

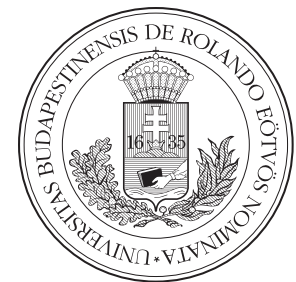
# Suppression of anisotropic flow without viscosity

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Emberi Erőforrások  
Minisztériuma

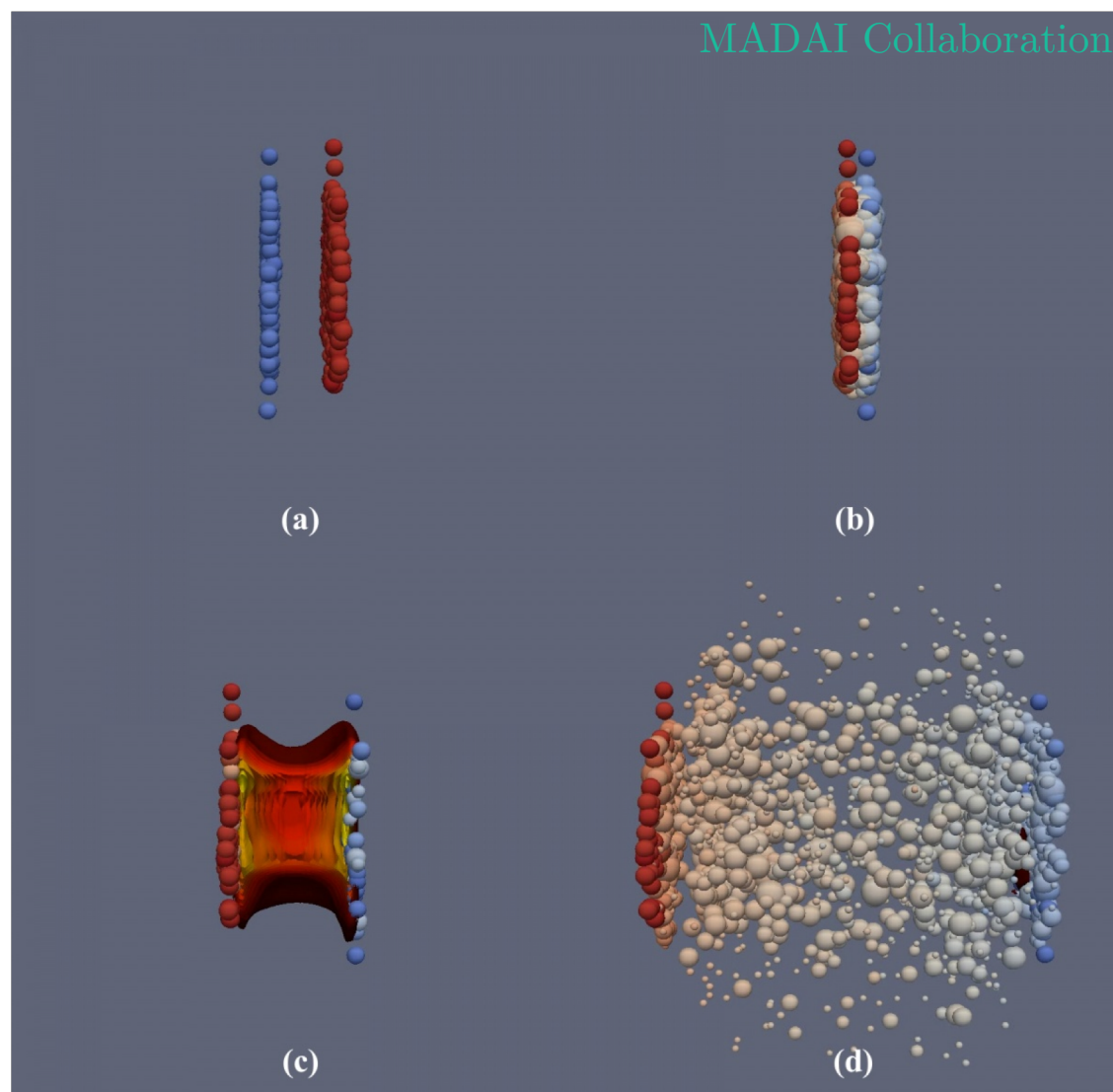
Supported by the 18-3 New National Excellence Program of the  
Ministry of Human Capacities and OTKA K120660.



# Outline

1. Motivation:  
relativistic hydrodynamics to understand heavy ion collisions
2. Particlization:  
basics, shortcomings, influence on observables
3. How much does  $f$  matter?
4. Results: 4-source model and 2+1D hydro
5. Conclusion and outlook

# Motivation



**For students:** see Gabriel Denicol's talk about hydro from Hot Quarks 2018  
<https://indico.cern.ch/event/703015/contributions/3095199/>

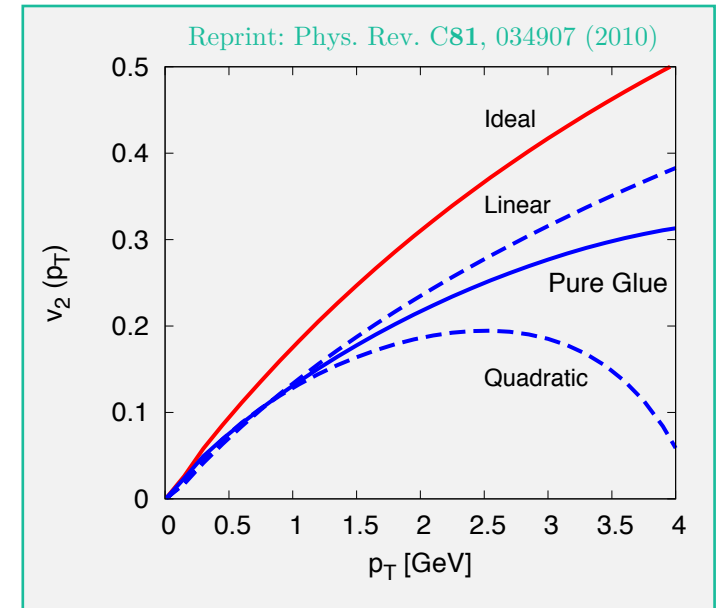
# Motivation

D. Molnar & P. Huovien, J.Phys. G**35** 104125 (2008)

K. Dusling, G.D. Moore, D. Teaney, Phys. Rev. C**81**, 034907 (2010)

D. Molnar & Z. Wolff, Phys. Rev. C**95**, 024903 (2017)

- Shear viscosity is sensitive to the particlization method
- There is no unique way to do the particlization
- Thermal equilibrium is an assumption



