\frac{x_a}{z}, t)}{f(x_a, t)} P(z)$$

(We've also replaced the parton luminosity factor!) The splitting kernels are



$$P_{qq} = C_F \frac{1+z^2}{1-z} \qquad P_{qg} = T_R\left(z^2 + (1-z)^2\right) \qquad P_{gg} = C_A \frac{\left(1 - z(1-z)\right)^2}{z(1-z)}$$

With explicit real-virtual cancellations, approximate an observable $\mathcal{O}$ by

$$\langle\mathcal{O}\rangle \approx d\sigma(pp\to X)\Big(\mathcal{O}(\Phi_X) \;-\; \mathcal{O}(\Phi_X)\int \frac{dt}{t}\frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{f(\frac{x_a}{z},t)}{f(x_a,t)}P(z) + \int \frac{dt}{t}\frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{f(\frac{x_a}{z},t)}{f(x_a,t)}P(z)\;\mathcal{O}(\Phi_X\Phi_g)\;\Big)$$

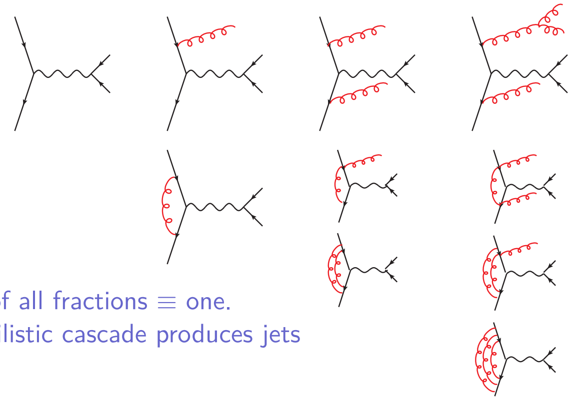
Imagine we re-insert this approximation $n\to\infty$ times. Then, we get

$$\langle\mathcal{O}\rangle \approx d\sigma(pp\to X)\Big[\mathcal{O}(\Phi_X)\exp\left(-\int\frac{dt}{t}d\Gamma(t)\right) + \int\frac{dt}{t}\exp\left(-\int_t\frac{d\bar{t}}{\bar{t}}d\Gamma(\bar{t})\right)d\Gamma(t)\;\;(\mathcal{O}(\Phi_X\Phi_g) + \text{2 or more emissions})\;\Big]$$

with $d\Gamma(t) = \frac{dz}{z}\frac{\alpha_s}{2\pi}\frac{f(x_a/z,t)}{f(x_a,t)}P(z)$

Like nuclear decay: A fraction of original configurations ($\mathcal{O}(\Phi_X)$) stay intact while others undergo transitions to radiative states ($\mathcal{O}(\Phi_X\Phi_g)$ etc.)

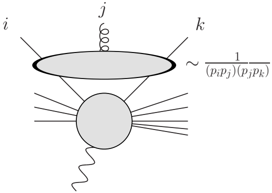
A fraction configurations "stay intact", another fraction accumulate radiation ...and then stay intact, or accumulate more radiation...



The sum of all fractions $\equiv$ one.
$\rightarrow$ Probabilistic cascade produces jets

Questions when constructing a parton shower:
- ◇ how detailed can we include perturbative IR structure?
- ◇ order of sequence of state changes?
- ◇ kinematics of changed states?

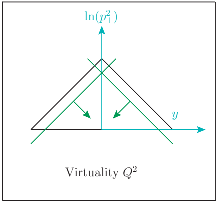
Low-energy (soft) emissions & IR correlations



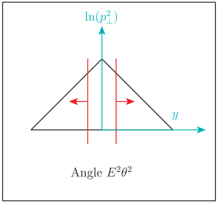
Soft gluons induce IR correlations between partons. They can be approximated through

- ◇ improved splitting kernels
- ◇ suitable ordering conditions
- ◇ clever kinematics reconstruction

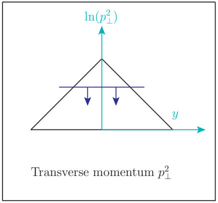
Modern showers contain a mix of these. Typical ordering criteria are:



Largest distortions "early". Coherence requires explicit additional vetoes.



Large distortions "late"? Integrated coherence by construction.



Largest distortions "early". Differential coherence for simple states. Many choices for $p_\perp$

$\hookrightarrow$ Later: Different jet algorithms mirror these different choices!

