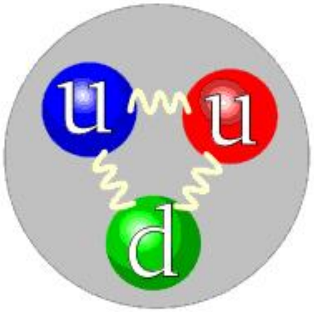
QGP from the quantum ground-state of QCD?

beautiful math or "New Physics" of QCD?

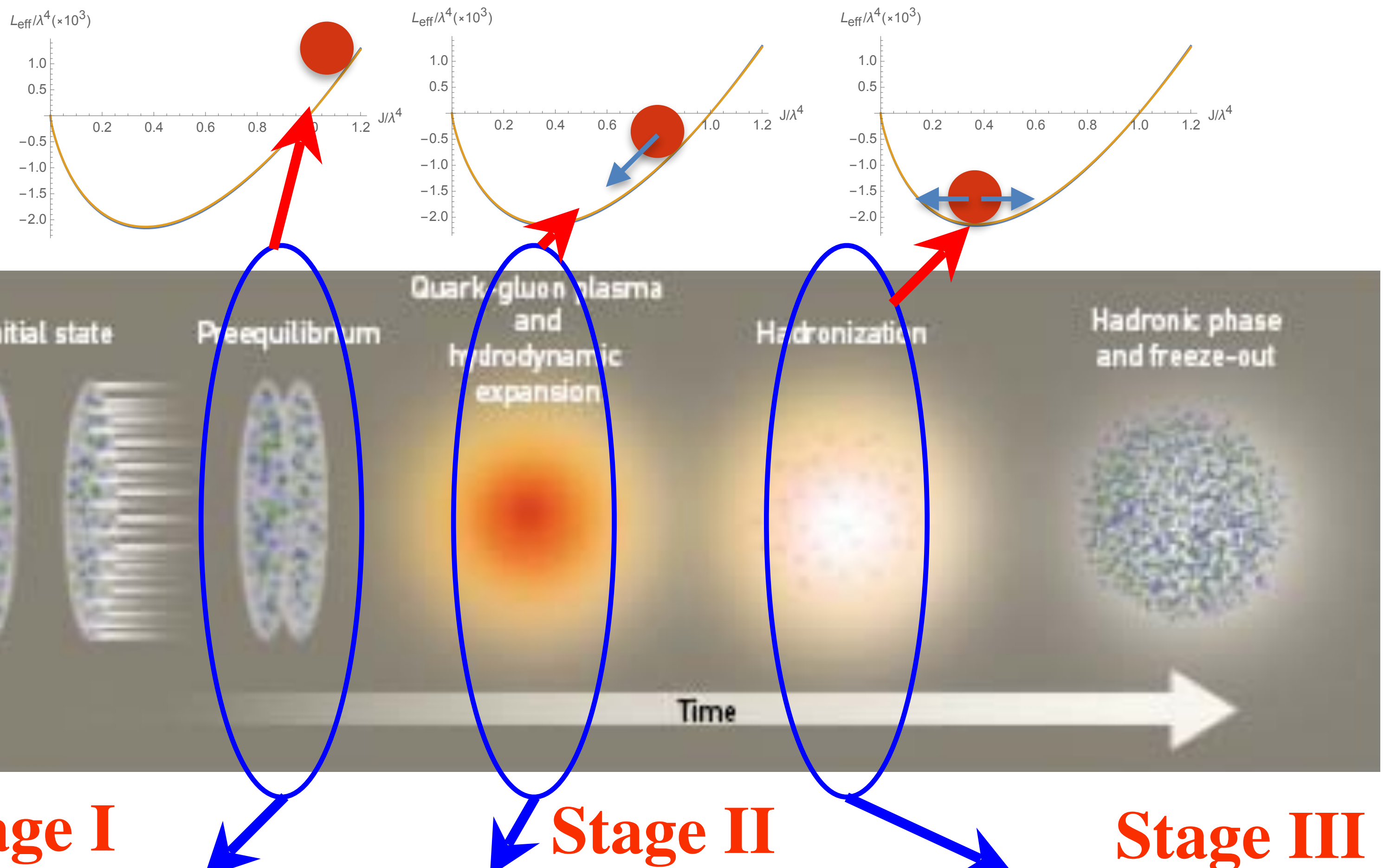


Roman Pasechnik

"The greatest adventure my generation will ever have
- the confined field theory of QCD"

Bo Andersson

Summary: stages of the "micro Big Bang"



Stage I

- (almost) classical homogeneous CE gluon condensate evolution
- small inhomogeneities
- perturbative regime (short distances)

Stage II

- energy "swap" from condensate to the fluctuations (quasiclassical pic.)
- large inhomogeneities (plasma modes)
- parametric resonance effect (particle production mechanism)

Stage III

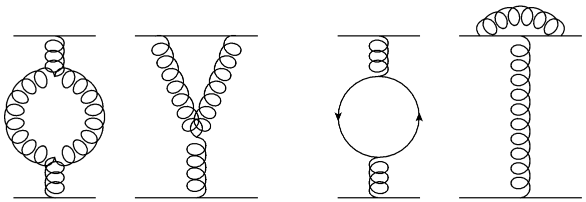
- quantum ground state formation (CE mostly + initiation of CM)
- large distances/essentially quantum dynamics
- domain-wall formation