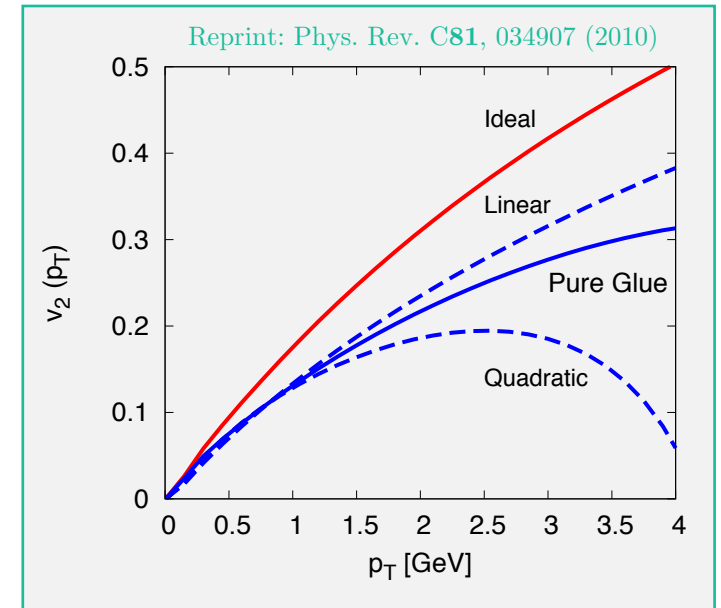
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- There is no unique way to do the particlization
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## Questions:

1. How sensitive is ideal hydro?
2. Is thermal equilibrium a legitimate assumption?

# Particlization

How is it done?

# Particlization: from hydro to particles

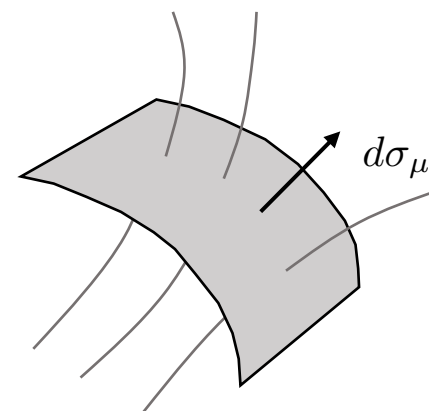
L. Csernai, *Introduction Relativistic Heavy Ion Collisions* (1994)

Conversion of fluid  $\rightarrow$  particles on a 3D hypersurface:  $d\sigma^\mu$

Inside: fluid  $(u^\mu(x), \varepsilon(x), P(x), n(x))$

$$N^\mu(x) = (n(x), \vec{j}(x))$$

$$T^{\mu\nu}(x) = [\varepsilon(x) + P(x)]u^\mu u^\nu - P(x)g^{\mu\nu}$$



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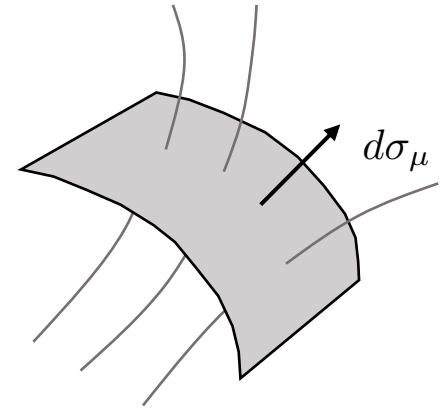
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Outside: particle  $f(x, p)$

$$N^\mu(x) = \int \frac{d^3p}{p^0} p^\mu f(x, p)$$

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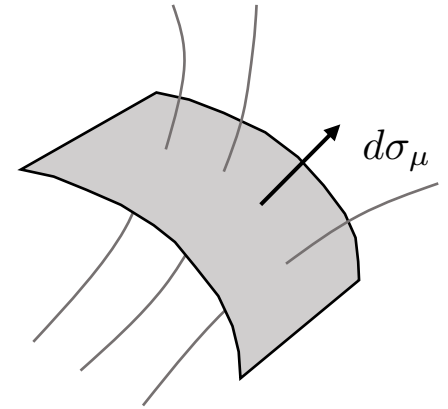
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Boundary: conservation laws

$$[N^\mu d\sigma_\mu]_{\text{in-out}} = 0$$

$$[T^{\mu\nu} d\sigma_\mu]_{\text{in-out}} = 0$$



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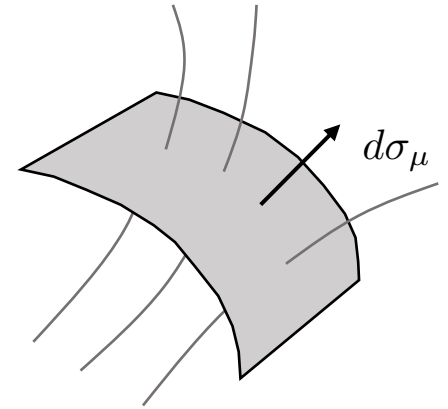
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Cooper-Frye formula: momentum spectrum (measurable)

$$E \frac{d^3N}{d^3p} = \int d\sigma_\mu p^\mu f(x, p)$$

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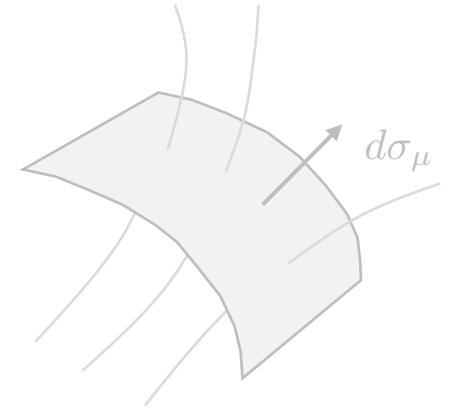
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What is  $f(x, p)$  ?

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- We don't know.

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- Described by some kinetic transport theory  
(assumption)

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- Described by the Boltzman Transport Equation  
(assumption of assumption)

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$f(x, p)$  - single-particle distribution

- 0<sup>th</sup> solution of the BTE

(assumption of assumption of assumption)

Relativistic Boltzmann (thermal) distribution:

$$f_0(x, p) = \frac{g}{(2\pi)^3} e^{-\frac{p^\mu u_\mu(x)}{T(x)}}$$

One field describes everything:  $T(x)$  temperature.



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- Change a little to „non-extensive“: corrections

$$f(x, p) = A \left[ 1 + \frac{\alpha}{T_\alpha(x)} p^\mu u_\mu(x) \right]^{-\frac{1}{\alpha}}$$

One field  $T_\alpha(x)$  „temperature“ + non-ext. parameter  $\alpha$ .