But wait, I see many well-separated separated jets!

Describing one/two collinear bunches won't be enough.
Need to improve calculation of partonic seed production.

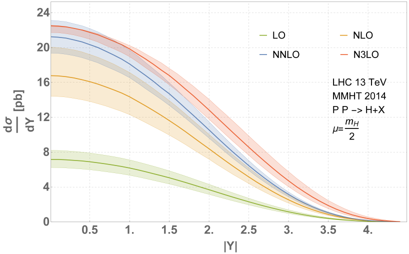
Fixed-order perturbation theory: Calculate few-parton x-section exactly:

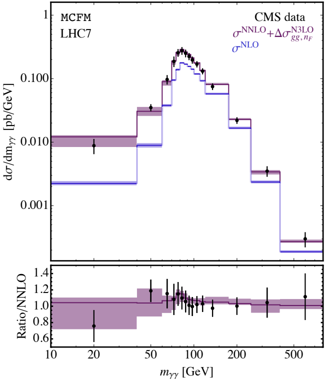
$$\begin{aligned}\langle\mathcal{O}\rangle &= \int d\Phi_B\,[B+V]\,\mathcal{O}(\Phi_B) + \int d\Phi_B d\Phi_1 R\mathcal{O}(\Phi_B\Phi_1)\\ &\Rightarrow \int d\Phi_B\left[B+V+\int d\Phi_1 B\otimes P_{PS}\right]\mathcal{O}(\Phi_B)+\int d\Phi_{B+1}\,[R-B\otimes P_{PS}]\mathcal{O}(\Phi_{B+1})\end{aligned}$$

where B=Born, V=virtual & R=real correction, and where we've already removed the known PS result in the second step.

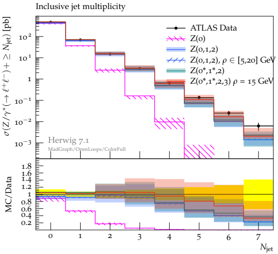
Using states distributed like this as shower's input, the real correction will give an improved model of well-separated jets ... but
this x-section might not be numerically integrable, and is shower-specific.
$\Rightarrow$ Large body of work, many methods with different strengths.

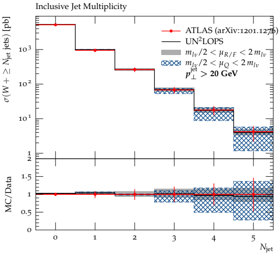
Perturbative records





Clockwise: N^{3}LO Higgs rapidity "pen & paper", NNLO numerically produced plots, NNLO events+shower, combining many NLO processes+shower





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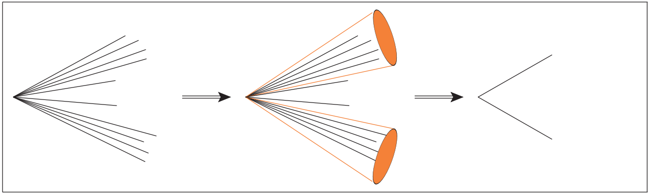
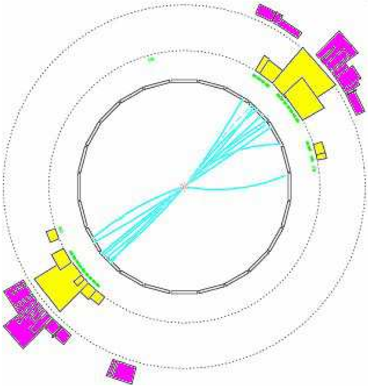
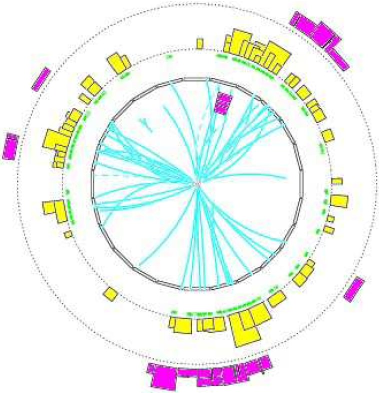
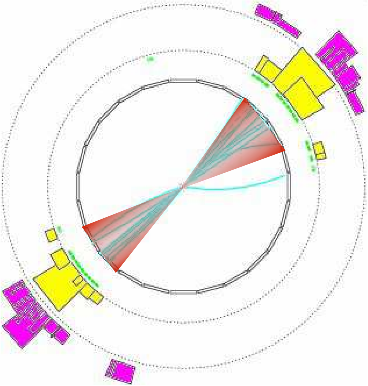
How should we analyze complex data to extract simple physics?

