

# Jet Quenching

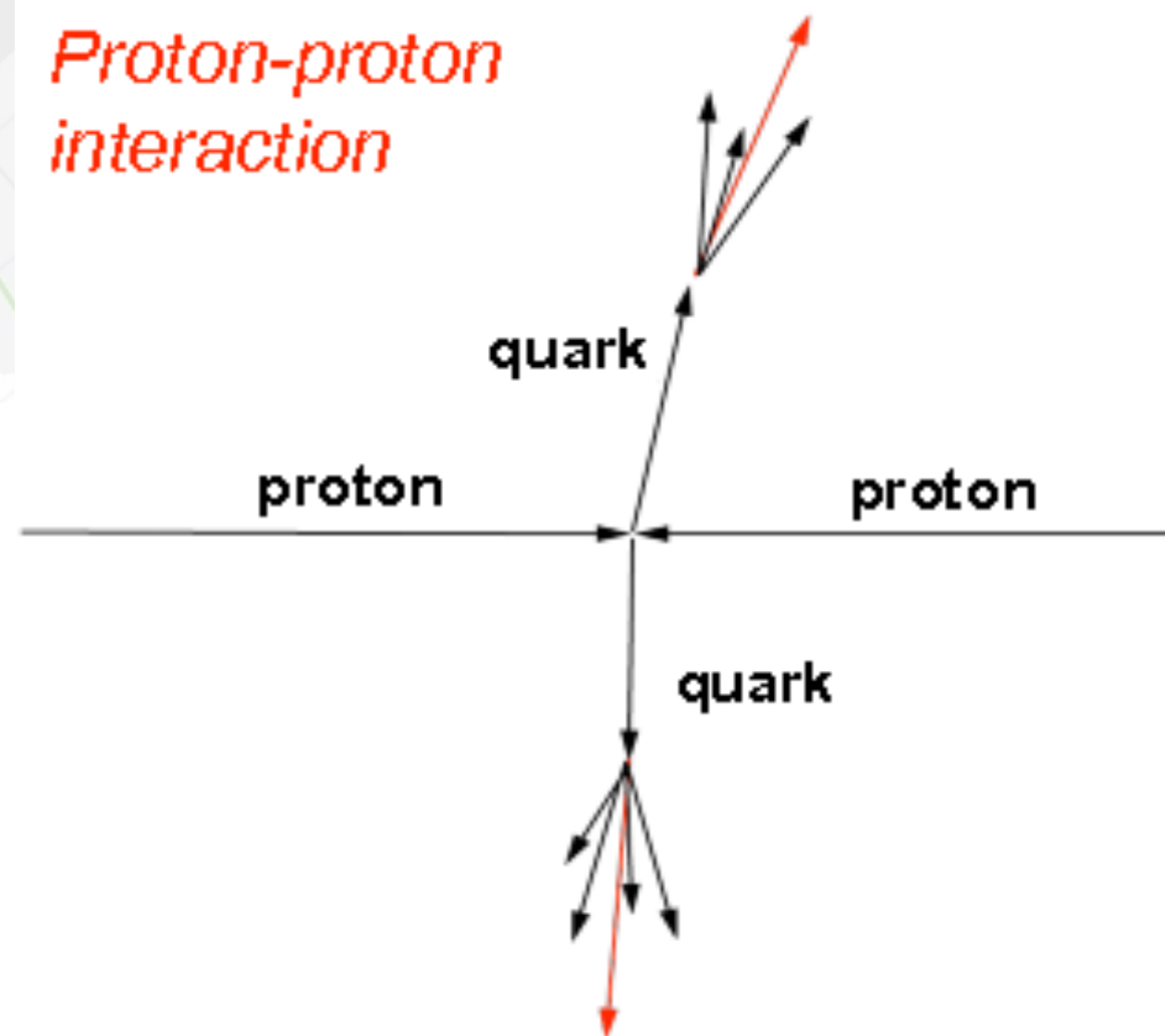
Liliana Apolinário





# Proton-Proton Collision

- ◆ pp collision: a “simple” (few particles) system

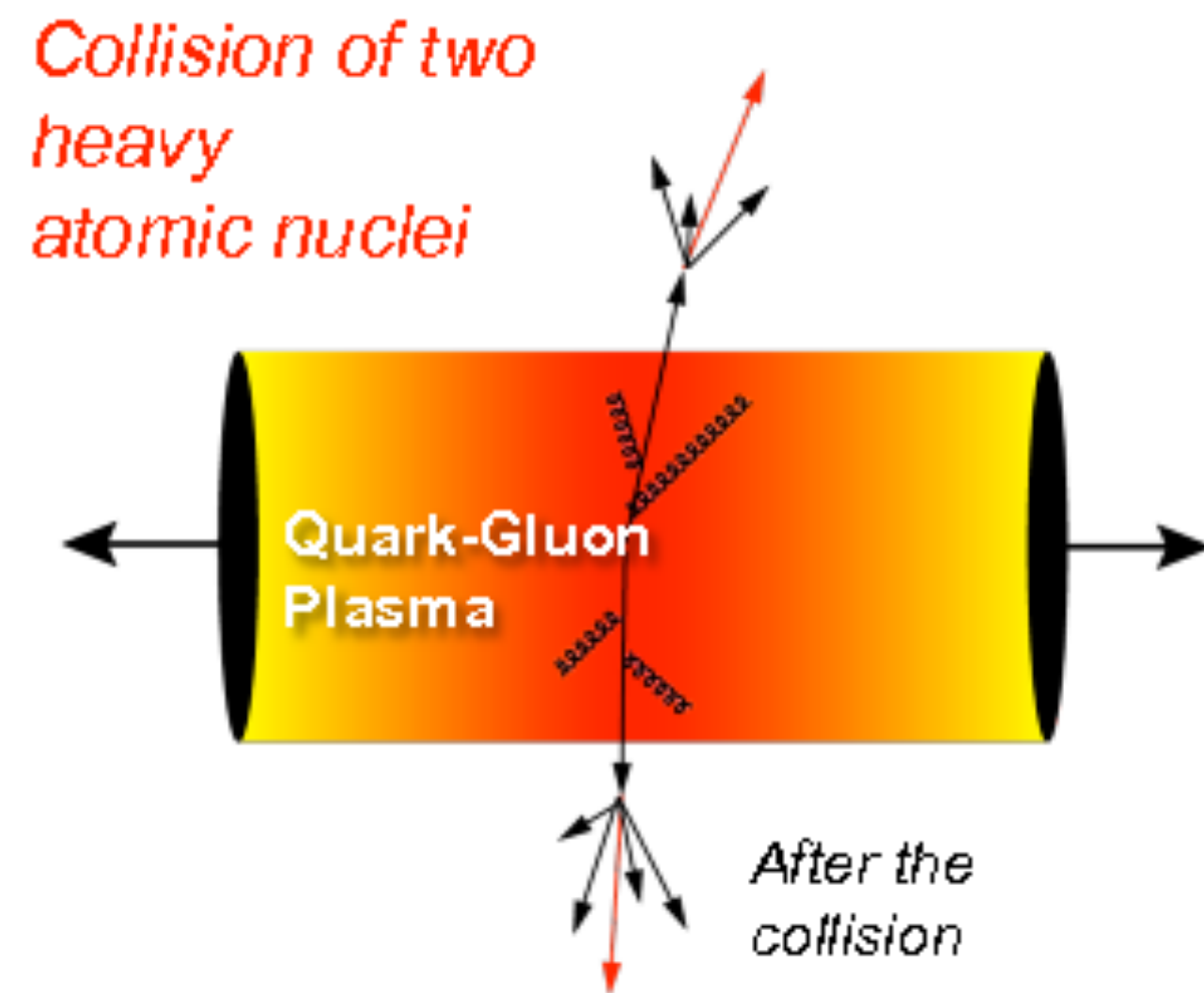


- ➔ Hard scattering:
- ➔ Process evolution can be described with Quantum Field Theory first principles

See talk “Jets” (S. Prestel) Monday

# Heavy-Ions Collision

- ◆ PbPb collision: a complex multi-particle system

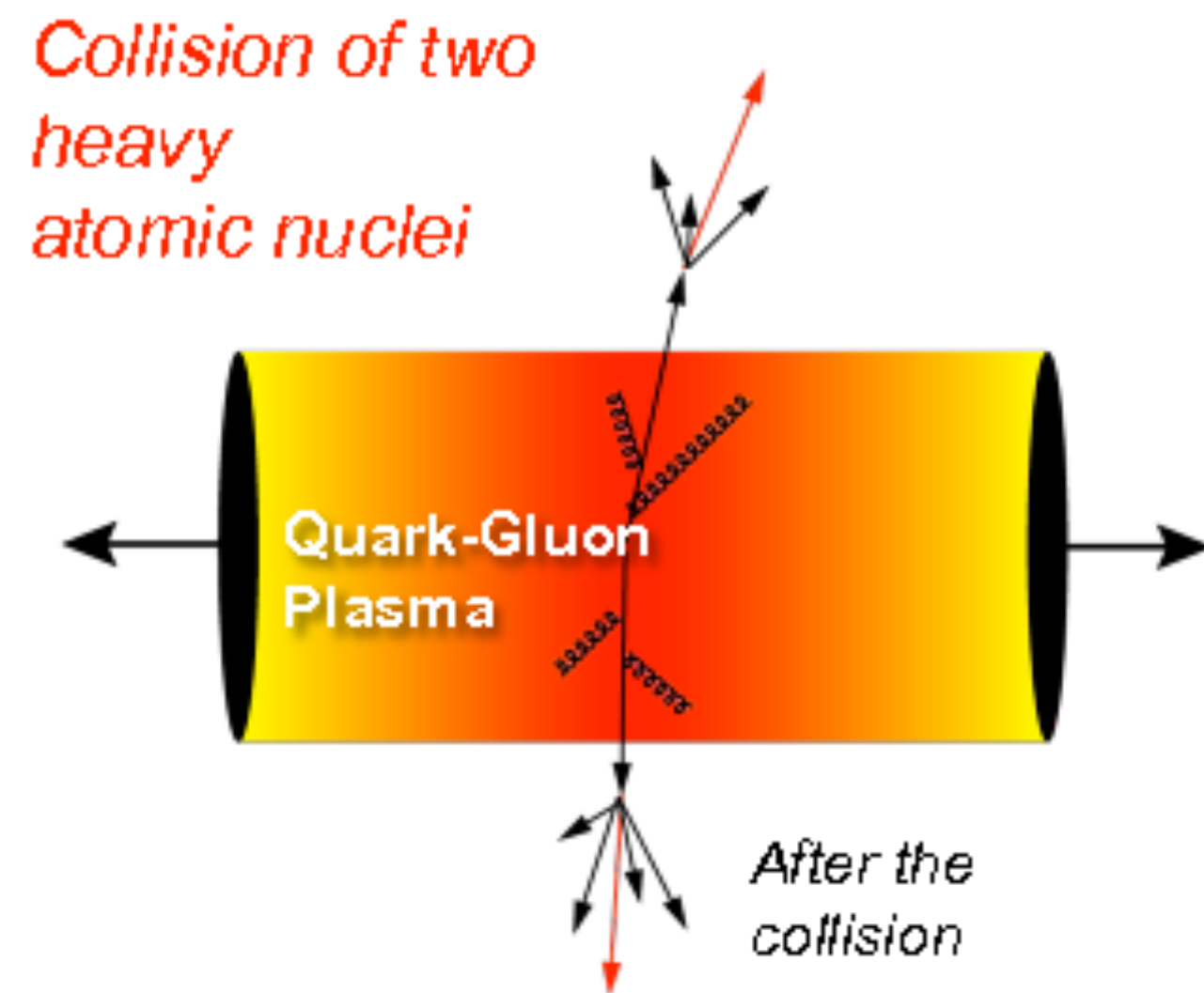


- ➔ Hard scattering

- ➔ Hot and dense medium (QGP)

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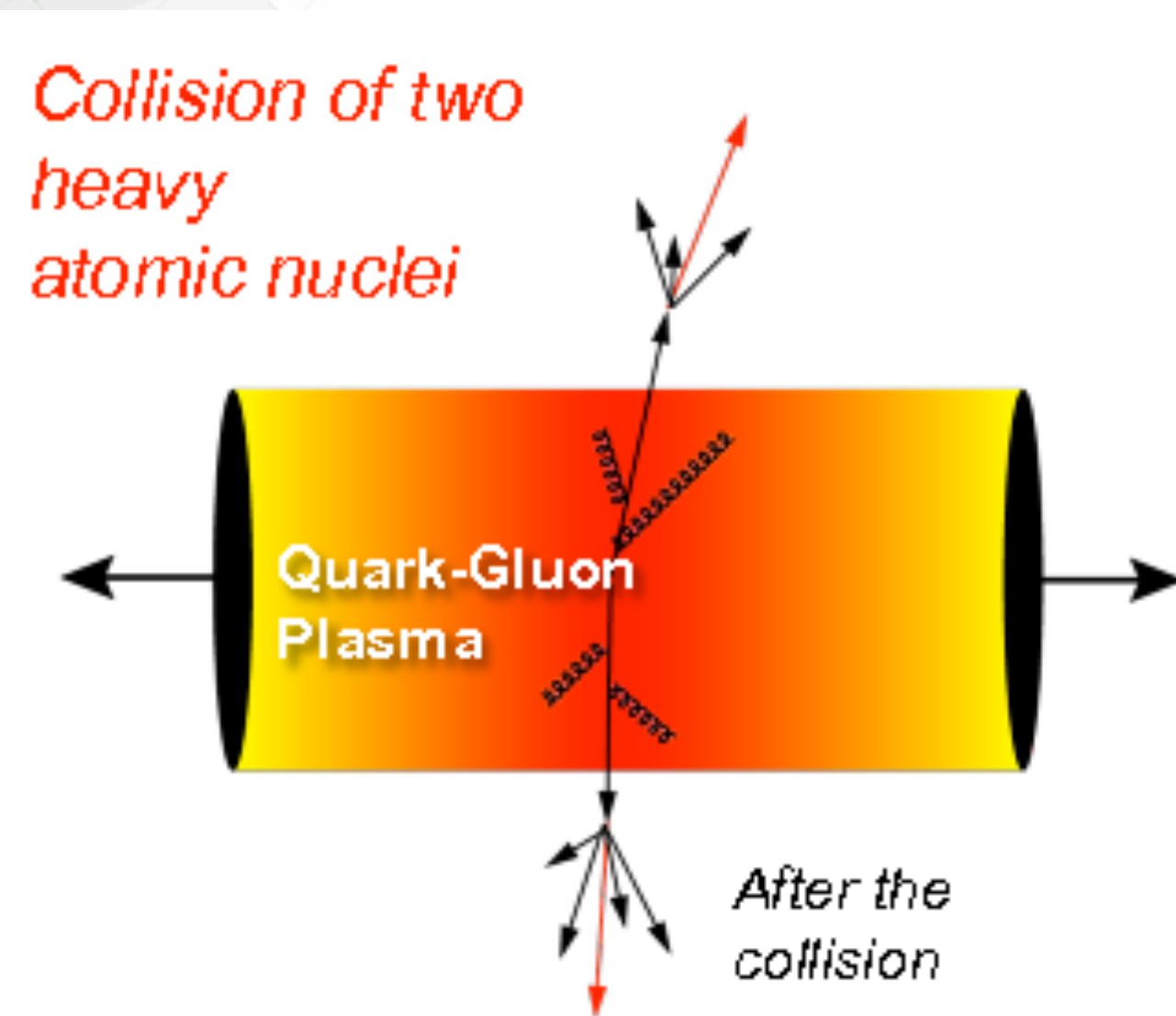


- ➔ Hard scattering
- ➔ Particles modified w.r.t pp: Jet Quenching effects
- ➔ Hot and dense medium (QGP)
- ➔ Fluid with collectivity phenomena
- ➔ Also QCD system, but strongly interacting!
- ➔ How collectivity emerge from a QFT?



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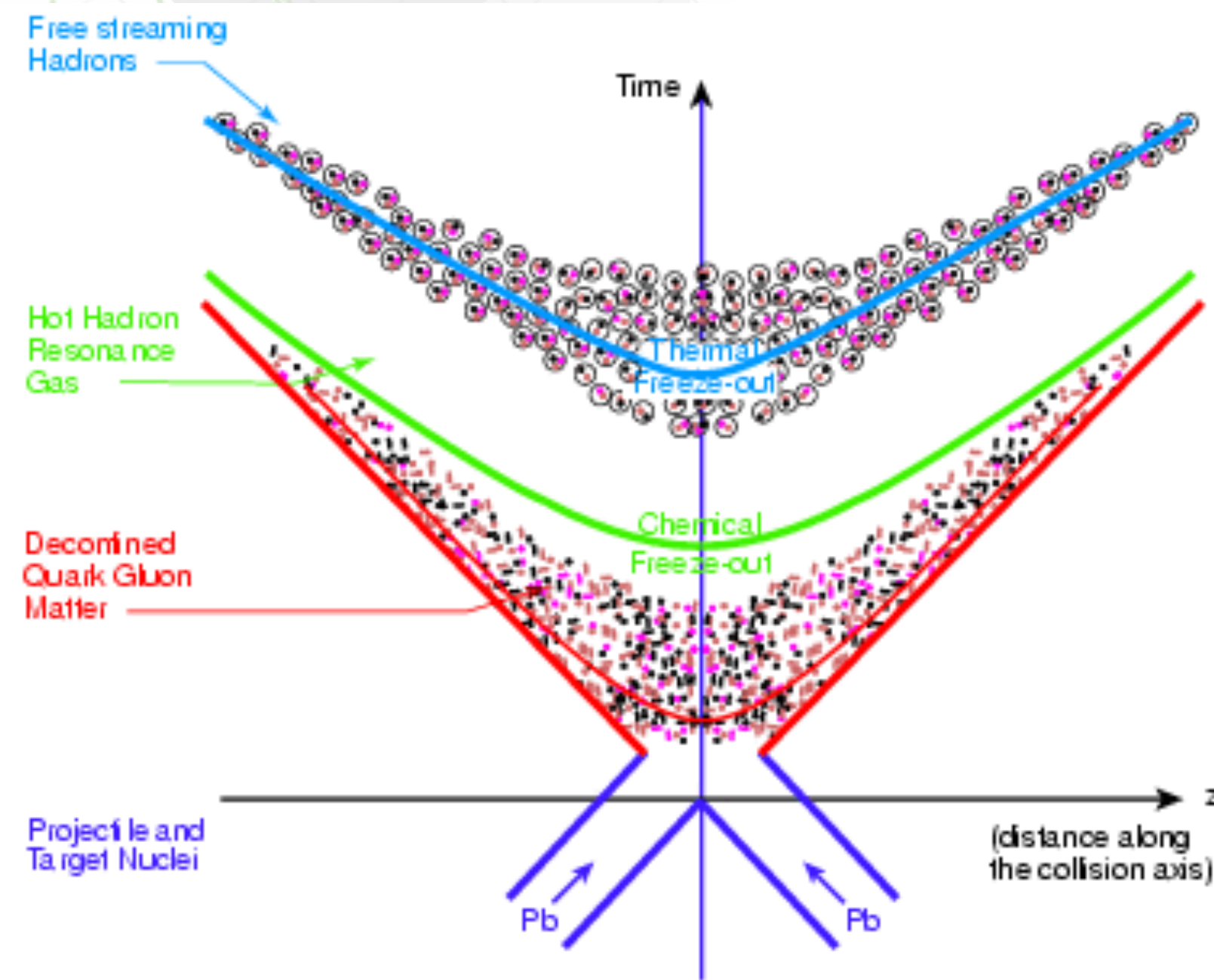


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See talk "Flow" (C. Plumberg) today

# Heavy-Ions: Open Questions

## ◆ PbPb collision evolution



➔ Final state particles (what we measure)

➔ Is the QGP strongly coupled?

➔ How is thermalized?

➔ QCD description in all energy range

➔ Quasi-particles?

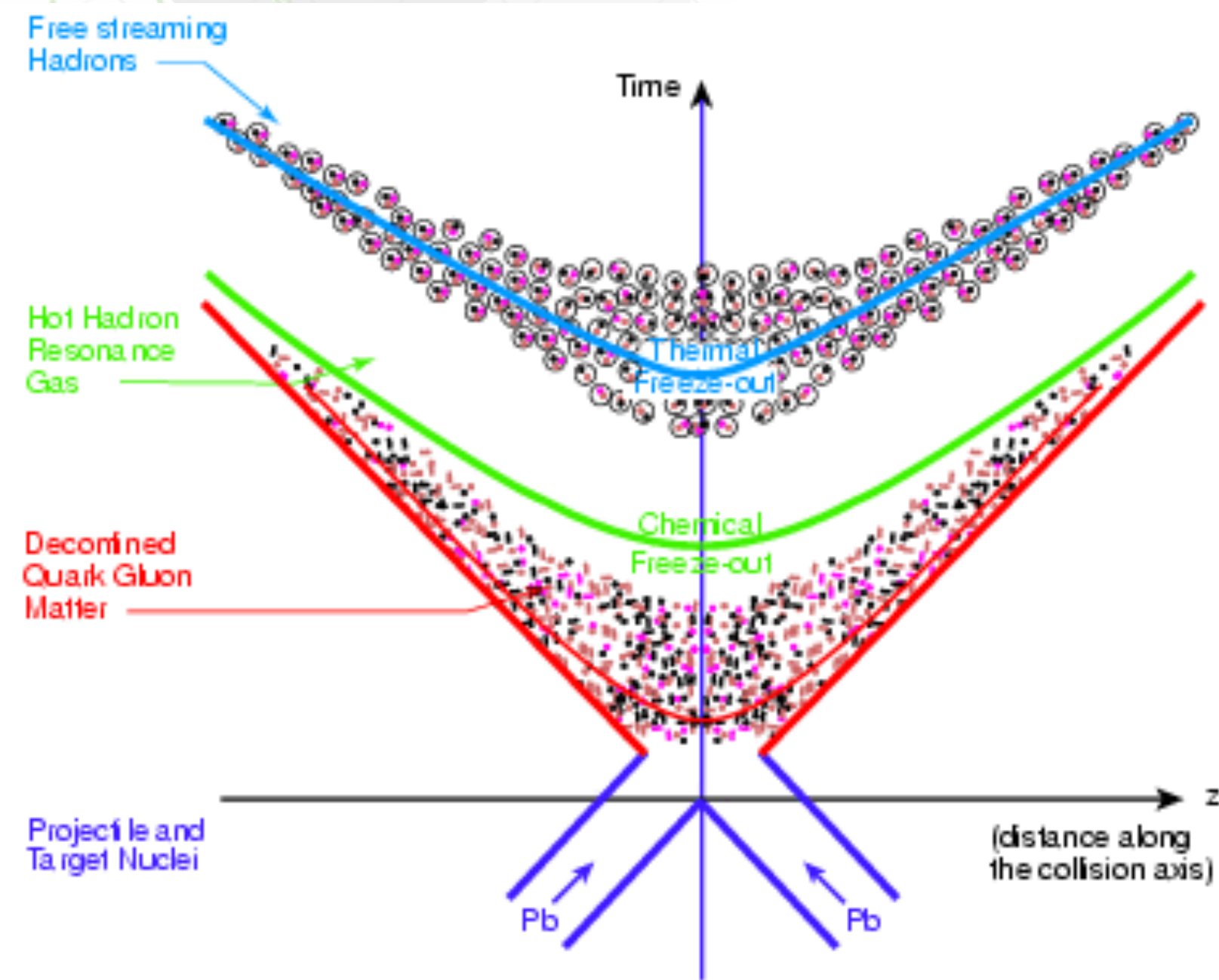
➔ Initial state (incoming nuclei) ?