If  $\alpha \rightarrow 0$  we get back Boltzmann!

Starts in exponential, end in power law!

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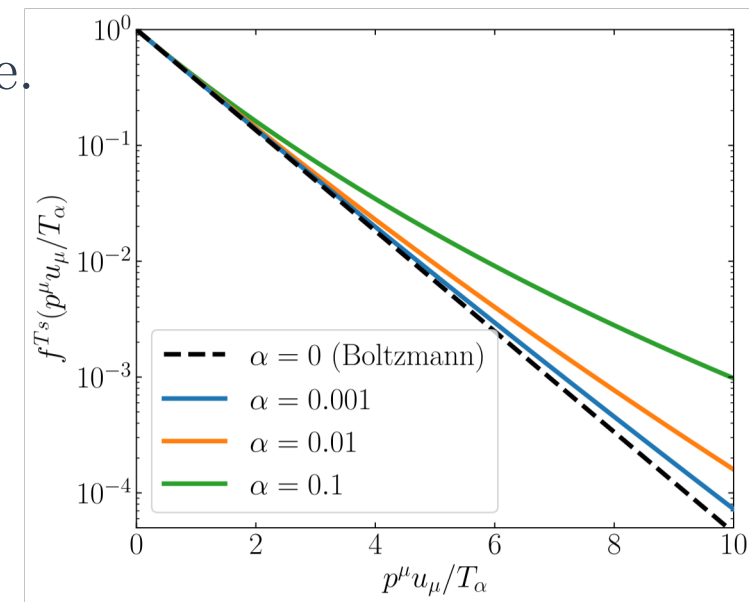
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$$v_n(p) = \frac{\int d\phi \cos(n\phi) E \frac{d^3 N}{d^3 p}}{\int d\phi E \frac{d^3 N}{d^3 p}}$$

Boltzmann:

$$v_n(p) \sim \frac{e^{-ap}}{e^{-bp}} = e^{-(a-b)p} \rightarrow 1$$

Tsallis:

$$v_n(p) \sim \frac{(ap)^{-1/\alpha}}{(bp)^{-1/\alpha}} \rightarrow \left(\frac{a}{b}\right)^{-\frac{1}{\alpha}} \leq 1 \text{ (looks interesting!)}$$

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- Spectrum: exponential like  $\rightarrow$  power law tail

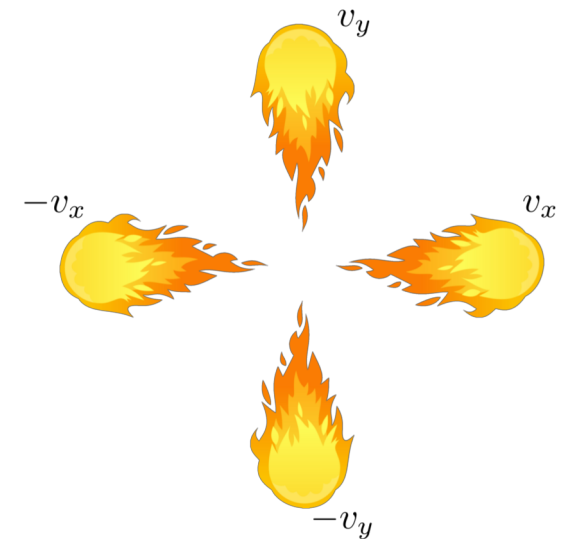
- Flow:

$$v_n(p) = \frac{\int d\phi \cos(n\phi) E \frac{d^3 N}{d^3 p}}{\int d\phi E \frac{d^3 N}{d^3 p}}$$

- Simple 4-source model:

4-uniform fireballs, no long. expansion,  
boosted sym.  $(\pm v_x, \pm v_y)$  and  $T=const$  freeze-out

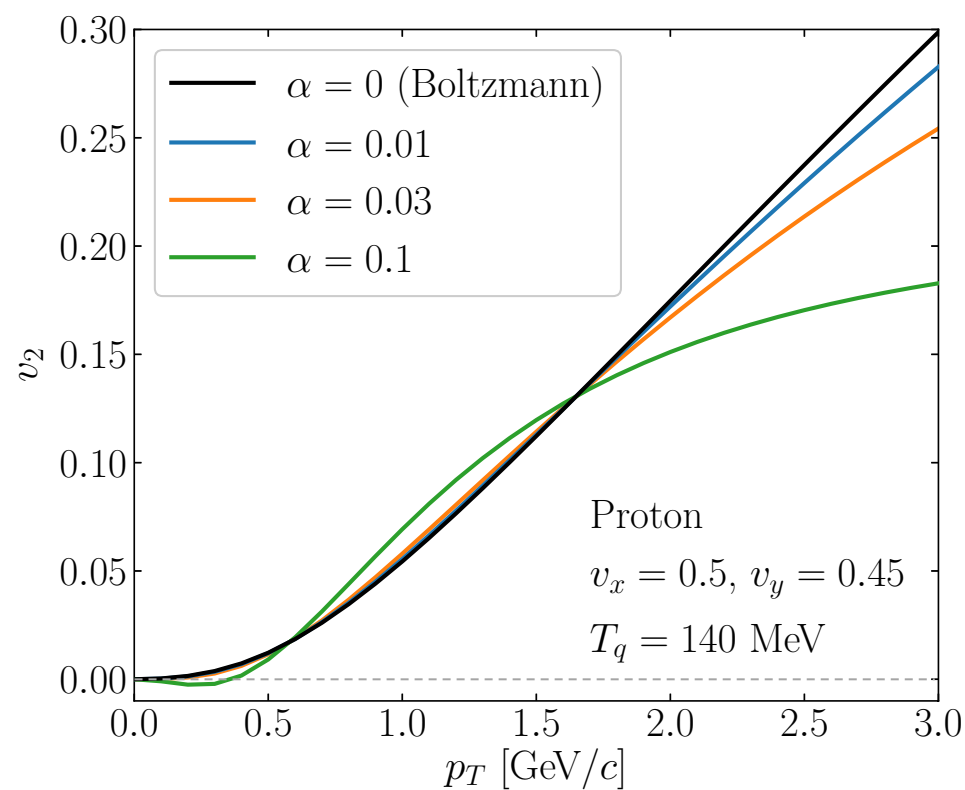
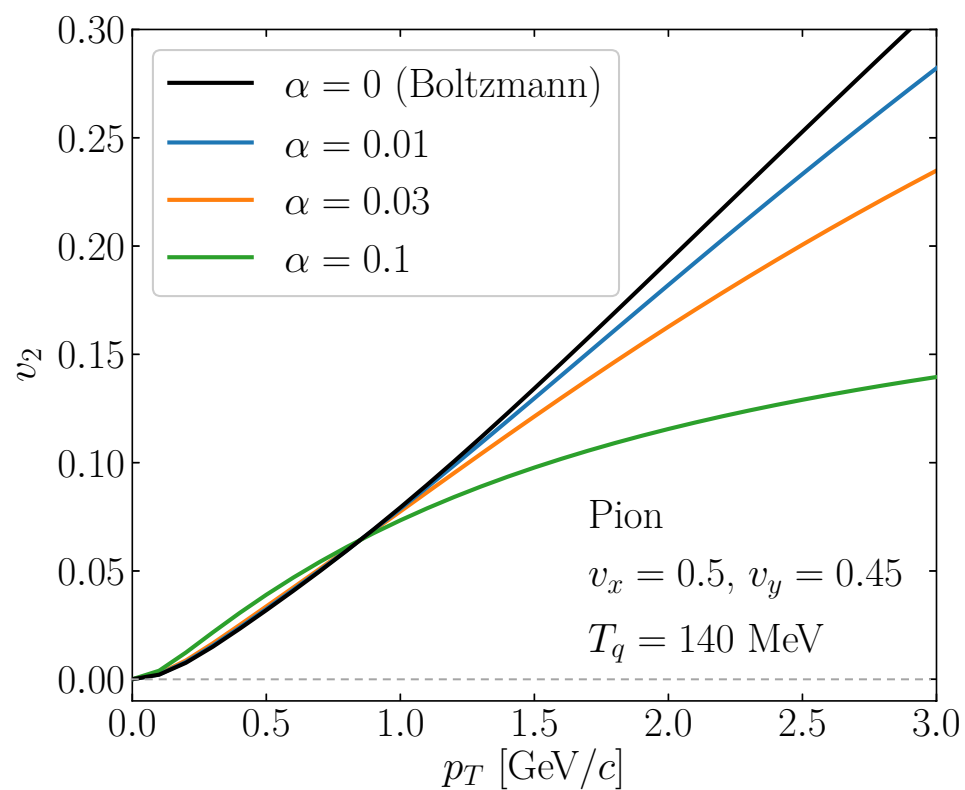
$$f^{4s}(p) = f|_{v_x} + f|_{-v_x} + f|_{v_y} + f|_{-v_y}$$



