Statistical thermal model

a.k.a. statistical hadronization model (SHM) or hadron resonance gas (HRG)

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COST Workshop on Interplay of hard and soft QCD probes for collectivity in heavy-ion collisions

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LUND UNIVERSITY

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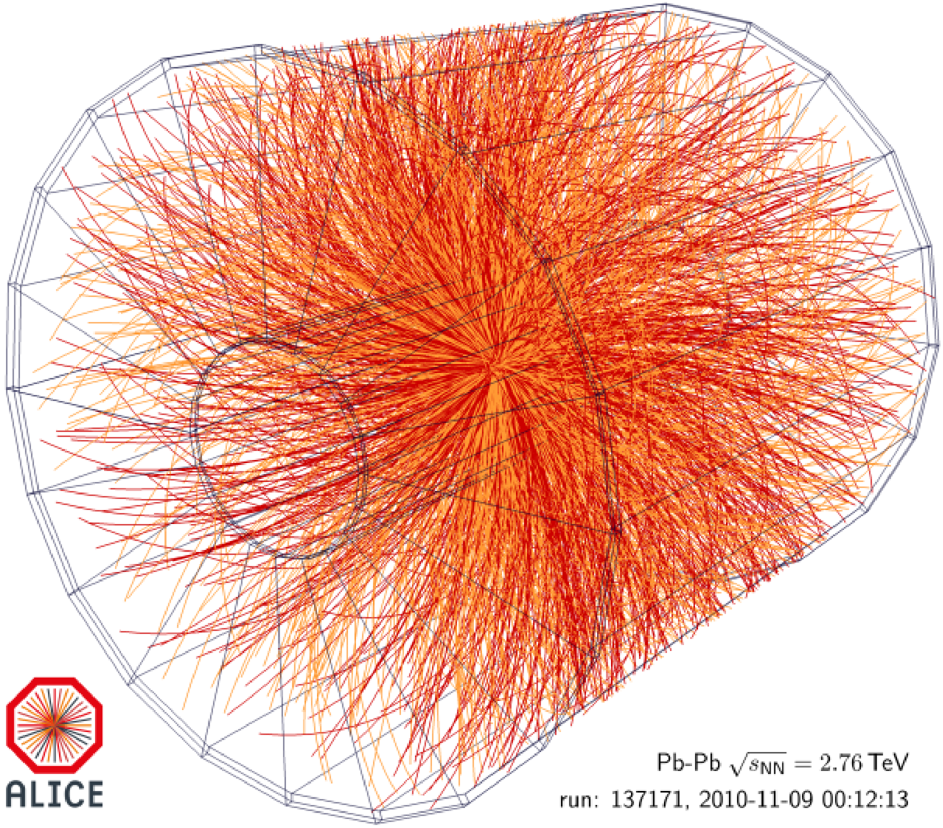
Relativistic heavy-ion collisions



Event display of a Pb-Pb collision in ALICE at LHC

Thousands of particles created in relativistic heavy-ion collisions

Apply concepts of statistical mechanics to describe particle production

Historical perspective

1951-1953: Early applications of statistical concepts to particle production [Fermi; Landau; Pomeranchuk]

О МНОЖЕСТВЕННОМ ОБРАЗОВАНИИ ЧАСТИЦ ПРИ СТОЛКНОВЕНИЯХ БЫСТРЫХ ЧАСТИЦ
Изв. АН СССР, серия физ., 17, 51, 1953

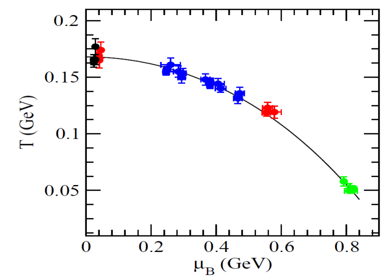
1965-1975: Hagedorn's model (statistical bootstrap), applications to high-energy collisions

$$\rho(m) = A\, m^{-\alpha} \exp(m/T_H)$$

1969: S-matrix formulation of statistical mechanics, *the basis of the thermal model*
Dashen, Ma, Bernstein, PRC 187, 45 (1969)

$$\left(\mathrm{Tr}\, A S^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S\right)_c$$

~1975: QCD as accepted theory of strong interactions

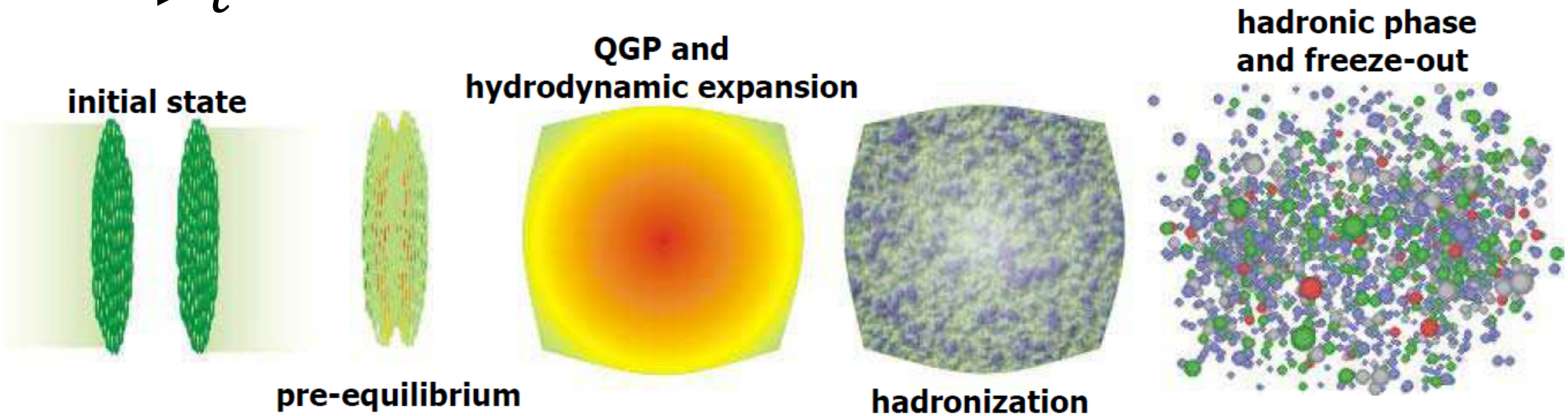


1992-...: Thermal fits to heavy-ion hadron yield data, mapping HIC to the QCD phase diagram
Cleymans, Satz; Braun-Munzinger, Stachel; Rafelski; Redlich; Becattini;...

2003-...: Open-source implementations of the thermal model: SHARE (Rafelski+), THERMUS (Cleymans+), the Thermal-FIST package (**V.V.**)

Relativistic heavy-ion collisions: Thermal model

→ t



dynamical models (transport, hydro,…)

macroscopic approach
(thermal model)

Pros:

- Simplest model with very few free parameters (T, μ_B,…)
- Connection to QCD phase diagram
- Easier to test new ideas

Cons:

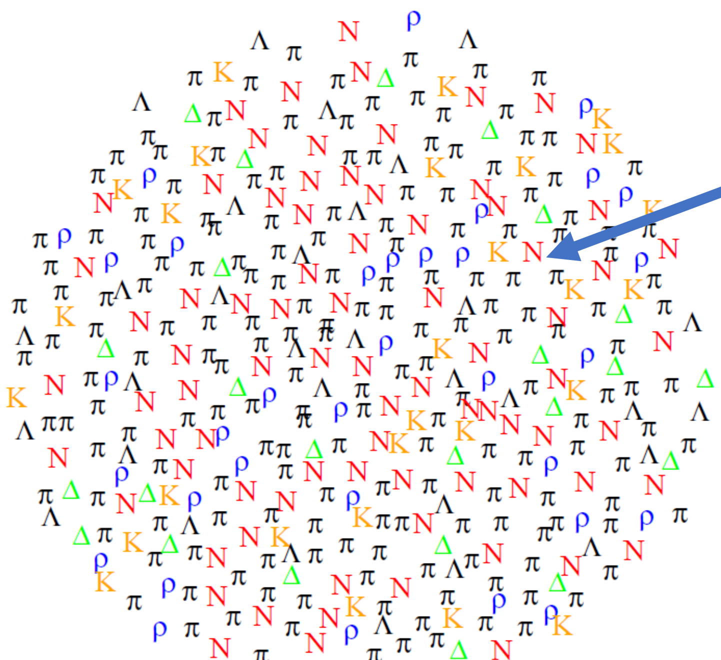
- No dynamics
- Describes only yields
- Thermal parameters fitted to data at each energy

Hadron resonance gas (HRG) at freeze-out

HRG: Equation of state of hadronic matter as a multi-component non-interacting gas of known hadrons and resonances

$$\ln Z \approx \sum_{i \in M,B} \ln Z_i^{id} = \sum_{i \in M,B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm \exp\left(\frac{\mu_i - E_i}{T}\right)\right]$$

Grand-canonical ensemble: $\mu_i = b_i \mu_B + q_i \mu_Q + s_i \mu_S$ ***chemical equilibrium***



© J. Cleymans

Thermal model:
Equilibrated hadron resonance gas at the chemical freeze-out stage of high-energy collisions

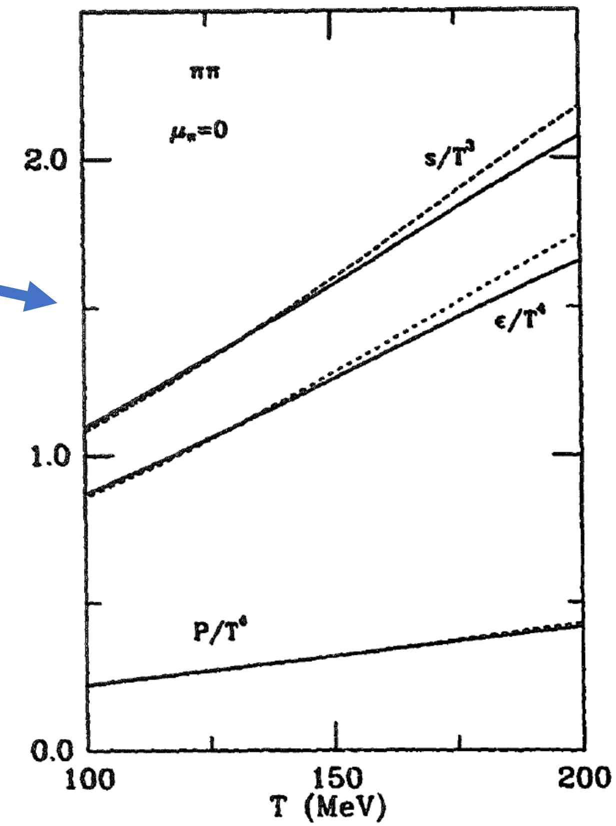
Model parameters:
T – temperature
μ_B, μ_Q, μ_S – chemical potentials
V – system volume

The basis of the HRG model

Dashen, Ma, Bernstein (1969): Inclusion of narrow resonances as free, point-like particles models attractive interactions where they are being formed   [S-matrix formulation of statistical mechanics, PRC 187, 345 (1969)]

Example: interacting pion system

Interacting pion gas within the S-matrix approach agrees with the **non-interacting** π+ρ gas