➔ See talk “Nuclear PDFs” (I.Heleniums) today



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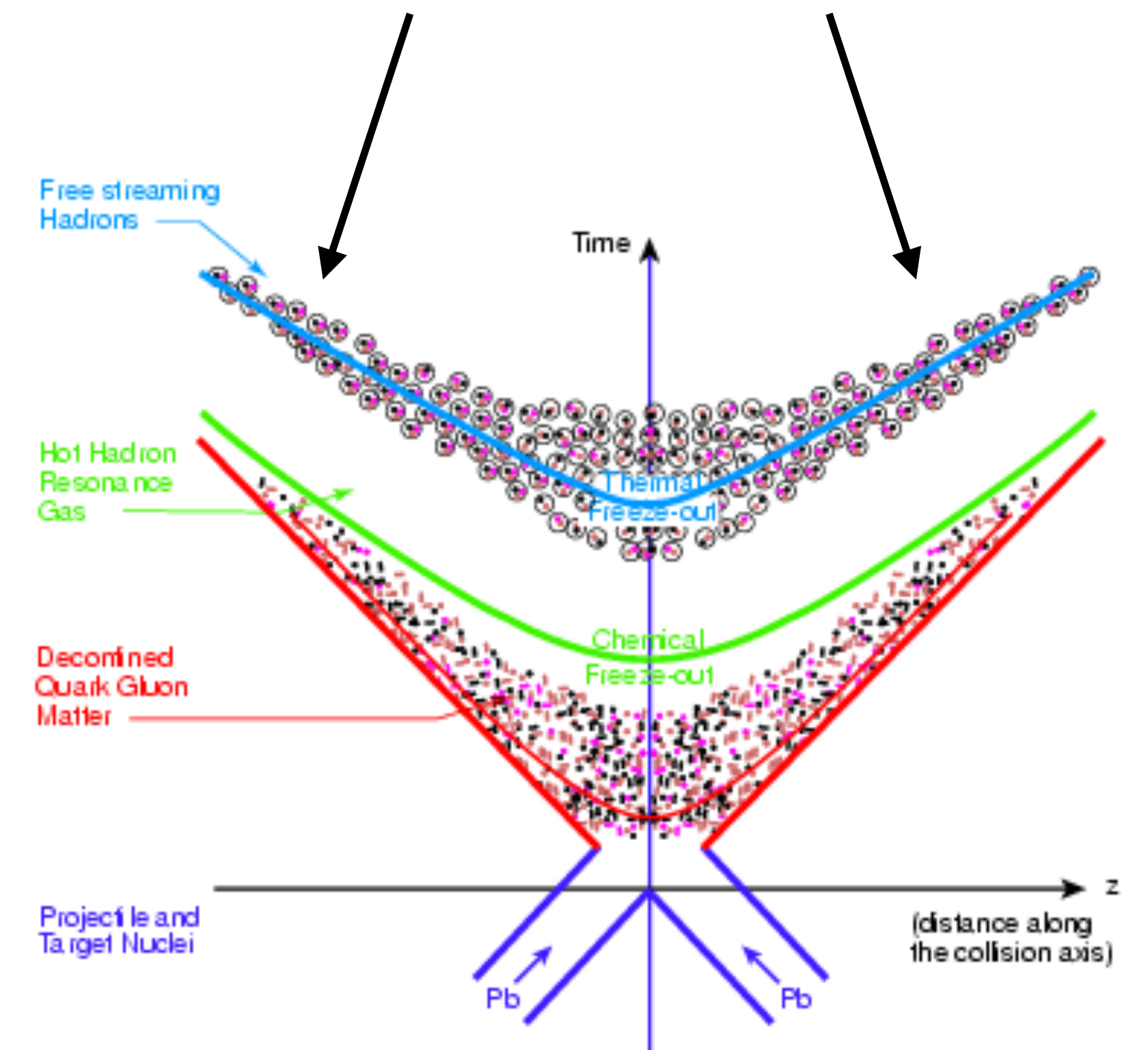
➔ Identify well controlled observables/  
probes to assess QGP properties!!

➔ Initial state (incoming nuclei) ?

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# QGP Probes

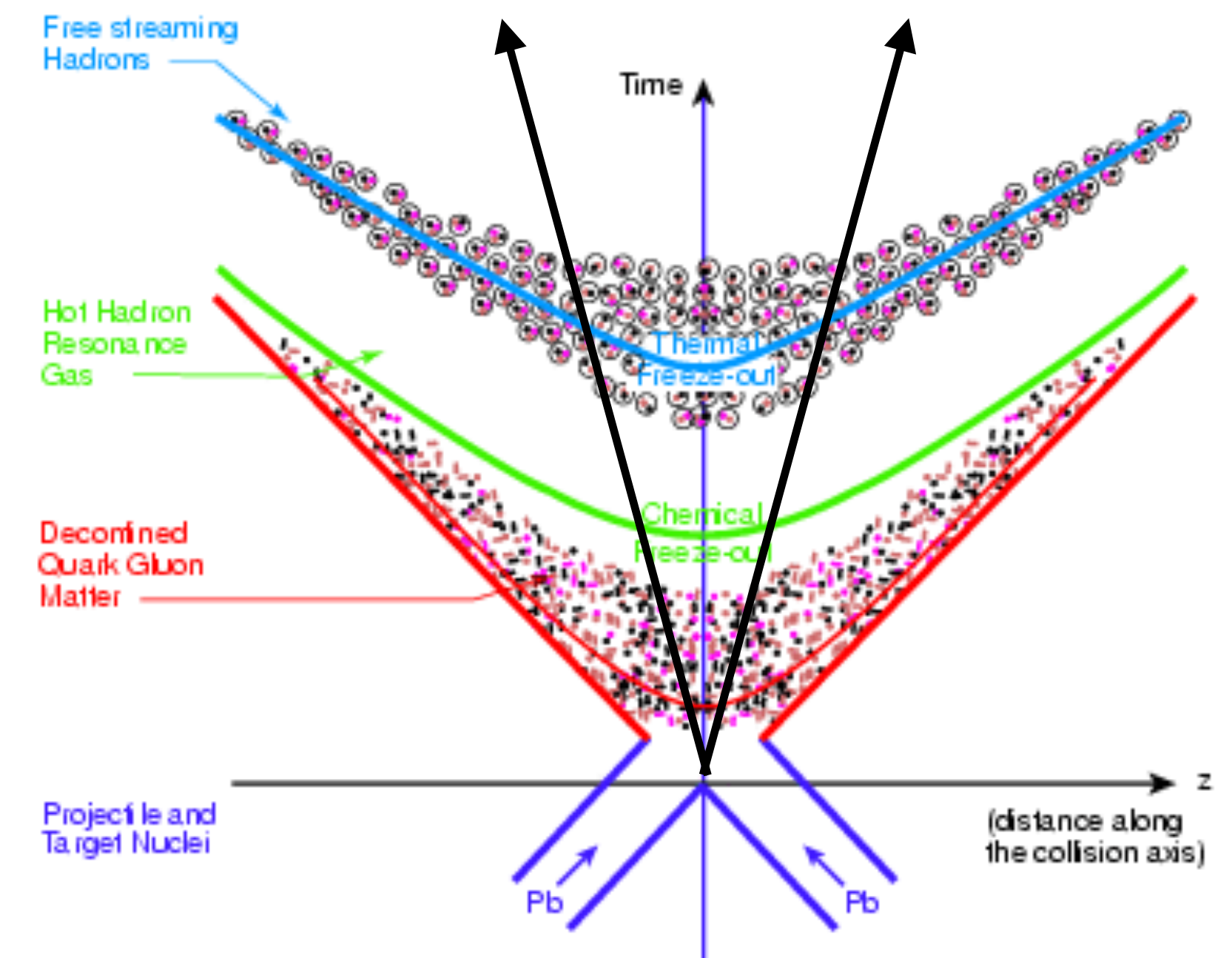
- ◆ Soft probes: flow, hydrochemistry, ...
- ◆ Direct result of the QGP evolution
- ➔ Collective properties and hydrodynamical evolution of the medium





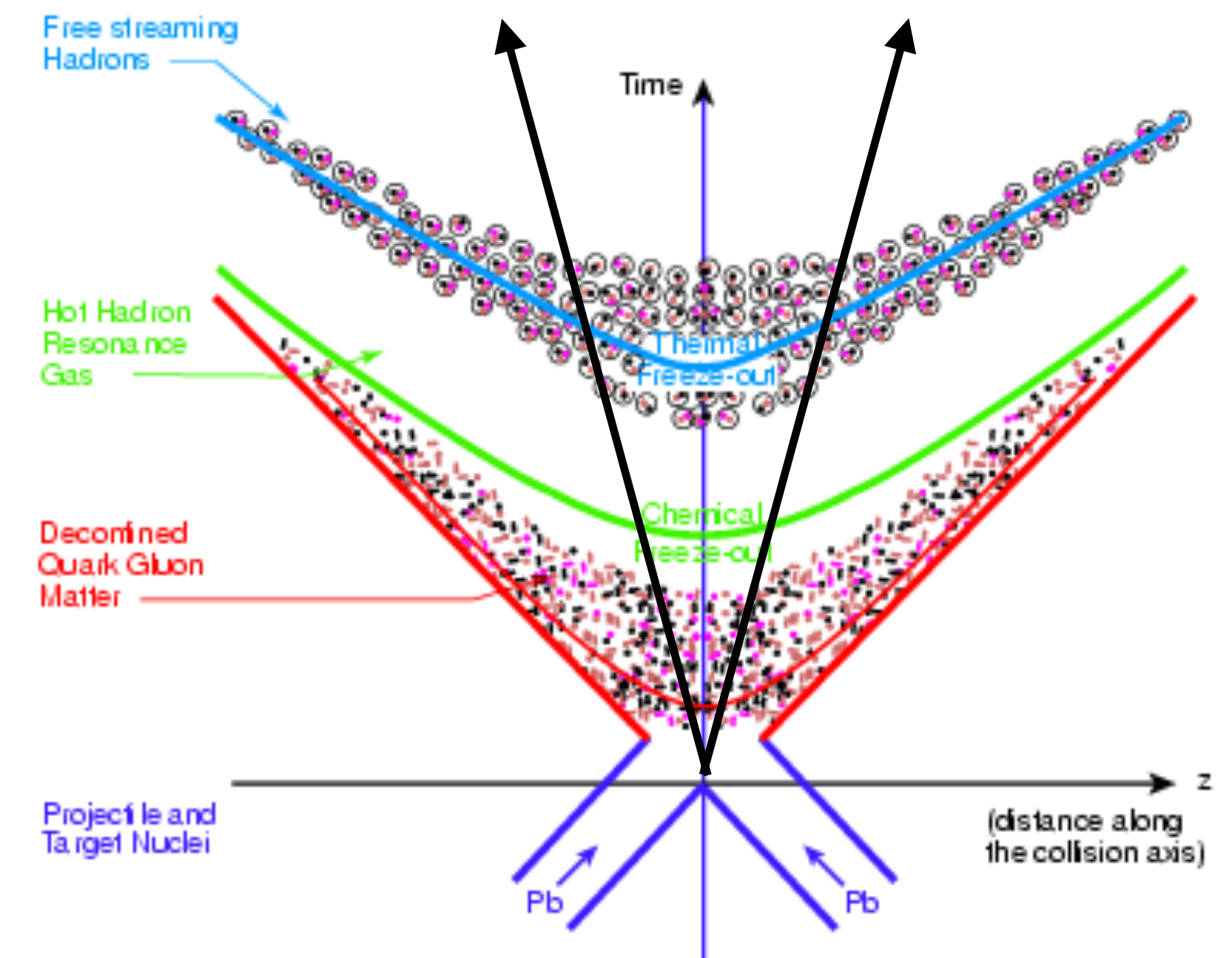
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  - ➔ Observe the evolution of the QGP (temperature, density,...)



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# Why Hard Probes?

- ◆ Better theoretical and experimental control!
- ➔ Like in pp, less sensitive to (unknown) details of incoming nucleus (nPDFs)

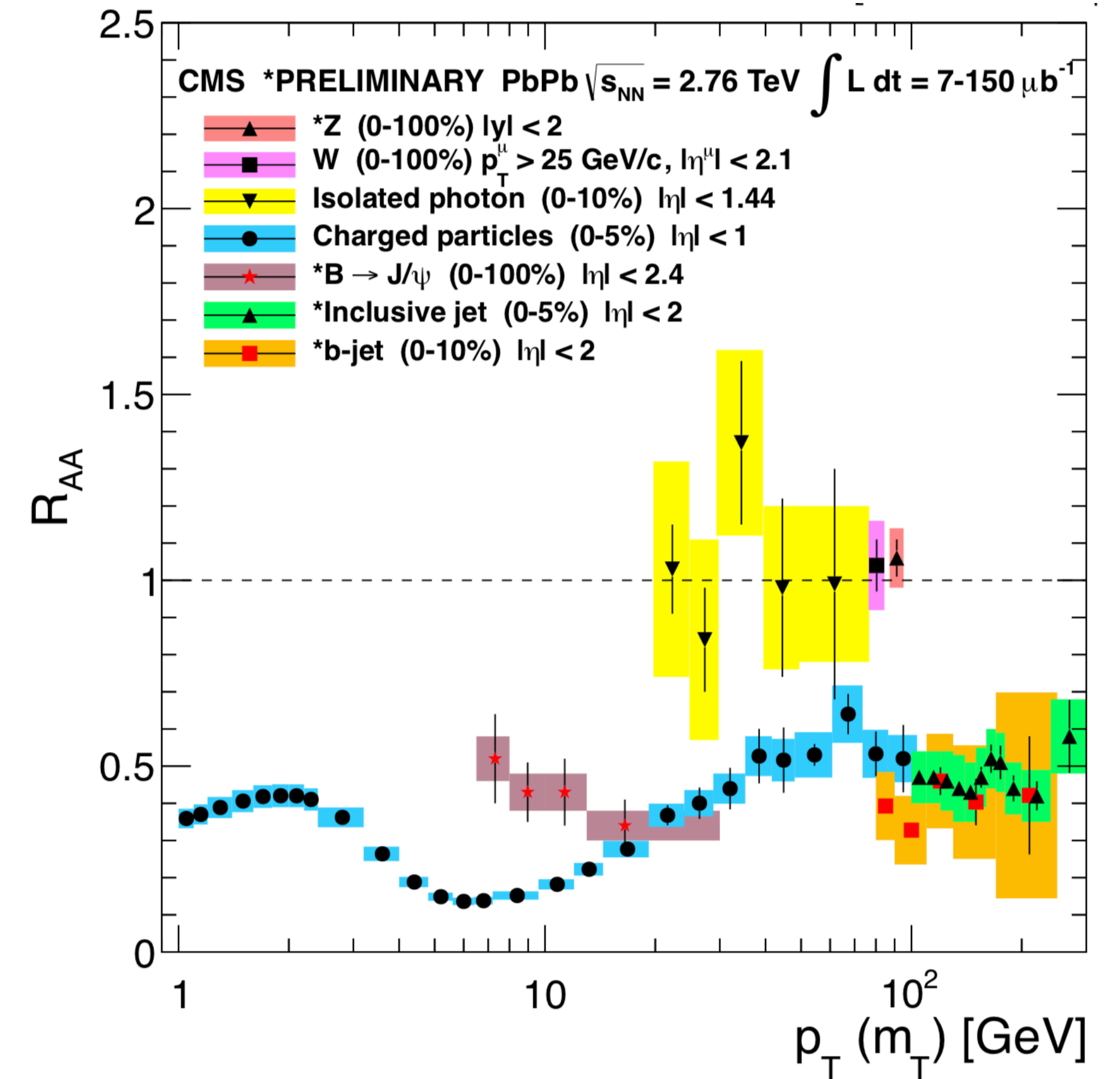
$$d\sigma_{(\text{vac})}^{AA \rightarrow h + \text{rest}} = \sum_{ijk} f_{i/A}(x_1, Q^2) \otimes f_{j/A}(x_2, Q^2) \otimes \hat{\sigma}_{ij \rightarrow f+k} \otimes D_{f \rightarrow h}^{(\text{vac})}(z, \mu_F^2).$$

Unmodified by the QGP!

Elementary "Hard" cross-section

$$d\sigma_{(\text{med})}^{AA \rightarrow h + \text{rest}} = \sum_{ijk} f_{i/A}(x_1, Q^2) \otimes f_{j/A}(x_2, Q^2) \otimes \hat{\sigma}_{ij \rightarrow f+k} \otimes P_f(\Delta E, L, \hat{q}, \dots) \otimes D_{f \rightarrow h}^{(\text{vac})}(z, \mu_F^2).$$

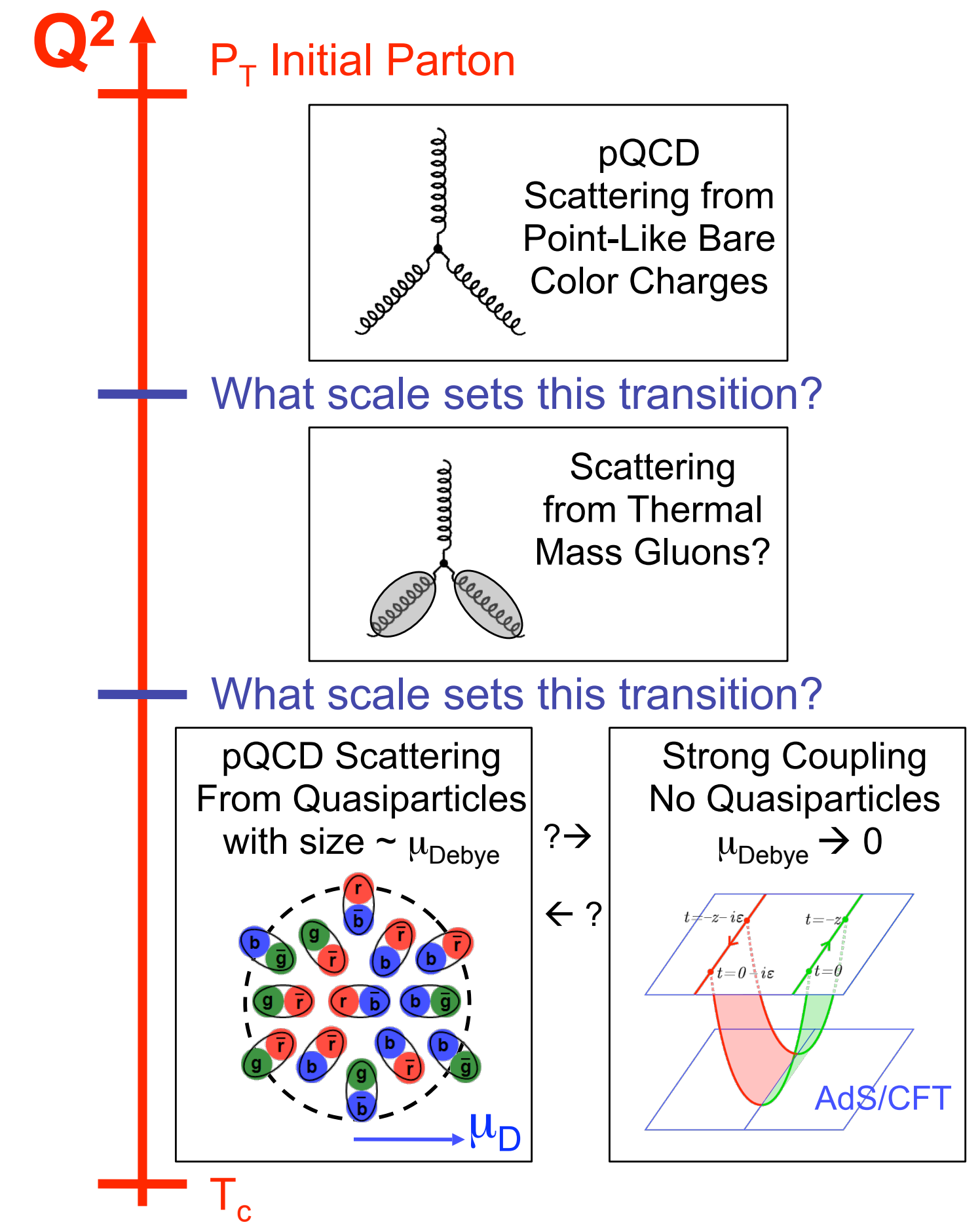
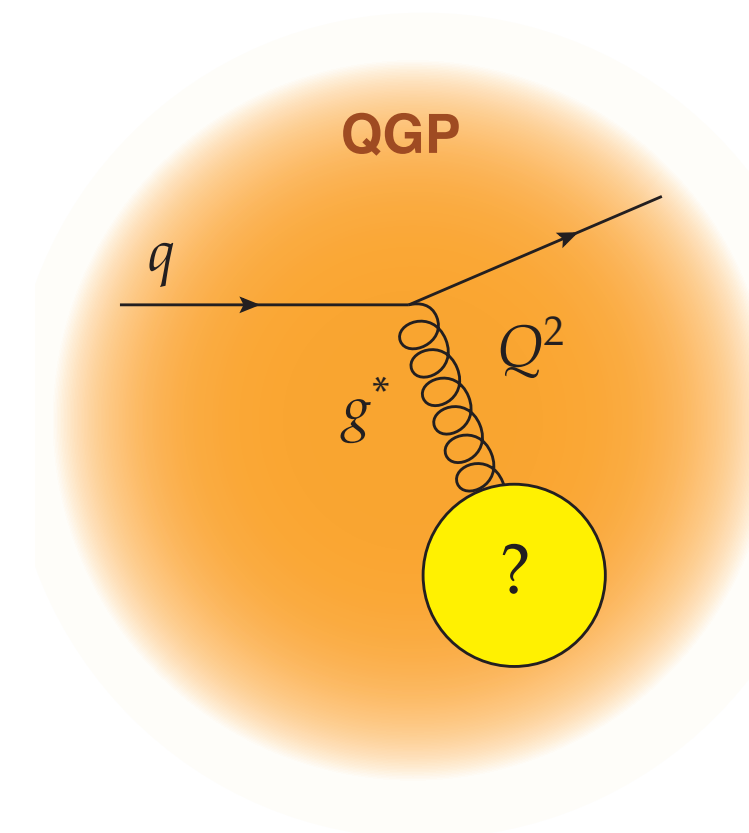
"Vacuum" Hadronization (?)