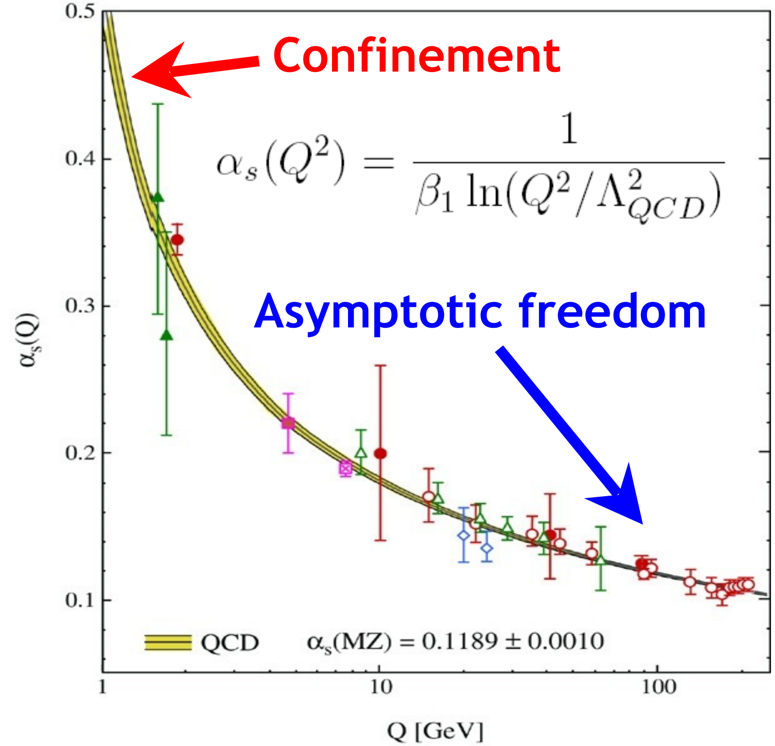
QCD vacuum: short vs long distances

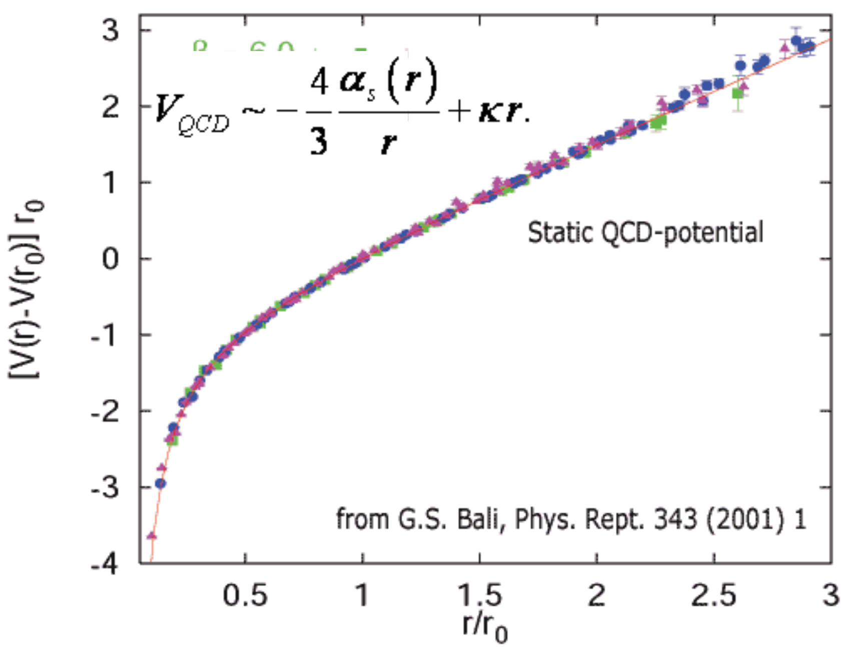
Running QCD coupling

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta(\alpha_s). \qquad \beta(\alpha_s) = -\left(11 - \frac{2n_f}{3}\right)\frac{\alpha_s^2}{2\pi}$$

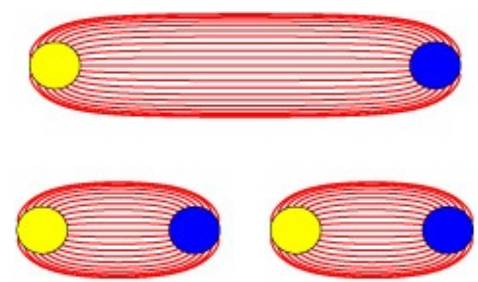
Color charge anti-screening





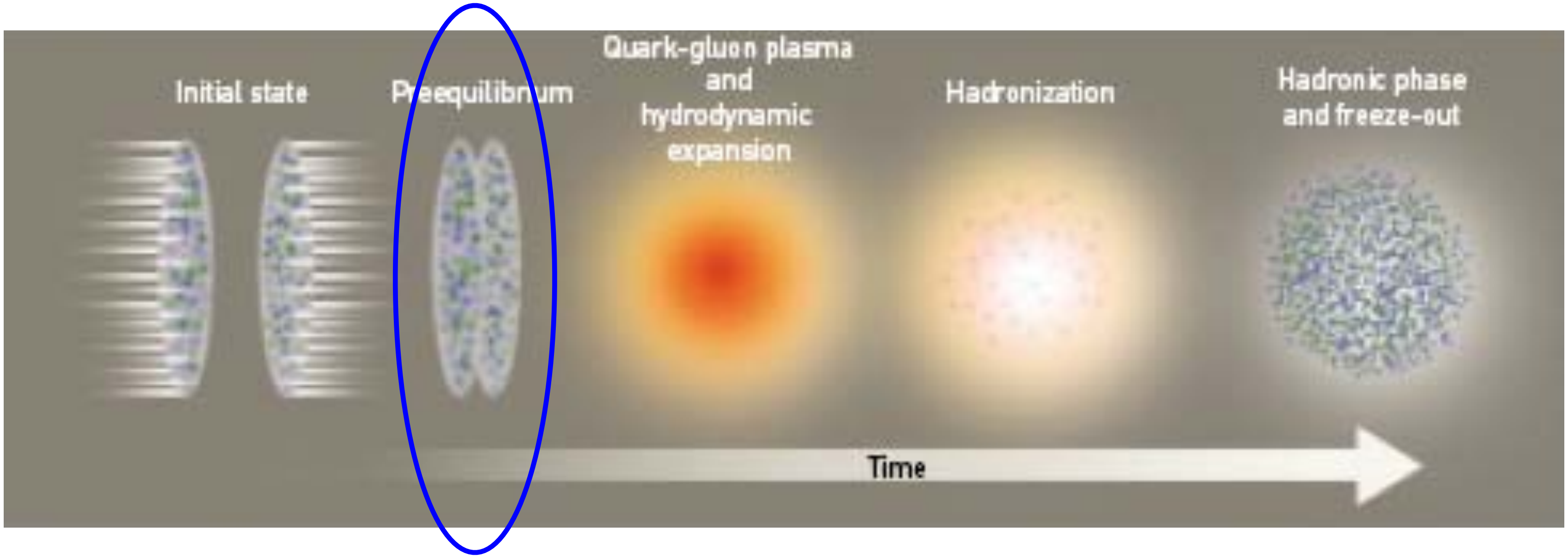


Short range strong interactions



Color confinement!

Stage I


Initial state
Preequilibrium
Quark-gluon plasma and hydrodynamic expansion
Hadronization
Hadronic phase and freeze-out
Time

Homogeneous gluon condensate: semi-classics

Corrections are small for gYM<<1 (short distances!)

Classical YM Lagrangian:

$$\mathcal{L}_{\rm cl} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_{\rm YM}\, f^{abc} A^b_\mu A^c_\nu$$

Basis for canonical (Hamiltonian) quantisation of “condensate+waves” system:

temporal (Hamilton) gauge $\quad A^a_0 = 0 \qquad e^a_i A^a_k \equiv A_{ik} \qquad e^a_i e^a_k = \delta_{ik} \qquad e^a_i e^b_i = \delta_{ab}$

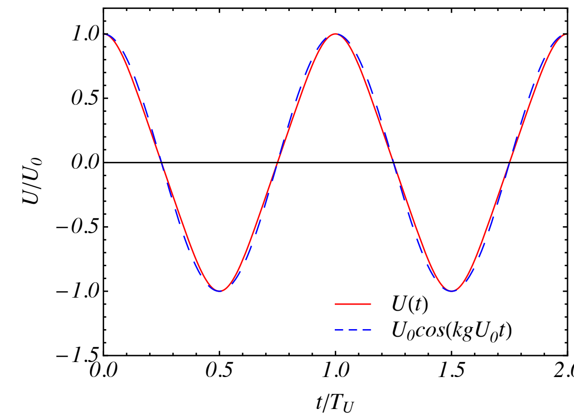
due to local SU(2) ~ SO(3) isomorphism $\quad A_{ik}(t,\vec{x}) = \delta_{ik} U(t) + \widetilde{A}_{ik}(t,\vec{x})$

$$U(t) \equiv \frac{1}{3} \delta_{ik} \langle A_{ik}(t,\vec{x}) \rangle_{\vec{x}}\,, \qquad \langle A_{ik}(t,\vec{x}) \rangle_{\vec{x}} = \frac{\int_\Omega d^3x\, A_{ik}(t,\vec{x})}{\int_\Omega d^3x}$$