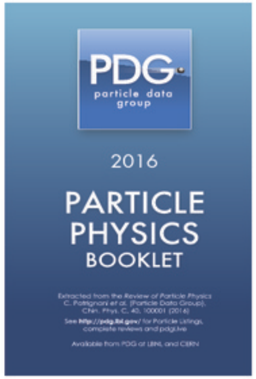
Venugopalan, Prakash, NPA (1992)

Include *all* resonances as free, point-like particles

HRG model

List of hadrons and resonances

$$\ln Z^{\mathrm{hrg}} = \sum_{i\in M,B} \frac{d_i V}{2\pi^2} \int_0^\infty \pm p^2 dp \ln\left[1 \pm \exp\left(\frac{\mu_i - E_i}{T}\right)\right]$$



Particle list in the thermal model usually includes all hadrons and resonances listed as established in the PDG listing

~400 species

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	****	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	***	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$N(1700)$	$3/2^-$	***	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*
$N(1710)$	$1/2^+$	****	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**
$N(1720)$	$3/2^+$	****	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**
$N(1860)$	$5/2^+$	**	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*
$N(1875)$	$3/2^-$	***	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***			
$N(1880)$	$1/2^+$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****
$N(1895)$	$1/2^-$	****	$\Delta(2200)$	$7/2^-$	***	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$		***
$N(1900)$	$3/2^+$	****	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$		**
$N(1990)$	$7/2^+$	**	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$		**

• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$	• $K^\pm$	$1/2(0^-)$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0	$1/2(0^-)$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*_0(700)$	$1/2(0^+)$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$	• $K^*(892)$	$1/2(1^-)$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(0^{-+})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$	• $K^*_0(1430)$	$1/2(0^+)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K(1630)$	$1/2(?^?)$

Connecting model to experiment

$$N_i^{hrg} = \frac{d_i V}{2\pi^2} \int_0^\infty p^2 dp \left[\exp\left(\frac{E_i - \mu_i}{T}\right) \pm 1\right]^{-1} \quad \propto \quad e^{-m_i/T}$$

Particle decays: Unstable resonances decay before being detected

$\Delta \to N\,\pi$ $\qquad K^* \to K\,\pi$ $\qquad \rho \to \pi\,\pi$ $\qquad$ etc.

Take into account feeddown: $N_i^{\text{fin}} = N_i^{\text{hrg}} + \sum_j BR(j \to i) N_j^{\text{hrg}}$

60-70% of π, p, etc. are from feeddown

Conservation laws:

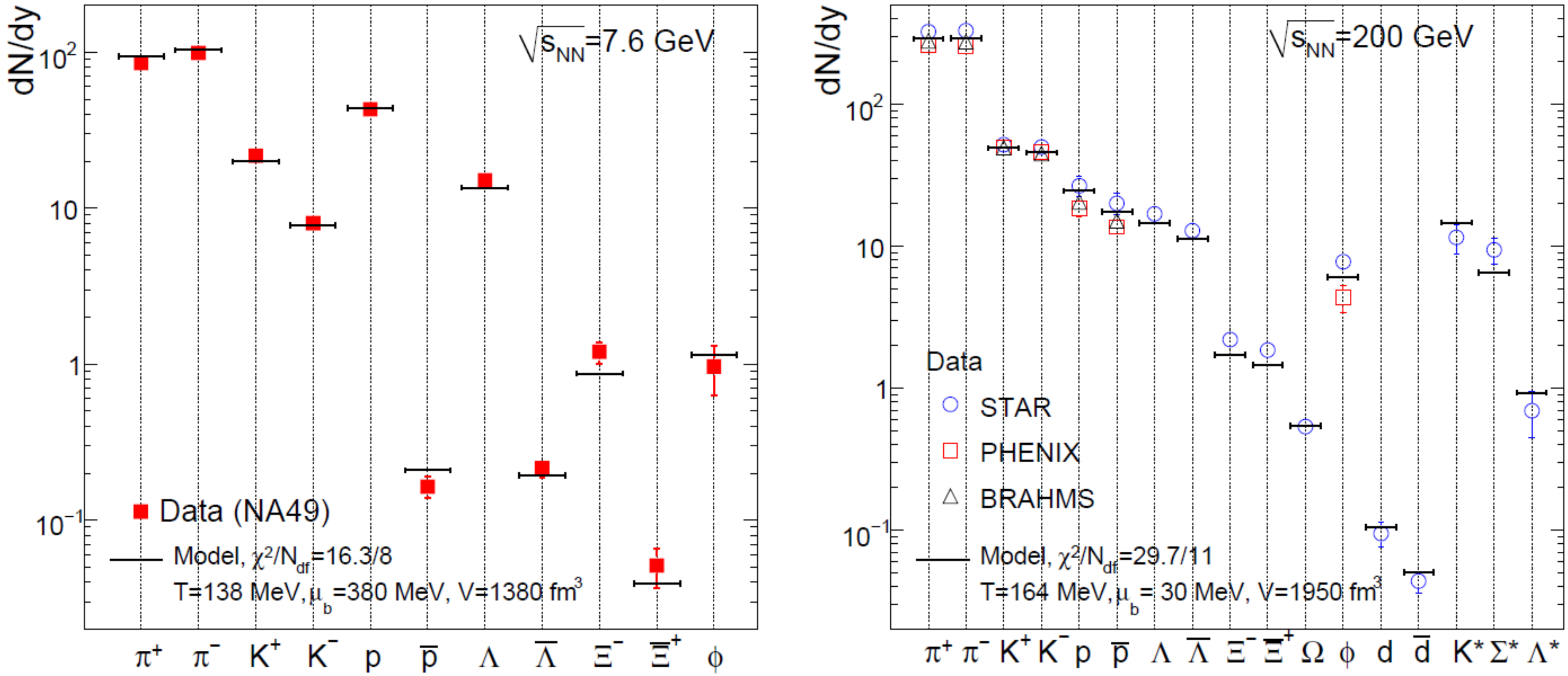
Zero net strangeness $\rightarrow$ $\boldsymbol{\mu_S}$

Electric-to-baryon ratio Q/B = 0.4-0.5 $\rightarrow$ $\boldsymbol{\mu_Q}$

Freeze-out parameters T, μ_B, V extracted through χ^2 **minimization**

$$\chi^2 = \sum_i \frac{(N_i^{\text{fin}} - N_i^{\text{exp}})^2}{(\sigma_i^{\text{exp}})^2}, \qquad i = \pi, K, p, \Lambda, \ldots$$

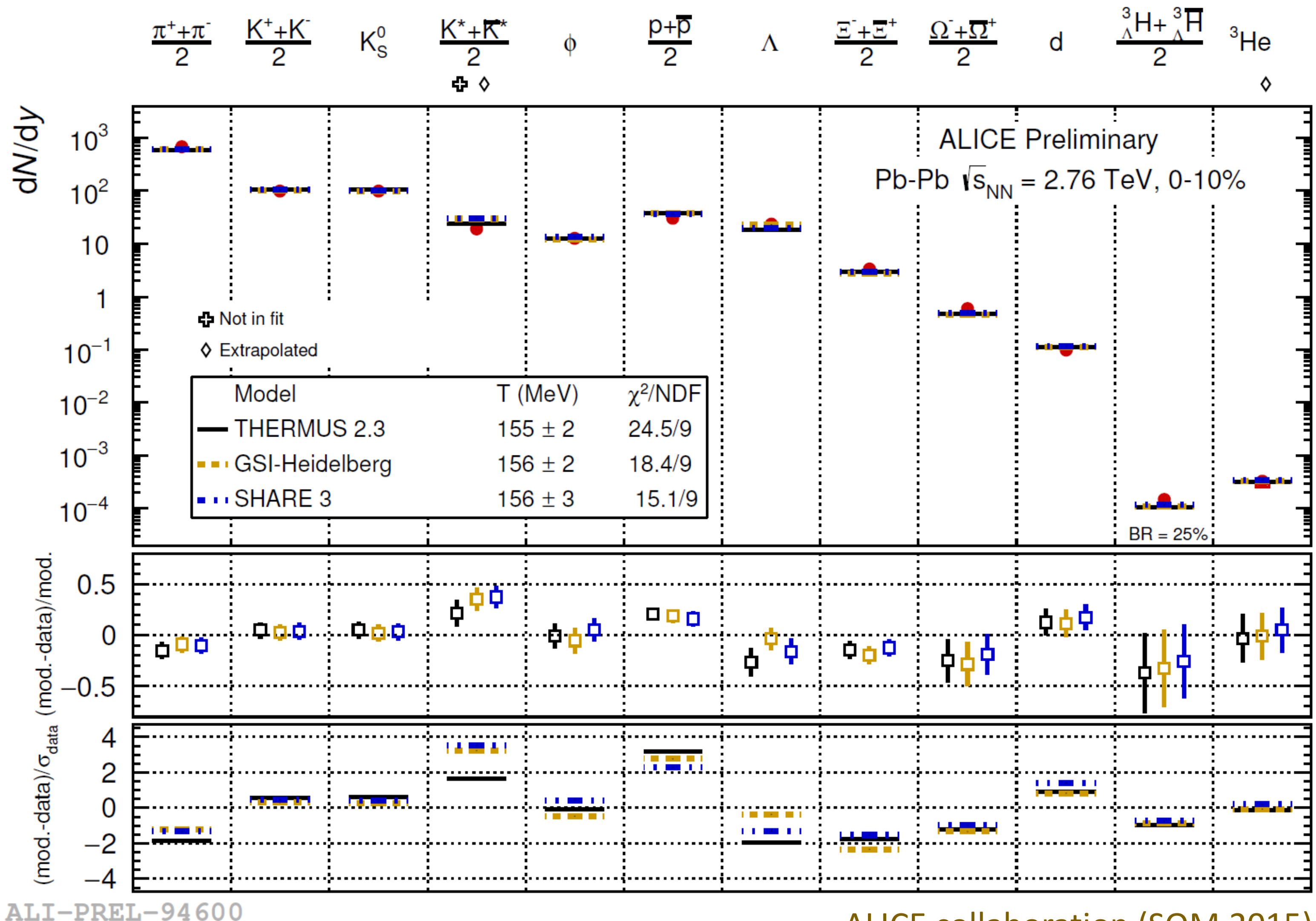
Thermal fits at SPS and RHIC energies



Andronic, Braun-Munzinger, Stachel, PLB (2009)

- Fair data description across several orders of magnitude
- Evidence for chemical equilibration of matter