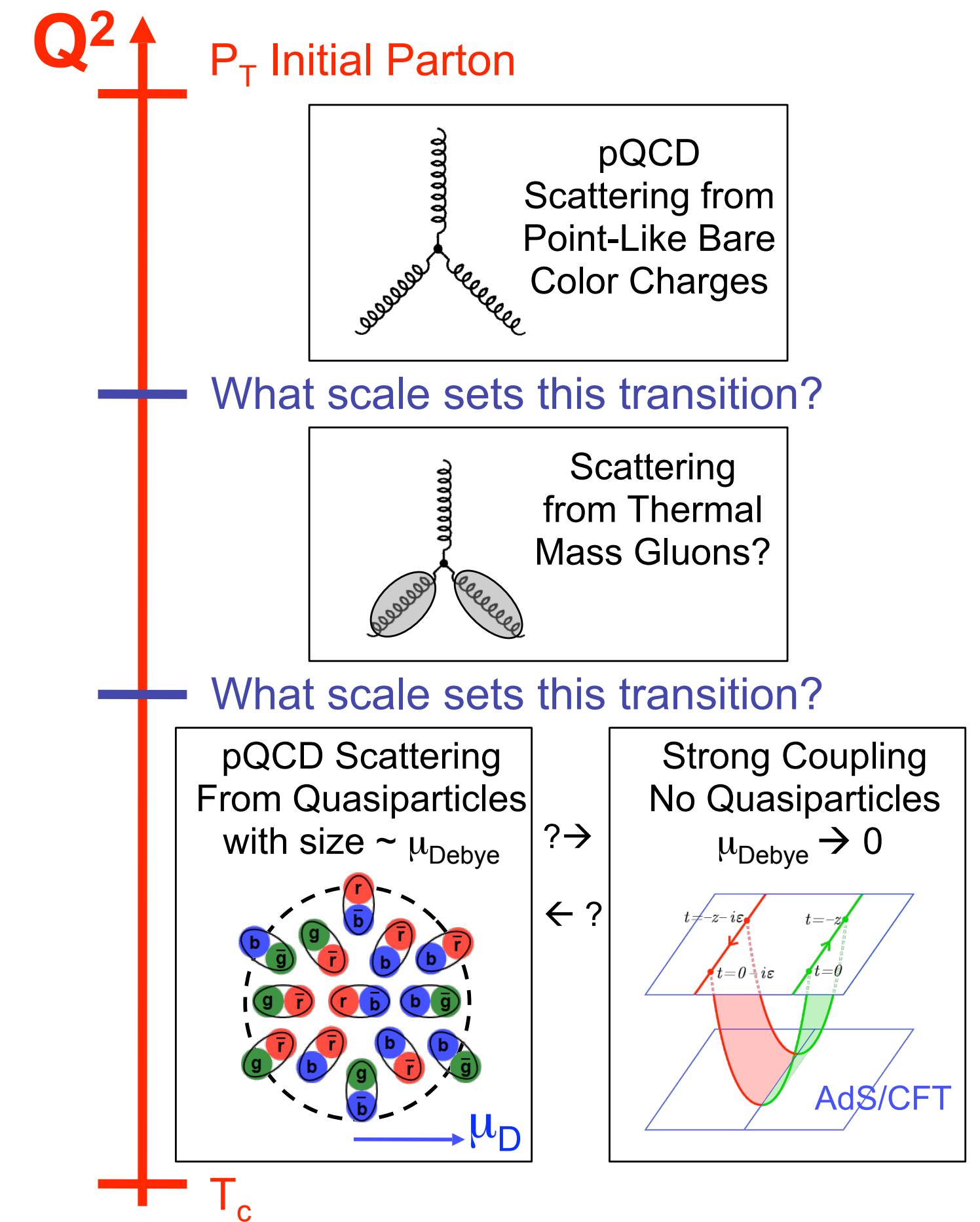
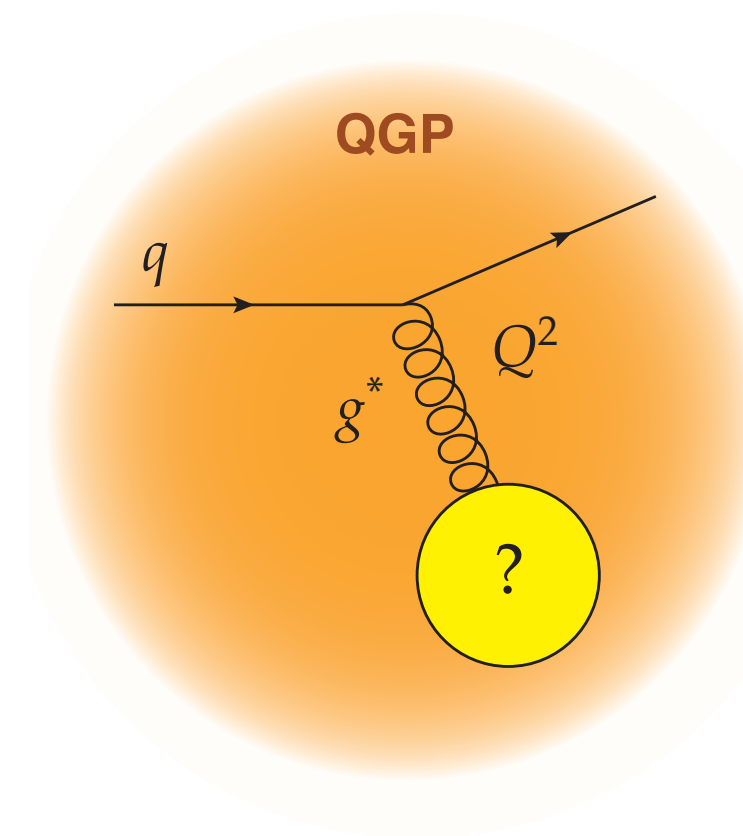
# Why Jets?

- ◆ Formed in the beginning of the collision:
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  - ➔ QGP evolution (E.g: thermalisation process)
- ◆ Formed by collection of soft to hard particles
  - ➔ Allow QGP probing by different scales
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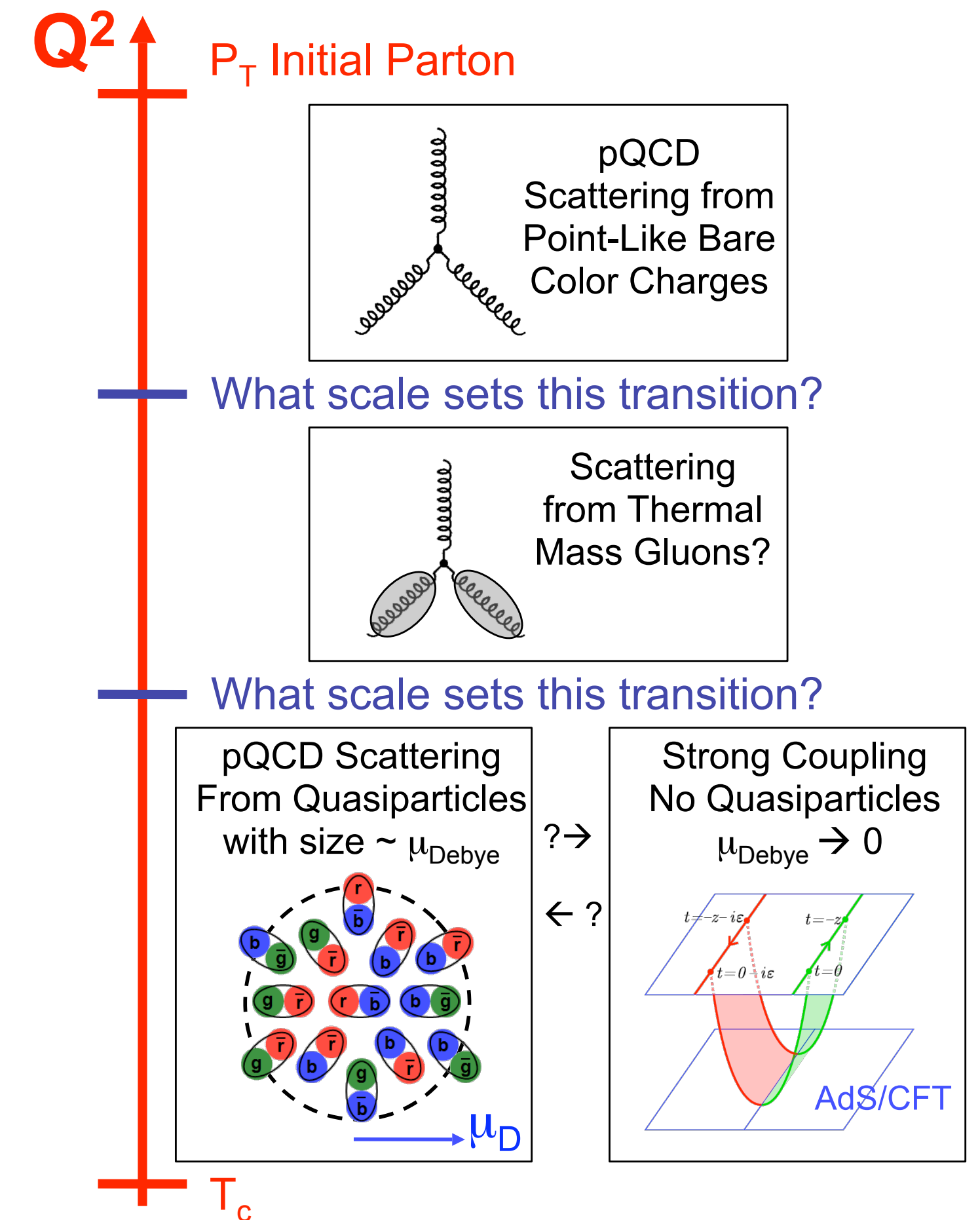
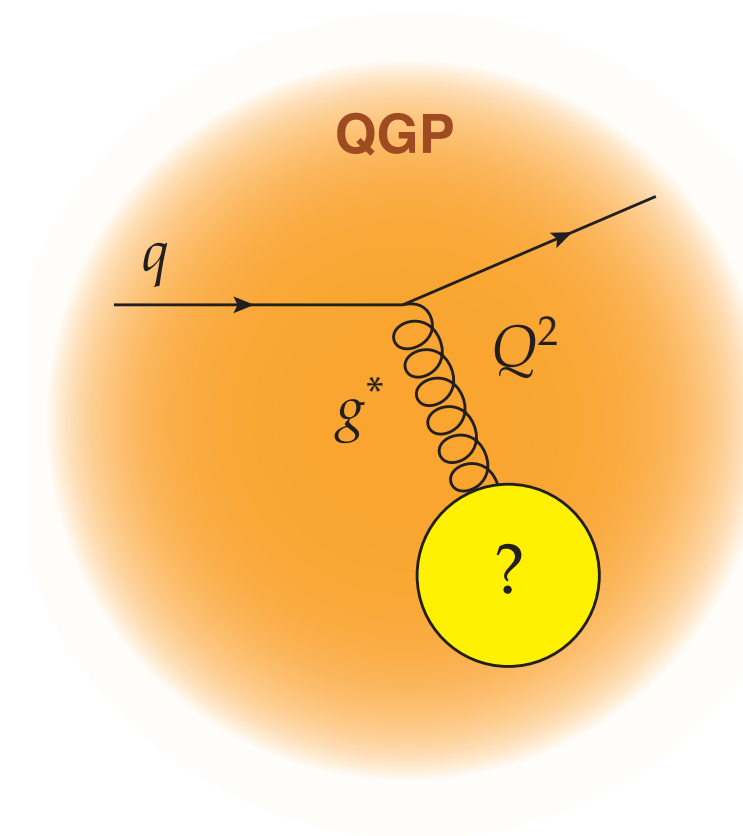


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Welcome to the field of jet quenching!

But before we start...



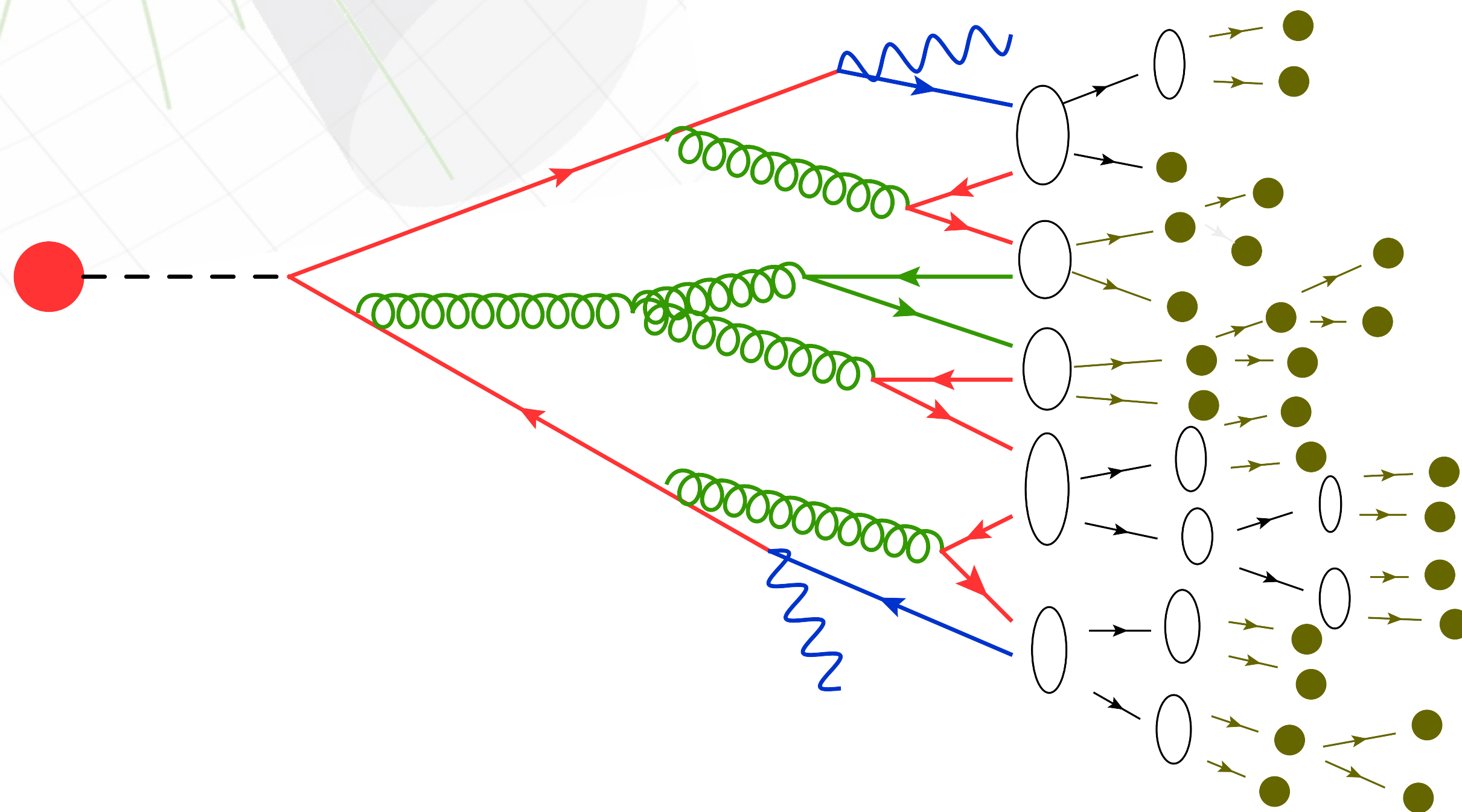
The background features a light gray grid pattern. Overlaid on this are several abstract elements: a 3D-style olive green cube in the top left; a series of purple lines, some solid and some dotted, that curve and converge towards the right side of the frame; and a collection of overlapping, semi-transparent geometric shapes in shades of olive green and light green on the left side.

**Revisiting our  
baseline: pp**



# From parton to jets in pp

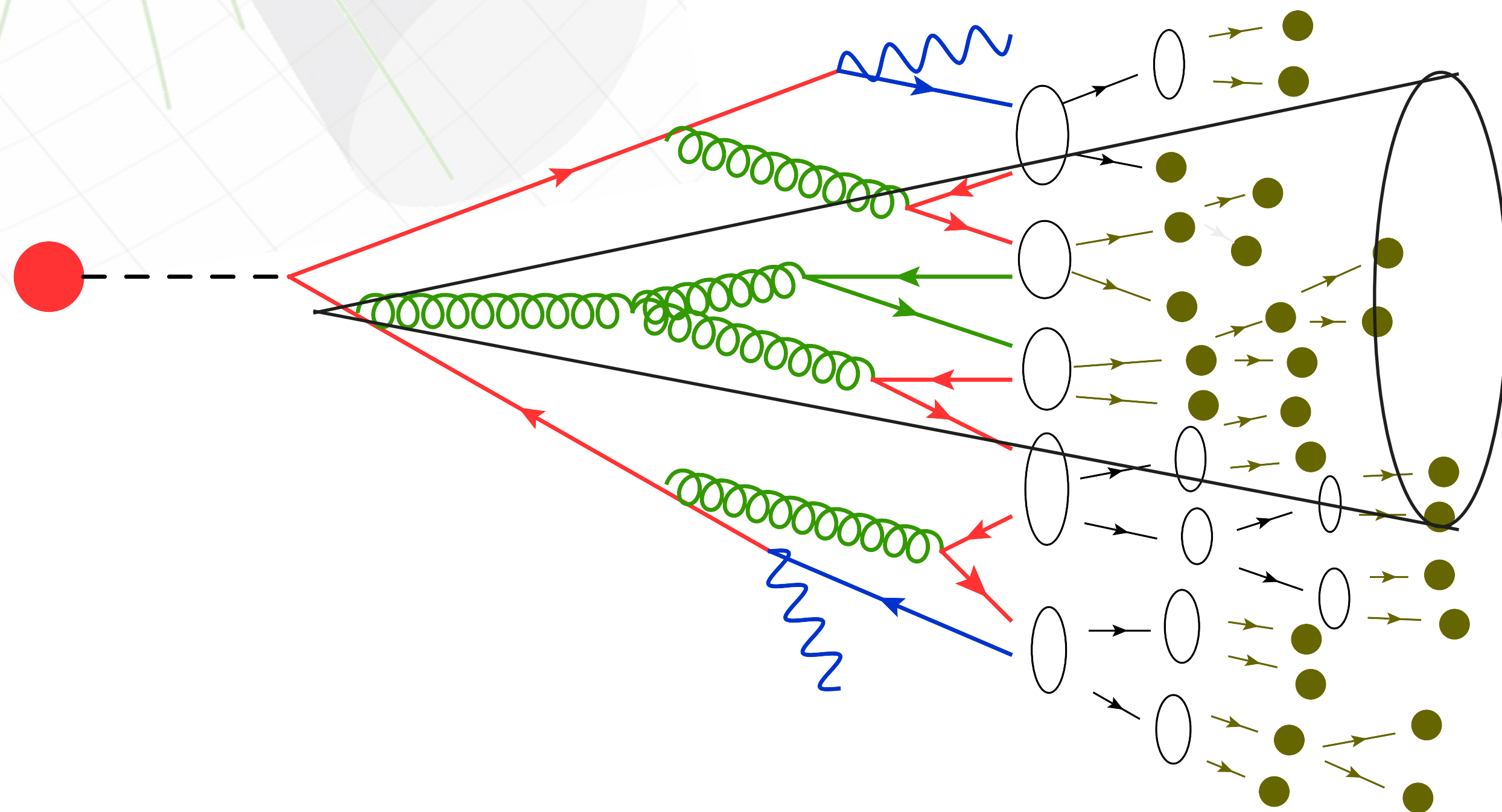
◆ “Vacuum” parton shower:



- ➔ A jet is a subsequent process of single parton emissions
- ➔ Avoid non perturbative effects (e.g: hadronization)

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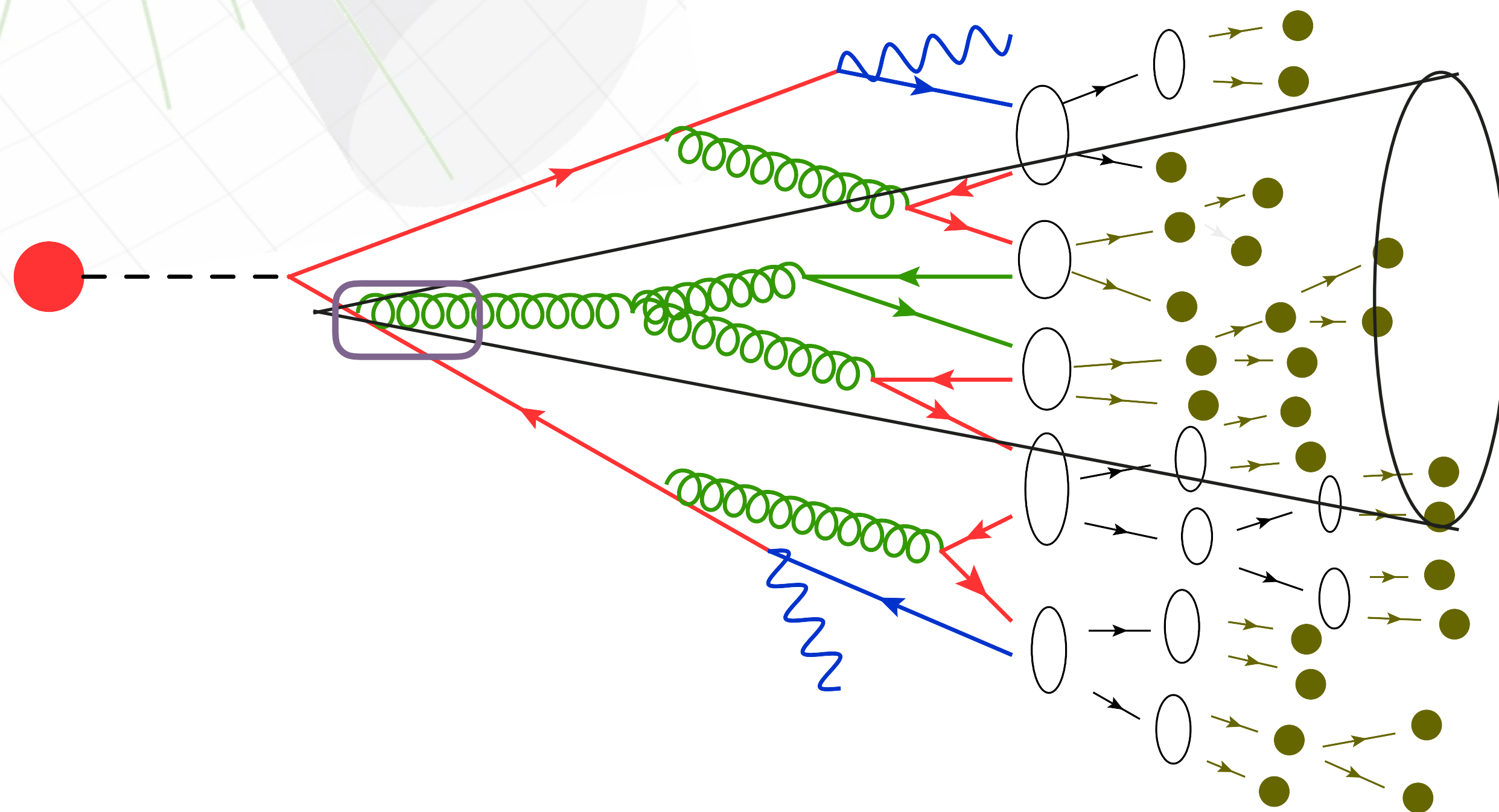


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Revisiting building blocks!

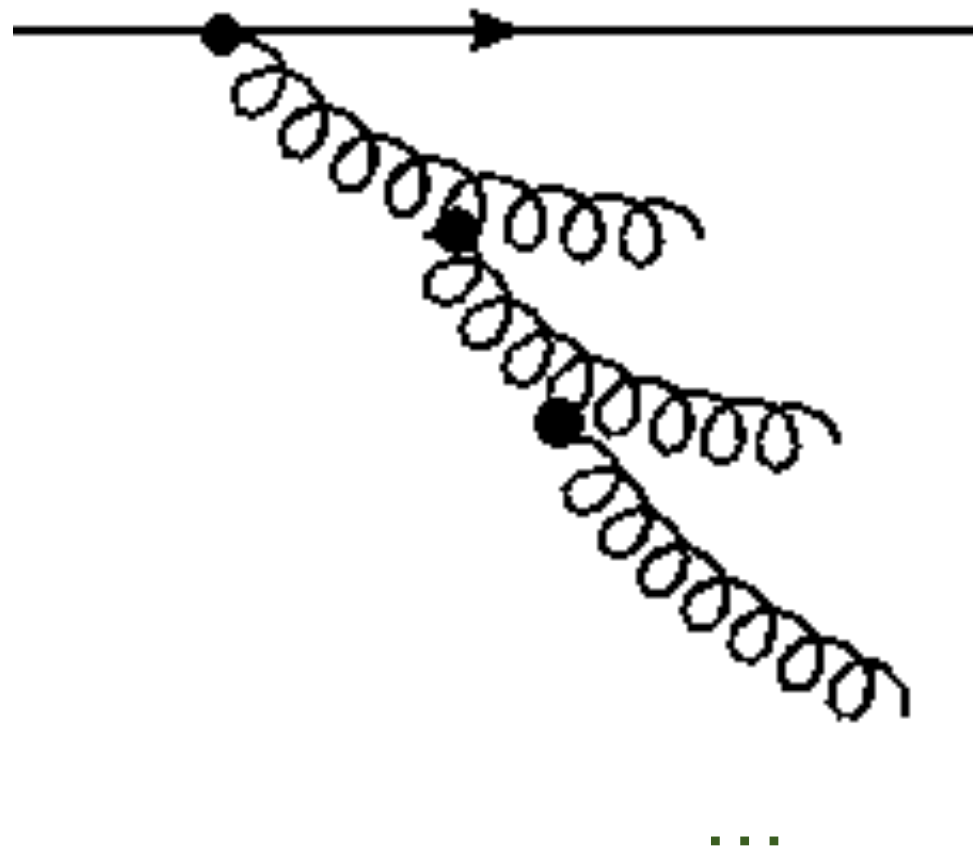
# Single parton emission

- ◆ Gluon bremsstrahlung from an off-shell (virtual) quark:



$$dP^{q \rightarrow qg} \sim \alpha_s C_R \frac{d\omega}{\omega} \frac{dk_{\perp}^2}{k_{\perp}^2}$$

- ➔ Soft and collinear divergent
- ➔ Needs re-summation: evolution equations (DGLAP, MLLA,...)





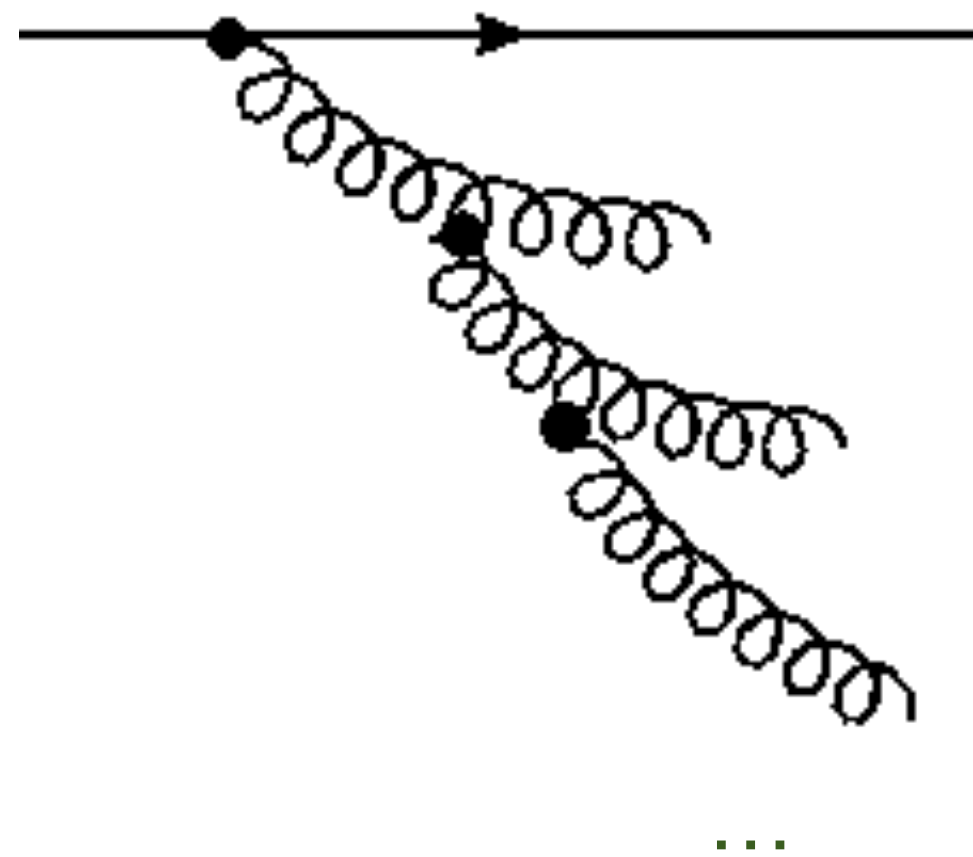
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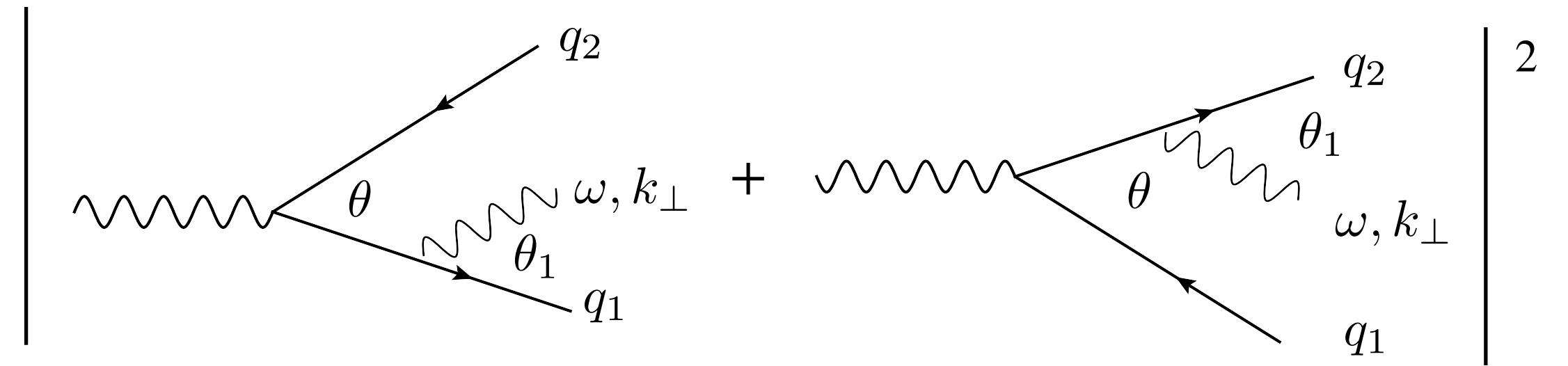
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Are they all independent?

# Jet Coherence

- ◆ Soft gluon radiation from a quark - anti-quark pair:
- ◆ color singlet configuration:

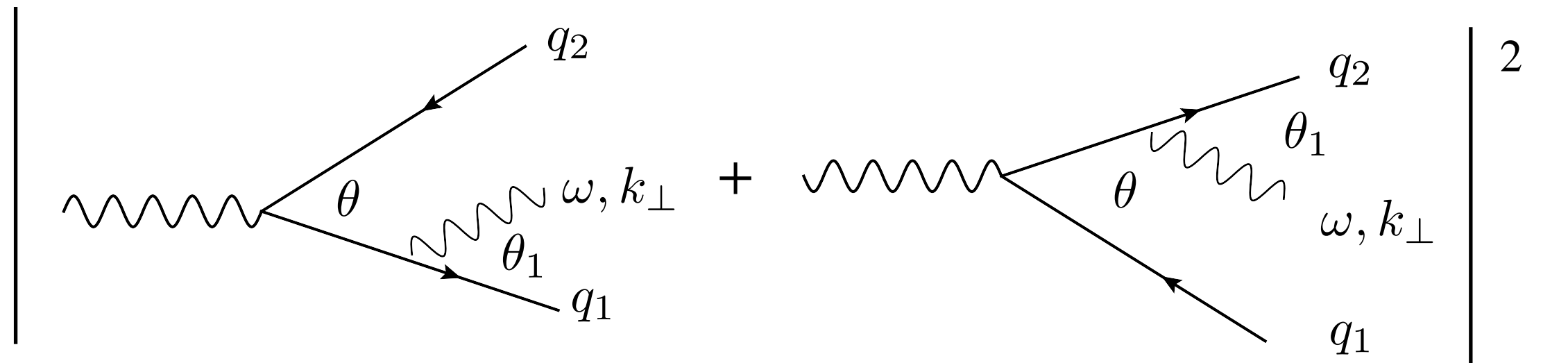


$$\frac{dI}{d\Omega_k} = R_q + R_{\bar{q}} - 2J = R_{coh}$$



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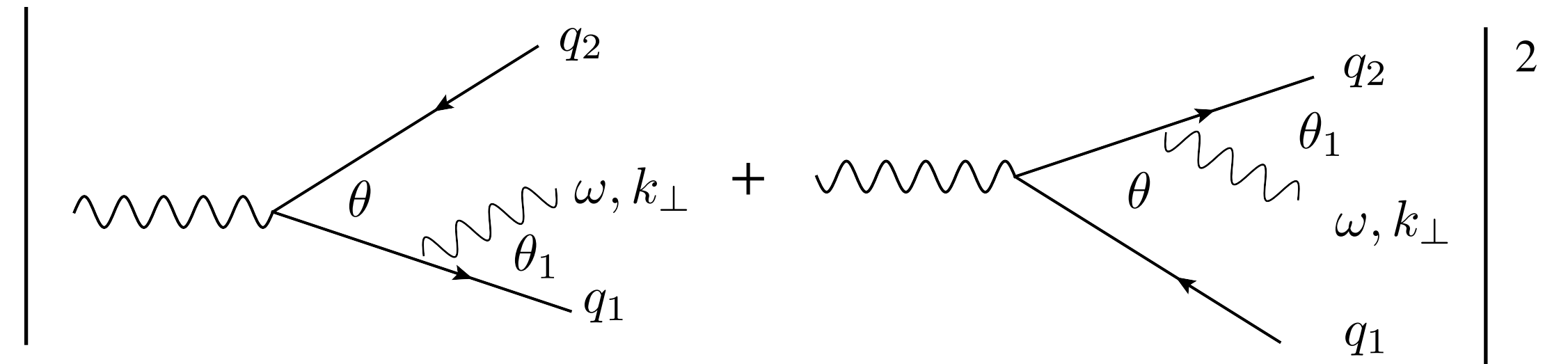


$$\frac{dI}{d\Omega_k} = R_q + R_{\bar{q}} - 2J = R_{coh} \quad \text{Integrating in azimuthal angle...}$$

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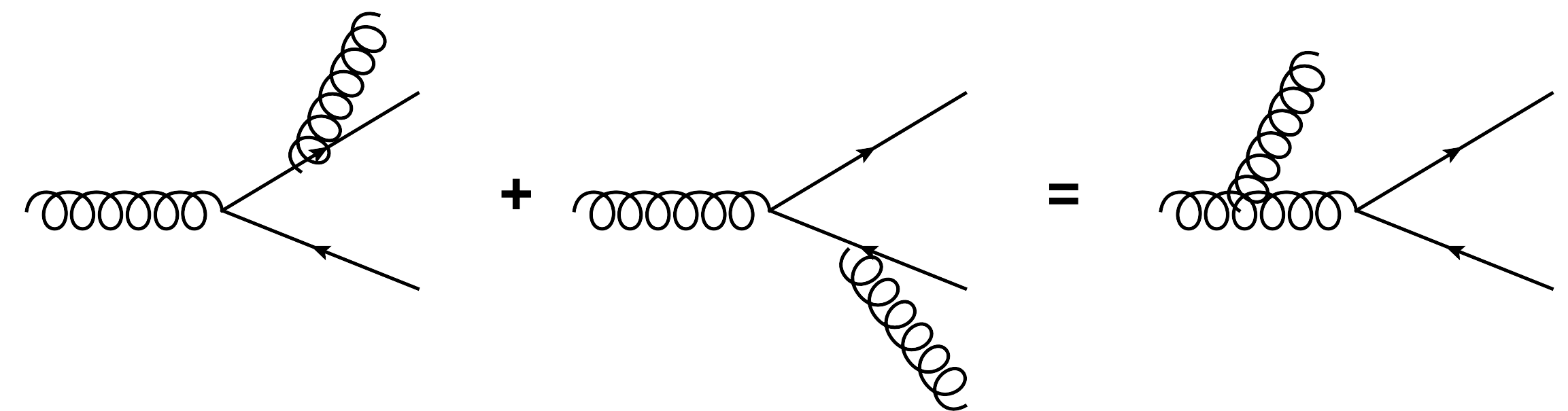
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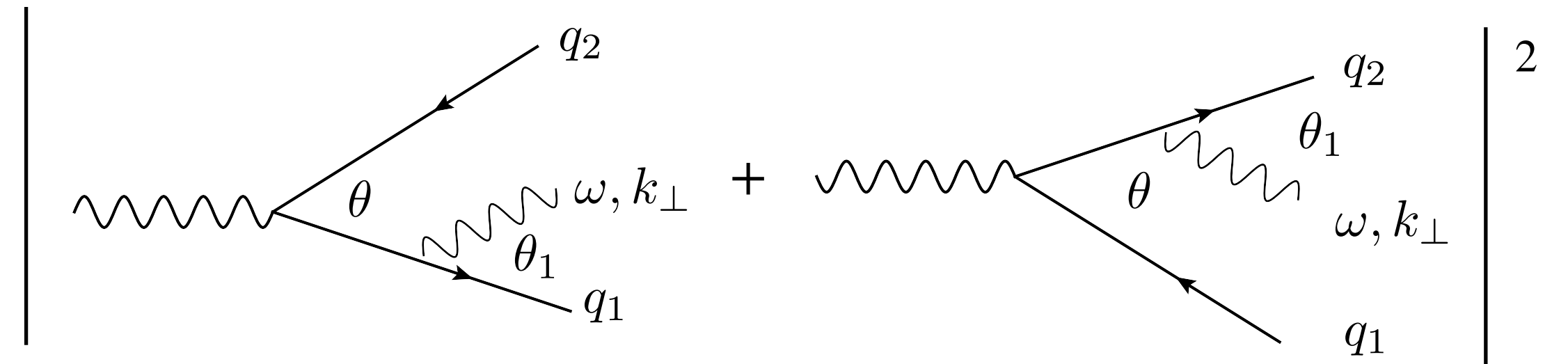
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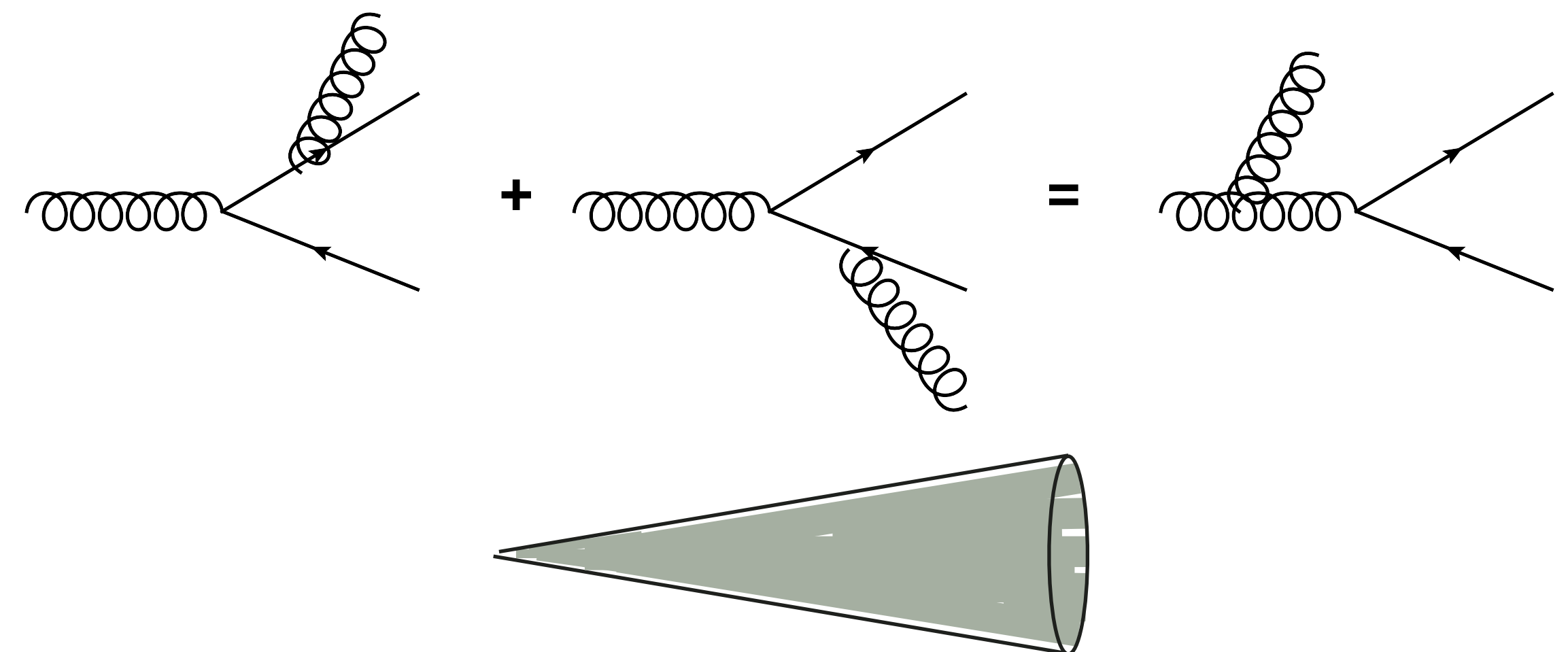
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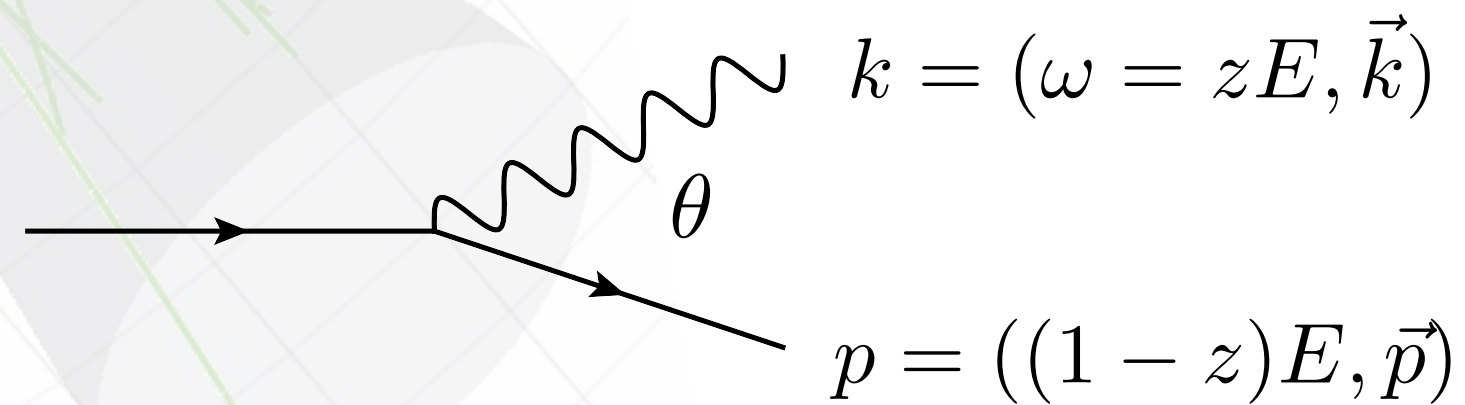
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# Vacuum formation time

