

Nuclear Parton Distribution Functions

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Motivation

Jets in collinear factorization

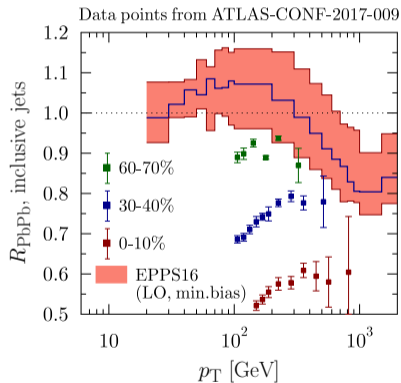
$$d\sigma^{AB \rightarrow jet+X} = \sum_{ij} f_i^A \otimes f_j^B \otimes d\sigma^{ij \rightarrow jet+X}$$

- $d\sigma^{ij \rightarrow jet+X}$: partonic cross section
- f_i^A : Parton distribution functions (PDFs) describing the partonic structure of the beam particle

Nuclear PDFs

- $f_i^A \neq f_i^P$
- Determined from exp. data
- Provides pQCD baseline for jet quenching effects

$$R_{\text{PbPb}} = \frac{d\sigma^{\text{PbPb}}}{208^2 d\sigma^{\text{pp}}}$$



[Figure: H. Paukkunen]

Outline for the lecture

1. Theoretical framework for PDF analyses

- Deep Inelastic scattering
- DGLAP evolution
- Uncertainty analysis

2. Nuclear PDF analyses

- Data types
- Current status and uncertainties

3. New constraints from

- LHC experiments
- Future facilities

4. Loose ends

Theoretical framework for PDF analyses



How to probe nucleon structure?

Deep Inelastic Scattering (DIS)

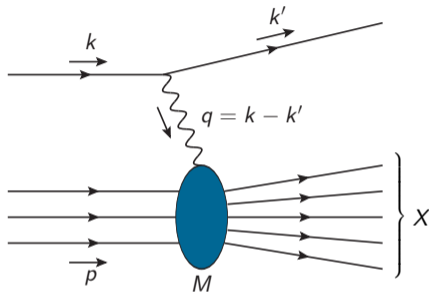
- Lepton scatters from a nucleon via a virtual photon with $Q^2 = -q^2$
- High $Q^2 \Rightarrow$ Small length scales
 \Rightarrow Sensitive to nucleon structure
- Invariant variables:

$$x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

- Cross section in terms of structure functions $F_{1,2}(x, Q^2)$

$$\frac{d\sigma^{\text{DIS}}}{dx dQ^2} = \frac{4\pi\alpha_{EM}^2}{Q^4} \frac{1}{x} \left[xy^2 F_1(x, Q^2) + \left(1 - y - \frac{xyM^2}{s - M^2} \right) F_2(x, Q^2) \right]$$

- \Rightarrow Scattered lepton provides information on $F_{1,2}(x, Q^2)$



Parton model in leading order

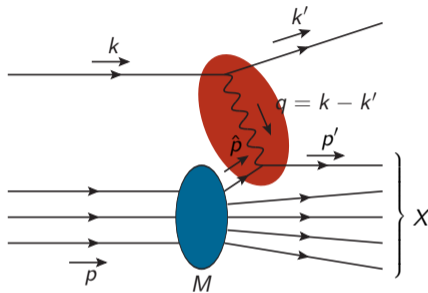
Assume that nucleon consists of partons

- Each parton carries a certain fraction x of the nucleon momentum ($\hat{p} = x p$)
- Lepton scatters off from a parton

$$d\sigma^{\text{DIS}} = \sum_i \int_0^1 dx d\hat{\sigma} f_i(x)$$

where

- $d\hat{\sigma}$: Partonic cross section of $2 \rightarrow 2$ scattering
- $f_i(x)$: Number density of partons i inside nucleon (=PDF)
 - Leading-order parton model: $2xF_1(x, Q^2) = F_2(x, Q^2) = \sum_i e_i^2 f_i(x)$
 - Process-independent (unlike $F_{1,2}$)



Scale dependence

- At higher orders in α_s the PDFs become scale dependent
⇒ The partonic structure depends on the scale at which the nucleon is probed

Scale evolution from DGLAP equations [Dokshitzer-Gribov-Lipatov-Altarelli-Parisi]

- Derived by resumming logarithmically divergent terms from collinear emissions to all orders

$$\frac{\partial f_{q_i}(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \left[P_{q_i q_j} \otimes f_{q_j}(Q^2) + P_{q_i g} \otimes f_g(Q^2) \right]$$

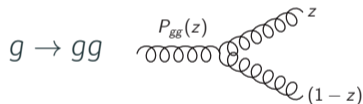
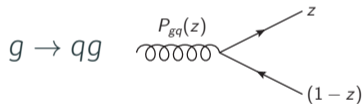
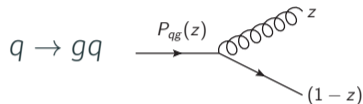
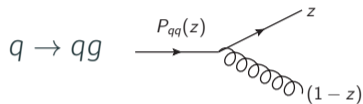
$$\frac{\partial f_g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \left[P_{gg} \otimes f_g(Q^2) + P_{gq_j} \otimes f_{q_j}(Q^2) \right]$$