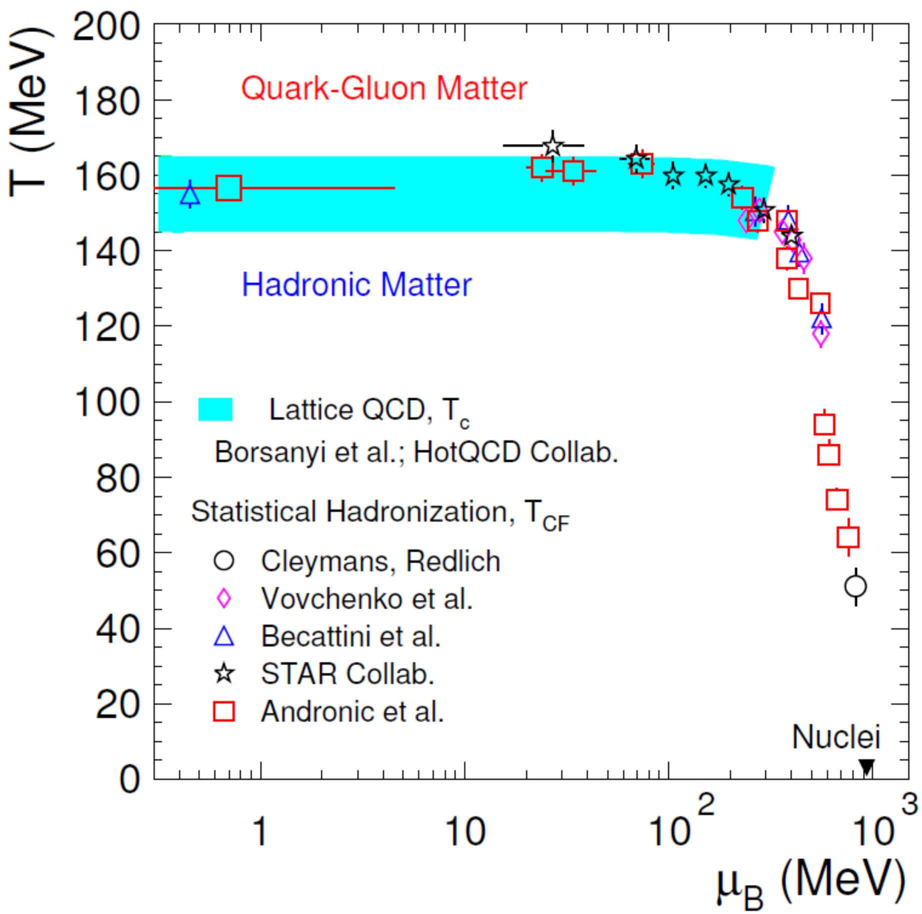
Thermal fits at LHC



ALICE collaboration (SQM 2015)

Heavy-ion collisions and the QCD phase diagram

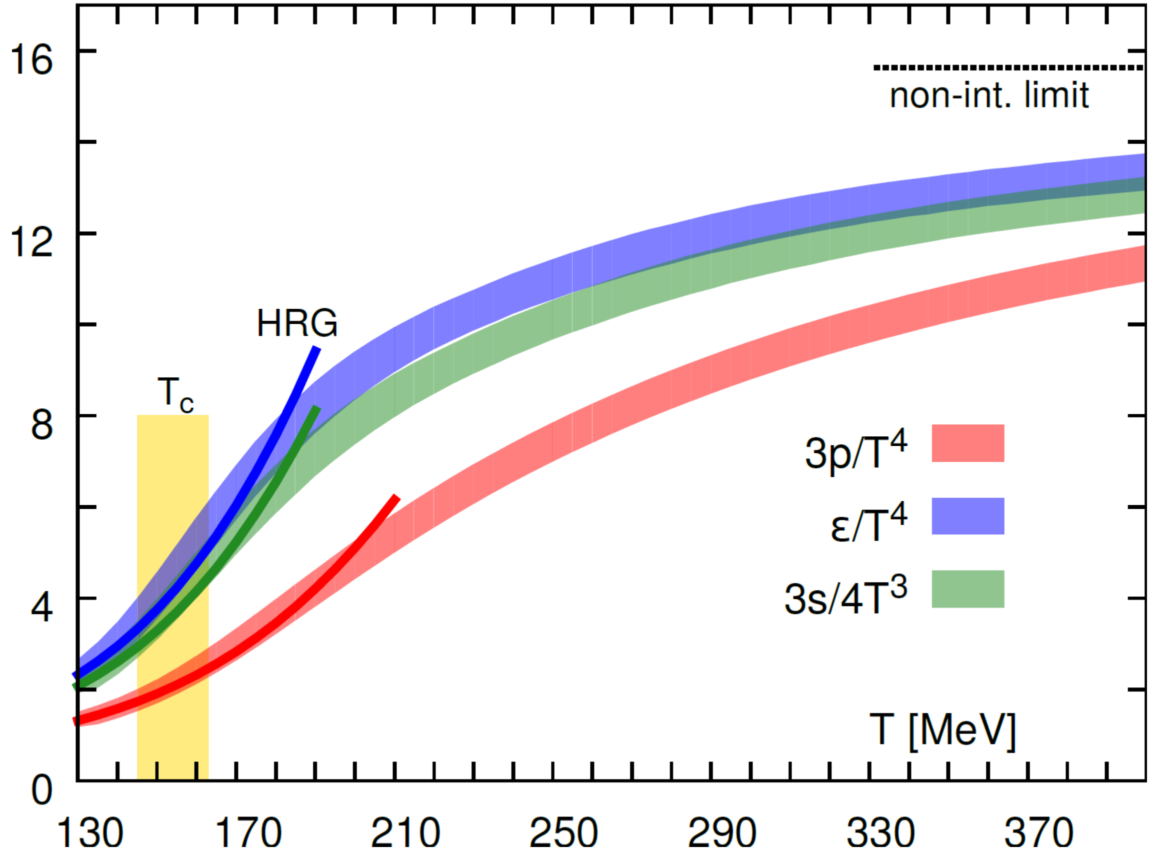
Thermal fits for systems at different collision energies map chemical freeze-out stage in heavy-ion collisions to the QCD phase diagram



A. Andronic et al., Nature 561, 321 (2018)

- **Chemical freeze-out curve** in T-μ_B plane
- $T_{ch}(\mu_B) \approx a - b\mu_B^2 - c\mu_B^4$
  J. Cleymans et al., PRC (2006)
- Energy per particle $E/N \approx 1$ GeV
  J. Cleymans, K. Redlich, PRL (1998)

HRG model and lattice QCD equation of state



[HotQCD collaboration, 1407.6387; similar results from Wuppertal-Budapest collab., 1309.5258]

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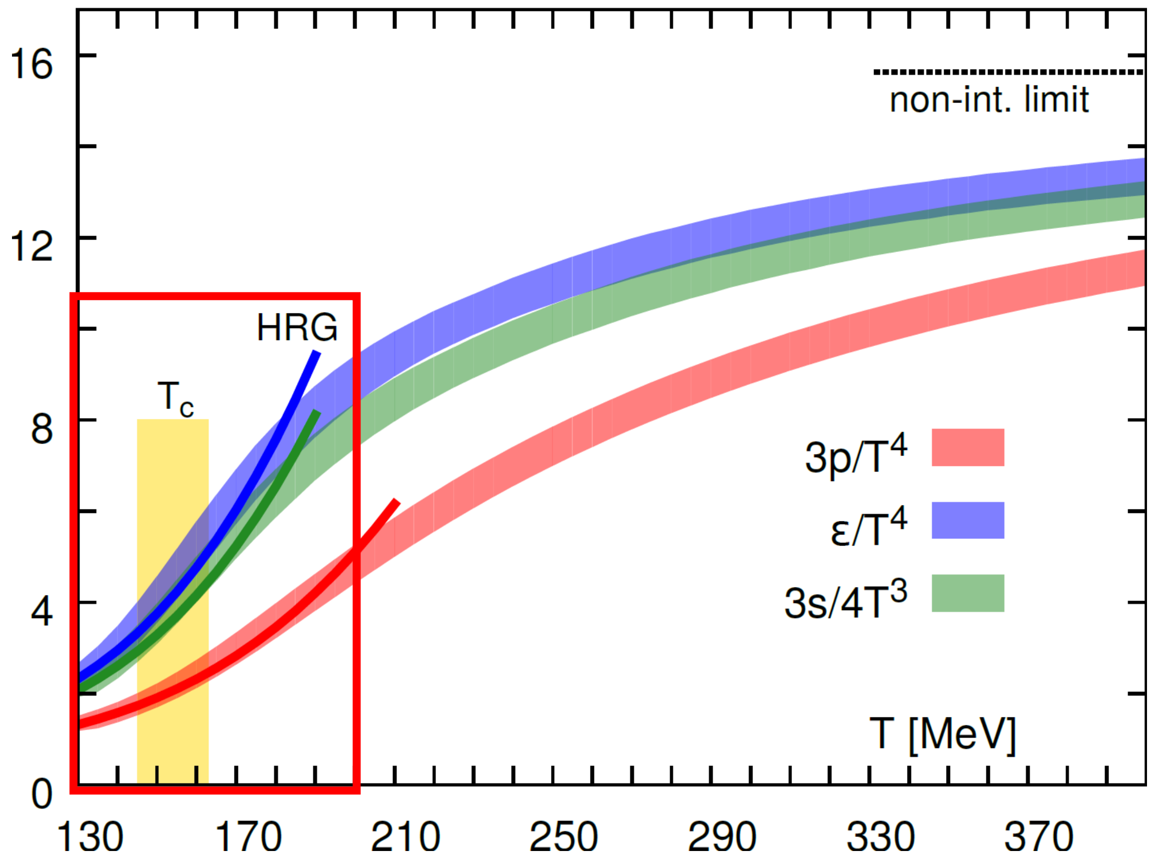

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HRG describes quite well LQCD thermodynamic functions below and in the vicinity of the pseudocritical temperature

Thermal model and radial flow

$$N_i^{\mathrm{hrg}} = V \frac{d_i}{2\pi^2} \int_0^\infty p^2 \, dp \left[\exp\left(\frac{E_i - \mu_i}{T} \right) \pm 1 \right]^{-1}$$

In thermal model yields are computed in **local rest frame**, i.e. no flow
But matter in HIC appears to have a substantial collective flow, so how can the model be applied to data?

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Hydro: $$N_i = \int_\sigma d\sigma_\mu u^\mu \underbrace{\int \frac{d^3 p_i}{p^0} p_\mu\, u^\mu \frac{d_i}{(2\pi)^3} \left[\exp\left(\frac{p_i^\mu u_\mu - \mu_i}{T}\right) \pm 1\right]^{-1}}_{n_i^{\mathrm{hrg}}}$$

“Freeze-out” across space-time hypersurface $\sigma(x)$ with collective velocity profile $u^\mu(x)$. If T and μ_i uniform across the hypersurface then

$$N_i = n_i^{\mathrm{hrg}} \underbrace{\int_\sigma d\sigma_\mu u^\mu}_{V_{\mathrm{eff}}} \qquad \text{and} \qquad \frac{\mathbf{N_i}}{\mathbf{N_j}} = \frac{\mathbf{N_i^{\mathrm{hrg}}}}{\mathbf{N_j^{\mathrm{hrg}}}}$$

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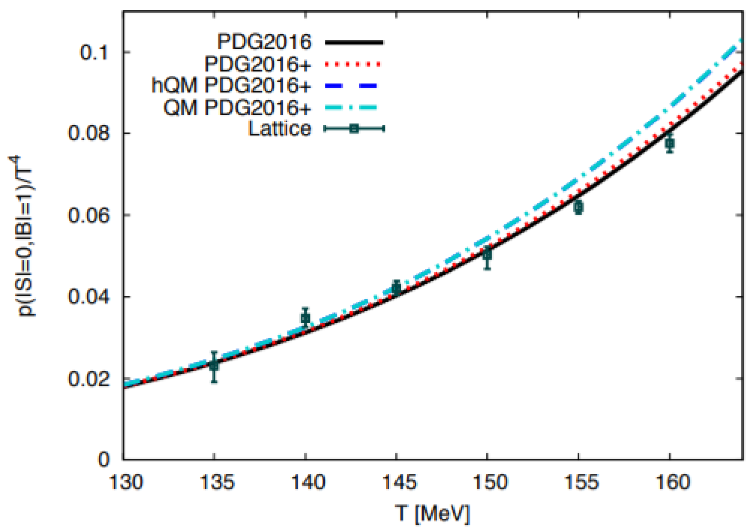
Effects of collective motion largely cancel out in yield ratios