See: Basics of Perturbative QCD, 1991

- ◆ Formation time of an emission:



Uncertainty principle:  $\Delta E \Delta t = 1$

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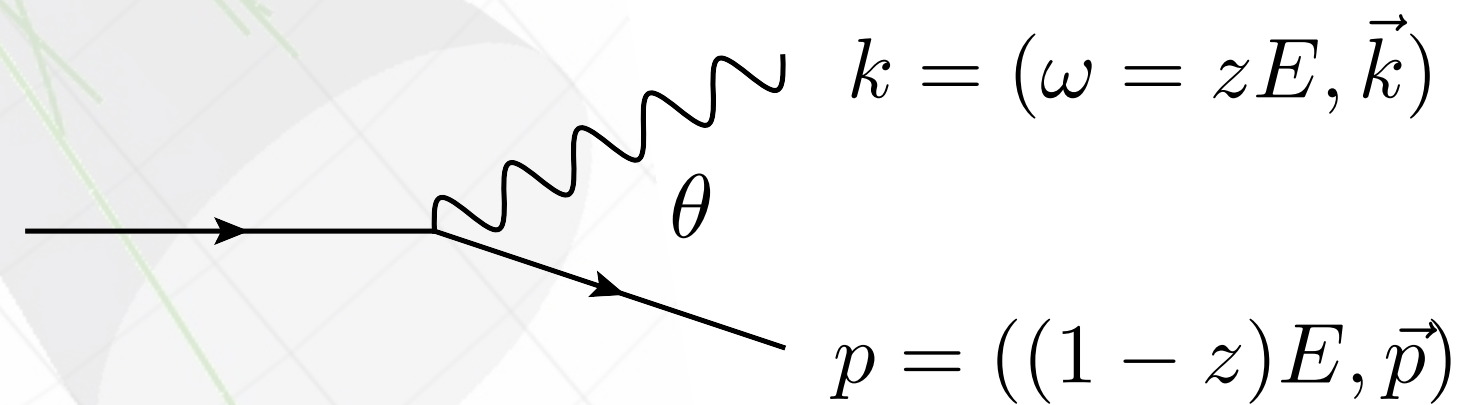
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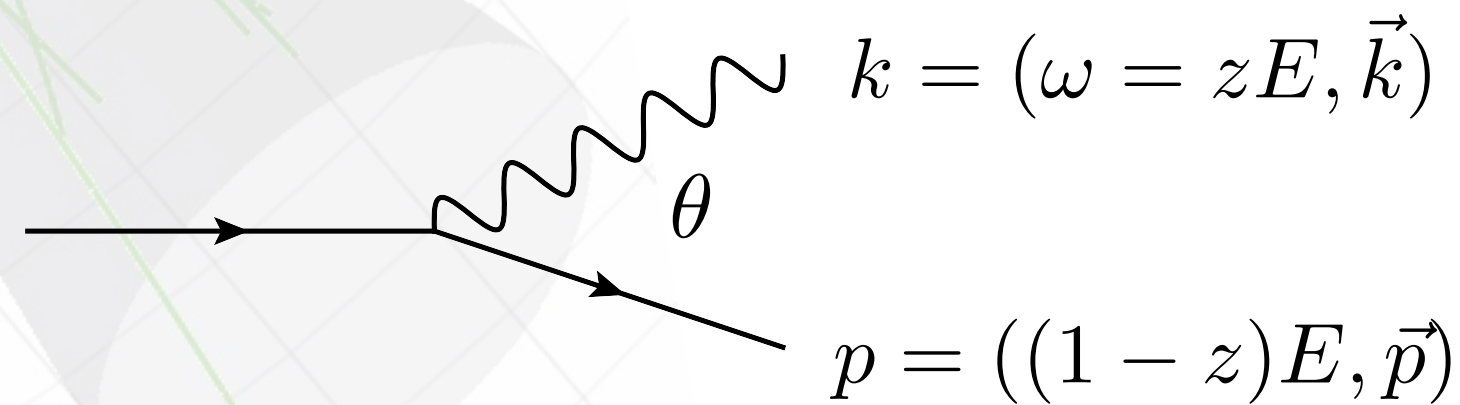
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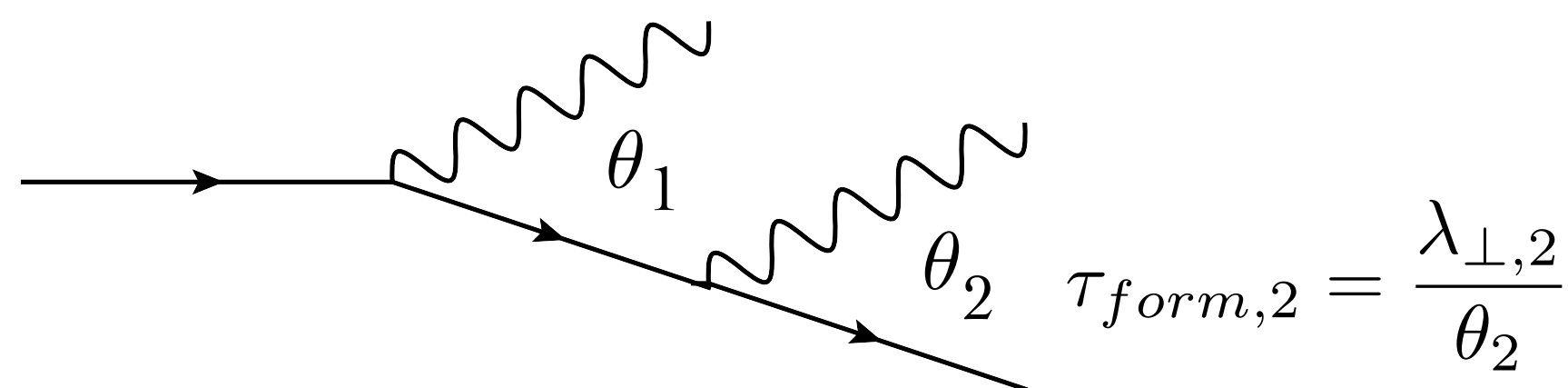
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During this time, the previous “antenna” separated:

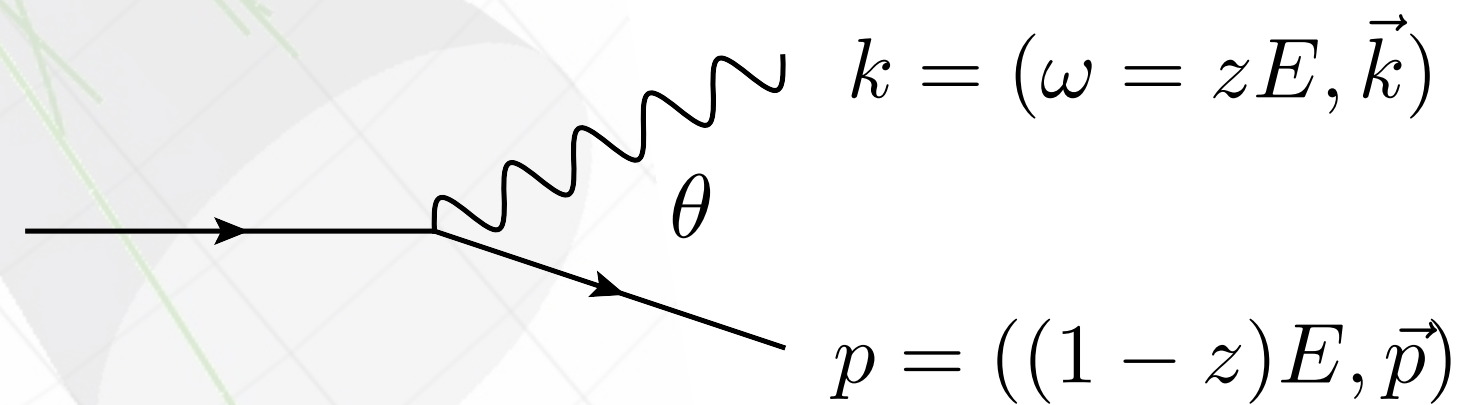
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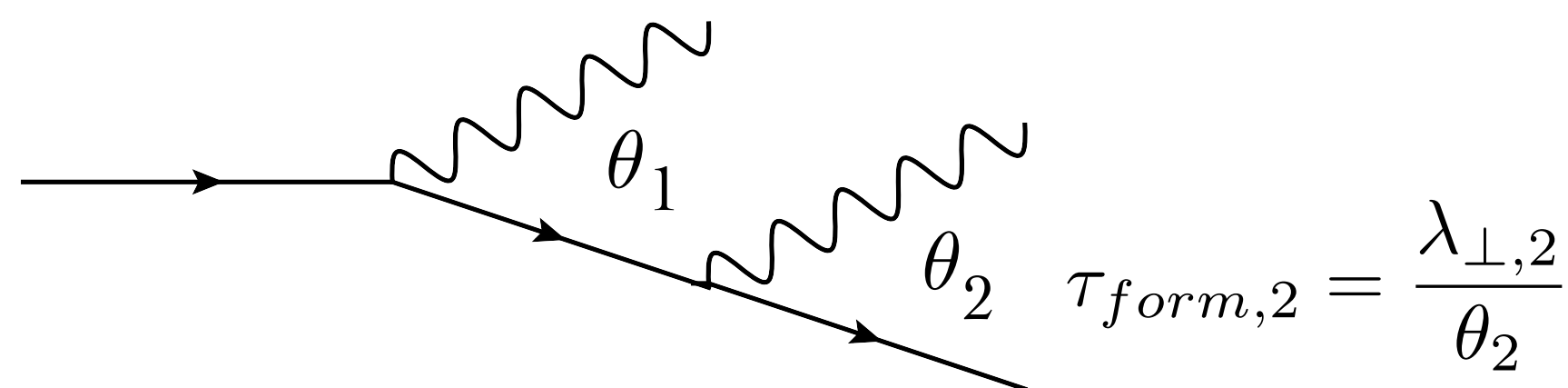
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$\theta_2 > \theta_1 \Rightarrow r_{\perp} < \lambda_{\perp} \rightarrow$  Sensitive to the “antenna” charge

$\theta_2 > \theta_1 \Rightarrow r_{\perp} > \lambda_{\perp} \rightarrow$  Sensitive to the “leg” charge



The background features a light gray grid pattern. Overlaid on this are several purple lines of varying thickness and style, including solid, dotted, and dashed lines. Some lines are straight, while others are curved or form loops. In the bottom right corner, the text "Now back to Heavy-Ions" is displayed in a bold, black, sans-serif font. The overall aesthetic is technical and scientific.

**Now back to  
Heavy-Ions**



# First Considerations

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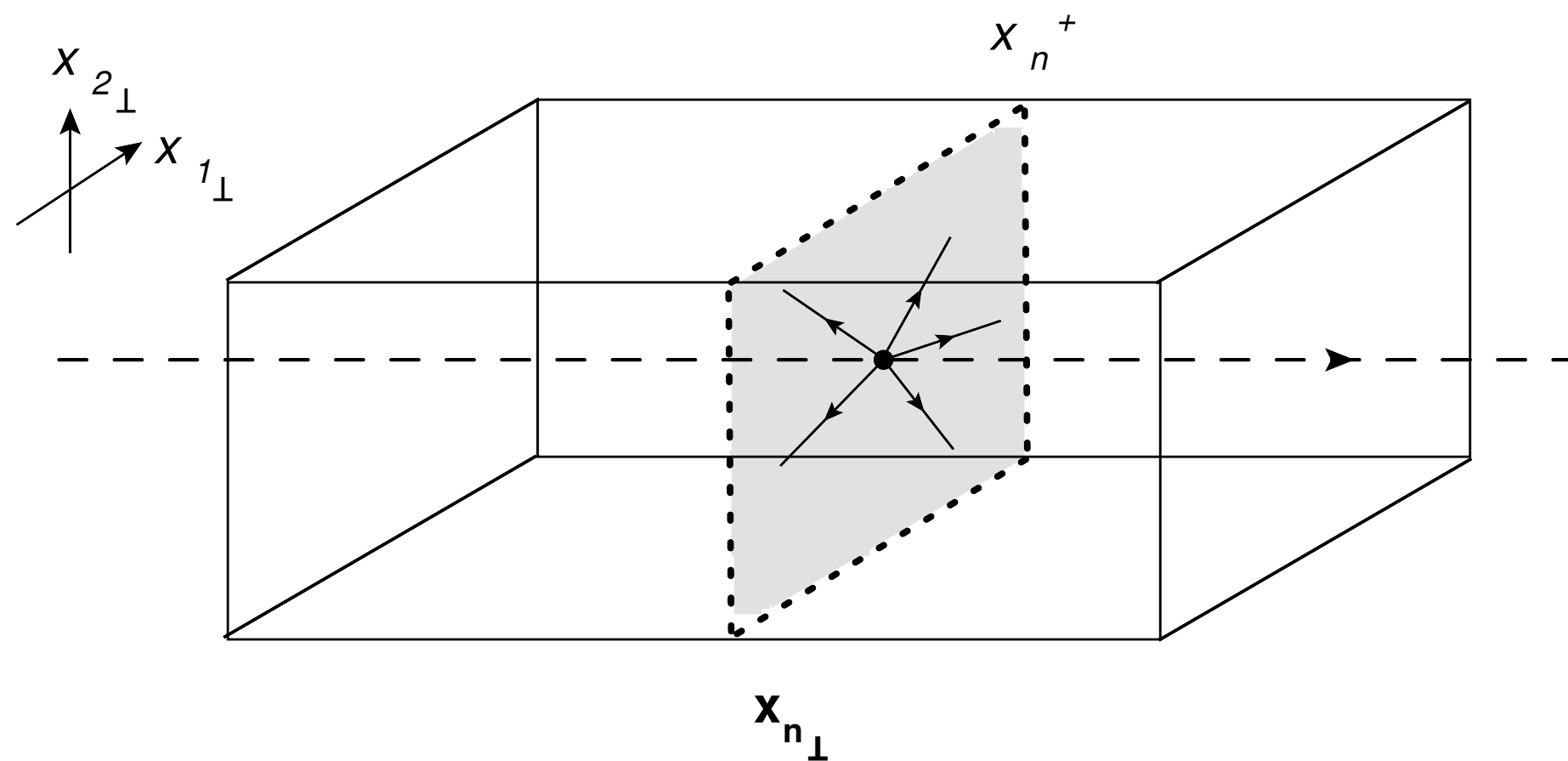
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- ◆ High-energy particles propagating through a medium:
  - ◆ Particle propagation time  $<$  timescale for changes in the medium fields
    - ➔ Medium can be considered in a static configuration
    - ➔ Only transverse momentum exchange

# Light Cone Gauge

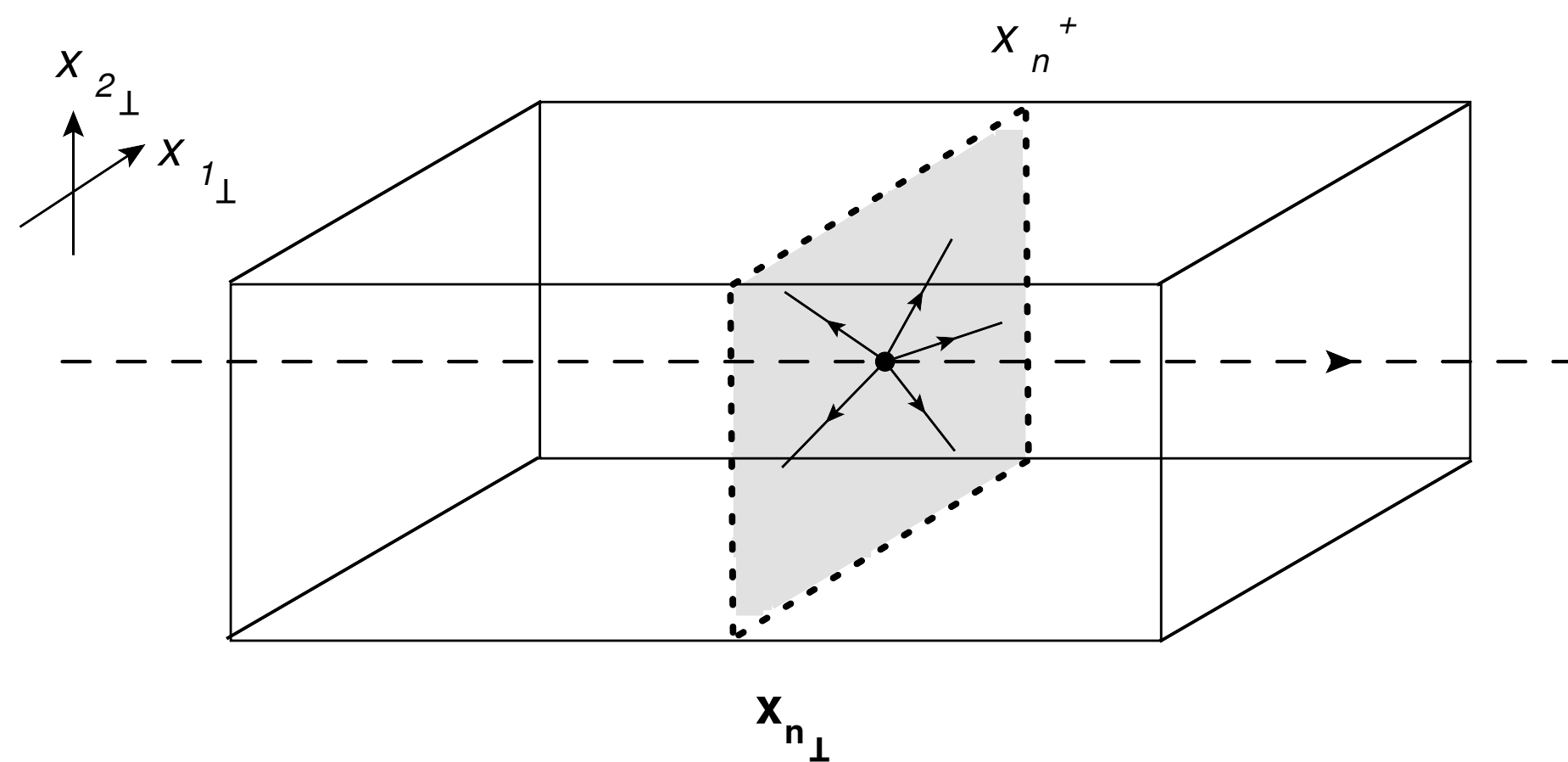
- ◆ Particle moving in the  $x_3$  direction: Light Cone Gauge  $A_+ = 0$
- ◆ Light Cone Coordinates:  $p_+ \gg p_T \gg p_-$
- ◆  $x_{\pm} = \frac{x_0 \pm x_3}{\sqrt{2}}$  and  $x_{\perp} = (x_1, x_2)$
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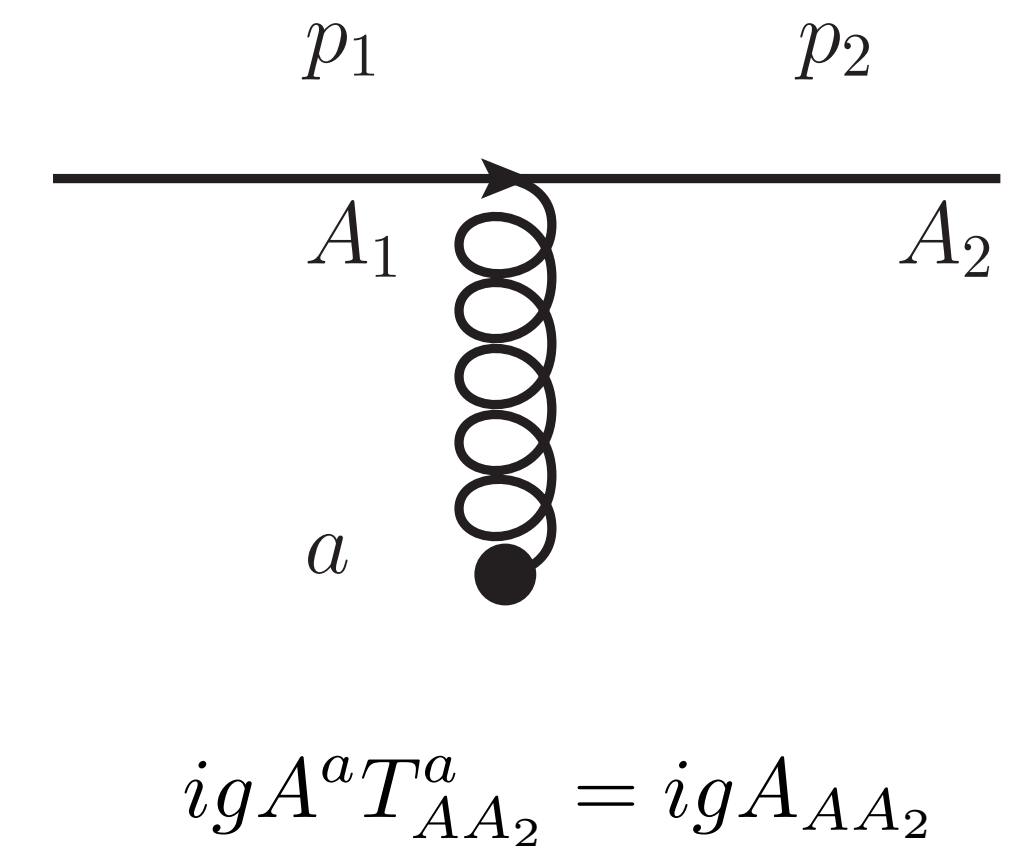