- Convolution defined as

$$P_{ij} \otimes f_j = \int_x^1 \frac{dz}{z} P_{ij}(x/z) f_j(z)$$

Splitting functions

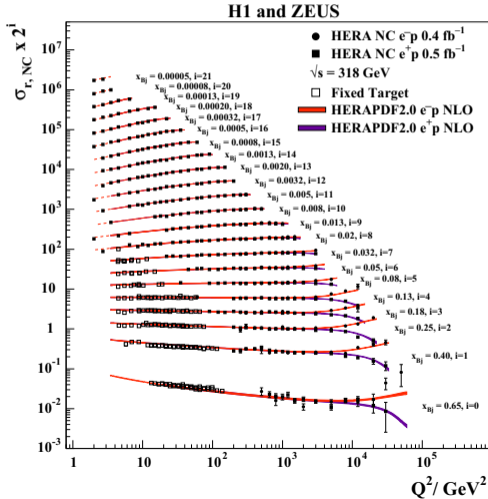
Splitting function can be interpreted as probabilities for partonic splittings



- Splitting functions P_{ij} calculable within pQCD, currently know up to α_S^2 (NNLL)
- PDFs for different partons mix during evolution

DGLAP evolution

Scale evolution of $d\sigma_{\text{reduced}}^{\text{DIS}} (\propto F_2 \approx \sum_i e_i^2 f_i)$ from HERA [Eur. Phys. J. C (2015) 75]



- **Small x :**
 F_2 increase with Q^2
 - **Intermediate- x :**
 Weak dependence on Q^2
 - **Large x :**
 F_2 decrease with Q^2
- \Rightarrow DGLAP evolution shifts partons from high x to low x
 \Rightarrow Very good description of the data in broad range of x and Q^2

Problem:

- PDFs cannot be calculated from perturbative QCD

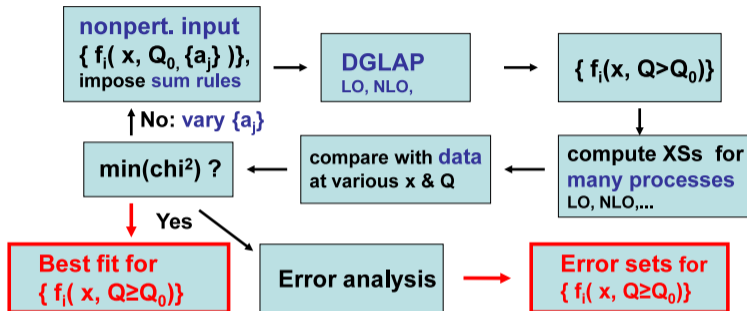
Solution:

- Parametrize the x dependence of PDFs at an initial scale Q_0^2 ($\mathcal{O}(1 \text{ GeV}^2)$)

$$f_i(x, Q_0^2) = x^{-a_i}(1-x)^{b_i}F_i(x; c_i, \dots)$$

- Use DGLAP equations to evolve PDFs and use data to fix the parameters
- Parametrization should be flexible enough to accommodate the physical features of the data
- PDF analyses also test the universality of the PDFs and QCD factorization

Fitting procedure



[from K.J.Eskola]

PDF fitting requires

- Effective DGLAP solving
- Fast evaluation of the cross sections
- Robust minimization algorithm
- Data with broad reach in x and Q^2 and flavour sensitivity

PDF properties

A few parameters may be fixed by sum rules

- Number of valence quarks

$$\int_0^1 dx [f_q(x, Q^2) - f_{\bar{q}}(x, Q^2)] = N_q,$$

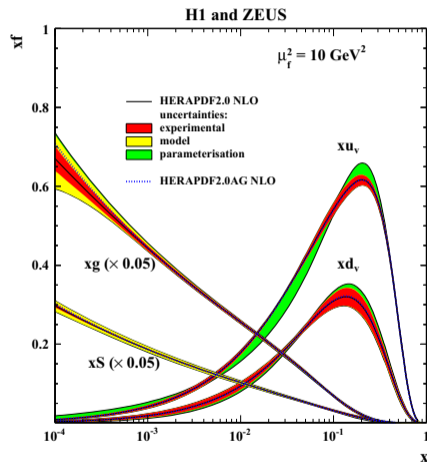
where $N_u = 2$, $N_d = 1$ for protons

- Momentum sum rule

$$\sum_{i=q,\bar{q},g} \int_0^1 dx x f_i(x, Q^2) = 1$$

⇒ The total momentum of all partons equals the momentum of the nucleon

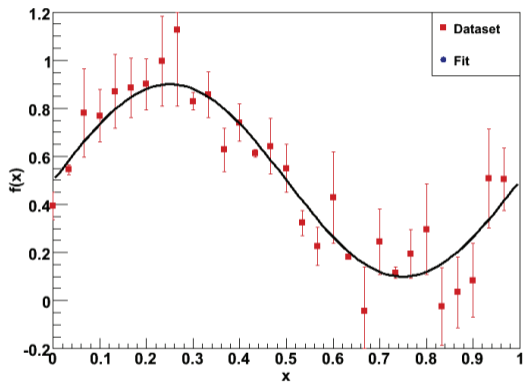
Example of a proton PDF analysis



[Eur. Phys. J. C (2015) 75]

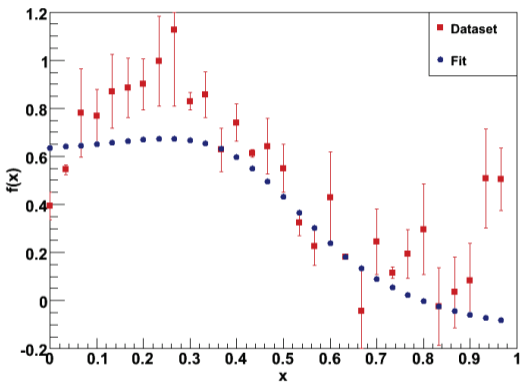
PDF uncertainties

The PDFs are fitted to finite number of points with some experimental uncertainty



[Figures: A. Guffanti]