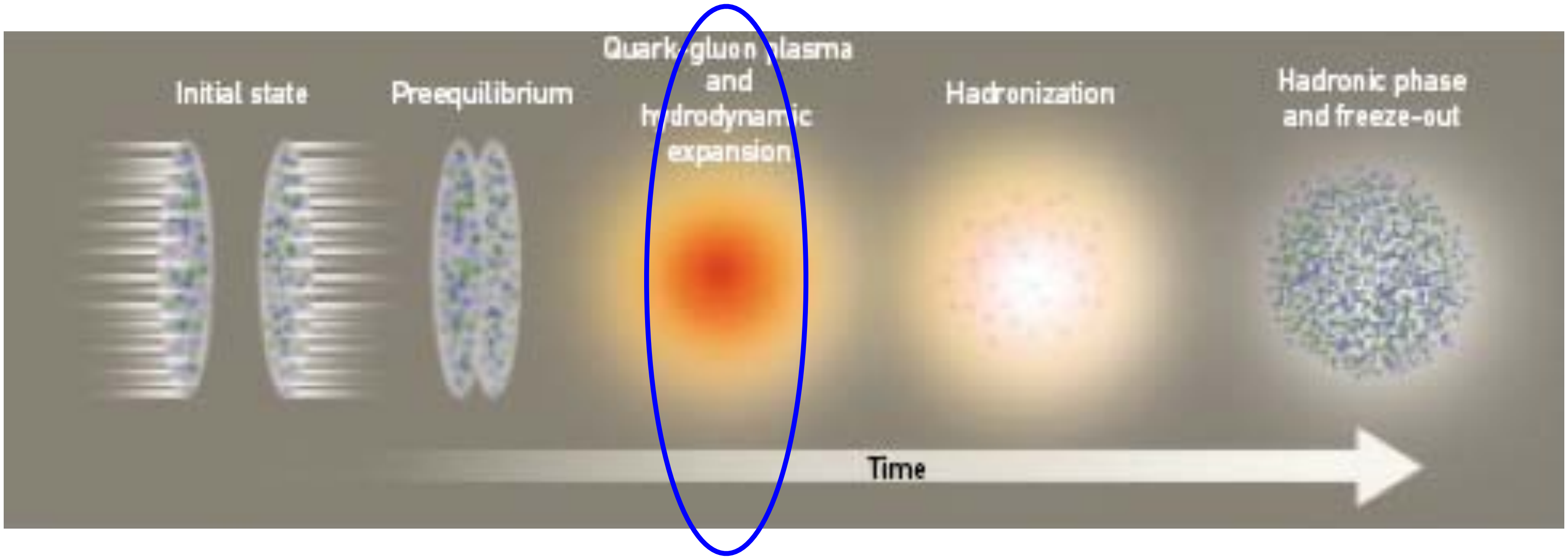
Zeroth-order in waves = “pre-quilibrium state”?

$$\mathcal{H}_{\rm YM} \simeq \mathcal{H}_{\rm YMC} = \frac{3}{2}\Big[(\partial_0 U)^2 + g^2 U^4\Big]\,, \qquad \partial_0\partial_0 U + 2g^2\, U^3 = 0$$

$$t = -\int_{U_0}^{U} \frac{dU}{\sqrt{g^2 U_0^4 - g^2 U^4}}\,, \qquad U(0) = U_0\,, \qquad U'(0) = 0$$



Stage II


Initial state
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Quark-gluon plasma
and
hydrodynamic
expansion
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Hadronic phase
and freeze-out
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Condensate+waves semi-classical system

"condensate+waves" system evolution:

$$
\begin{aligned}
&-\delta_{lk}(\partial_0\partial_0 U + 2g^2U^3) + (-\partial_0\partial_0\widetilde{A}_{lk} + \partial_i\partial_i\widetilde{A}_{lk} - \partial_i\partial_k\widetilde{A}_{li} - g e_{lmk}\partial_i\widetilde{A}_{mi}U - 2g e_{lip}\partial_i\widetilde{A}_{pk}U \\
&- g e_{lmi}\partial_k\widetilde{A}_{mi}U + g^2\widetilde{A}_{kl}U^2 - g^2\widetilde{A}_{lk}U^2 - 2g^2\delta_{lk}\widetilde{A}_{ii}U^2) + (-g e_{lmp}\partial_i\widetilde{A}_{mi}\widetilde{A}_{pk} \\
&- 2g e_{lmp}\widetilde{A}_{mi}\partial_i\widetilde{A}_{pk} - g e_{lmp}\partial_k\widetilde{A}_{mi}\widetilde{A}_{pi} + g^2\widetilde{A}_{li}\widetilde{A}_{ik}U + g^2\widetilde{A}_{li}\widetilde{A}_{ki}U + g^2\widetilde{A}_{ik}\widetilde{A}_{il}U \\
&- 2g^2\widetilde{A}_{ii}\widetilde{A}_{lk}U - g^2\delta_{lk}\widetilde{A}_{pi}\widetilde{A}_{pi}U) + g^2(\widetilde{A}_{li}\widetilde{A}_{pk}\widetilde{A}_{pi} - \widetilde{A}_{pi}\widetilde{A}_{pi}\widetilde{A}_{lk}) = 0
\end{aligned}
$$

tensor basis decomposition

$$\chi_l^{\vec{p}} = s_l^{\sigma}\eta_{\sigma}^{\vec{p}} + n_l\lambda^{\vec{p}}$$

$$\widetilde{A}_{ik} = \psi_{ik} + e_{ikl}\chi_l$$

$$\psi_{ik}^{\vec{p}} = \psi_{\lambda}^{\vec{p}}Q_{ik}^{\lambda} + \varphi_{\sigma}^{\vec{p}}(n_i s_k^{\sigma} + n_k s_i^{\sigma}) + (\delta_{ik} - n_i n_k)\Phi^{\vec{p}} + n_i n_k\Lambda^{\vec{p}}$$

Full Hamiltonian

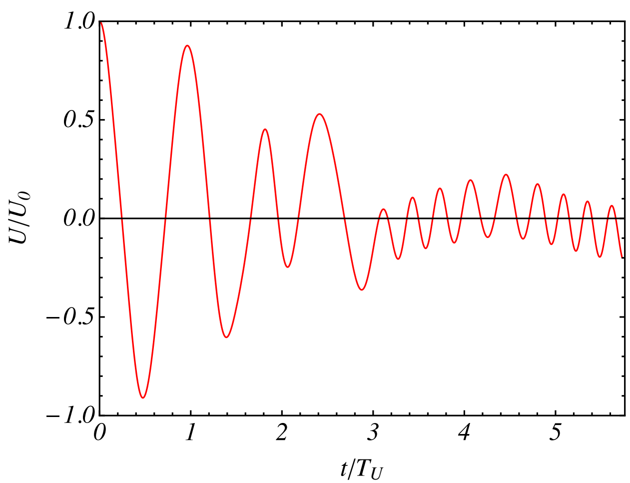
$$
\begin{aligned}
\mathcal{H}_{\rm YM}^{\rm waves} = \; & \frac{1}{2}\Big\{\partial_0\psi_\lambda\,\partial_0\psi_\lambda^\dagger + \partial_0\phi_\sigma\,\partial_0\phi_\sigma^\dagger + \partial_0\Phi\,\partial_0\Phi^\dagger + \frac{1}{2}\,\partial_0\Lambda\,\partial_0\Lambda^\dagger + \partial_0\eta_\sigma\,\partial_0\eta_\sigma^\dagger \\
&+ \partial_0\lambda\,\partial_0\lambda^\dagger + p^2\,\psi_\lambda\psi_\lambda^\dagger + \frac{p^2}{2}\,\phi_\sigma\phi_\sigma^\dagger + p^2\,\Phi\Phi^\dagger + \frac{p^2}{2}\,\eta_\sigma\eta_\sigma^\dagger + p^2\,\lambda\lambda^\dagger \\
&- \frac{p^2}{2}\,e^{\gamma\sigma}(\eta_\sigma\phi_\gamma^\dagger + \phi_\gamma\eta_\sigma^\dagger) + igp\,U\,e^{\sigma\gamma}\eta_\sigma\eta_\gamma^\dagger - igp\,U\,Q^{\lambda\gamma}\psi_\lambda\psi_\gamma^\dagger \\
&- igpU\,e^{\sigma\gamma}\phi_\sigma\phi_\gamma^\dagger - igp\,U\,(2\Phi\lambda^\dagger - 2\lambda\Phi^\dagger + \Lambda\lambda^\dagger - \lambda\Lambda^\dagger) \\
&+ 2g^2\,U^2\,\eta_\sigma\eta_\sigma^\dagger + 2g^2\,U^2\,\lambda\lambda^\dagger + g^2\,U^2\,(4\Phi\Phi^\dagger + 2\Phi\Lambda^\dagger + 2\Lambda\Phi^\dagger + \Lambda\Lambda^\dagger)\Big\}
\end{aligned}
$$