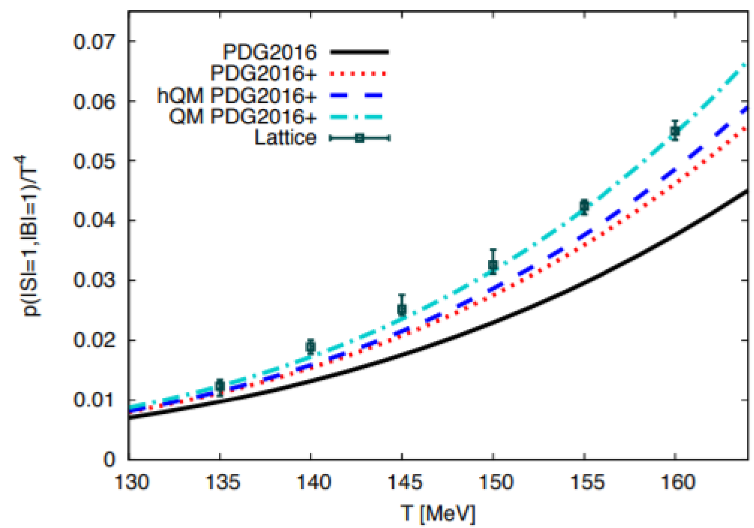
Many aspects of the thermal model

- particle list and decay properties
- finite resonance widths
- loosely bound states
- chemical non-equilibrium (γ_q, γ_s)
- excluded volume/van der Waals interactions
- exact conservation of conserved charges (canonical ensemble)
- particle number fluctuations
- statistical hadronization of charm

Different particle lists

- Established (*** & ****) hadrons from PDG **(the standard option)**
- Include unconfirmed (* & **) or theoretical (quark model) states





[Alba et al., 1702.01113; see also 1404.6511 (HotQCD)]

Evidence for extra strange baryons from lattice QCD

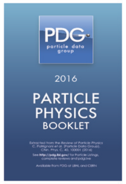
- Exponential Hagedorn mass spectrum $\rho(m) = A\, m^{-\alpha} \exp(m/T_H)$

New phenomena: “limiting” temperature, (phase) transition to QGP etc.

[Gallmeister et al., 1712.04018; **V.V.** et al., 1811.05737]

Decay properties

Decay properties of many resonances are not very well established
This affects determination of feeddown contributions



$K_1(1400)$ DECAY MODES	Fraction (Γ_i/Γ)
$K^*(892)\pi$	(94 ± 6) %
$K\rho$	(3.0 ± 3.0) %
$K f_0(1370)$	(2.0 ± 2.0) %
$K\omega$	(1.0 ± 1.0) %

$N(1650)$ DECAY MODES	Fraction (Γ_i/Γ)
$N\pi$	50–70 %
$N\eta$	15–35 %
ΛK	5–15 %
$N\pi\pi$	8–36 %

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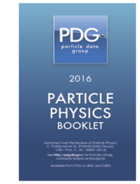

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$\Xi(1690)$ DECAY MODES	Fraction (Γ_i/Γ)
$\Lambda\overline{K}$	seen
$\Sigma\overline{K}$	seen
$\Xi\pi$	seen
$\Xi^-\pi^+\pi^-$	possibly seen

$\Xi(1820)$ DECAY MODES	Fraction (Γ_i/Γ)
$\Lambda\overline{K}$	large
$\Sigma\overline{K}$	small
$\Xi\pi$	small
$\Xi(1530)\pi$	small

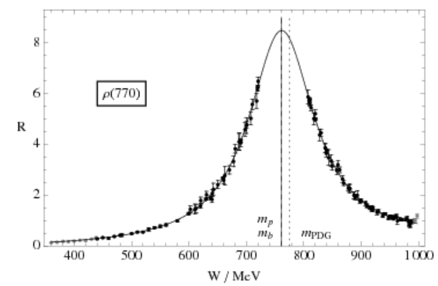
That's not very helpful

"Educated" guesses sometimes needed to calculate feeddown

Source of a systematic uncertainty ~10%

Finite resonance widths

$$n_i(T,\mu;m_i) \to \int_{m_i^{\min}}^{m_i^{\max}} dm\, \rho_i(m)\, n_i(T,\mu;m)$$



1) Zero-width approximation

Simplest possibility, used commonly in LQCD comparisons

2) Constant Breit-Wigner (BW) in $\pm 2\Gamma_i$ interval

Popular choice in thermal fits

Enhances resonance yields

$$\rho_i(m) = A_i \frac{2\, m\, m_i\, \Gamma_i}{(m^2 - m_i^2)^2 + m_i^2\, \Gamma_i^2}$$

3) Energy-dependent Breit-Wigner (eBW)

$$\Gamma_{i\to j}(m) = b_{i\to j}\,\Gamma_i \left[1 - \left(\frac{m_{i\to j}^{\mathrm{thr}}}{m}\right)^2\right]^{l_{ij}+1/2}$$

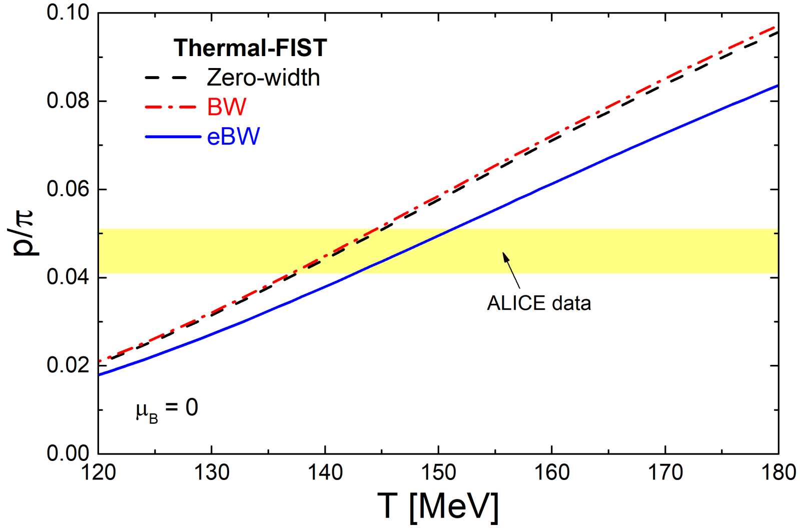
suppression at the threshold

Suppresses resonance yields

4) Phase shifts within the S-matrix approach $\rho_i(m) \propto \frac{\partial \delta(m)}{\partial m}$

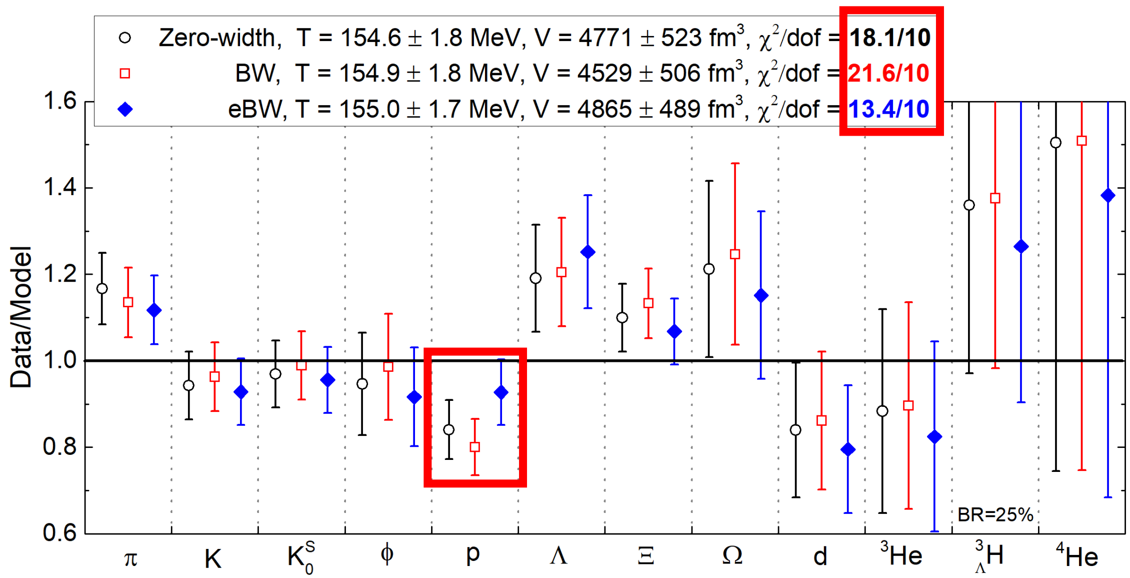
Usually based on measured scattering phase shifts

Finite resonance widths: effect on thermal fits



Energy-dependent Breit-Wigner leads to a 15% suppression of proton yields

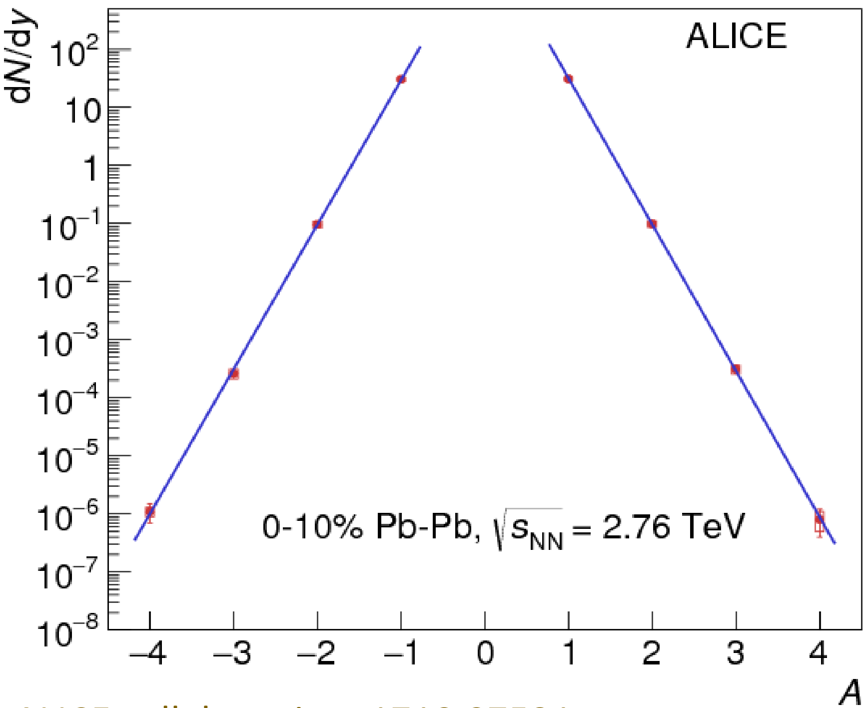
This is enough to describe the 'proton yield anomaly' at the LHC