P. Hoven et al Phys. Lett. B **503**, 58 (2001)

# How does spectrum and flow change?

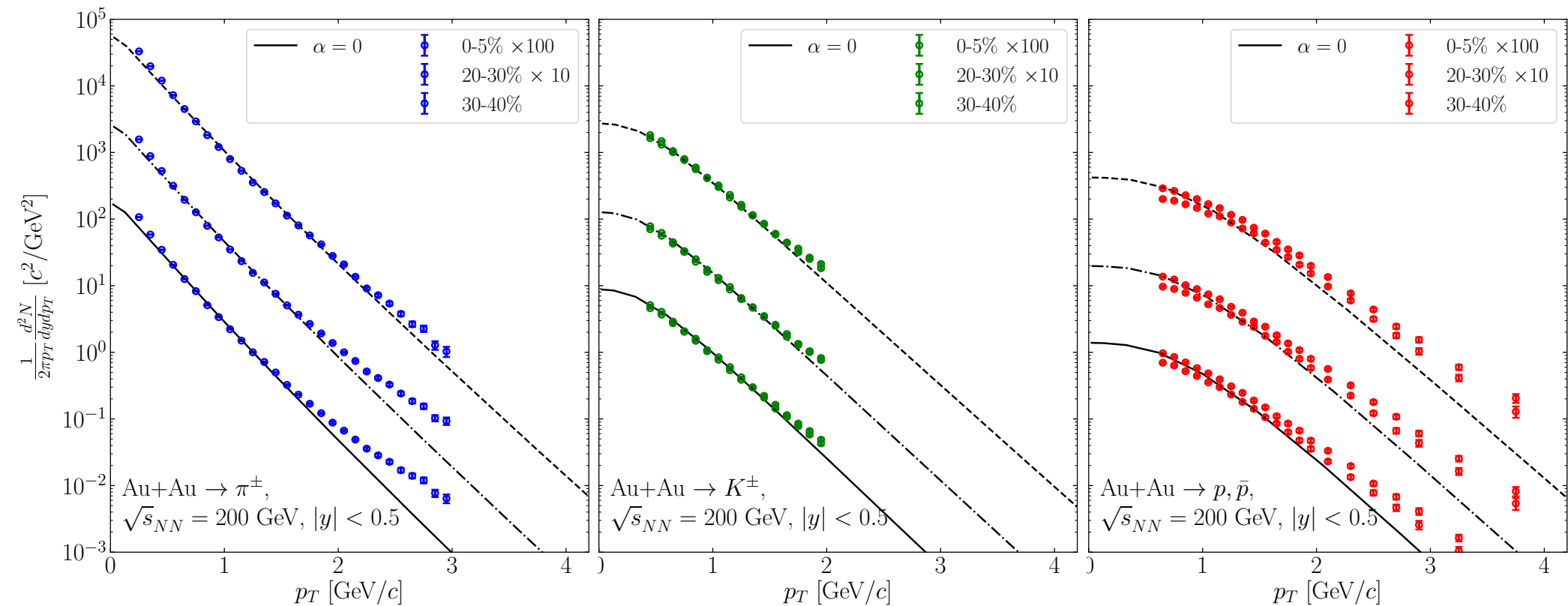
Result from 4-source model: **suppression in  $v_2$ !**