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Additional notation:



# Eikonal Approximation

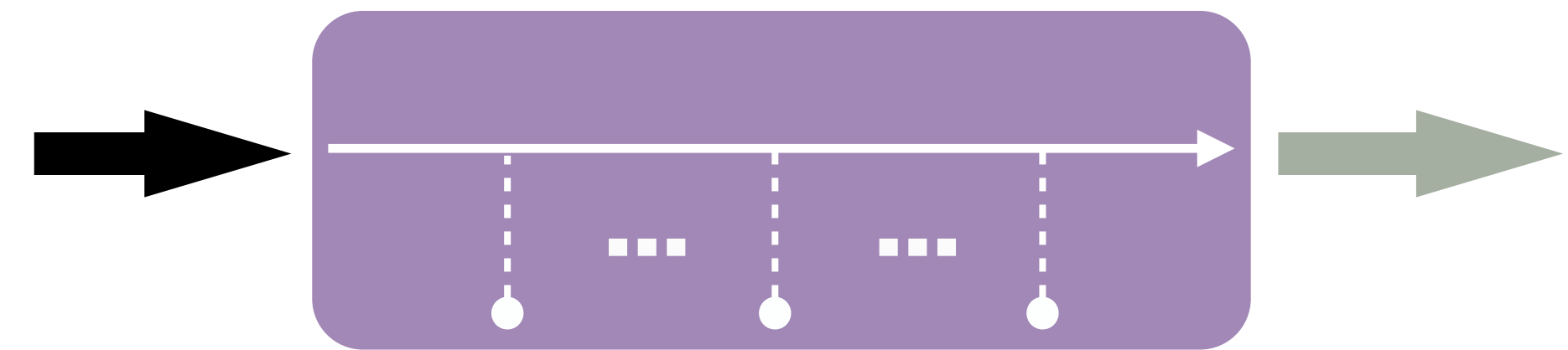
- ◆ Consider a high energetic particle propagating through a collection of static scattering centres:
- ◆ Result will be only a color phase rotation:
- ➔ In-medium propagator: Wilson Line

$$W(x_{0+}, L_+; \mathbf{x}_\perp) = \mathcal{P} \exp \left\{ ig \int_{x_{0+}}^{L_+} dx_+ A_-(x_+, \mathbf{x}_\perp) \right\}$$

Path-ordering

Medium colour field

Transverse coordinate





# Eikonal Approximation

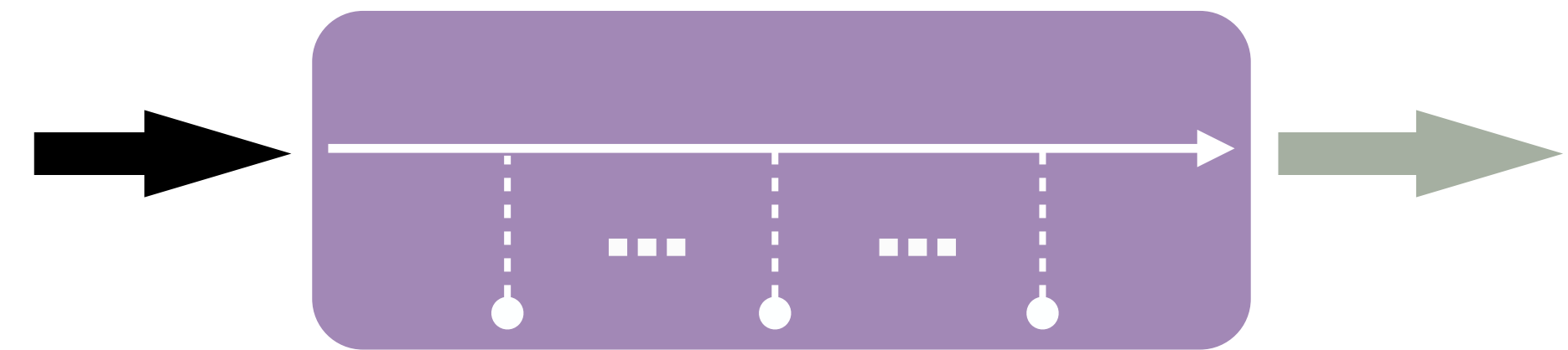
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Let's re-derive this propagator!

Starting by two scatterings...

# Eikonal Approximation

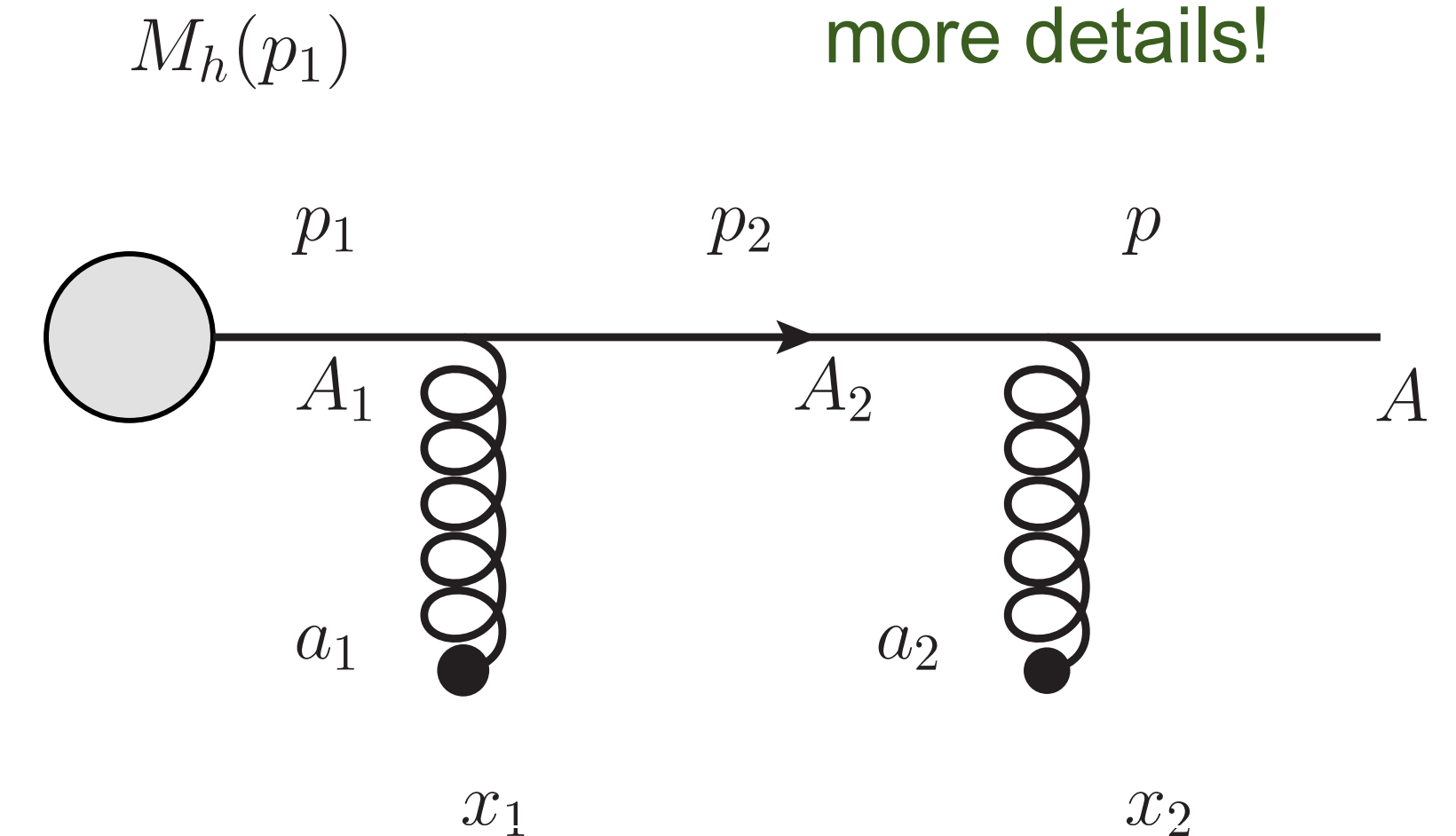
See: arXiv:0712.3443

- Consider a high energetic particle propagating interacting twice with the medium:

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See backup slides for more details!



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# Eikonal Approximation

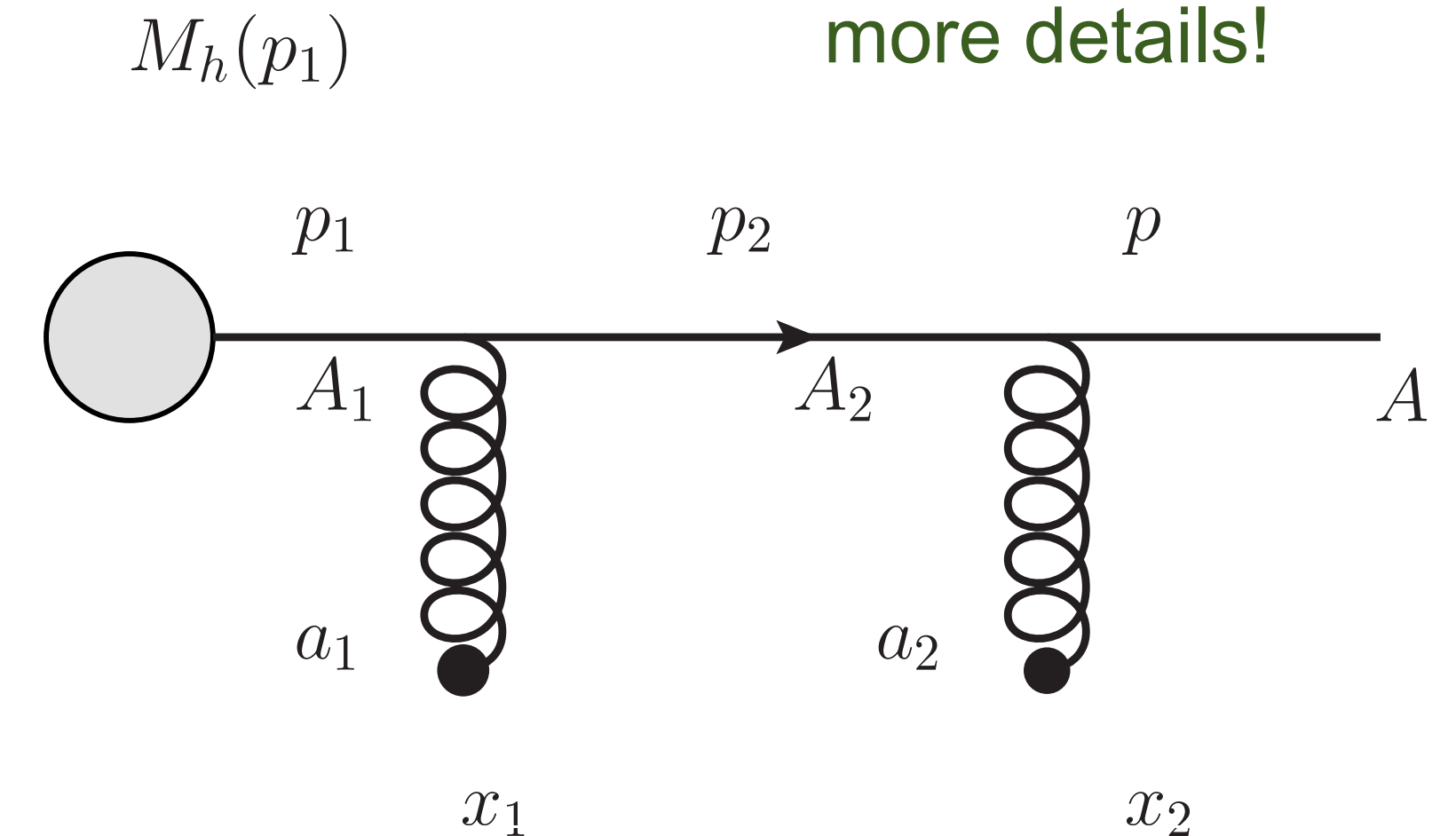
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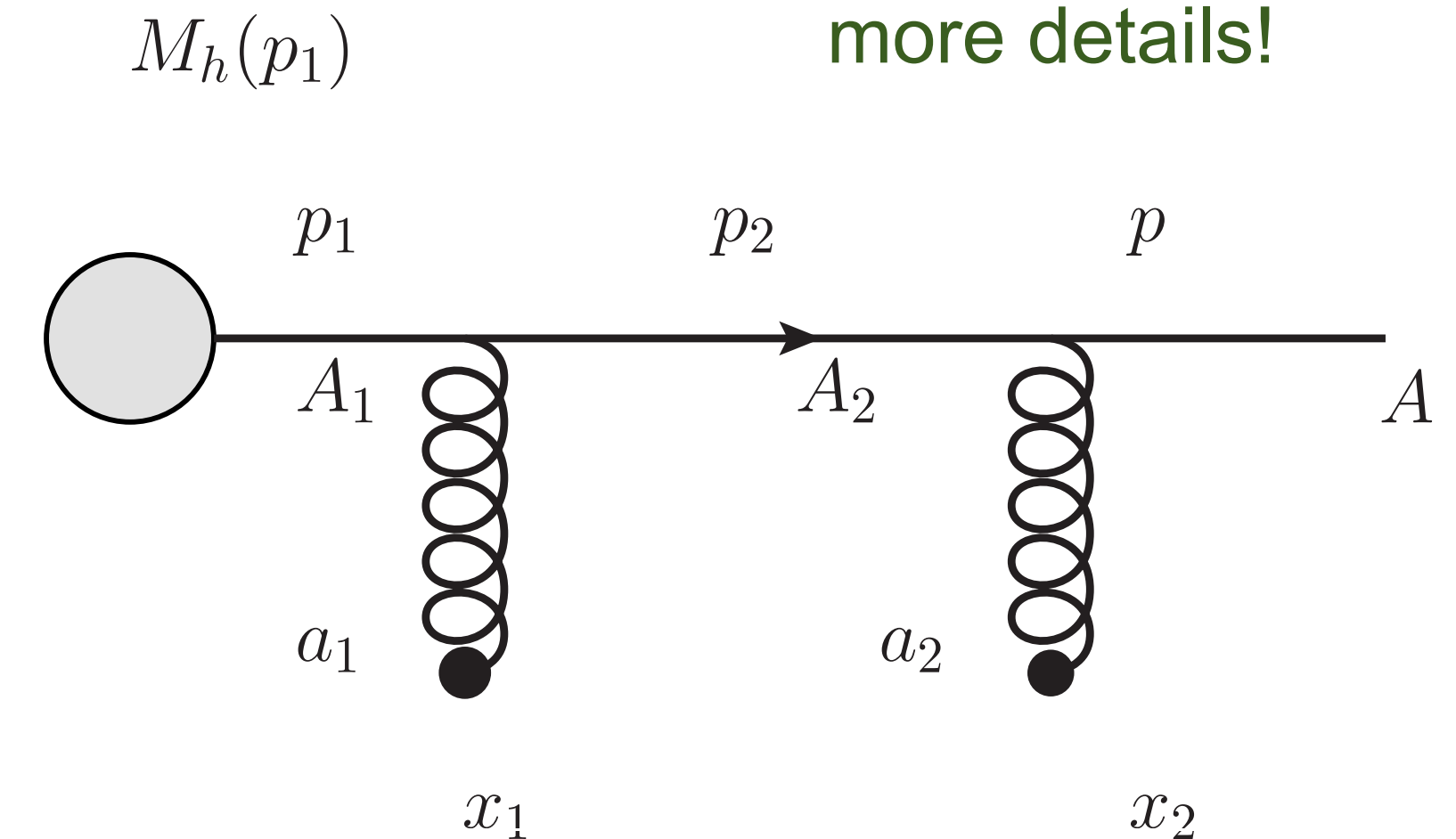
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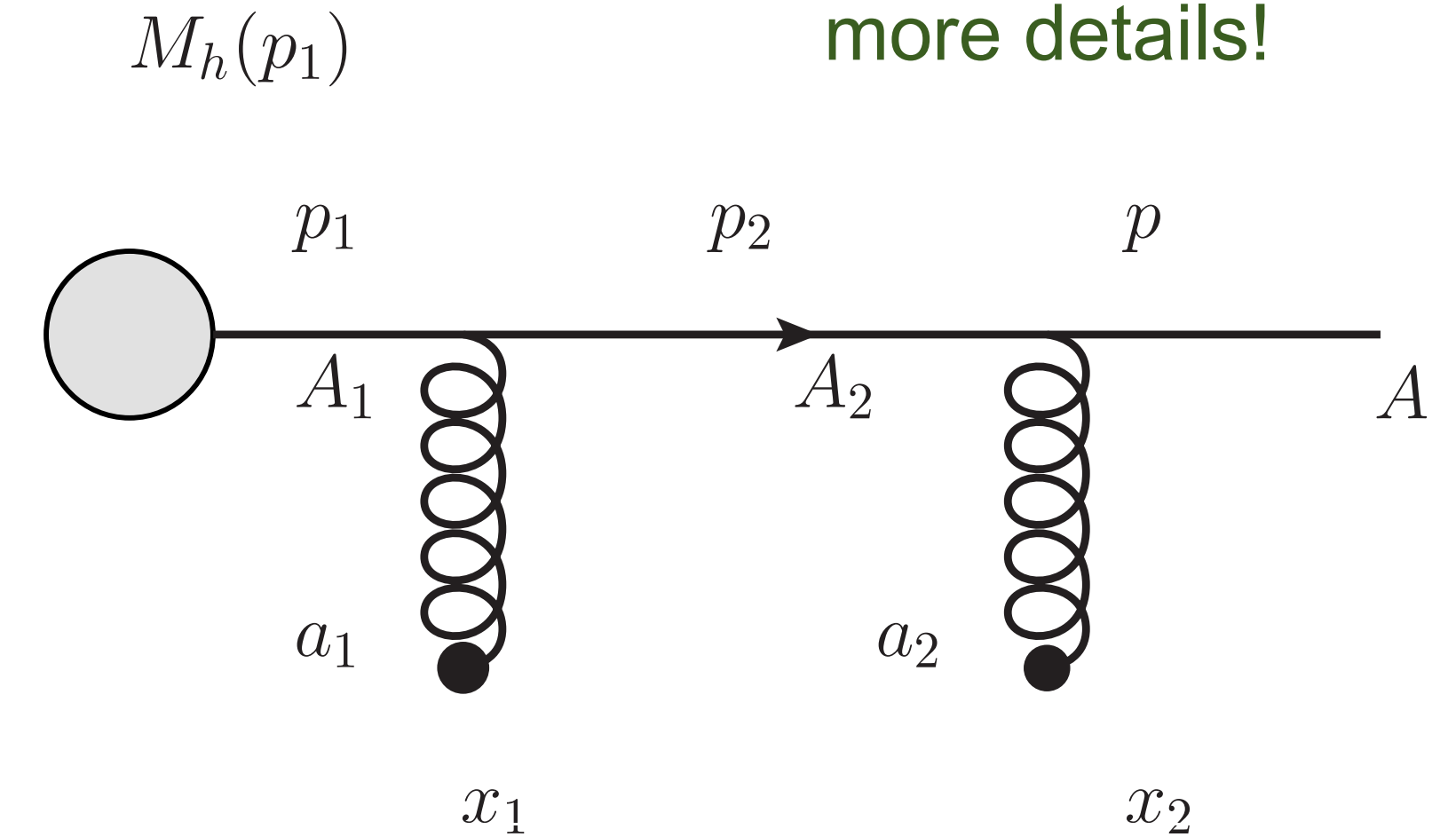
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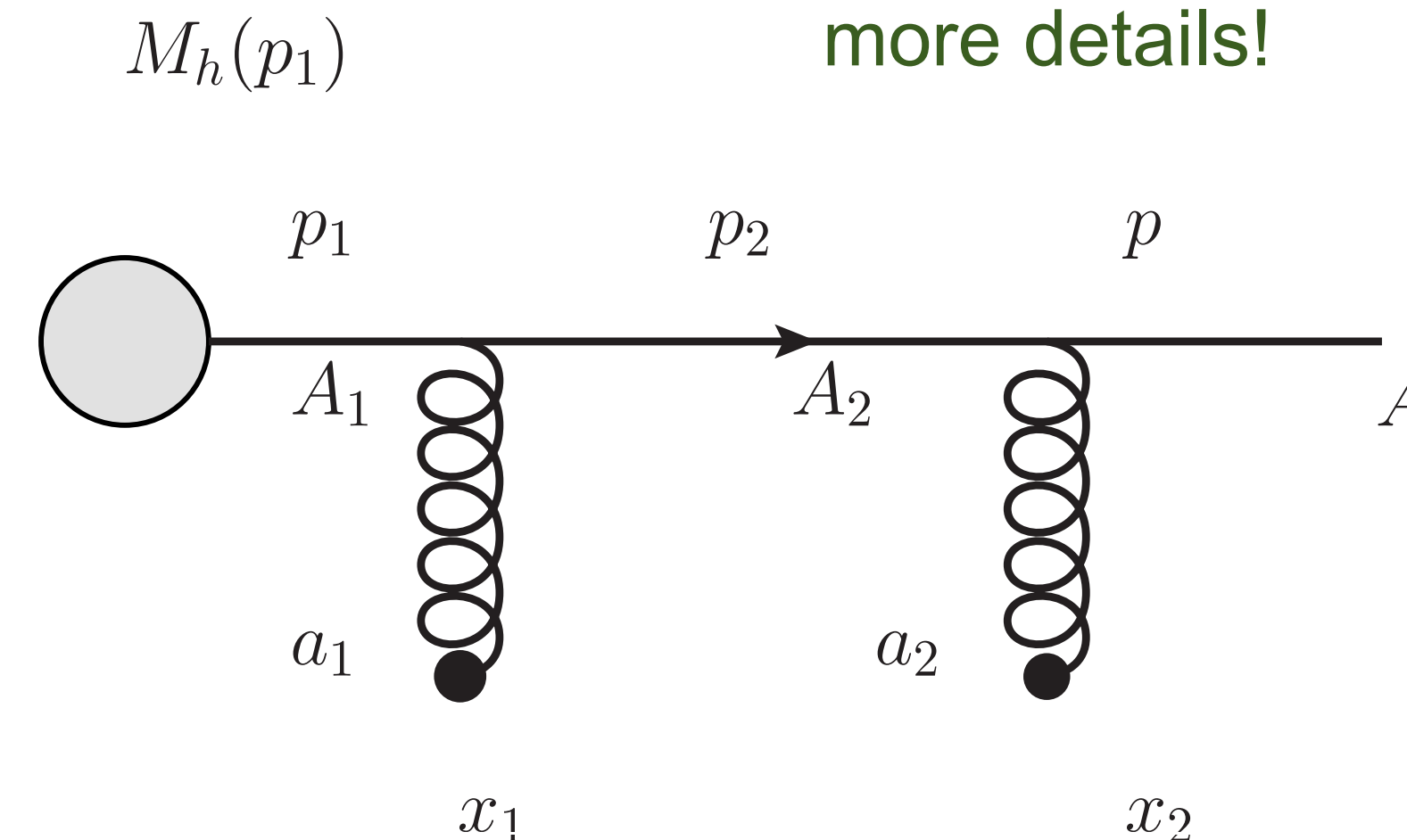
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Integrals on  $p_{it}$  will just set:

$$\Rightarrow x_{1\perp} = x_{2\perp} = 0$$



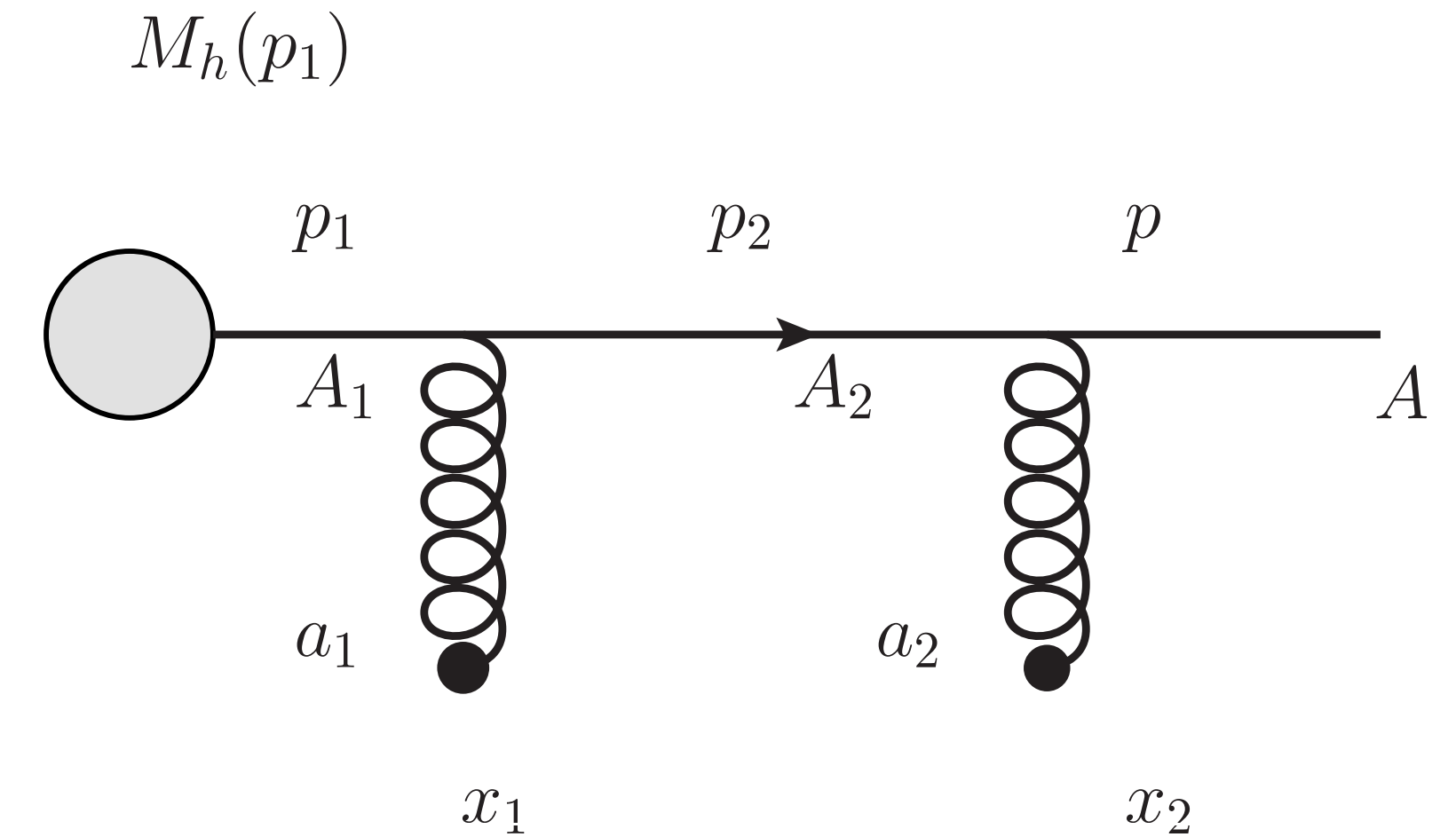
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After some work...

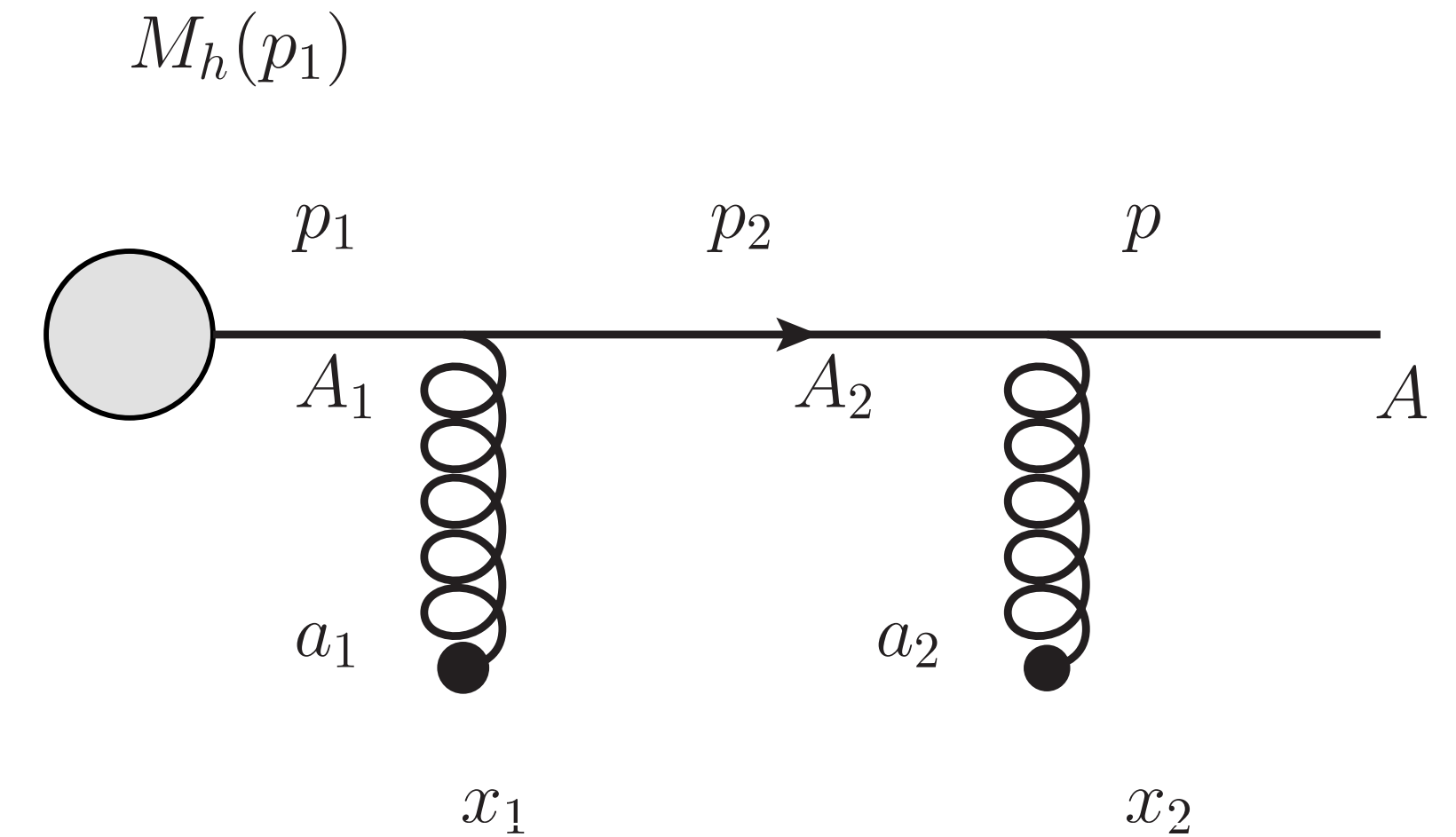
$$\simeq \int dx_{1+} dx_{2+} ig(A_-)_{AA_2}(x_{2+}, 0_\perp) \theta(x_2 - x_1) + ig(A_-)_{A_2A_1}(x_{1+}, 0_\perp) \theta(x_1)_+ \bar{u}(p) M_h(p)$$

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No interaction term



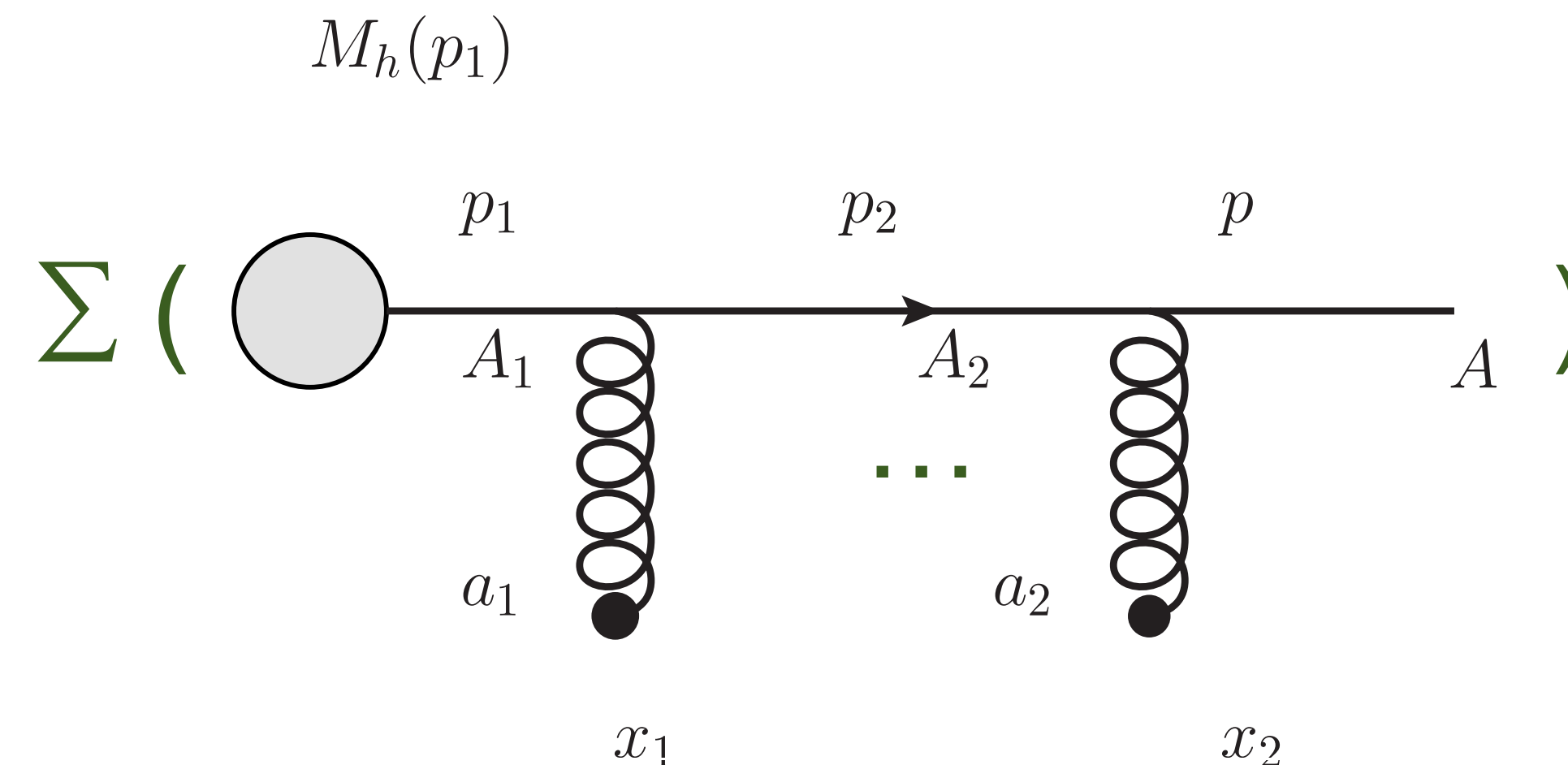
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summing over all 'n' interaction terms

$$\mathcal{P} \exp \left\{ ig \int_{x_{1+}}^{x_{n+}} dx_+ A_-(x_+, x_\perp = 0) \right\} = W(x_{n+}, x_{1+}; x_\perp = 0)$$

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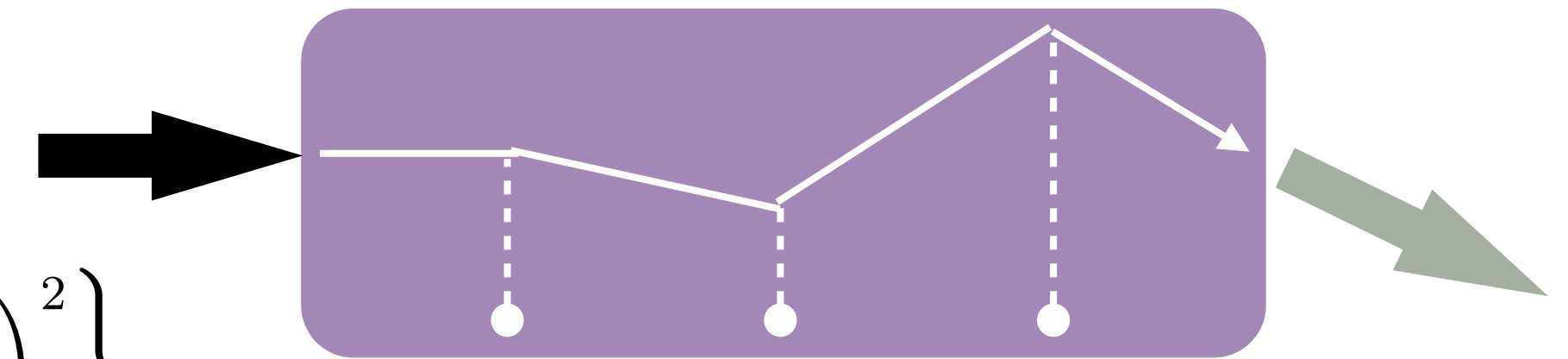
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See: arXiv:0712.3443

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Initial/Final coordinates





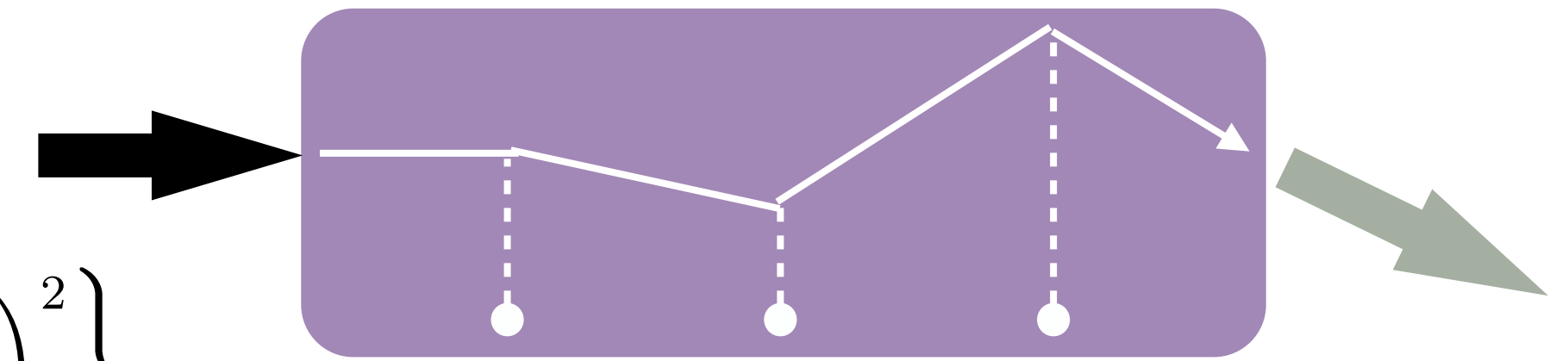
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How to get this in-medium propagator?

sub-leading terms in denominator

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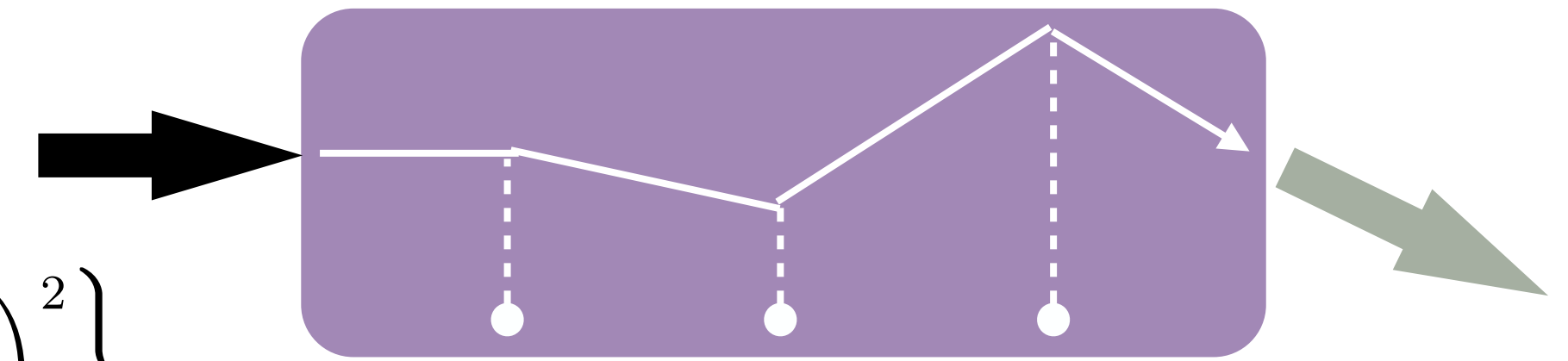
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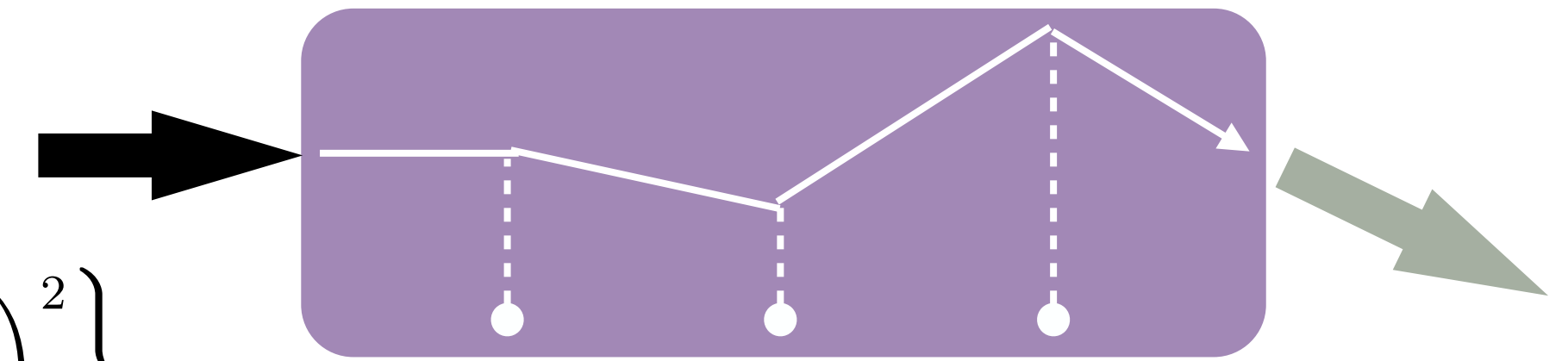
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How to get this in-medium propagator?