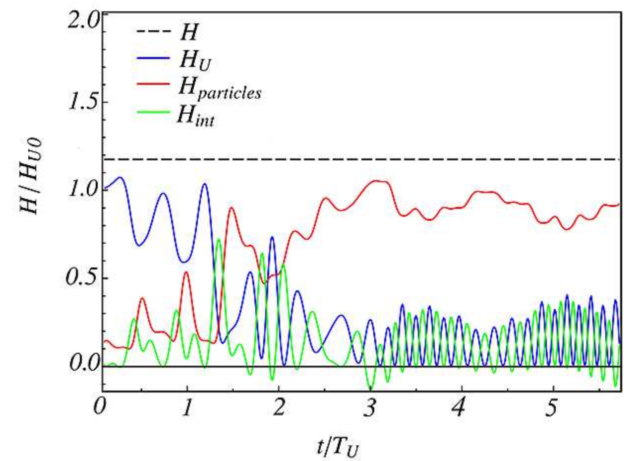
$$\mathcal{H}_{\rm YM} = \mathcal{H}_{\rm YMC} + \sum_{\vec{p}}\mathcal{H}_{\rm YM}^{\rm waves}$$

Longitudinally polarised (plasma) mode becomes physical due to interactions with the homogeneous condensate!

Decay of the homogeneous condensate

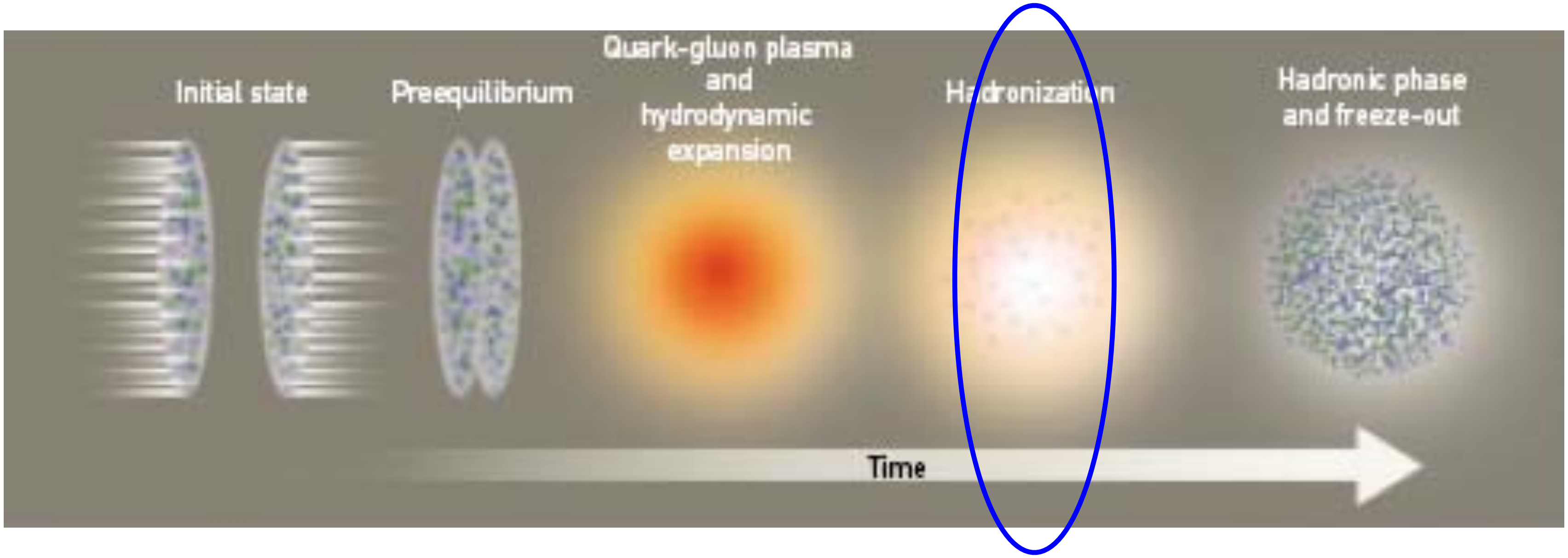
$$
\begin{aligned}
\mathcal{H}_{\mathrm{U}} &= \frac{3}{2}\left(\partial_0 U \partial_0 U + g^2 U^4\right), \\
\mathcal{H}_{\mathrm{particles}} &= \frac{1}{2}\sum_{\vec{p}} \Big(\partial_0\psi_\lambda\,\partial_0\psi_\lambda^\dagger + \partial_0\phi_\sigma\,\partial_0\phi_\sigma^\dagger + \partial_0\Phi\,\partial_0\Phi^\dagger + \frac{1}{2}\,\partial_0\Lambda\,\partial_0\Lambda^\dagger + \partial_0\eta_\sigma\,\partial_0\eta_\sigma^\dagger \\
&+ \partial_0\lambda\,\partial_0\lambda^\dagger + p^2\,\psi_\lambda\psi_\lambda^\dagger + \frac{p^2}{2}\,\phi_\sigma\phi_\sigma^\dagger + p^2\,\Phi\Phi^\dagger + \frac{p^2}{2}\,\eta_\sigma\eta_\sigma^\dagger + p^2\,\lambda\lambda^\dagger \\
&- \frac{p^2}{2}\,e^{\gamma\sigma}\left(\eta_\sigma\phi_\gamma^\dagger + \phi_\gamma\eta_\sigma^\dagger\right)\Big), \\
\mathcal{H}_{\mathrm{int}} &= \frac{1}{2}\sum_{\vec{p}} \Big[igp\,U\,e^{\sigma\gamma}\eta_\sigma\eta_\gamma^\dagger - igp\,U\,Q^{\lambda\gamma}\psi_\lambda\psi_\gamma^\dagger \\
&- igpU e^{\sigma\gamma}\phi_\sigma\phi_\gamma^\dagger - igp\,U\left(2\Phi\lambda^\dagger - 2\lambda\Phi^\dagger + \Lambda\lambda^\dagger - \lambda\Lambda^\dagger\right) \\
&+ 2g^2\,U^2\,\eta_\sigma\eta_\sigma^\dagger + 2g^2\,U^2\,\lambda\lambda^\dagger + g^2\,U^2\left(4\Phi\Phi^\dagger + 2\Phi\Lambda^\dagger + 2\Lambda\Phi^\dagger + \Lambda\Lambda^\dagger\right)\Big].
\end{aligned}
$$





Ultra-relativistic gluon plasma production!