# Hydrodynamic Simulation

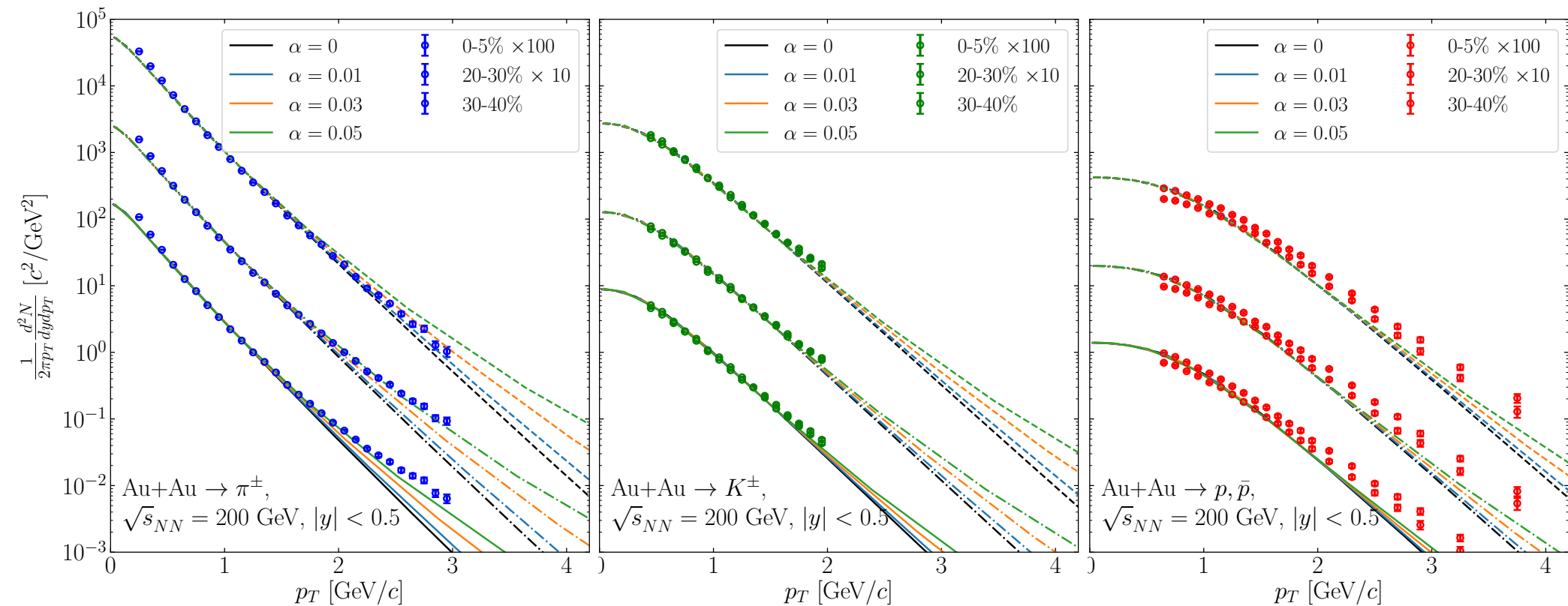
1. Initial condition: Au+Au @ 200 GeV optical Glauber model  $S_0=110 \text{ fm}^{-3}$ .
2. 2+1D numerical ideal hydro with Azhydro.
3. Cooper–Frye freeze-out with Tsallis  
 $\varepsilon(x)$ ,  $P(x)$  and  $q$  are fixed,  $T_f = 140 \text{ MeV}$
4. Resonance decays are included.

# Results: hydrodynamic simulation



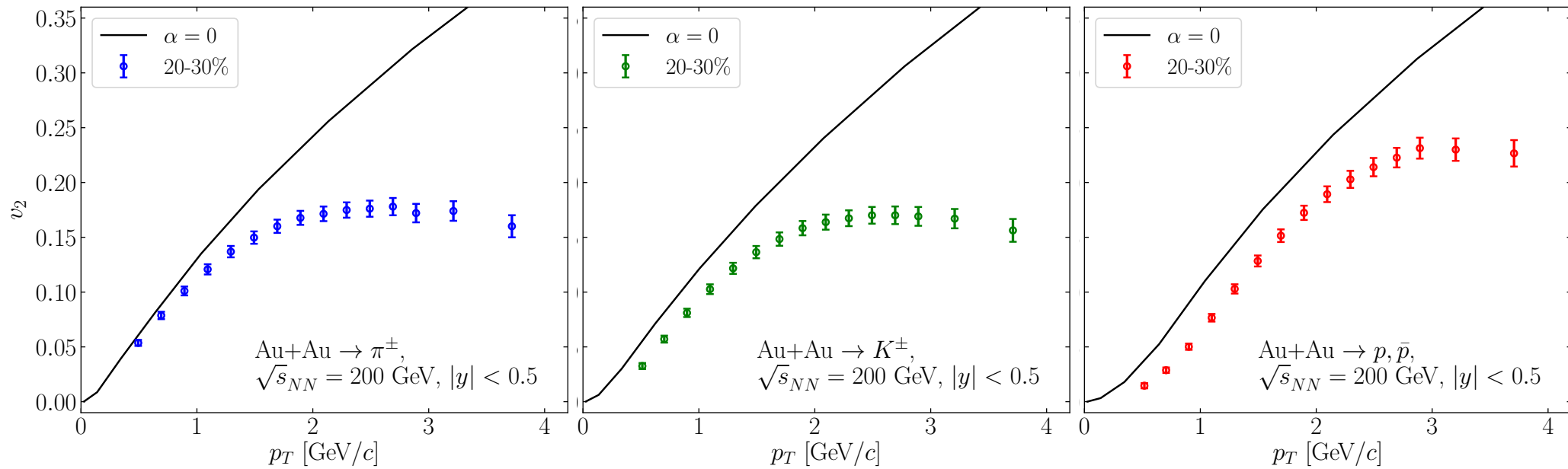


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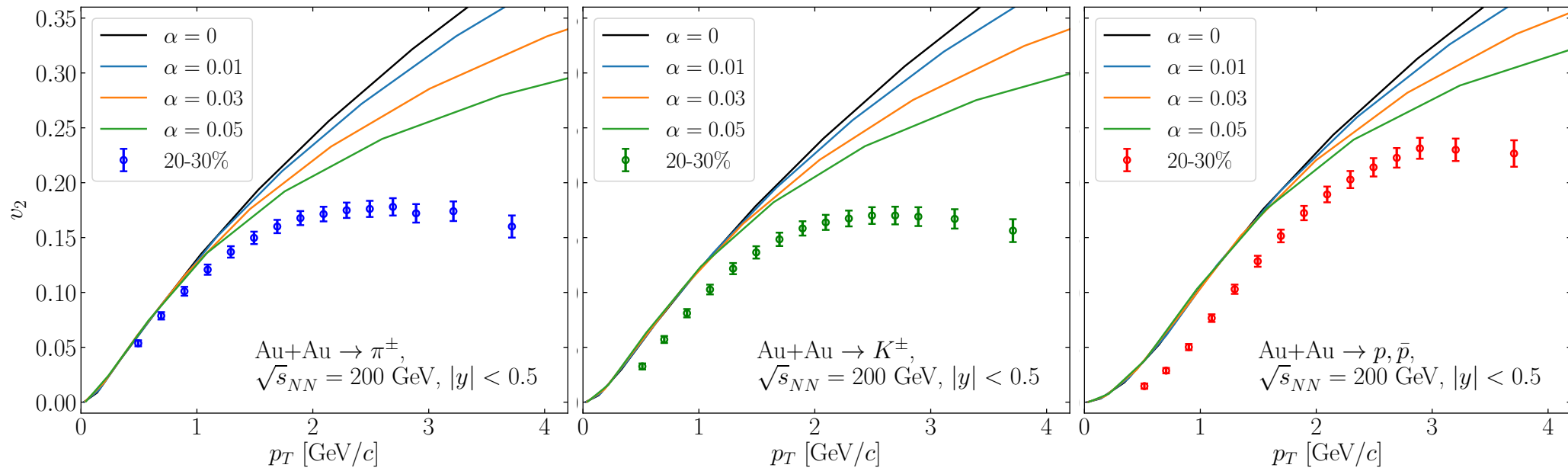


- Power-law tail appears
- Better agreement with data!