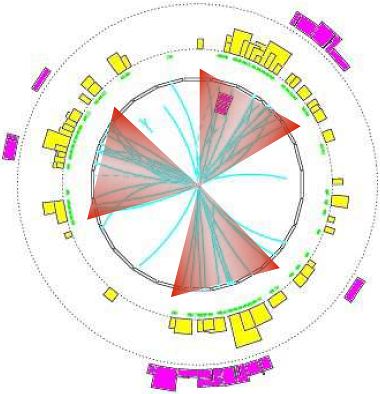




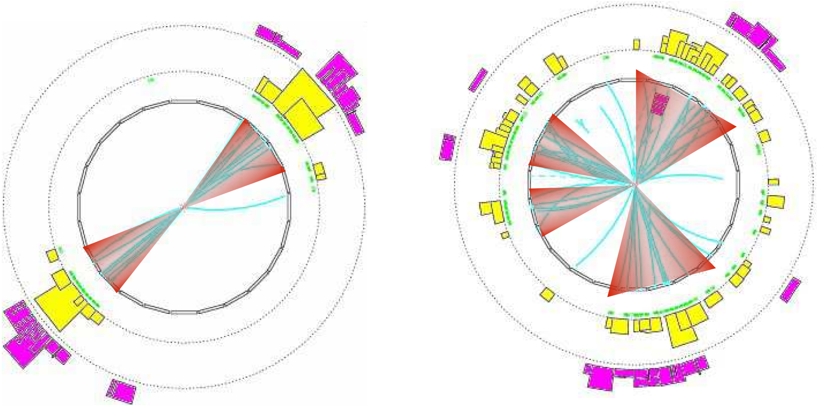


Two typical LEP-era detector-level events. How many jets?





Two typical LEP-era detector-level events. How many jets?
Two on the left? Three on the right?



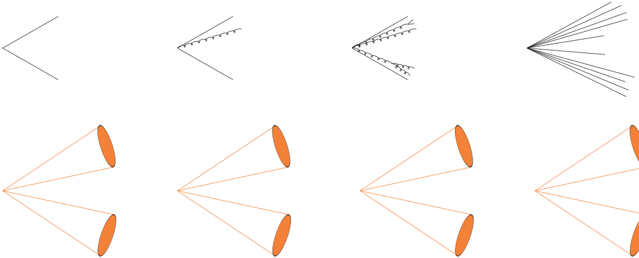
Two typical LEP-era detector-level events. How many jets?
Two on the left? Four on the right?

The reconstruction of jets is ambiguous. Thus, we need to agree

1. Which objects (particles, tracks, calorimeter towers...) can be combined? How many to recombine?
2. How do these objects recombine?
   What is the maximal recombination range? Which objects are recombined first? How is the recombined momentum constructed?

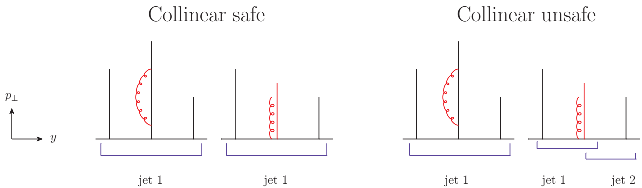
Note: Although important in practice for experiments, we'll assume here that we know what objects should be recombined. Also, we assume that we assume that we always recombine $2 \to 1$ object.

In the best of all worlds, jet reconstruction should be as insensitive as possible to the showering and hadronization process.



⇒ Jet definition allows to project complex measurement onto simple objects that allow comparison to straight-forward (even simplistic) calculations.

To be able to compare reconstructed jets to (precise) perturbative calculations, the jet recombination needs to ensure that IR cancellations are intact!



For example, starting the recombination with the highest energy particle is problematic because of collinear splittings!

Sequential recombination algorithms fulfill these requirements:

1. Define a distance d_{ij} between the objects i and j.
   The overall behavior is governed by what we call "distance".
2. Recombine pair ij with smallest d_{ij} into a new object.
   Most current jet algorithms simply add the 4-momenta.
3. Iterate until all $d_{ij} > d_{cut}$

This will lead to stable, yet often irregularly shaped jets. But the result is very theorist-friendly.

d_{cut} is a resolution parameter governing the level of coarse-graining.