PDF uncertainties

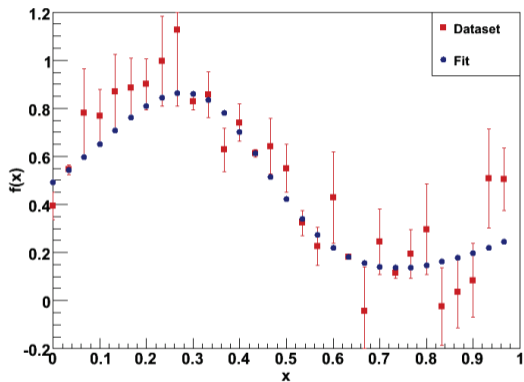


[Figures: A. Guffanti]

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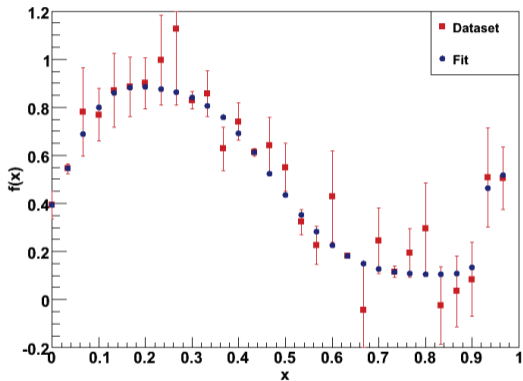
PDF uncertainties

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[Figures: A. Guffanti]

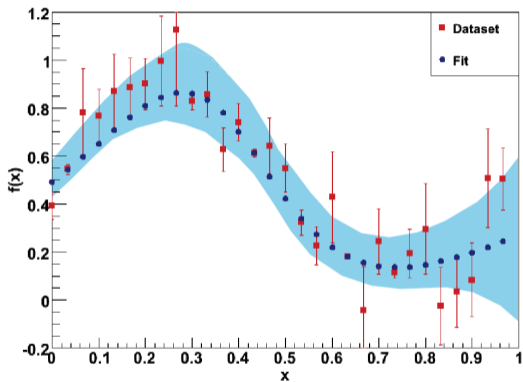
PDF uncertainties



[Figures: A. Guffanti]

The PDFs are fitted to finite number of points with some experimental uncertainty

- Acceptable agreement with different parameters and parametrizations



[Figures: A. Guffanti]

The PDFs are fitted to finite number of points with some experimental uncertainty

- Acceptable agreement with different parameters and parametrizations
 - Fits comprise some amount of uncertainty
- ⇒ How to quantify this uncertainty and how it propagates to observables?

- The fit minimizes χ^2 defined as

$$\chi^2 = \sum_i^N \left[\frac{D_i - T_i(\{a_j\})}{\delta_i} \right]^2$$

where D_i are data points, δ_i their (statistical) uncertainty and T_i corresponding theory points with given set of parameters $\{a_j\}$

- Expand in terms of parameters a_i around minimum χ_0^2

$$\chi^2 \approx \chi_0^2 + \sum_{ij} \frac{1}{2} (a_i - a_i^0)(a_j - a_j^0) \left. \frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right|_{a=a^0}$$

where parameter set $\{a^0\}$ gives the χ_0^2

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where parameter set $\{a^0\}$ gives the χ_0^2

- Diagonalize to find an orthonormal basis $\{z_i\}$
- Define error sets S_k^\pm by allowing certain $\Delta\chi^2 = \chi^2 - \chi_0^2$ when moving along z_k in positive and negative directions ($\Delta\chi^2 \sim 1 - 100$)

- Generate replicas of the data as

$$D_i \rightarrow D_i(1 + \delta_i R_i)$$

where δ_i is the experimental uncertainty and R_i a random number from a gaussian distribution

- Perform a refit to each prepared replica data
- The uncertainty of an observable is estimated from the variance of replica sets
- Requires $\mathcal{O}(1000)$ replicas for sufficient statistics

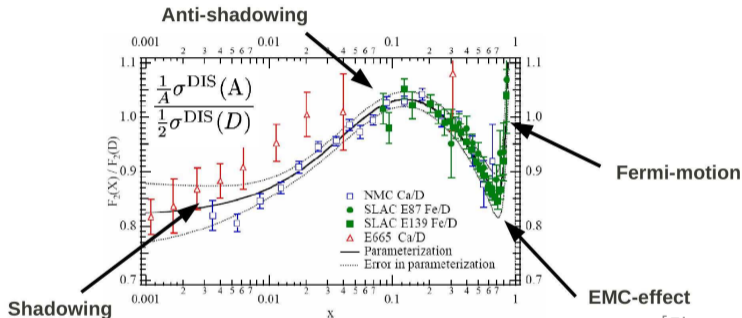
Further uncertainties

- These methods quantify only the uncertainty originating from the data
- Also some theoretical uncertainties due to parametrization etc.