# Results: hydrodynamic simulation

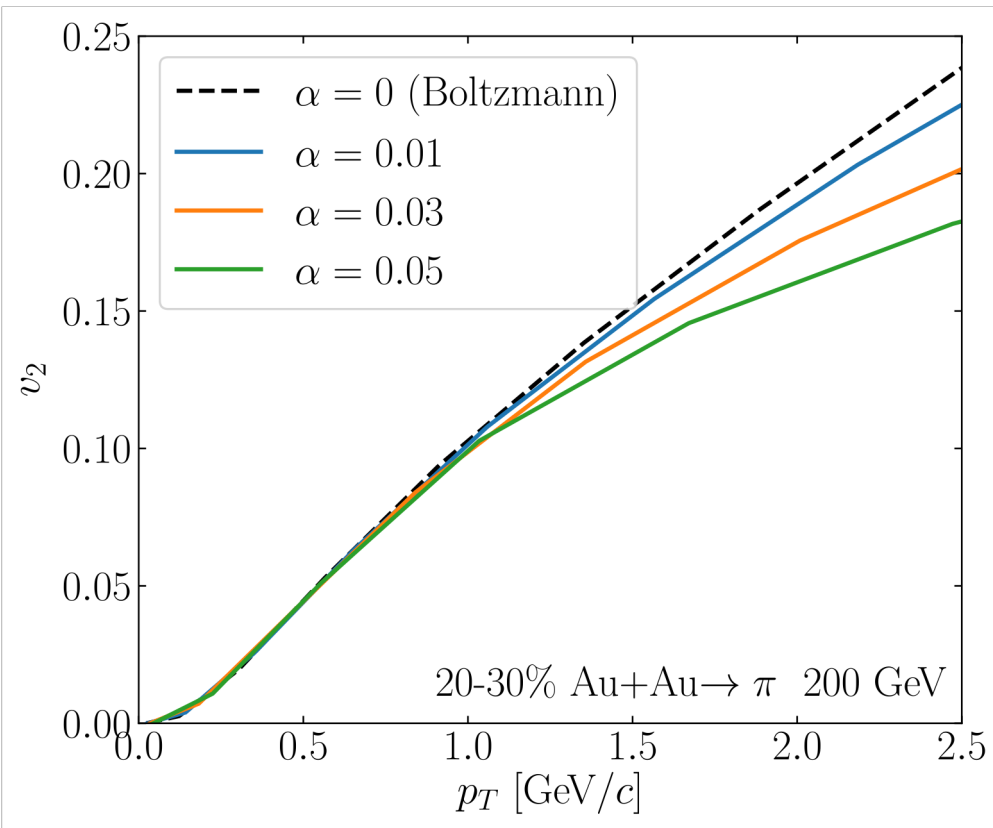


# Results: hydrodynamic simulation

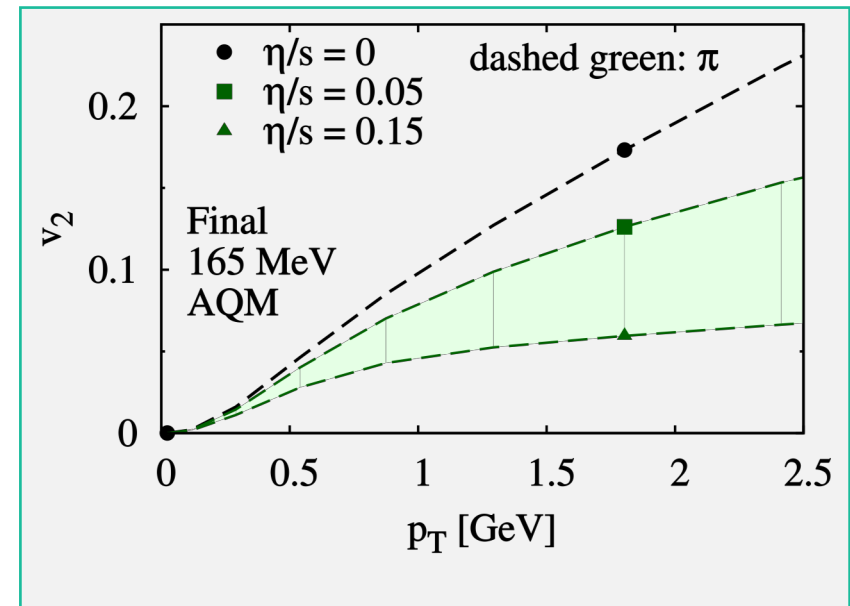


- Suppression in flow!
- Better agreement with data!
- Mass ordering in the suppression, like shear viscosity!

# Results: hydrodynamic simulation



## Viscous calculation:



Reprint: D.Molnar & Z.Wolff, Phys.Rev. **C95**, 024903 (2017)

- Non-ext. behaves like shear viscosity (no viscosity in the model)!
- Comparison to viscous calculation:  $\alpha \approx 0.05 \leftrightarrow \eta/s = 0.05$
- From fits to spectra:  $\alpha = 0 - 0.07$

K.Urmosy, G.G.Barnafoldi, T.S.Biro,  
J.Phys. Conf.Ser. 612 012048 (2015)