Stage III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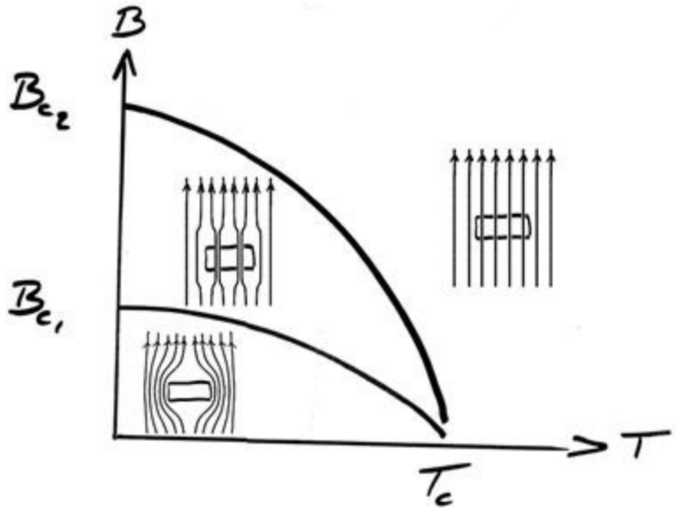
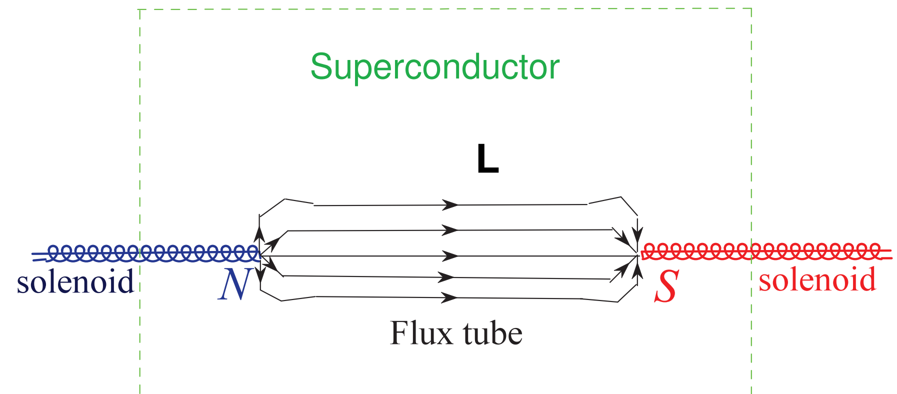
QCD confinement as a dual Meissner effect

type-II superconductor



usual Meissner effect



Energy of the magnetic “monopole-antimonopole” pair is proportional to L (string potential)

Magnetic field cannot penetrate through a superconductor, except by burning out a narrow tube where the superconductivity is destroyed (the Abrikosov vortex)

The dual Meissner effect in QCD (analogous to that in dual superconductors):

- **the QCD vacuum as a condensate of chromo-magnetic monopoles (c.f. condensation of BCS pairs in usual superconductors)**
- **quarks are sources of chromo-electric field**
- **inside the quark-antiquark tube the chromo-magnetic condensate is destroyed**
- **electric field is squeezed inside the tube (the Abrikosov-Nielsen-Olesen vortex)**

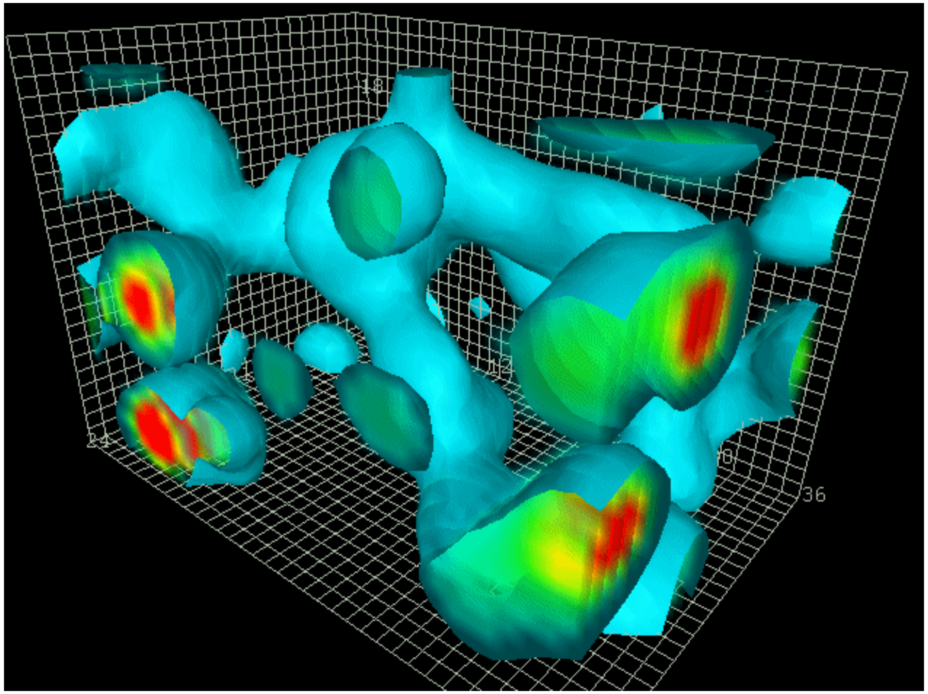
Long distances: chromo-magnetic condensate

Quantum-topological (chromomagnetic) vacuum in QCD

$$\varepsilon_{vac(top)} = -\frac{9}{32}\langle 0| : \frac{\alpha_s}{\pi} F^a_{ik}(x) F^{ik}_a(x) : |0\rangle + \frac{1}{4}\left(\langle 0| : m_u \bar{u}u : |0\rangle + \langle 0| : m_d \bar{d}d : |0\rangle + \langle 0| : m_s \bar{s}s : |0\rangle\right)$$
$$\simeq -(5 \pm 1) \times 10^9 \ \mathrm{MeV}^4.$$

<u>CM condensate:</u>

$\epsilon_{vac} \sim 10^{-2} \mathrm{GeV}^4$



<u>Ground-state at long distances:</u>

$\Lambda_{\rm cosm} \sim 10^{-47}\,\mathrm{GeV}^4$

Vacuum in QCD has incredibly wrong energy scale… or

We must be missing something very important!?

Effective YM action approach

At least, for SU(2) gauge symmetry, the all-loop and one-loop effective Lagrangians are practically indistinguishable (by FRG approach)

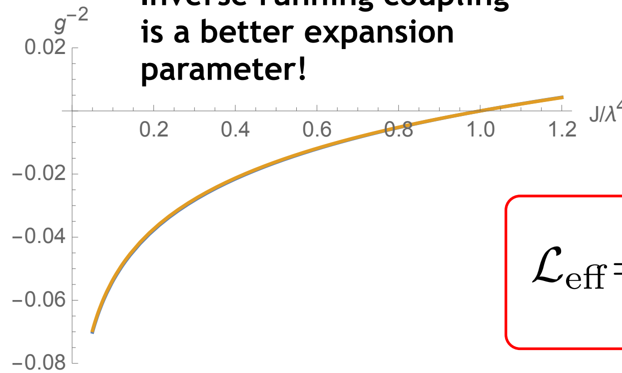
P. Dona, A. Marciano, Y. Zhang and C. Antolini, Phys. Rev. D **93** (2016) no.4, 043012.