Nuclear PDFs

Motivation for nPDFs

Observation: Differences in structure functions between different nuclei



[Figure: H. Paukkunen]

Factorization of nuclear modifications:

- Absorb the observed modifications into universal nuclear PDFs, $f_i^A(x, Q^2)$
- Perform a global QCD analysis with the same framework as for proton PDFs

- Nuclear PDFs (nPDFs) are the sum of proton and neutron PDFs:

$$f_i^A(x, Q^2) = Z f_i^{p/A}(x, Q^2) + (A - Z) f_i^{n/A}(x, Q^2)$$

for nucleus with a mass number A and Z protons

- The PDFs for neutrons can be obtained from proton PDFs using *isospin symmetry*:

$$f_u^{n/A}(x, Q^2) = f_d^{p/A}(x, Q^2) \quad \text{and} \quad f_d^{n/A}(x, Q^2) = f_u^{p/A}(x, Q^2)$$

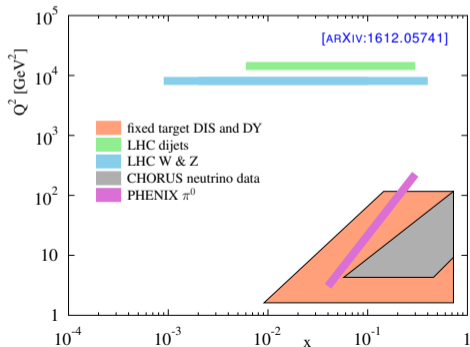
- Often one considers the nuclear modification of the PDFs defined as

$$R_i^A(x, Q^2) = \frac{f_i^{p/A}(x, Q^2)}{f_i^p(x, Q^2)}$$

Overview on applied data sets

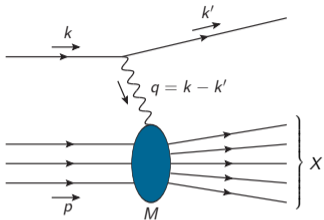
- Should include only data where factorization of nuclear effects valid and no final state energy loss expected
- Measurements with several different A required to constrain A dependence

Kinematic reach in x and Q^2 of applied data



- (neutrino) DIS with nuclear target (fixed target only)
- Drell-Yan (DY) dilepton production in p+A (fixed target)
- Inclusive π^0 production in d+Au
- Dijets in p+Pb at the LHC
- W^\pm and Z production in p+Pb

Nuclear DIS data



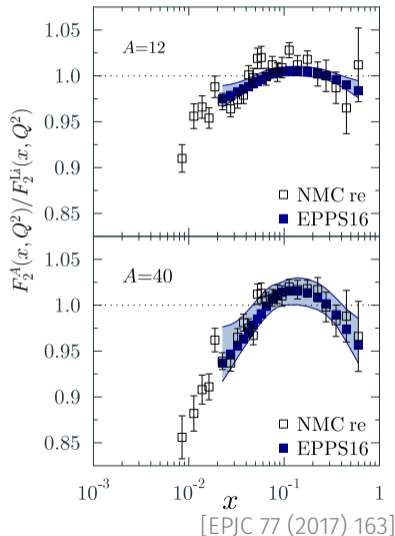
Kinematics from k'

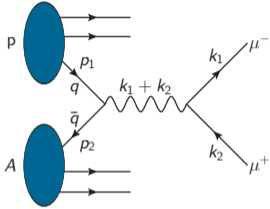
- $Q^2 = -(k - k')^2$
- $x = \frac{Q^2}{2p \cdot q}$
- $y = \frac{p \cdot q}{p \cdot k}$

LO cross section

$$\frac{d\sigma^{\text{DIS}}}{dx dQ^2} \approx \frac{\pi \alpha_{\text{em}}^2}{x Q^4} [1 + (1 - y)^2] \sum_q e_q^2 (f_q^A(x, Q^2) + f_{\bar{q}}^A(x, Q^2))$$

- x and Q^2 can be directly related to observable
- Probes gluons only at NLO (and via scale evolution)
- Only fixed-target experiments with nuclear targets
⇒ Limited reach in x and Q^2





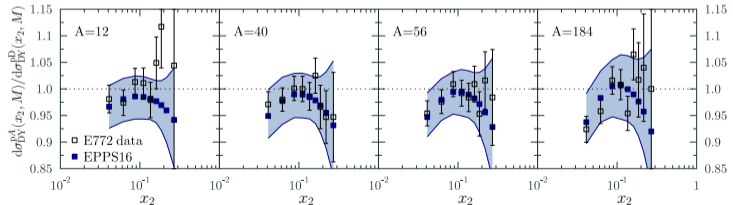
LO cross section

$$\frac{d\sigma^{\text{DY}}}{dM^2 dy_R} = \frac{4\pi\alpha_{EM}^2}{9sM^2} \sum_q e_q^2 \left[f_q^P(x_1, Q^2) f_{\bar{q}}^A(x_2, Q^2) + f_{\bar{q}}^A(x_2, Q^2) f_q^P(x_1, Q^2) \right]$$

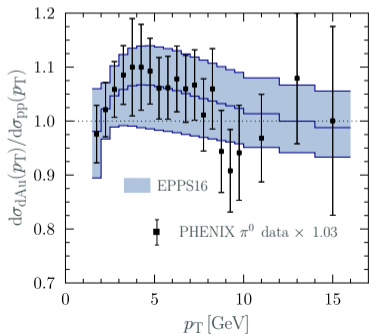
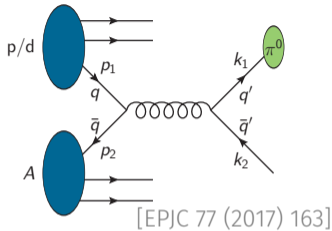
- x_1 and x_2 can be related to measured quantities
- Enhanced sensitive to sea quark PDFs
- Gluons appear only at NLO (and DGLAP)
- Fixed-target pA for nuclear targets, LHC soon

Kinematics from k_1, k_2

- $M^2 \equiv (k_1 + k_2)^2$
- $y_R = \frac{1}{2} \log \frac{E_{k_1+k_2} + p_{k_1+k_2}^z}{E_{k_1+k_2} - p_{k_1+k_2}^z}$
- $x_{1,2} = \frac{M}{\sqrt{S}} e^{\pm y_R}$