# Summary

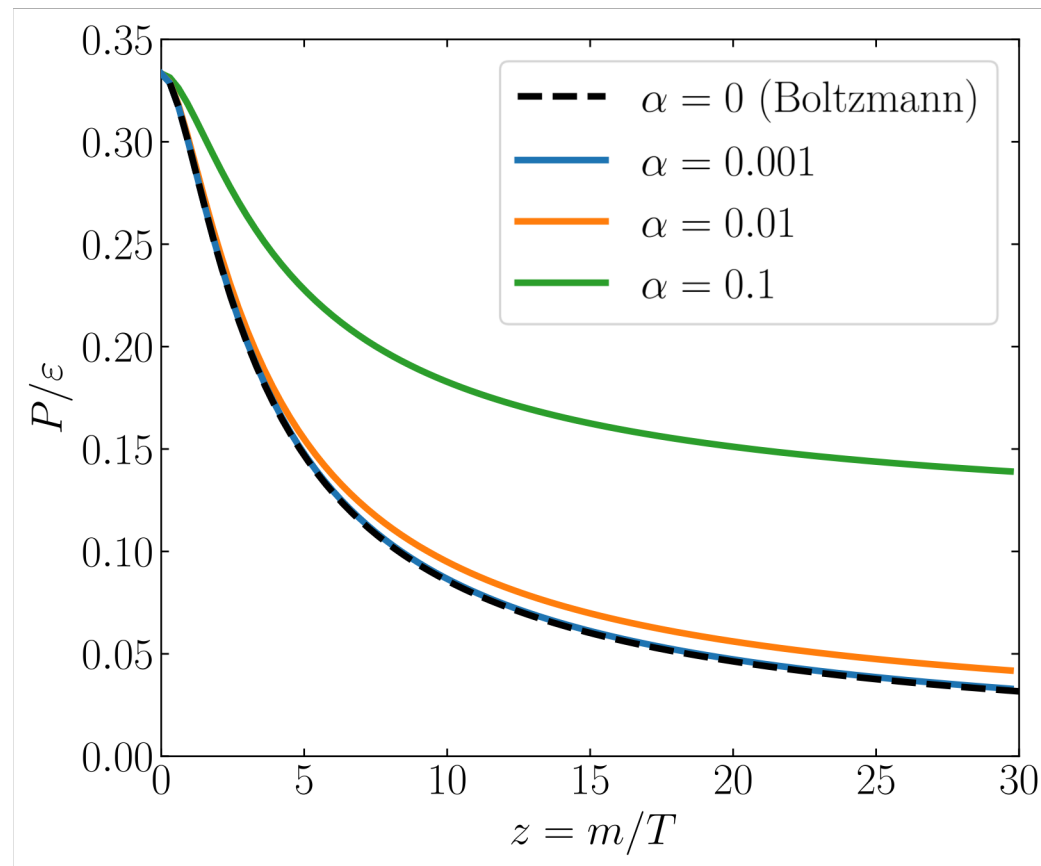
- Relativistic Boltzmann distribution (thermal equilibrium) is an approximation.
- To study a specific type of correction (using Tsallis distribution):
  1. Finite size effects and correlations.
  2. More suitable for the spectrum (power law tail) and the flow ( $v_n < 1$ ).
  3. Isotropization? Conformal theories do not require equilibrium.
- The correction has important effects:
  - Better spectra.
  - Suppressed flow
  - Mimic shear viscosity:  $\alpha \approx 0.05 \leftrightarrow \eta/s = 0.05$
- Future:
  - Extend to viscous calculation.
  - Study kinetic transport for an exact correction. Better understand  $\alpha$

P.Arnold, J.Lenaghan, G.D.Moore, L.G. Yaffe, Phys. Rev. Lett. **94**, 072302 (2005)

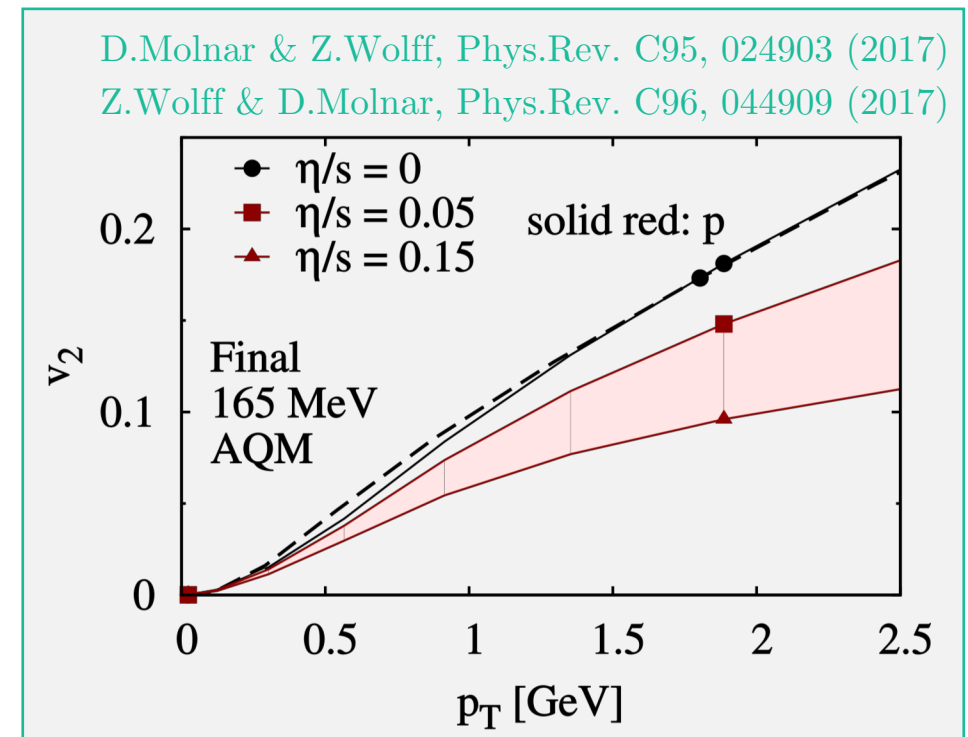
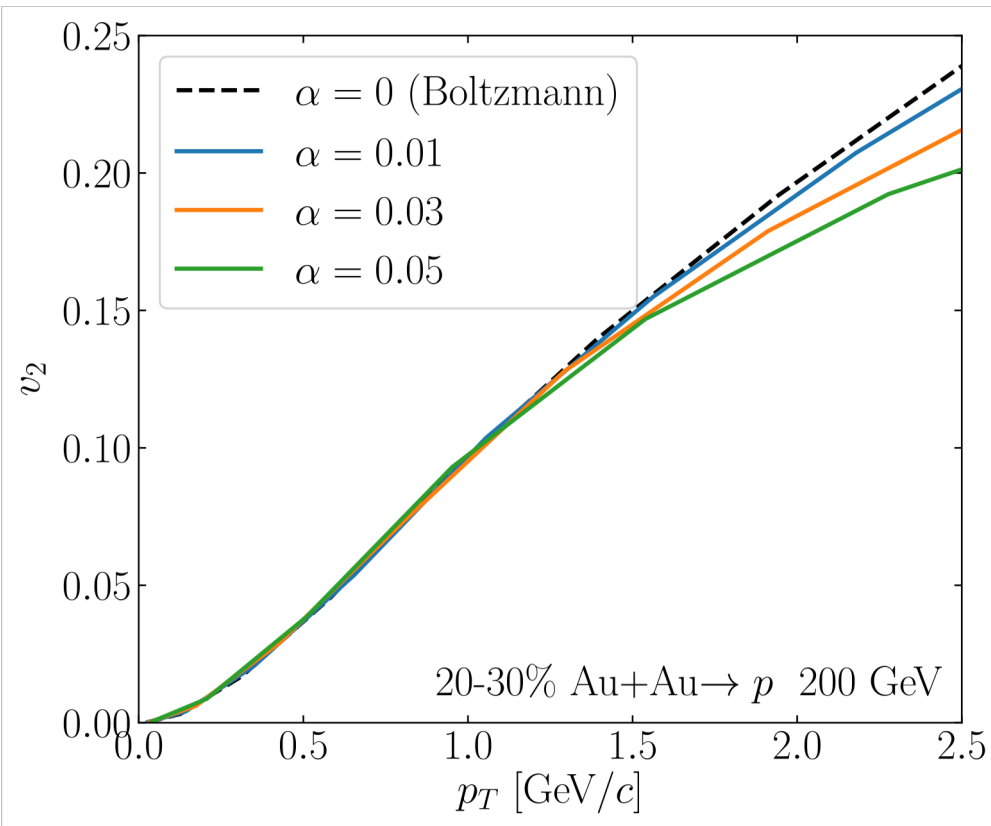
M.Luzum & P.Romatschke. Phys.Rev. **C78** 034915 (2008)