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D **83** (2011) 045014 [Phys. Rev. D **83** (2011) 069903].

H. Pagels and E. Tomboulis, Nucl. Phys. B **143**, 485 (1978).

$$\mathcal{L}_{\rm cl} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \qquad \mathcal{A}^a_\mu \equiv g_{\rm YM} A^a_\mu \qquad \mathcal{F}^a_{\mu\nu} \equiv g_{\rm YM} F^a_{\mu\nu}$$

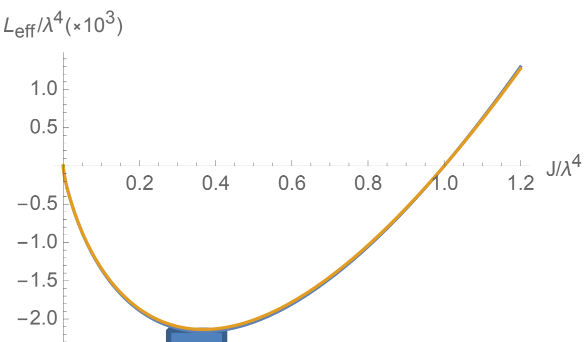
Inverse running coupling is a better expansion parameter!



Effective YM Lagrangian:

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{J}}{4\bar{g}^2(\mathcal{J})}, \qquad \mathcal{J} = -\mathcal{F}^a_{\mu\nu}\mathcal{F}^{\mu\nu}_a$$

Effective Lagrangian:



chromoelectric (CE) condensate (Savvidy vacuum) $\quad \mathcal{J}^* > 0$

G. K. Savvidy, Phys. Lett. **71B**, 133 (1977)

The energy-momentum tensor:

$$T^\nu_\mu = \frac{1}{\bar{g}^2}\left[\frac{\beta(\bar{g}^2)}{2} - 1\right]\left(\mathcal{F}^a_{\mu\lambda}\mathcal{F}^{\nu\lambda}_a + \frac{1}{4}\delta^\nu_\mu \mathcal{J}\right) - \delta^\nu_\mu \frac{\beta(\bar{g}^2)}{8\bar{g}^2}\mathcal{J}$$

Equations of motion:

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \left[\frac{\mathcal{F}^{\mu\nu}_b}{\bar{g}^2}\left(1 - \frac{\beta(\bar{g}^2)}{2}\right)\right] = 0,$$

$$\overrightarrow{\mathcal{D}}^{ab}_\nu \equiv \left(\delta^{ab}\overrightarrow{\partial}_\nu - f^{abc}\mathcal{A}^c_\nu\right),$$

trace anomaly:

$$T^\mu_\mu = -\frac{\beta(\bar{g}^2)}{2\bar{g}^2}\mathcal{J}$$

NOTE: the RG equation

$$\frac{d\ln|\bar{g}^2|}{d\ln|\mathcal{J}|/\mu_0^4} = \frac{\beta(\bar{g}^2)}{2}$$

appears to be invariant under

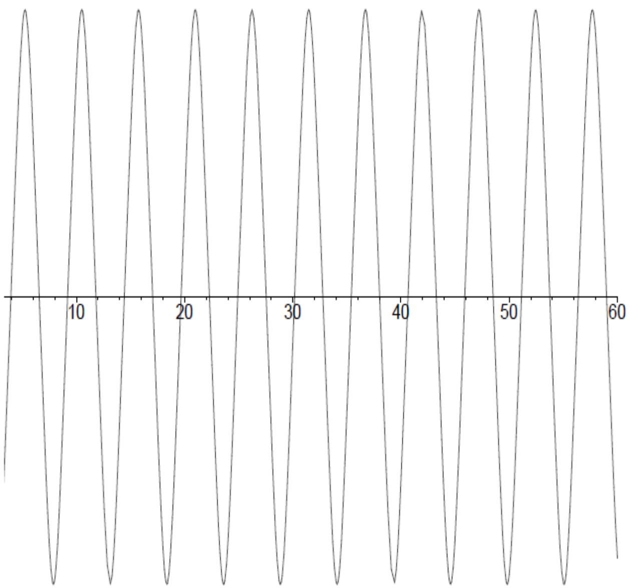
$$\mathcal{J} \longleftrightarrow -\mathcal{J}$$

$$\bar{g}^2 = \bar{g}^2(|\mathcal{J}|)$$

CE condensate on non-stationary (FLRW) background

$$Q \equiv \frac{32}{11}\pi^2 e(\xi\Lambda_{\rm QCD})^{-4} T^{\mu}_{\mu}[U]$$
$$= 6e\left[(U')^2 - \frac{1}{4}U^4\right] a^{-4}(\xi\Lambda_{\rm QCD})^{-4}$$

Classical YM condensate



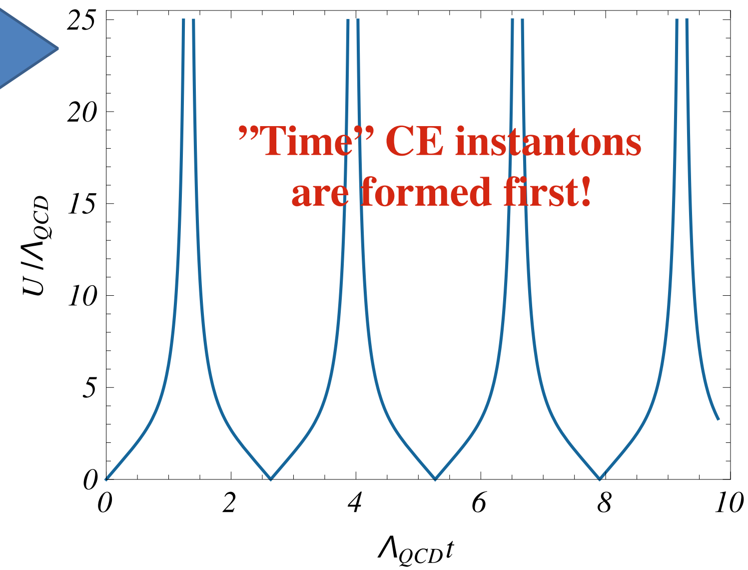
Exact partial solution:

$$|Q| = 1$$

Quantum corrections

Savvidy (CE) vacuum

$$Q(U) = 1$$



"Radiation" medium

$$\epsilon_{\rm YM} \propto 1/a^4$$

Unstable solution!

QCD vacuum:
a ferromagnetic undergoing spontaneous magnetisation (Pagels&Tomboulis)

Asymptotic tracker solution!

$$\epsilon_{\rm CE} \to +\text{const} \qquad t \to \infty$$

Stable solution!