Inclusive hadron production



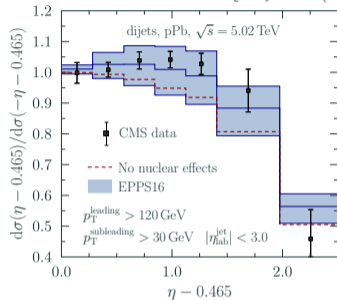
Partonic spectra convoluted with fragmentation functions (FFs)

$$\frac{d\sigma^{\pi^0}}{dP_T dY} = \sum_{i,j,k} f_i^P(x_1, Q^2) \otimes f_j^A(x_2, Q^2) \otimes \frac{d\hat{\sigma}^{ij \rightarrow k+X}}{dp_T dy} \otimes D_k^{\pi^0}(x_2, Q^2)$$

- Directly sensitive to gluon PDFs
- Convolutions smear the kinematics
 \Rightarrow Contribution from broad x_2 range
- Fragmentation not completely independent even in pA (baryon production, strangeness enhancement, ...)
- Now data also from LHC

Further data from LHC

[EPJC 77 (2017) 163]

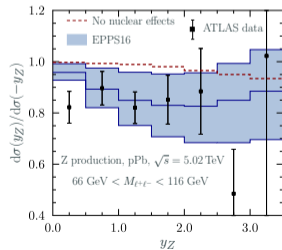
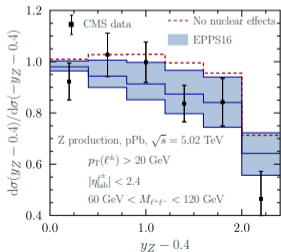


Z bosons in pPb

- Very clean process
- Does not couple to gluons
- High- Q^2 process

Dijets in pPb

- Sensitivity to gluons PDFs
- Theoretically well known and only small effects from hadronization
- Requires high- $p_T \Rightarrow$ difficult to study small- x



[EPJC 77 (2017) 163]₂₀

Recent nPDF analyses

	EPS09	DSSZ12	KA15	NCTEQ15	EPPS16
Order in α_s	NLO	NLO	NNLO	NLO	NLO
DIS in $\ell^- + A$	✓	✓	✓	✓	✓
Drell-Yan in p+A	✓	✓	✓	✓	✓
RHIC pions d+Au	✓	✓		✓	✓
Neutrino-nucleus DIS		✓			✓
Drell-Yan in $\pi + A$					✓
LHC p+Pb dijets					✓
LHC p+Pb W, Z					✓
Q cut in DIS	1.3 GeV	1 GeV	1 GeV	2 GeV	1.3 GeV
datapoints	929	1579	1479	708	1811
free parameters	15	25	16	16	20
error analysis	Hessian	Hessian	Hessian	Hessian	Hessian
error tolerance $\Delta\chi^2$	50	30	N.N	35	52
proton baseline PDFs	CTEQ6.1	MSTW2008	JR09	CTEQ6M-like	CT14NLO
Heavy-quark effects		✓		✓	✓
Flavour separation				partial	full
Reference	JHEP 0904 065	PR D85 074028	PR D93, 014026	PR D93 085037	EPJ C77 163

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Neutrino-nucleus DIS		✓			✓
Drell-Yan in $\pi + A$					✓
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Parametrizations

- EPPS16: piece-wise function for R_i^A

$$R_i^{\text{EPPS16}}(x, Q_0^2) = \begin{cases} a_0 + a_1(x - x_a)^2 & x \leq x_a \\ b_0 + b_1x^\alpha + b_2x^{2\alpha} + b_3x^{3\alpha} & x_a < x \leq x_e \\ c_0 + (c_1 - c_2x)(1 - x) & x_e < x \leq 1 \end{cases}$$

with A dependence on three parameters

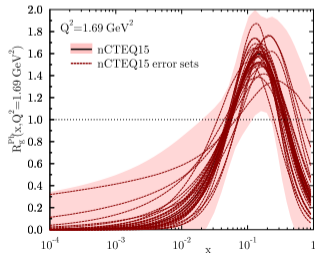
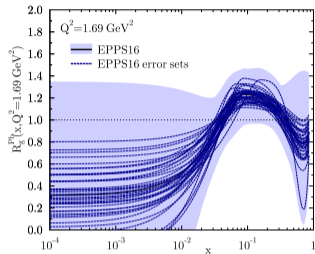
⇒ In total 20 parameters to fit

- nCTEQ15: CTEQ-like parametrization for $f_i^{p/A}$:

$$f_i^{\text{nCTEQ15}}(x, Q_0^2) = c_0x^{c_1}(1-x)^{c_2}e^{c_3x}(1+e^{c_4x})^{c_5},$$

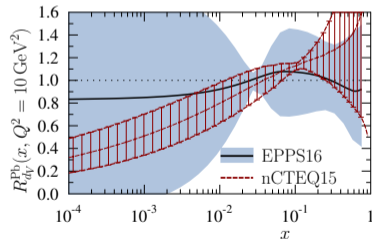
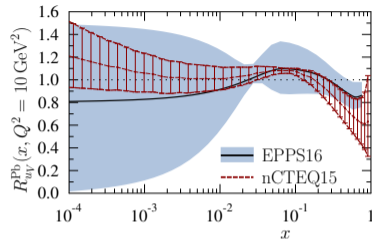
with $c_k(A) = c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}})$