Thank you for your attention!

# Backup: describing hydro fields with non-ext. distr.

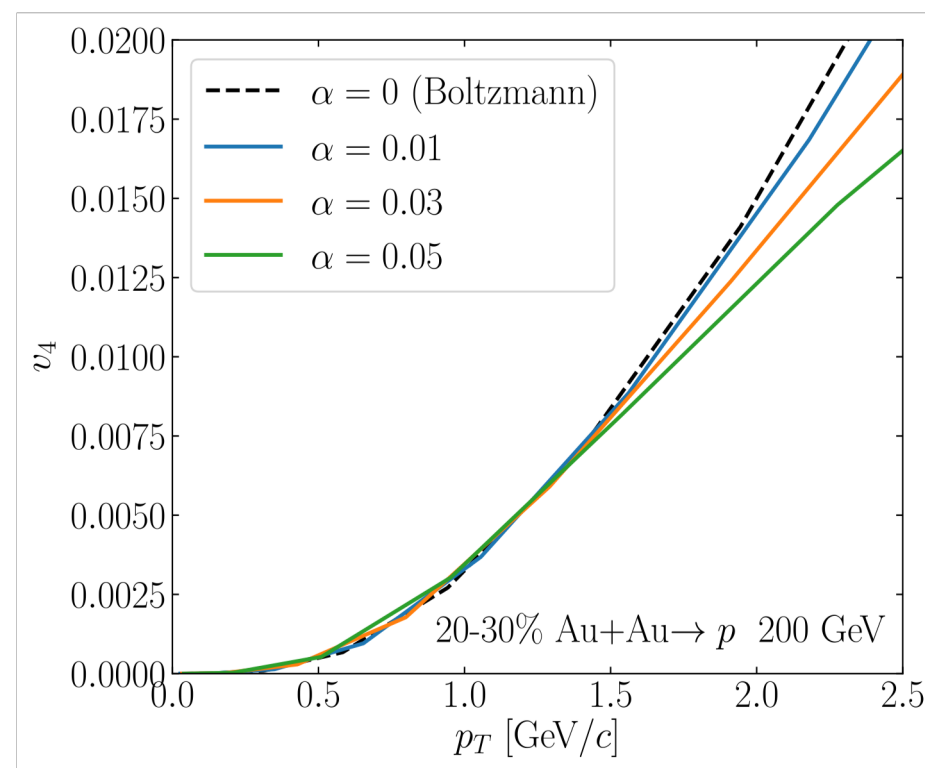
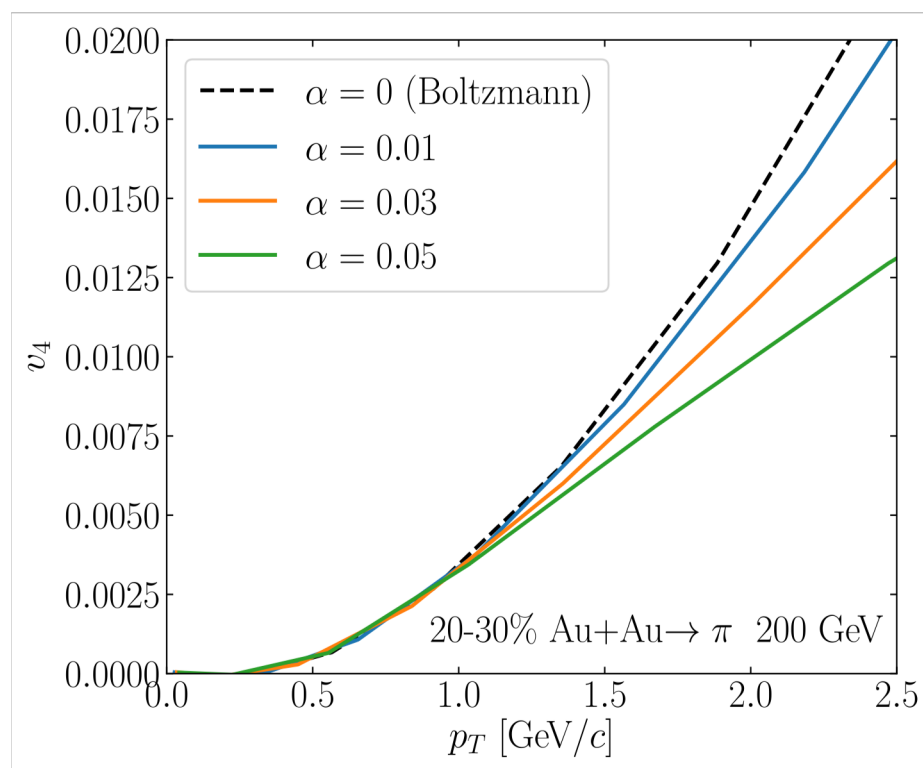


# Backup: non-ext. parameter vs. shear viscosity





# Backup: $v_4$ with non-ext. $f$



# Backup: changes in other observables