- **In fact, both chromoelectric and chromomagnetic condensates are stable on non-stationary (FLRW) background of expanding Universe**

"Mirror" symmetry of the ground state

In a vicinity of the ground state, the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{\mathcal{J}}{4\bar{g}^2} \qquad \mathcal{J} \simeq \mathcal{J}^*$$

is invariant under

$$\boxed{\mathbb{Z}_2: \qquad \mathcal{J}^* \longleftrightarrow -\mathcal{J}^*, \quad \bar{g}^2(\mathcal{J}^*) \longleftrightarrow -\bar{g}^2(\mathcal{J}^*), \quad \beta(\bar{g}_*^2) \longleftrightarrow -\beta(\bar{g}_*^2)}$$

For pure gluodynamics at one-loop: $\beta_{(1)} = -\frac{bN}{48\pi^2}\,\bar{g}^2_{(1)} \qquad b = 11$

$$\alpha_s = \frac{\bar{g}^2}{4\pi} \qquad \alpha_{\rm s}(\mu^2) = \frac{\alpha_{\rm s}(\mu_0^2)}{1 + \beta_0\,\alpha_{\rm s}(\mu_0^2)\ln(\mu^2/\mu_0^2)} \qquad \mu^2 \equiv \sqrt{|\mathcal{J}|}$$

Choosing the ground state value of the condensate $\mu_0^2 \equiv \sqrt{|\mathcal{J}^*|}$ **as the physical scale**

we observe that the mirror symmetry, indeed, holds provided

$$\mathcal{J} \simeq \mathcal{J}^* \qquad \alpha_s(\mu_0^2) \longleftrightarrow -\alpha_s(\mu_0^2)$$

i.e. in the ground state only!

Heterogenic quantum YM ground state: two-scale vacuum

The running coupling at one-loop

$$\bar{g}_1^2(\mathcal{J}) = \frac{\bar{g}_1^2(\mu_0^4)}{1+\frac{bN}{96\pi^2}\bar{g}_1^2(\mu_0^4)\ln(|\mathcal{J}|/\mu_0^4)} = \frac{96\pi^2}{bN\ln(|\mathcal{J}|/\lambda_\pm^4)}$$

$$\mathcal{L}_{\rm eff}^{(1)} = \frac{bN}{384\pi^2}\mathcal{J}\ln\left(\frac{|\mathcal{J}|}{\lambda_\pm^4}\right)$$

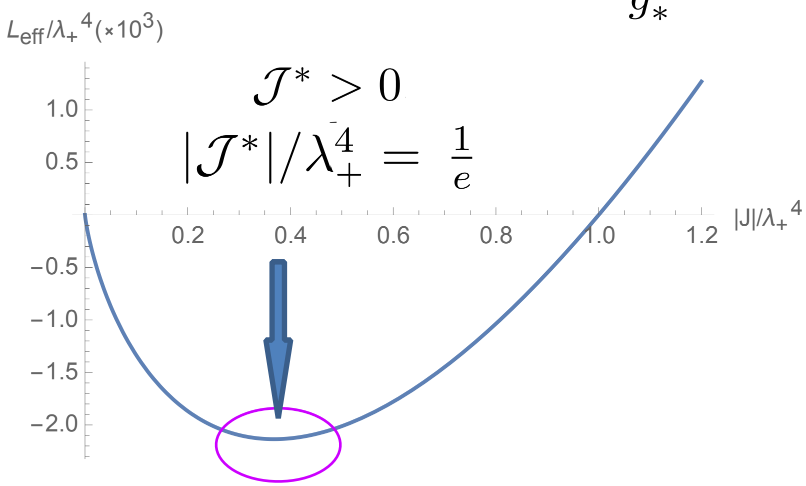
with two energy scales

$$\boxed{\lambda_\pm^4 \equiv |\mathcal{J}^*|\exp\left[\mp\frac{96\pi^2}{bN|\bar{g}_1^2(\mathcal{J}^*)|}\right] \qquad |\mathcal{J}^*| = \lambda_+^2\lambda_-^2}$$

$$\boxed{\textbf{CE vacuum:} \quad \beta(\bar{g}_*^2) = 2}$$

e.o.m. is automatically satisfied!

Trace anomaly: $T^\mu_{\mu,\rm CE} = -\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CE attractor

Mirror symmetry

One-loop:

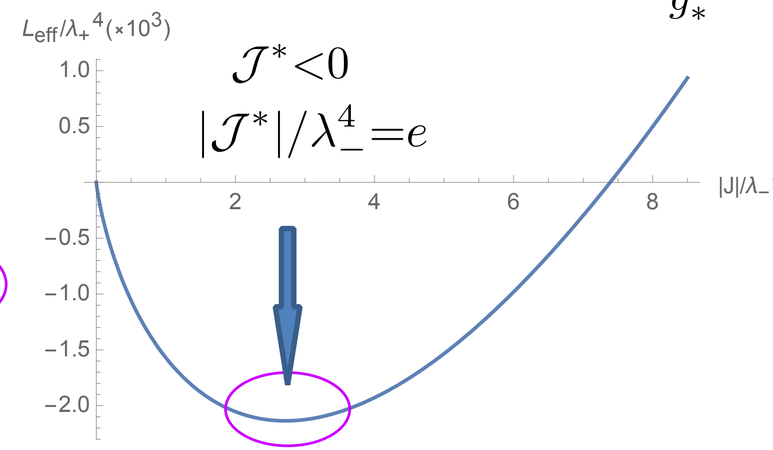
$$\lambda_+^2/\lambda_-^2 = e$$

$$\boxed{\textbf{CM vacuum:} \quad \beta(\bar{g}_*^2) = -2}$$

Reduces to the standard YM e.o.m. discussed in e.g. in instanton theory

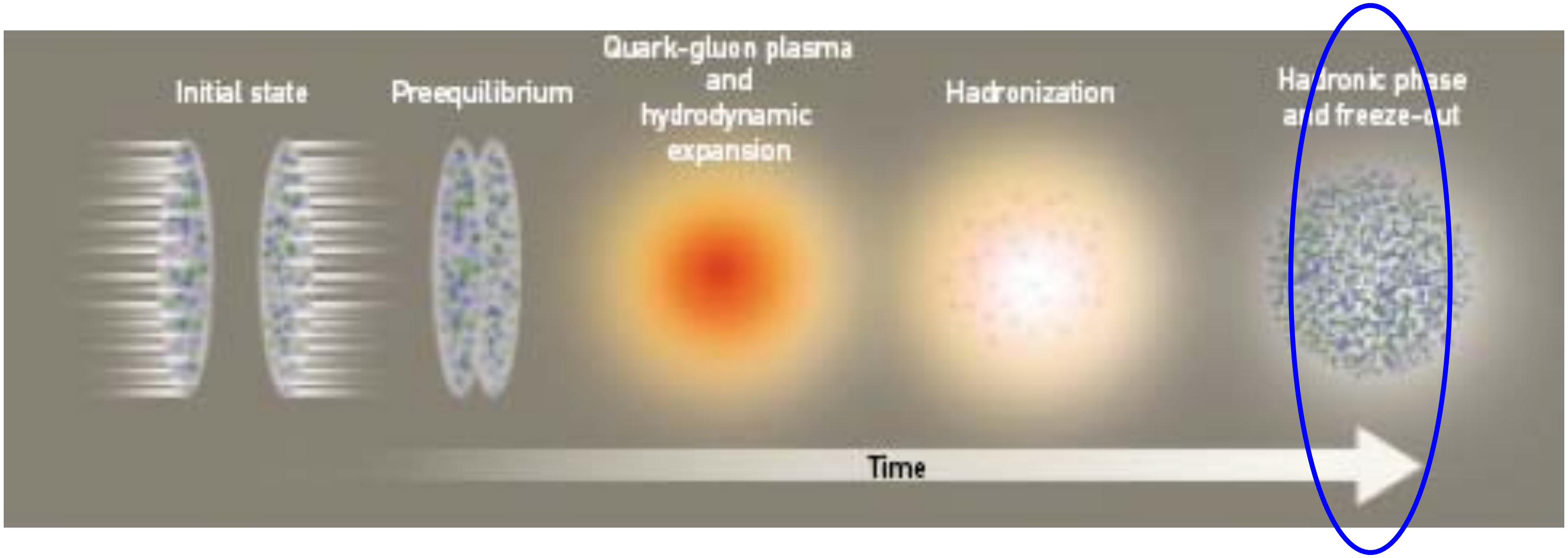
$$\overrightarrow{\mathcal{D}}_\nu^{ab}\left[\frac{\mathcal{F}_b^{\mu\nu}}{\bar{g}^2}\right] = 0, \quad \bar{g}^2 \simeq \bar{g}_*^2$$