[**V.V.**, Gorenstein, Stoecker, 1807.02079; see also phase shift analysis P. Lo et al., 1808.03102]

Thermal model and loosely-bound states

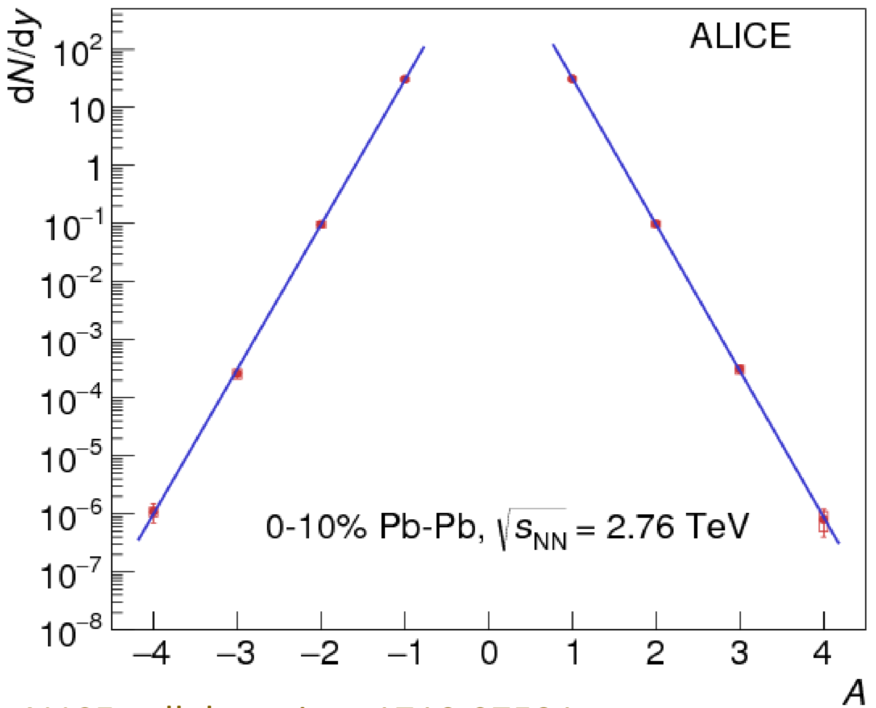
Yields of light nuclei at LHC decrease exponentially with mass



ALICE collaboration, 1710.07531

Thermal model and loosely-bound states

Yields of light nuclei at LHC decrease exponentially with mass, agree well with thermal model at T = 155 MeV



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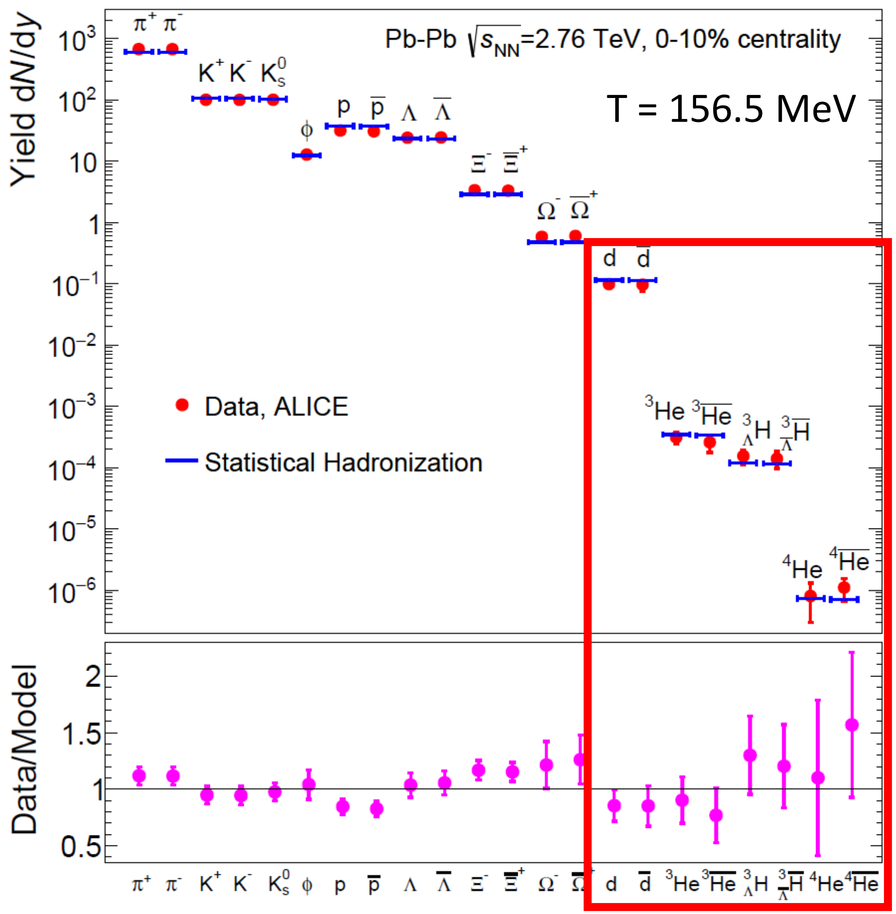

Andronic et al., 1710.09425

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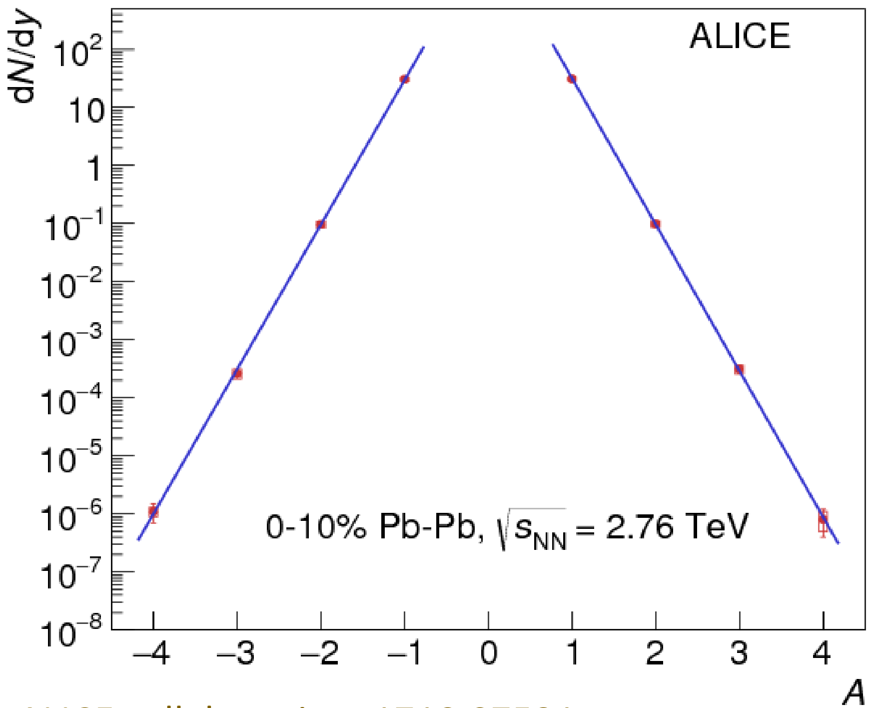

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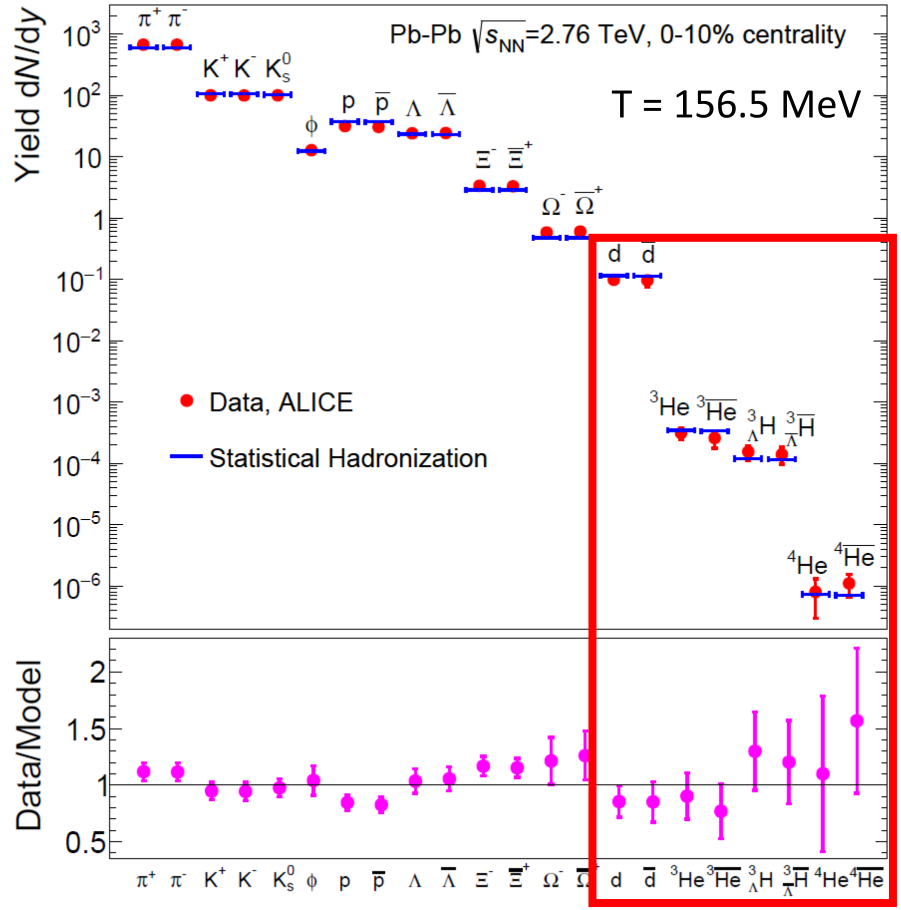


Andronic et al., 1710.09425

Loosely-bound states (few MeV or less binding energy) expected to be immediately destroyed at T = 155 MeV. Why the thermal model works so well for the yields of light nuclei remains not fully understood

Incomplete chemical equilibrium of strangeness

A reasonable description of strangeness production often requires introduction of strangeness saturation parameter γ_S, which in thermal picture interpreted as an incomplete equilibration of strangeness