16 free parameters, $R_i^A = f_i^{p/A} / f_i^p$



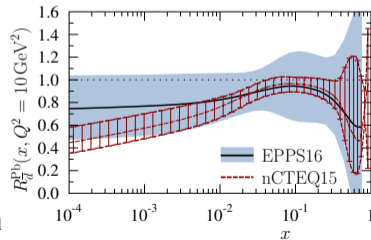
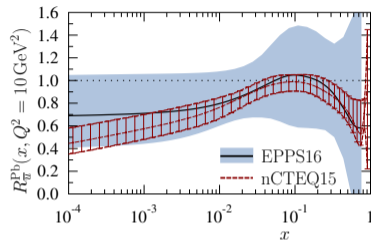
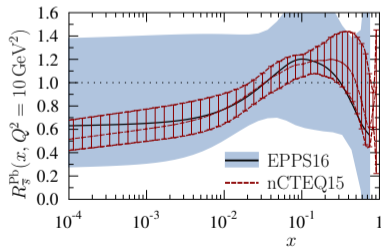
Nuclear modification

- In both analyses u_V and d_V parameters independent
 - However, nCTEQ15 finds large differences whereas in EPPS16 behaviour similar
 - EPPS16 use also neutrino-DIS data sensitive to flavour separation
 - Remember: $f_i^{p/A} \neq f_i^A$
- ⇒ Total uncertainty reduced compared to individual ones as uncertainties correlated



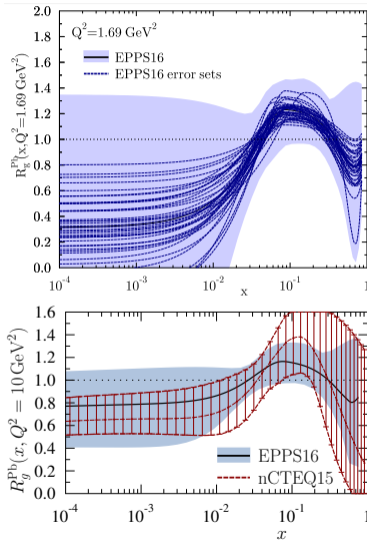
Nuclear modification

- In nCTEQ15 the sea quark parametrization flavor independent
- In EPPS16 $R_{\bar{u}}$, $R_{\bar{d}}$, $R_{\bar{s}}$ free
- ⇒ Larger uncertainties in EPPS16 but less biased
- Uncertainties in s-quark nPDFs large due to lack of data

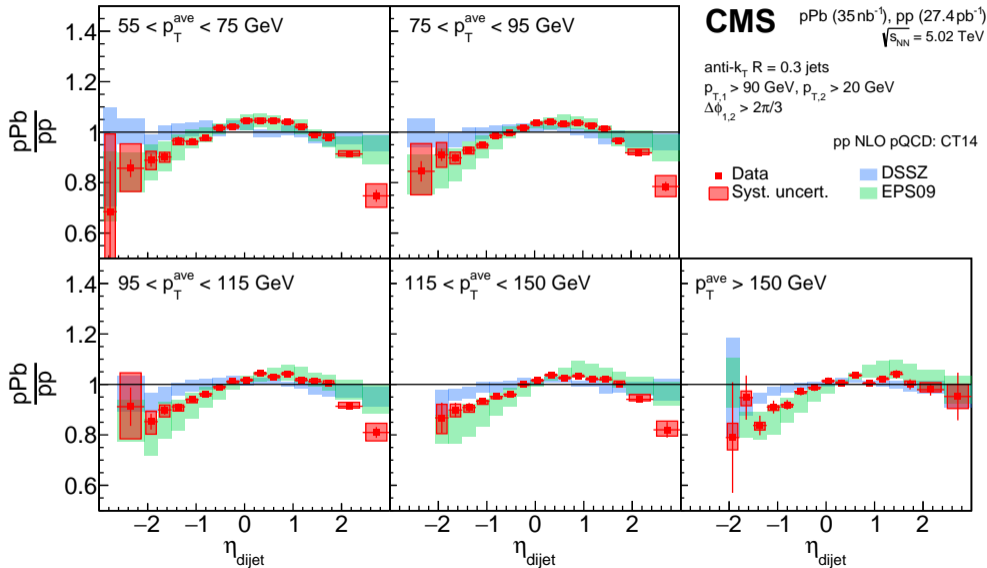


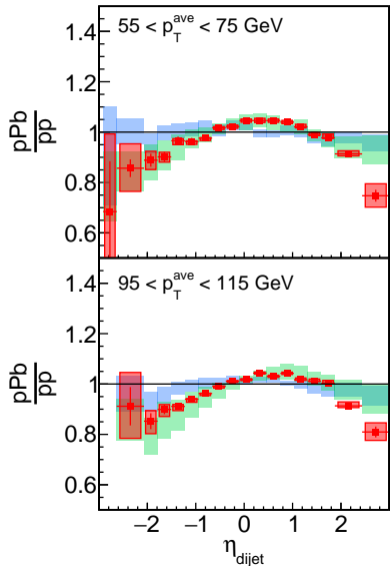
Nuclear modification

- Data constrain gluons only around $x \sim 0.1$
⇒ Large uncertainties at small- x
- However, scale evolution rapidly shrinks the uncertainties at small- x as these originate from well-constrained quarks at higher x
- Reasonable agreement between the analyses
 - **Intermediate x** : Smaller uncertainty in EPPS16 due to dijet data
 - **Small x** : Smaller uncertainty in nCTEQ15 probably due to more restrictive parametrization

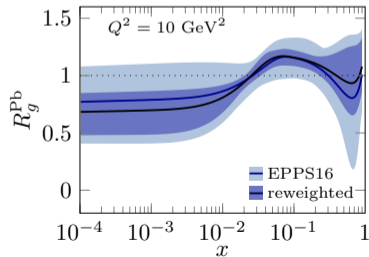


New constraints from LHC



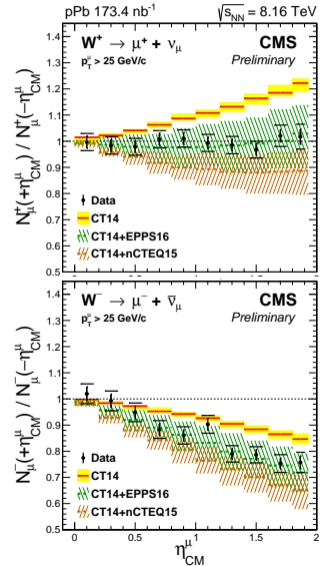


- Increased statistics, More data points
- Impact quantified with reweighting method
- Reduces gluon uncertainty
- Favours enhanced shadowing