Modification for hadron beams, as “beam jets” down the beam pipe cannot be observed:

1. Define a distance d_{ij} between the final-state objects i and j.
   Define a distance d_{iB} between a final-state i and the beam.
2. Find the smallest distance d_{ij} or d_{iB}.
   If min$= d_{ij}$, recombine and proceed
   If min$= d_{iB}$, call i a jet and proceed w/o i
3. Iterate until no objects left.
   Only use jets with $p_\perp > p_{\perp cut}$

Further, it is sensible to introduce a “catchment parameter” (the jet radius R) as a handle on contamination from beam remnants.

Most of today's recombination measures are of the form

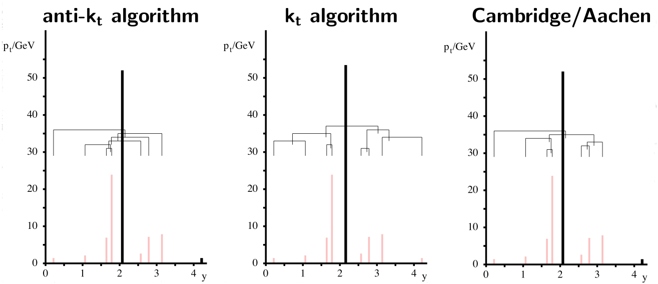
$$\boxed{\begin{array}{l} d_{ij} = \min\left(p_{ti}^{2k}, p_{tj}^{2k}\right) \dfrac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = p_{ti}^{2k} \\ \text{where} \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 \end{array}}$$

- ΔR_{ij}-dependence: To combine collinear particles early on
- $\min\left(p_{ti}^{2k}, p_{tj}^{2k}\right)$ to combine soft particle early on
- k determines competition of soft/collinear clusterings

Note that such distance measures are infrared/collinear safe.
Note the use of quantities invariant under boosts along beam axis.

Widespread recombination algorithms

Anti-kT distance	kT distance	Cambridge/Aachen
$d_{ij}=\frac{\Delta R_{ij}^2}{R^2}/\min\left(p_{ti}^2,p_{tj}^2\right)$ $d_{iB}=1/p_{ti}^2$	$d_{ij}=\min\left(p_{ti}^2,p_{tj}^2\right)\frac{\Delta R_{ij}^2}{R^2}$ $d_{iB}=p_{ti}^2$	$d_{ij}=\frac{\Delta R_{ij}^2}{R^2}$
Hard collinear privileged	Democratic soft/collinear	"Ignores" soft divergence
Clustering somewhat inverse to Q^2-ordered PS	Clustering somewhat inverse to $p_\perp$-ordered PS	Cluster small angle first $\approx$ Inverse of θ-ordered PS



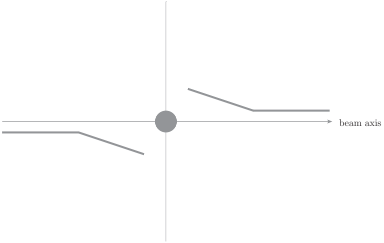
Hadron-collider jet algorithms contain a jet radius R parameter of $\mathcal{O}(1)$:

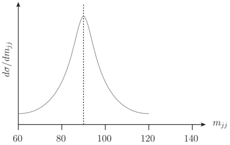
$$d_{ij} = \min\left(p_{ti}^{2k}, p_{tj}^{2k}\right)\frac{\Delta R_{ij}^2}{R^2}$$

This allows to minimize contamination from beam remnants.

⇒ Find value that best suits experimental conditions & analysis needs.

(Values in Fig. are just examples!)





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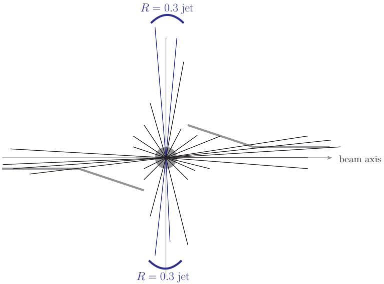

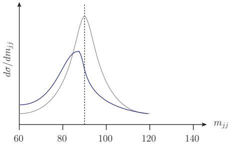

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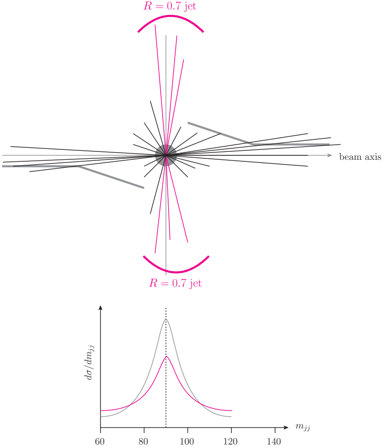