Trace anomaly: $T^\mu_{\mu,\rm CM} = +\frac{1}{\bar{g}_*^2}\mathcal{J}^*$



Cosmological CM attractor

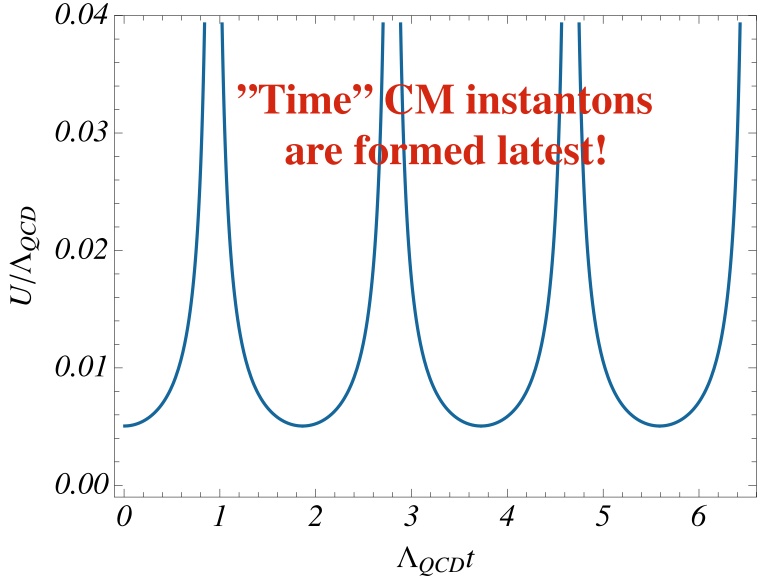
Post-confinement: stage IV



- Both CE and CM reach their attractors
- CM/CE domains "crystallisation"
- CC is formed

Macroscopic evolution and vacua cancellation

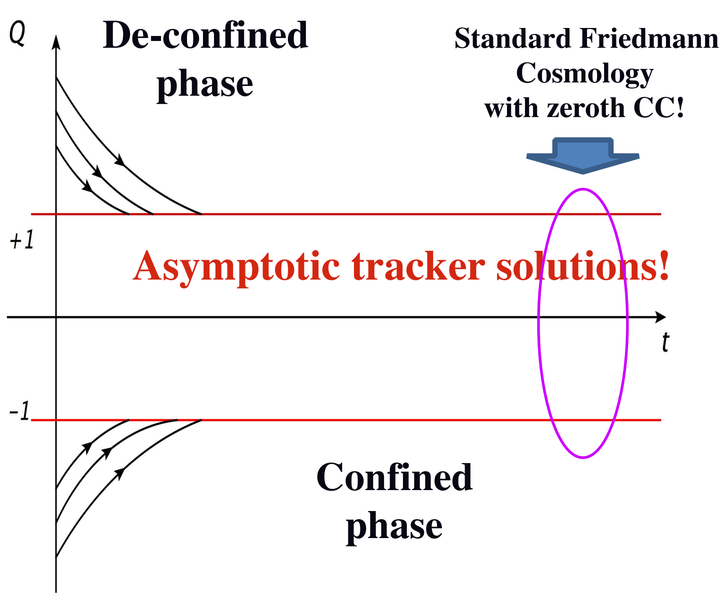
$Q(U) = -1$



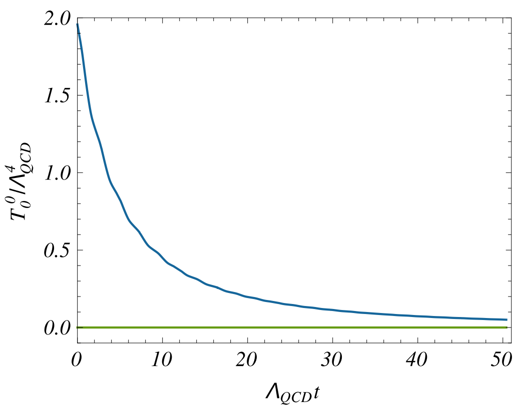
$$\epsilon_{\rm vac} \equiv \frac{1}{4}\langle T^{\mu}_{\mu}\rangle_{\rm vac} = \mp\mathcal{L}_{\rm eff}(\mathcal{J}^*)$$

$$\boxed{\epsilon^{\rm CE}_{\rm vac}\Big|_{\mathcal{J}^*>0} + \epsilon^{\rm CM}_{\rm vac}\Big|_{\mathcal{J}^*<0} \equiv 0}$$

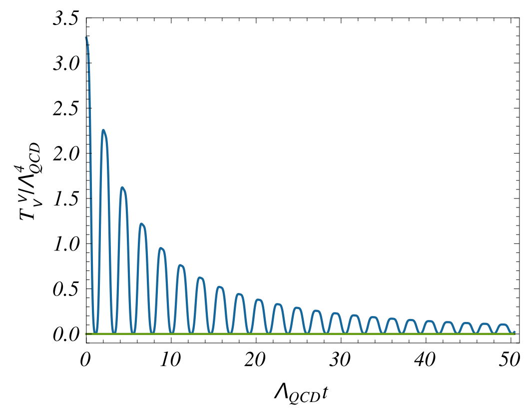
Exact compensation of CM and CE vacua as soon as the cosmological attractor is achieved!



CE energy density



CE EMT trace



System with very unusual dynamical properties!

Take-home facts on QCD ground-state “crystal”:

- No ghost problem associated with negative coupling due to:
  (i) only gauge invariant quantities are used
  (ii) local loss of Lorentz (e.g. rotational) invariance

- Nielsen-Olsen proof of instability of CE condensate on a rigid Minkowski in NOT in contradiction with our results: we consider YM evolution on a dynamical (FLRW) spacetime while equilibrium is achieved only asymptotically.

- A possible decay of CE condensate into an anisotropic vacuum after a cosmological relaxation time would be exponentially suppressed and is practically never realised

- Even starting from an initial non-zero energy-density, the evolution of localised 3-space “pockets” of the CE and CM condensates trigger a mutual screening, flowing towards a zero-energy density attractor and accompanying by a formation of the domain walls corresponding to an asymptotic restoration of the Z2 (Mirror) symmetry and effectively protecting the “false” CE vacua pockets from further decay (“time crystal” ground-state)

- The vacua cancellation mechanism seems to naturally marry the existing confinement pictures related to a formation of a network of t’Hooft monopoles or chromovortices. In this approach, the scalar kink profile may correspond the J-invariant whose change may be related to the presence of monopole or vortex solutions localise inside the space-time domain walls. This implies the existence of space-time solitonic objects of a new type.