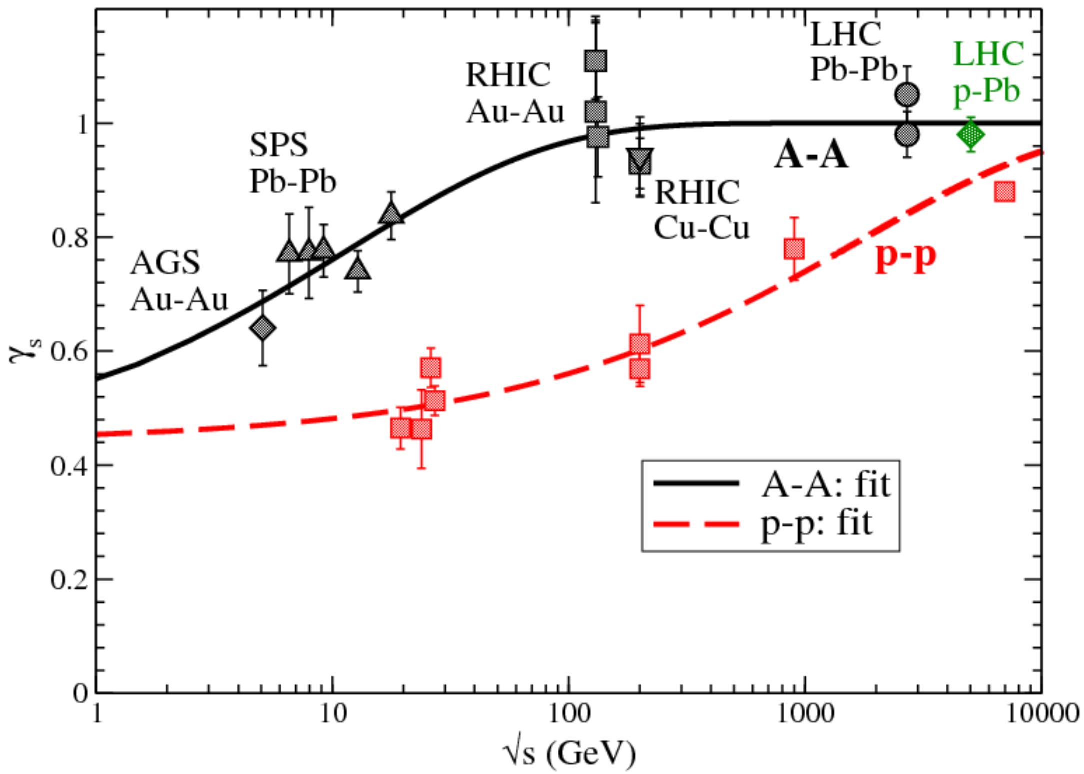


Figure from Castorina, Plumari, Satz, 1603.06529

$$N_i^{hrg} \rightarrow (\gamma_S)^{|s_i|} N_i^{hrg}$$

$|s_i|$ - strange quark content

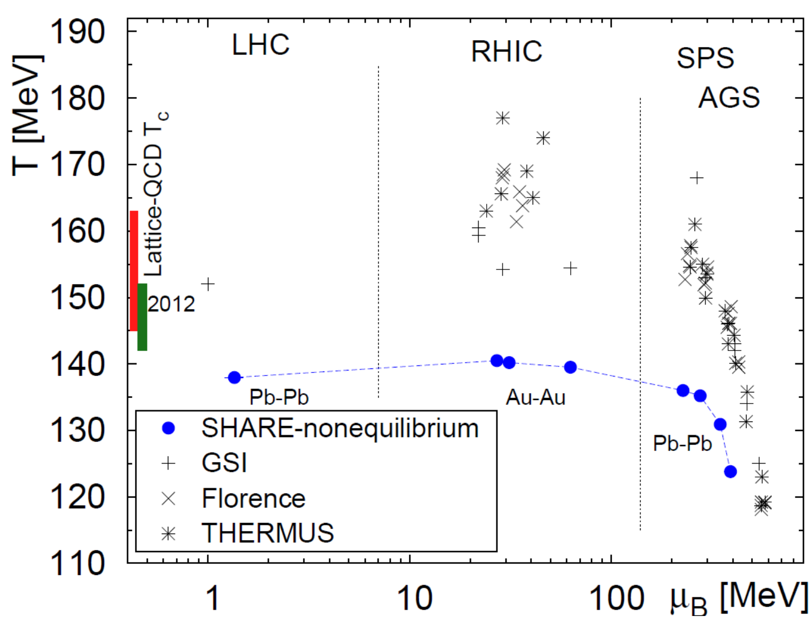
$\gamma_S < 1$ in p-p & A-A at AGS/SPS

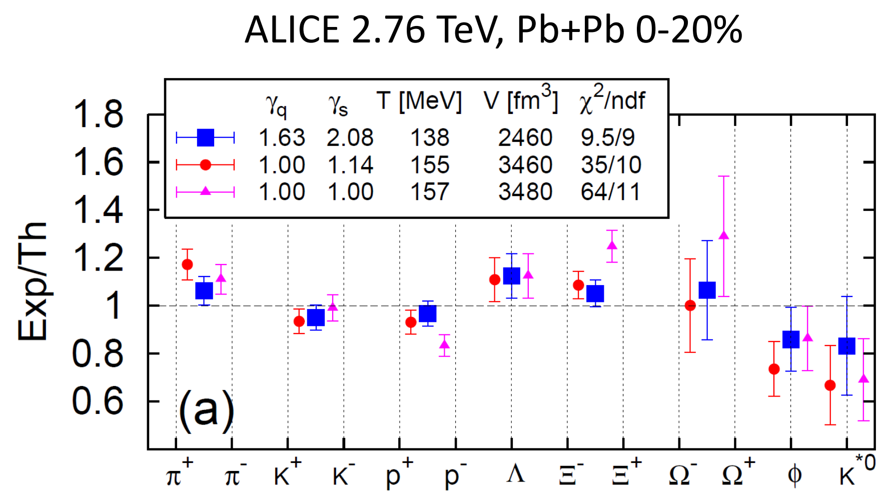
$\gamma_S \approx 1$ in A-A at RHIC and LHC

Chemical non-equilibrium scenario

In chemical non-equilibrium scenario $N_i^{\mathrm{hrg}} \to (\gamma_q)^{|q_i|}(\gamma_s)^{|s_i|}\, N_i^{\mathrm{hrg}}$
both light ($|q_i|$) and strange ($|s_i|$) quarks out of chemical equilibrium

Scenario: hadronization of chem. non-eq. supercooled QGP [Letessier, Rafelski, ‘99]

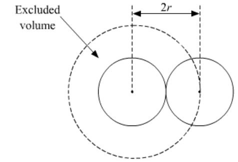




[M. Petran et al., 1303.2098]

- smaller reduced χ^2 compared to chem. equilibrium scenario
- $\gamma_q = 1.63$ => $\mu_\pi \approx 135\ MeV \approx m_\pi$ => pion BEC? [V. Begun et al., 1503.04040]
- However, $\gamma_q \approx \gamma_s \approx 1$ when light nuclei included in fit [M. Floris, 1408.6403]

Excluded volume corrections



Notion that hadrons have finite eigenvolume suggested awhile ago

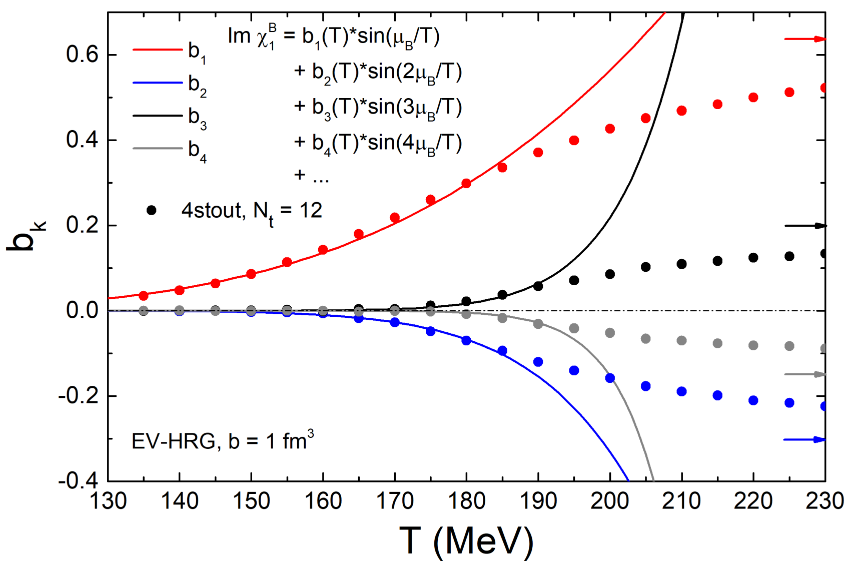
[R. Hagedorn, J. Rafelski, PLB '80]

Excluded volume model: $V \to V - bN \quad \Rightarrow \quad p(T,\mu) = p^{\rm id}(T, \mu - bp)$

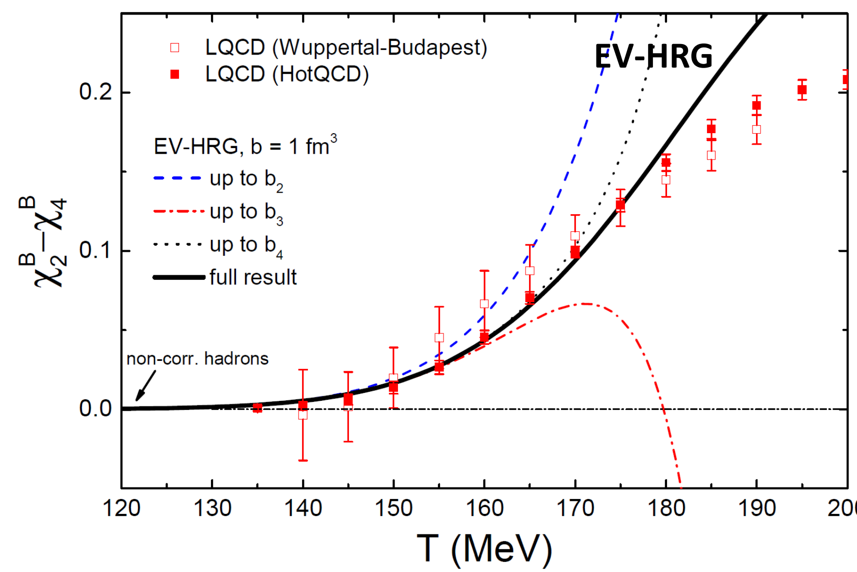
[D. Rischke et al., Z. Phys. C '91]

Recent lattice QCD data favor EV-like effects in baryonic interactions

Fourier coefficients at imaginary μ_B



Susceptibilities $\chi_B^k \sim \partial^k p / \partial \mu_B^k$



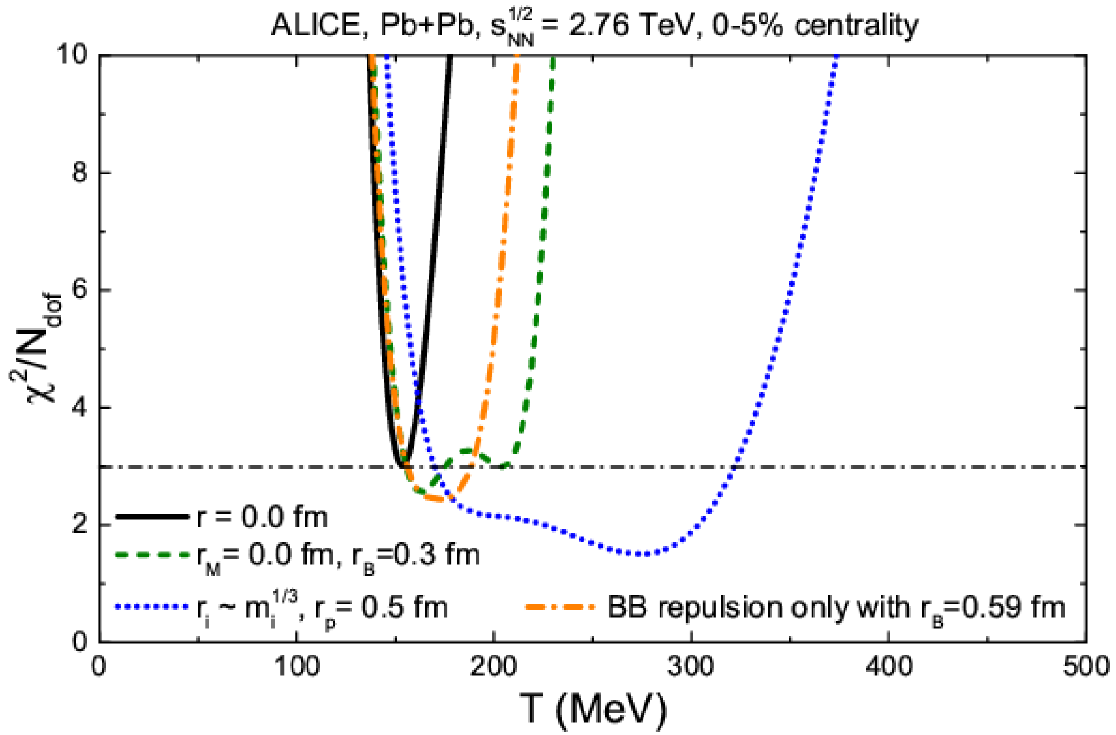
V.V., A. Pasztor, Z. Fodor, S.D. Katz, H. Stoecker, 1708.02852