Changing the jet radius

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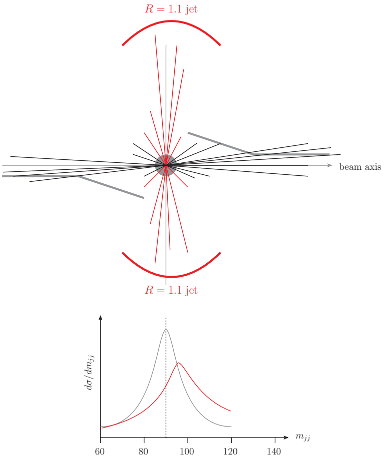
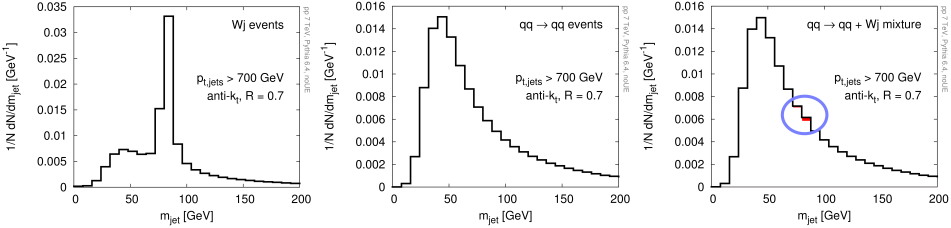


Plots taken from G. Salam's TASI lectures 2013

Sequential recombination algorithms open the door to studies of

Jet substructure: Once a jet has been constructed, and all its constituents are known, use the kinematic information of the clustering tree to distinguish jet production mechanisms.

Example: Distinguish hadronic W-boson decays from large QCD backgrounds.



Jet substructure example: W-boson tagging

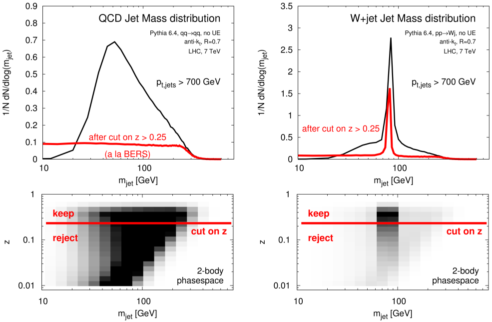
Plots taken from G. Salam's TASI lectures 2013

"QCD jets" obtain mass from soft/collinear radiation, with distribution

$$d\sigma(pp \to X + g) \propto d\sigma(pp \to X) \int \frac{dp_\perp^2}{p_\perp^2} \int_{z_{g,min}}^{z_{g,max}} \frac{dz_g}{z_g}$$

One parton's E hardly changes, the other takes a small fraction z_g.
In W-boson decays, the energy of partons is more evenly shared.

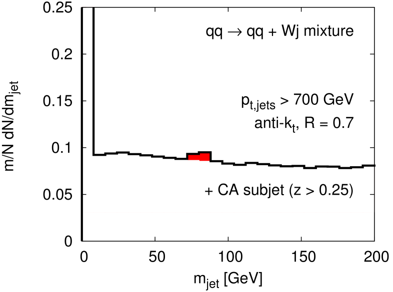
$\Rightarrow$ Use cut on energy fractions between two jet "prongs" in an intermediate clustering step can suppress QCD.



Jet substructure methods can often help to minimize QCD backgrounds.

Many different methods& ideas exist to exploit QCD knowledge exist. Of course, when digging into jets, need to be careful not to spoil IR cancellations!

arXiv:1901.10342 is a very good recent review.



Jets are a basic fact of high-energy physics.
Jets are remnants of the IR structure of massless gauge theories.

Hadronic jets arise via radiative cascades of soft and collinear quarks and gluons. The "ordering" of emissions is ambiguous.
The reconstruction of jets from data mirrors this ambiguity.

Jet reconstruction methods should be infrared safe.
Widespread algorithms rely on successive recombination using p_t- and angle-dependent jet distance measures.

Successive recombination algorithms also open the door to jet substructure methods to measure or suppress QCD backgrounds.

Enjoy the school & workshop!