New data from CMS

- Old 5 TeV data on W^\pm had only little impact due to small statistical weight
- Now data at 8 TeV with higher precision
- Strong preference for nPDFs
- Very good agreement with EPPS16
- Will provide further constraints for flavor decomposition



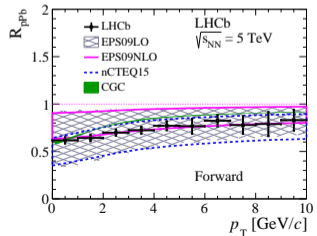
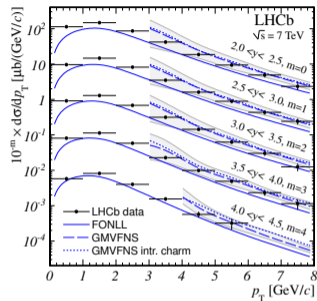
- Recent interest to constrain gluon PDFs in proton with D (and B) mesons

[Gauld, Rojo, PRL 118, 072001; PROSA, EPJ C75, 396]

- Now also data from pPb collisions

D -meson production

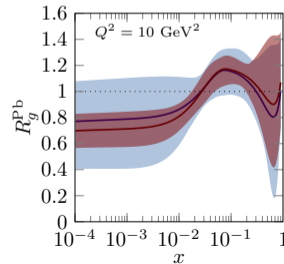
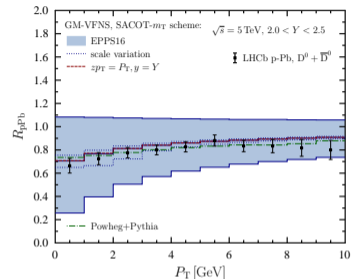
- Contains a c -quark
 \Rightarrow Always produced perturbatively
- Can probe very low x and Q^2 , especially with LHCb acceptance
- Some challenge in theoretical treatment



Heavy-flavour R_{pPb} impact on nPDFs

Strong constraints for gluons

- LHCb data up to $Y = 4$ [JHEP 1710 (2017) 090]
- Sensitivity down to $x \sim 10^{-5}$
- Impact studied with reweighting method
 - Assuming simplified kinematics [Kusina et al., PRL 121, 052004]
 - With full GM-VFNS NLO calculation [Paukkunen et al., *to appear*]
- Good agreement with collinear factorization
- Provides significant reduction of the small- x gluon uncertainty
- Shadowing consistent with the dijet R_{pPb}



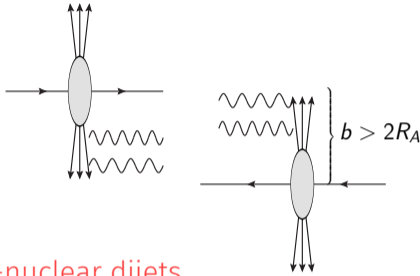
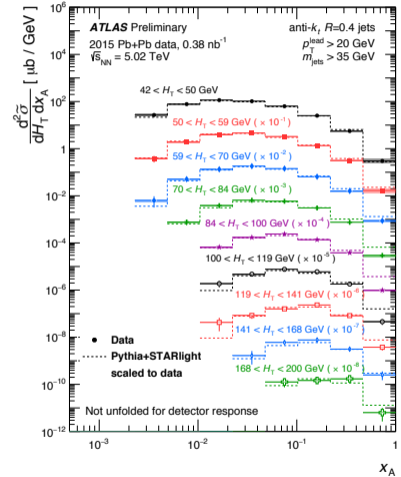


Photo-nuclear dijets

- Nuclei pass without strong interaction
- EM-field, described with quasi-real photons, interacts with another nucleus
 $\Rightarrow \gamma A$ collision that can produce jets
- First preliminary data by ATLAS



[ATLAS-CONF-2017-011]

Future facilities

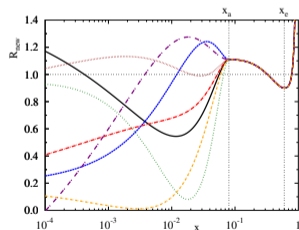
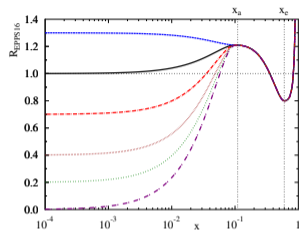
Electron-Ion collider (EIC)