Evidence for EV effects for mesons is less compelling

Excluded volume corrections and thermal fits

Excluded volume effect: $N_i \to \kappa\, e^{-\frac{v_i\, p}{T}}\, N_i$

This may have an effect on data description if v_i are different

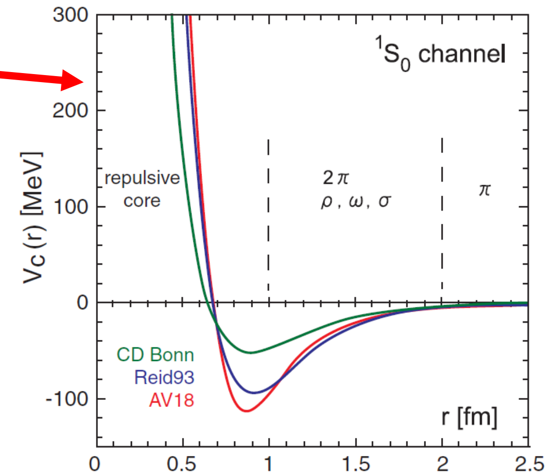


V.V., H. Stoecker, 1512.08046 & 1606.06218

Depending on v_i parameterization effects on fits are between being negligible to strong and controversial (χ^2 minima at very high *T*)

van der Waals interactions in HRG

NN potential: repulsive core and attraction



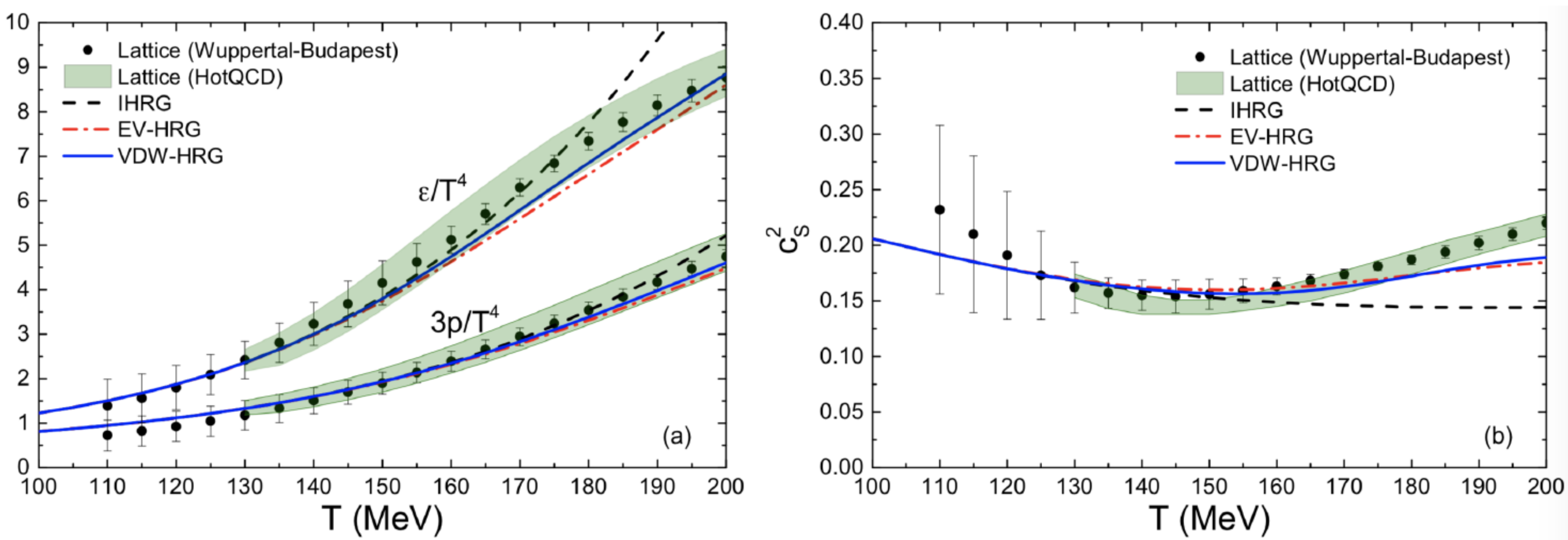
from Ishii et al., PRL (2007)

vdW-HRG: baryons described by the vdW equation

$$p = p_M^{\rm id} + p_B^{\rm vdW} + p_{\bar{B}}^{\rm vdW} \qquad p_B^{\rm vdW} \simeq \frac{T n_B}{1 - b\, n_B} - a\, n_B^2$$

a = 329 MeV fm^3, b = 3.42 fm^3 from fit to ground state

Critical point at T_c = 19.7 MeV, $\mu_c = 908$ MeV



V.V., M.I. Gorenstein, H. Stoecker, PRL 118, 182301 (2017)

Canonical statistical model

Grand-canonical ensemble: configurations with all possible quantum numbers

$$Z^{\mathrm{gce}}(\mu_B,\mu_Q,\mu_S) = \sum_{B=-\infty}^{\infty}\sum_{Q=-\infty}^{\infty}\sum_{S=-\infty}^{\infty} e^{\frac{B\mu_B+Q\mu_Q+S\mu_S}{T}} Z^{\mathrm{ce}}(B,Q,S)$$

including those not realized in heavy-ion collisions, e.g. $S \neq 0$

Thermodynamic equivalence of ensembles: $N_i^{\mathrm{gce}} = N_i^{\mathrm{ce}} + O(V^{-1})$

GCE justified for large systems, but canonical effects needed for smaller systems

[Rafelski, Danos, et al., PLB ’80; Hagedorn, Redlich, ZPC ‘85]

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Canonical partition function:

$$\mathcal{Z}(B, Q, S) = \int_{-\pi}^{\pi} \frac{d\phi_B}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_Q}{2\pi} \int_{-\pi}^{\pi} \frac{d\phi_S}{2\pi}\, e^{-i\,(B\phi_B + Q\phi_Q + S\phi_S)} \exp\left[\sum_j z_j^1\, e^{i\,(B_j\phi_B + Q_j\phi_Q + S_j\phi_S)}\right]$$

[Becattini et al., ZPC '95, ZPC '97]

$$z_j^1 = V_c \int dm\, \rho_j(m)\, d_j \frac{m^2 T}{2\pi^2} K_2(m/T) \qquad \langle N_j^{\rm prim} \rangle^{\rm ce} = \frac{Z(B - B_j, Q - Q_j, S - S_j)}{Z(B, Q, S)} \langle N_j^{\rm prim} \rangle^{\rm gce}$$

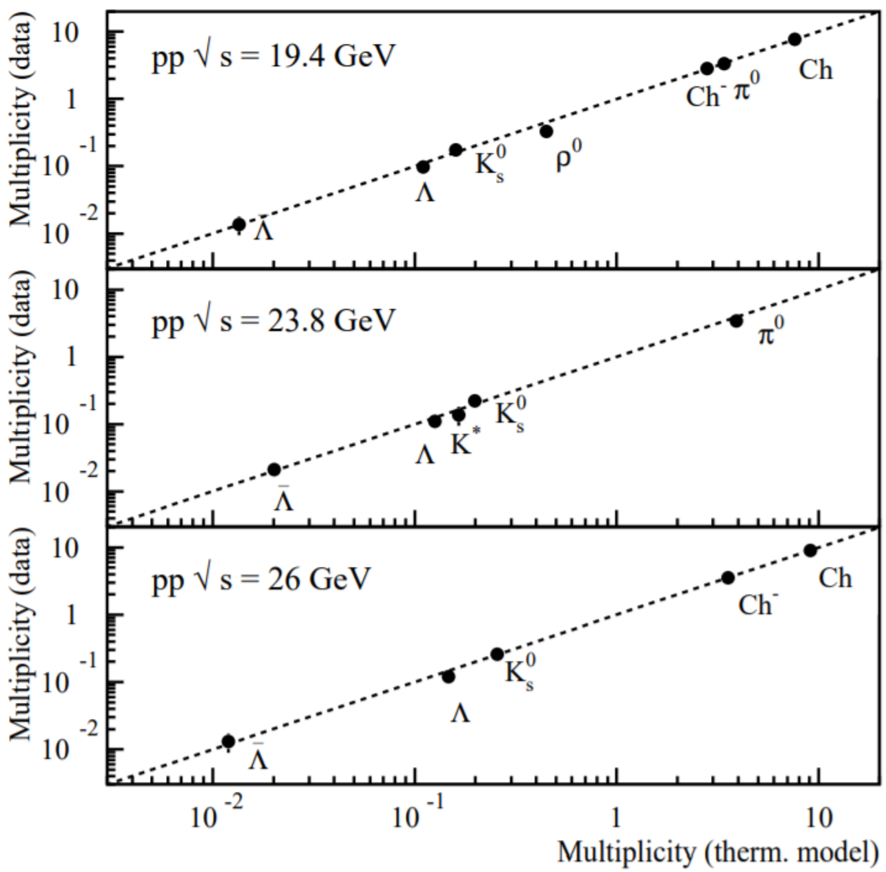
CE effects typically suppress yields relative to the GCE **(canonical suppression)**

Strangeness enhancement as a manifestation of an **absence of CE suppression**

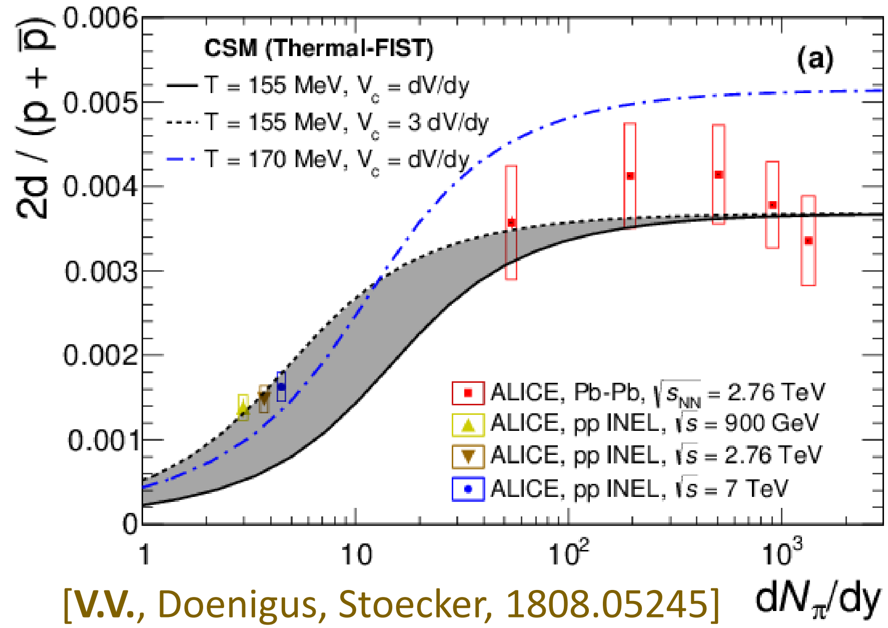
[Hamieh, Redlich, Tounsi, PLB (2000)]

Canonical statistical model and thermal fits

Canonical thermodynamics allows to use thermal model for small systems such as p-p, p-p̄, e^+e^-



[F. Becattini et al., ZPC '95, ZPC '97]



[**V.V.**, Doenigus, Stoecker, 1808.05245]

More on this on Wednesday, 15:40

Thermal model tools

Available thermal model codes:

1) **SHARE 3** [G. Torrieri, J. Rafelski, M. Petran, et al.] *Since 2003*
   Fortran/C++. Chemical (non-)equilibrium, fluctuations, charm, nuclei
   open source: http://www.physics.arizona.edu/~gtshare/SHARE/share.html

2) **THERMUS 4** [S. Wheaton, J. Cleymans, B. Hippolyte, et al.] *Since 2004*
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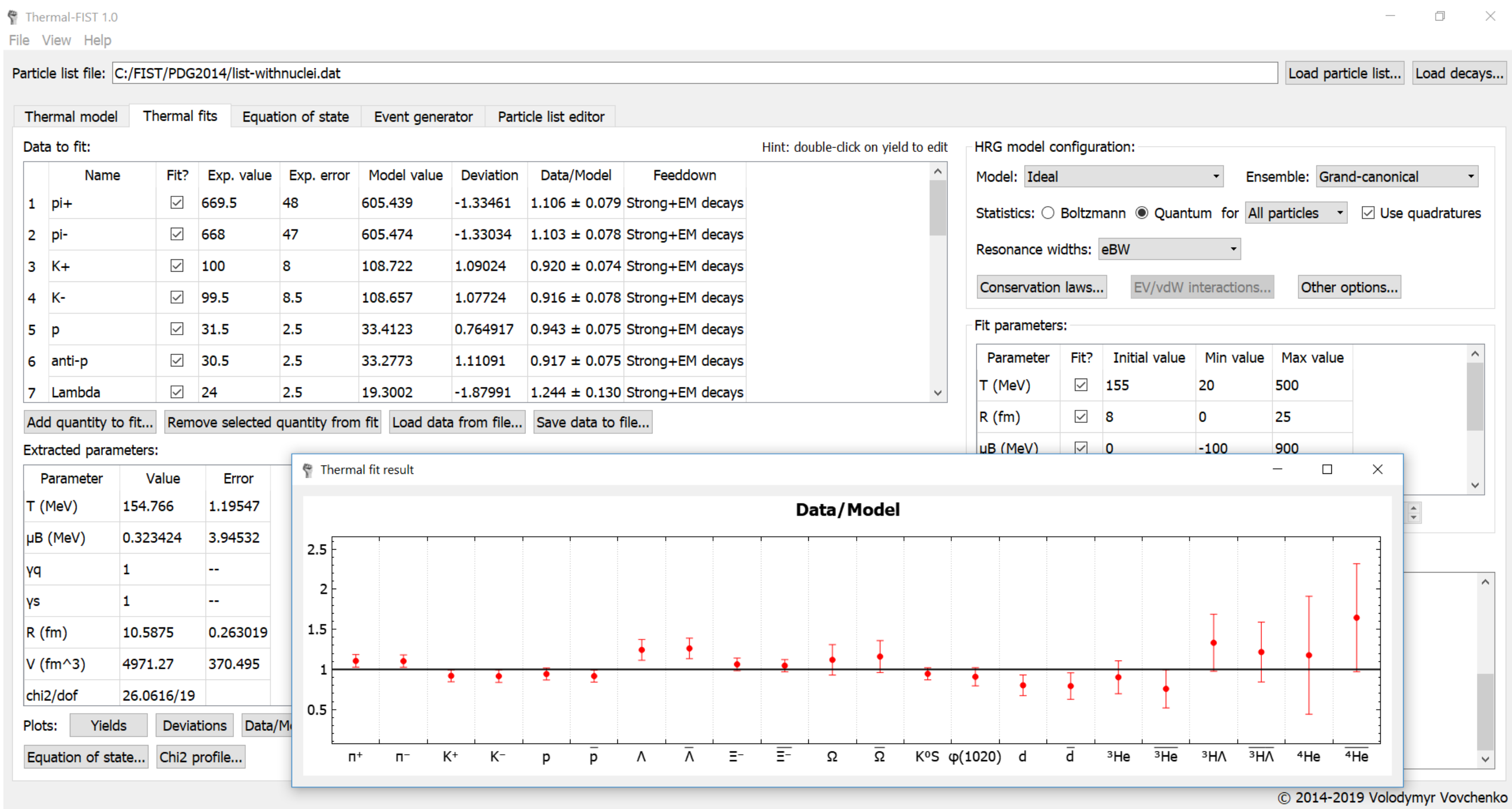
New development:

Thermal-FIST v1.1 (or simply "The FIST") [**V.V.**, H. Stoecker]
C++. Chemical (non-)equilibrium, EV/vdW corrections, Monte Carlo, (higher-order) fluctuations, canonical ensemble, combinations of effects
open source: https://github.com/vlvovch/Thermal-FIST *Since 2018*
physics manual: arXiv:1901.05249

Thermal-FIST

Graphical user interface for *general-purpose* thermal fits and more



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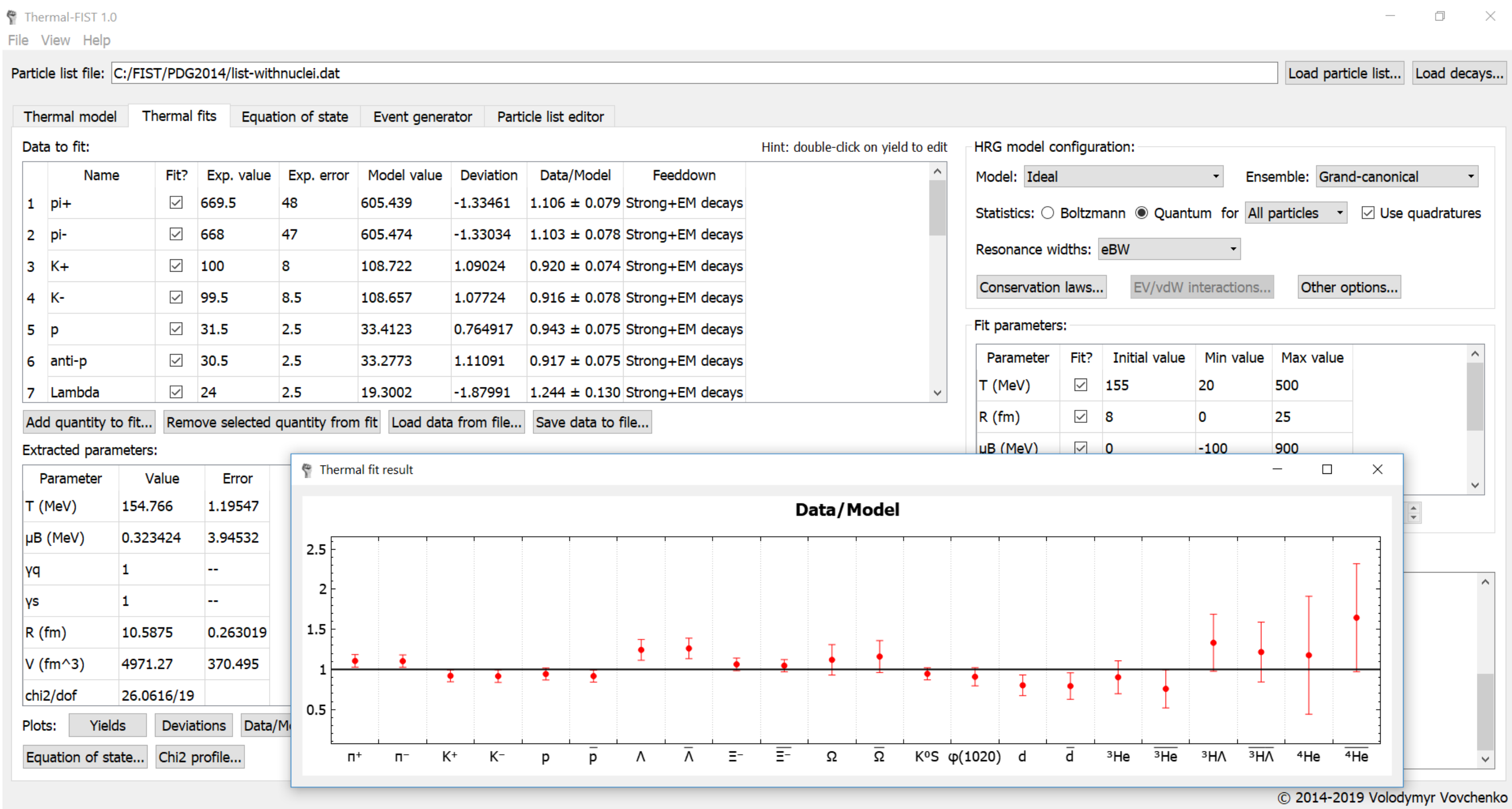

"So that's how you get your results so quickly!"

J. Cleymans

"Thanks for reproducing my results!"

F. Becattini

Using Thermal-FIST

*The package is **cross-platform*** (Linux, Mac, Windows, Android)

Installation using **git** and **cmake**

```
# Clone the repository from GitHub
git clone https://github.com/vlvovch/Thermal-FIST.git
cd Thermal-FIST

# Create a build directory, configure the project with cmake
# and build with make
mkdir build
cd build
cmake ../
make

# Run the GUI frontend
./bin/QtThermalFIST

# Run the test calculations from the paper
./bin/examples/cpc1HRGTDep
./bin/examples/cpc2chi2
./bin/examples/cpc3chi2NEQ
./bin/examples/cpc4mcHRG
```

GUI requires free Qt5 framework, the rest of the package has no external dependencies

Quick start guide *Documentation* *Physics manual*

Summary

- The statistical thermal model is the "simplest" model for particle production, which describes yields across many collision energies on a 10-15% level

- The model has many ambiguous details – sources of systematic uncertainty in the model – currently under investigation

- Model applications available through a number of open source codes. New **Thermal-FIST** package provides most of the features used in thermal model analysis in a convenient way.

https://github.com/vlvovch/Thermal-FIST

Summary

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Thanks for your attention!