Proposed ep/eA collider projects

- JLEIC@JLAB ($\sqrt{s} \sim 20 - 60$ GeV) [arXiv:1504.07961]
- eRHIC@BNL ($\sqrt{s} \sim 80 - 140$ GeV) [arXiv:1409.1633]

nPDF constraints

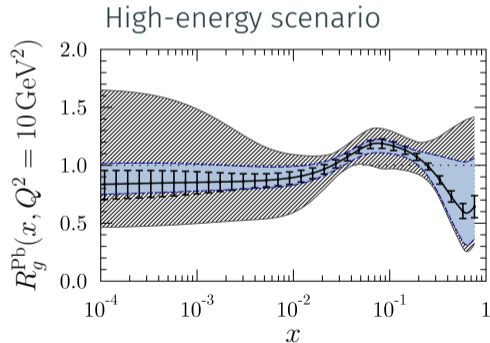
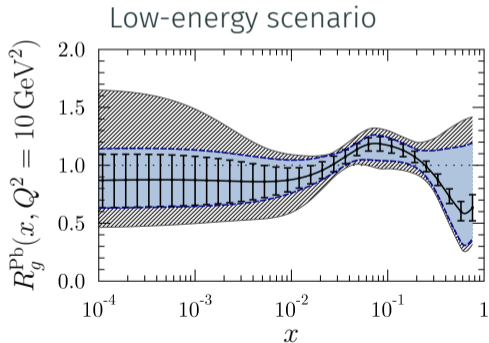
- DIS still the cleanest probe of (n)PDFs
- Need more flexible parametrization for unbiased studies
- $R_{\text{EPPS16}}^A(x < x_a) = a_0 + a_1(x - x_a)^2$
- $R_{\text{NEW}}^A(x < x_a) = a_0 + a_1(x - x_a)^2 + (x - x_a)^2 \sum_{k=1}^2 a_{k+2} x^{k/4}$



[E.C. Aschenauer, S. Fazio, M.A.C. Lamont, H. Paukkunen, P. Zurita, PRD96 (2017) no.11, 114005]

EIC impact on gluon nPDFs

- Increased freedom at small- x yields larger uncertainty (gray) than in EPPS16
- Estimated EIC potential provides significant constraints (blue)
- Some further effect when including also inclusive charm (bars)

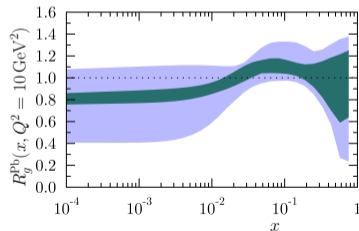
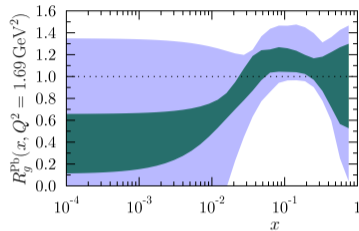


Large hadron-electron collider

- Use the proton/ion beam from LHC
- Construct a new lepton accelerator
 \Rightarrow Provides e+p/A collider with $\sqrt{s} \sim 1$ TeV

Estimated impact on nPDF precision

- Baseline: EPPS16 (blue)
 (Recall parameter-bias at small-x)
- After simulated LHeC data (green)
- A drastic reduction of gluon nPDF uncertainties

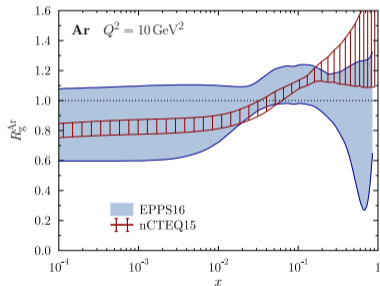
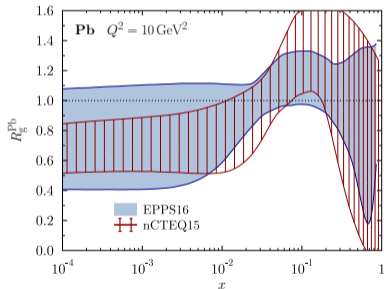


[arXiv:1709.08342]

Loose ends

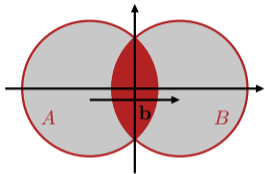
nPDFs for smaller nuclei

- Only fixed-target data for lighter than ^{197}Au , LHC data only for ^{208}Pb
- A-dependence not well in control (parametrization-bias)
- Relevant for astrophysical applications (^{16}O , ^{14}N)



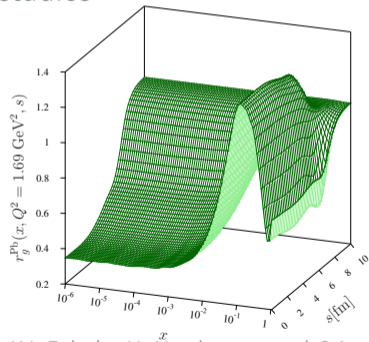
- Very different large- x A dependence in EPPS16 and nCTEQ15

Impact parameter dependence



- Different centralities probes different parts of nuclei
- Currently only min. bias (integrated over impact parameters) data in global analyses

- Only few studies on impact-parameter dependence
- Need more data for further studies



[I.H., K.J. Eskola, H. Honkanen and C.A Salgado, JHEP 1207 (2012) 073]

Nuclear PDFs

- Based on the well-established DGLAP framework
- Essential input for factorization-based cross-section calculations in nuclear collisions
- Currently well-constrained only at $x \gtrsim 0.01$

LHC impact

- Already some LHC data in nPDF analysis
- Further constraints will be provided by
 - W^\pm and Z boson production in pPb (maybe also PbPb)
 - Dijets in pPb collisions, possibly also UPCs
 - Inclusive D -meson production in pPb by LHCb
- So far no observation of factorization breaking at the LHC

Nuclear PDFs

- Add more data from LHC for improved constraints
- Current state-of-the-art NLO in pQCD \Rightarrow first NNLO fits to come
- Need also more data for different nuclei, only Pb at the LHC
- Reduce parametrization bias by releasing some assumptions
- Impact-parameter dependence?

New experimental facilities

- Projected electron-ion colliders can provide clean nPDF constraints (EIC, LHeC, also other proposals)
- Further possibilities with Future Circular Collider (FCC) at CERN ($\sqrt{s} \sim 40 - 100$ TeV)