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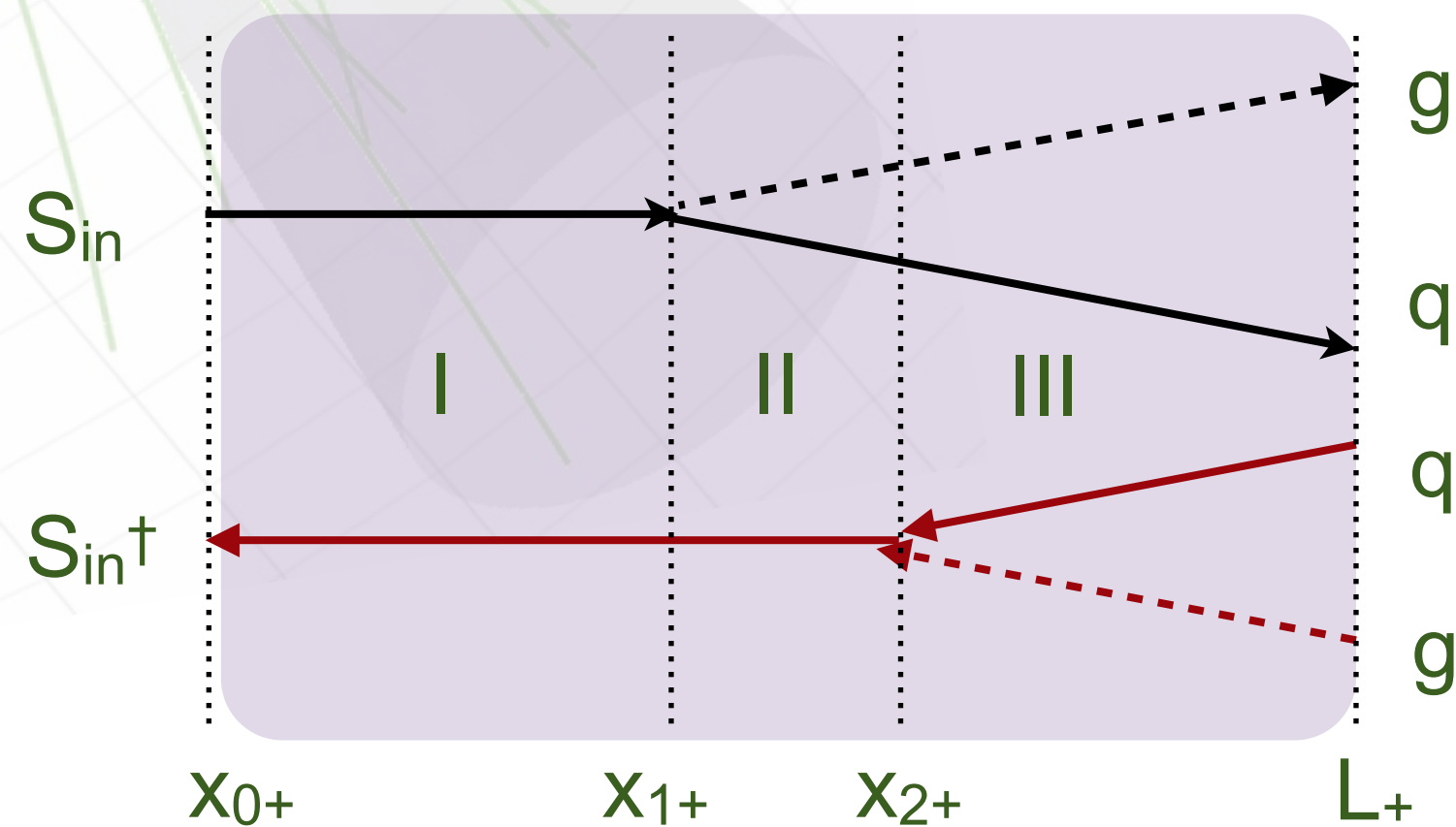
Integral in  $p_{T\perp}$ :

$$\int \frac{d^2 p_{i\perp}}{(2\pi)^2} e^{i\frac{p_{i\perp}^2}{2p_+}(x_{i-1}-x_i)_- - ip_{i\perp}(x_{i-1}-x_i)_\perp} = \frac{p_+}{2\pi i(x_{i-1} - x_i)_+} \exp \left\{ i\frac{p_+}{2} \frac{(x_{i-1} - x_i)_\perp^2}{(x_{i-1} - x_i)_+} \right\} = \int_{x_{(i-1)\perp}}^{x_{i\perp}} \mathcal{D}(x_\perp) \exp \left\{ \frac{ip_+}{2} \int_{x_{(i-1)+}}^{x_{i+}} \left( \frac{dx_\perp}{dx_+} \right)^2 \right\} = G_0(x_{i+}, x_{i\perp}; x_{(i-1)+}, x_{(i-1)\perp} | p_+)$$

# In-medium gluon emission

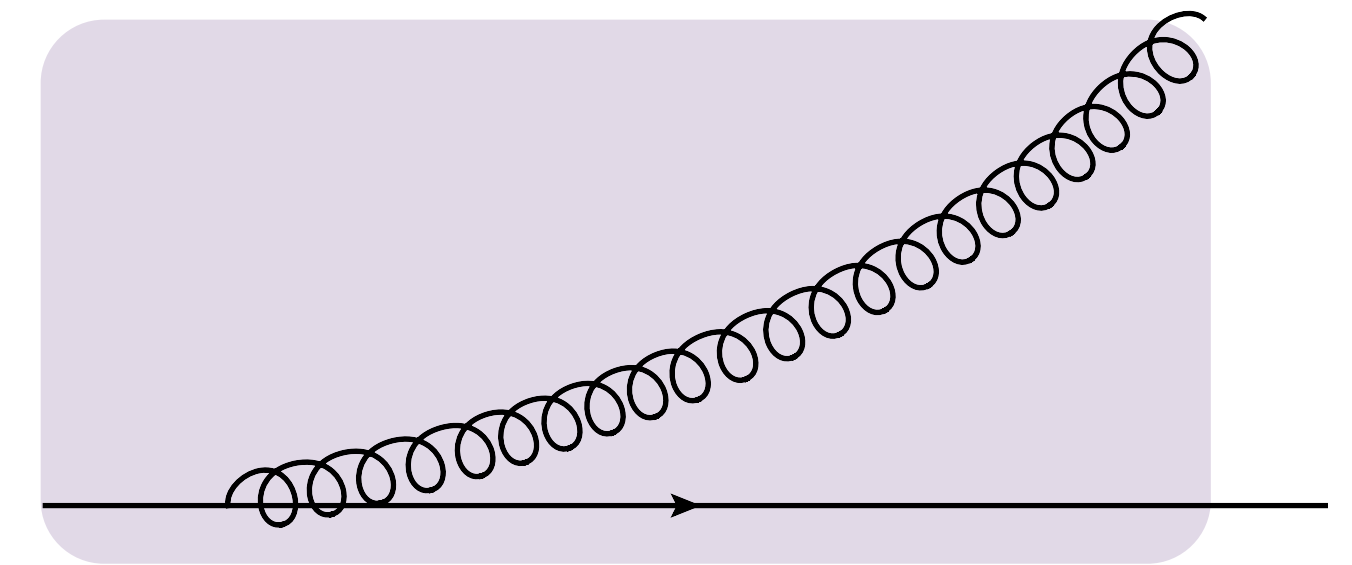
◆ Medium-induced gluon radiation:

Mathematical picture



High energy approximation:  
 ⇒ Decomposition with a  
 fixed number of propagators  
 ⇒ 3 different regions

Physical picture



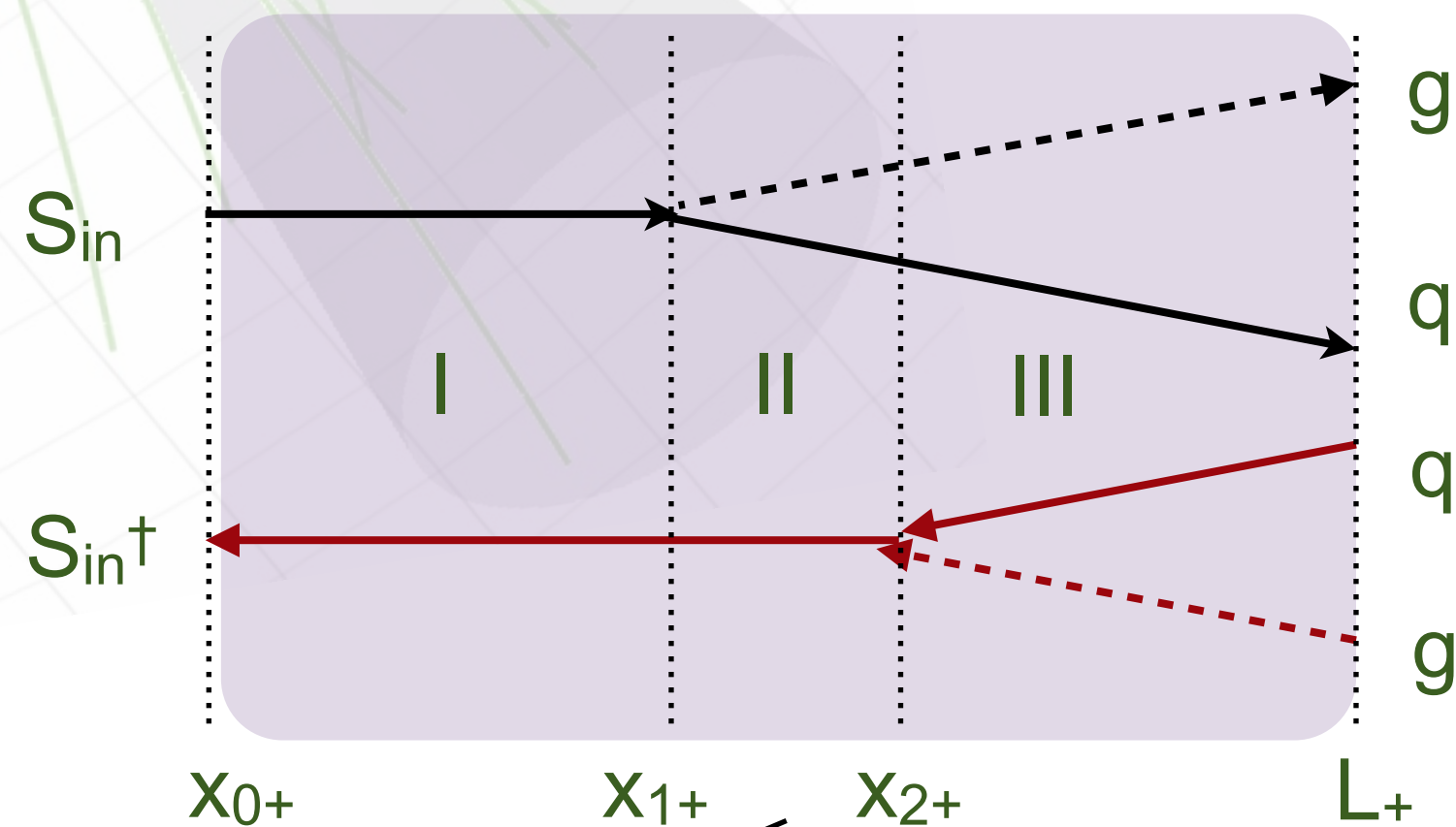
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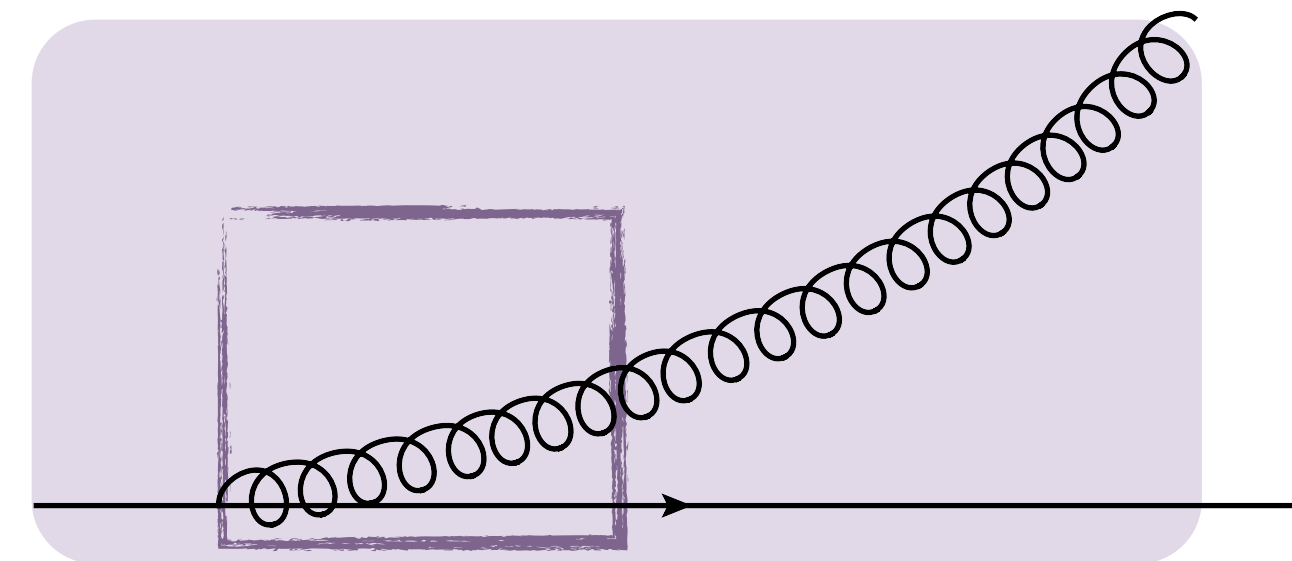
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Emission Kernel

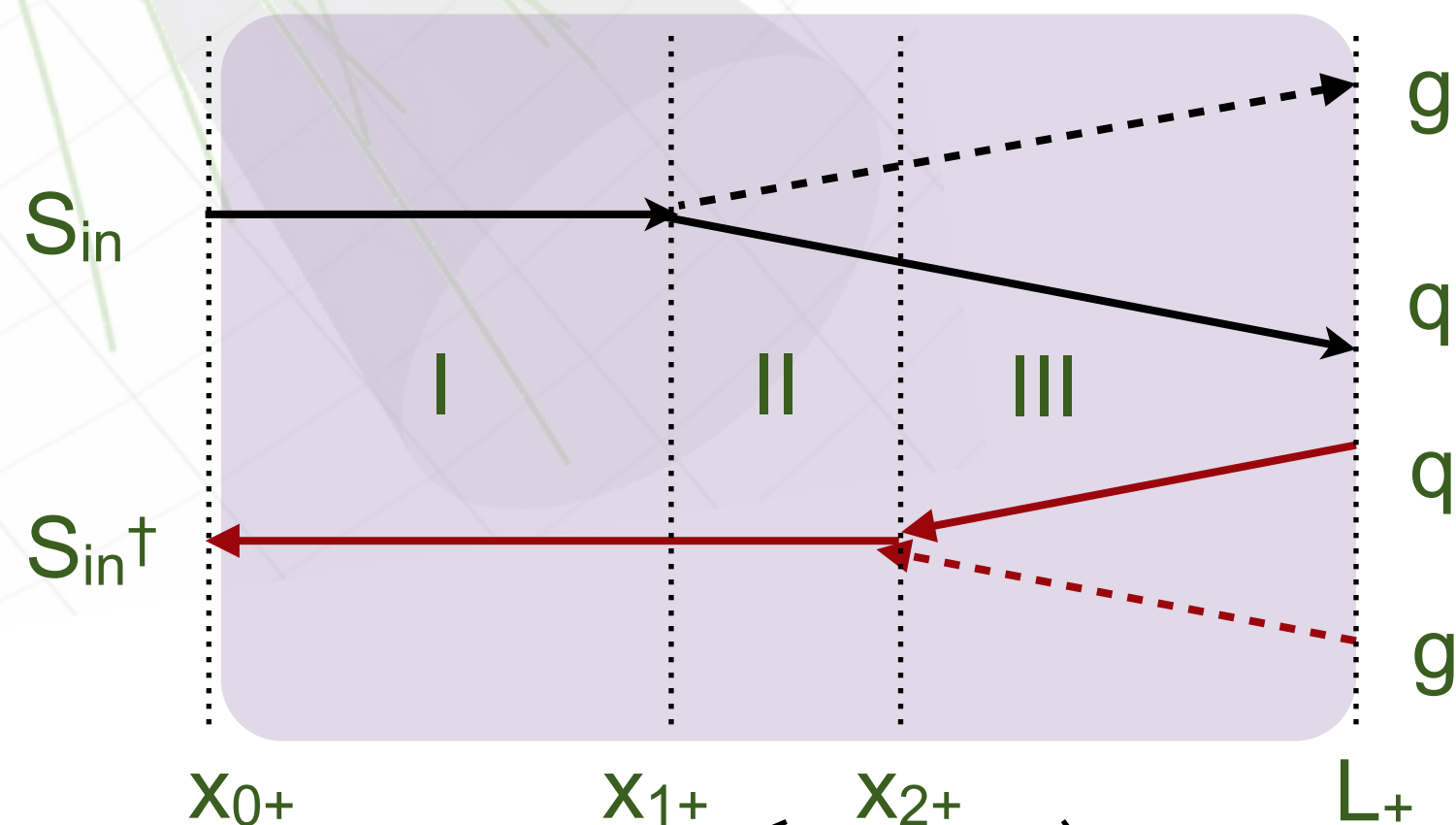
$$\text{Tr} \left\langle W_A(r_g) W_A^\dagger(0) \right\rangle$$

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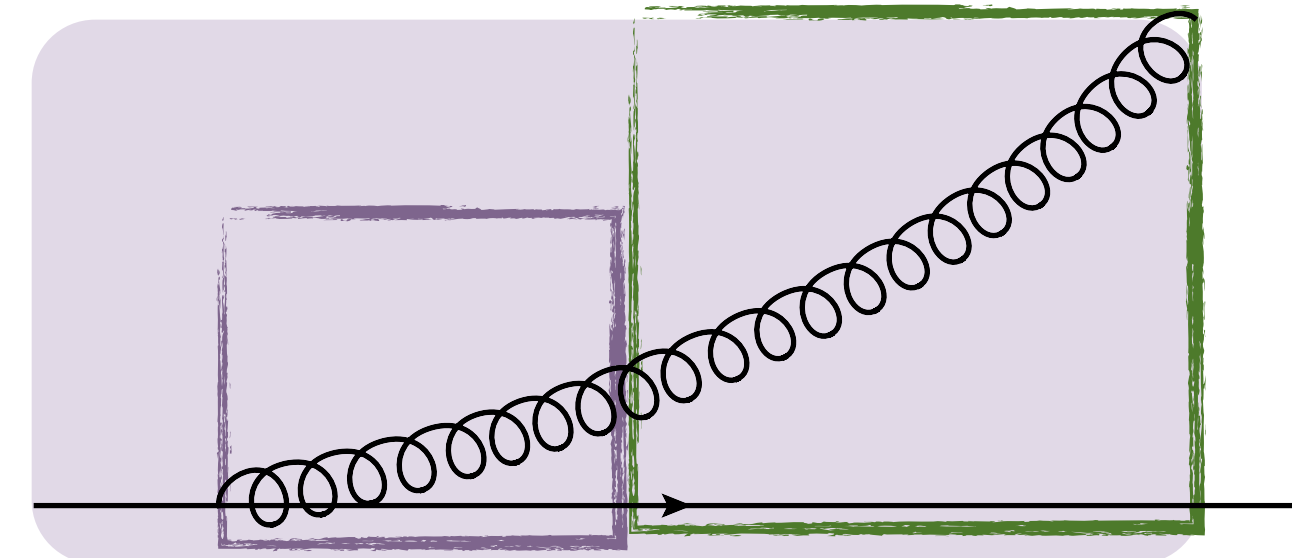
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Classical Broadening

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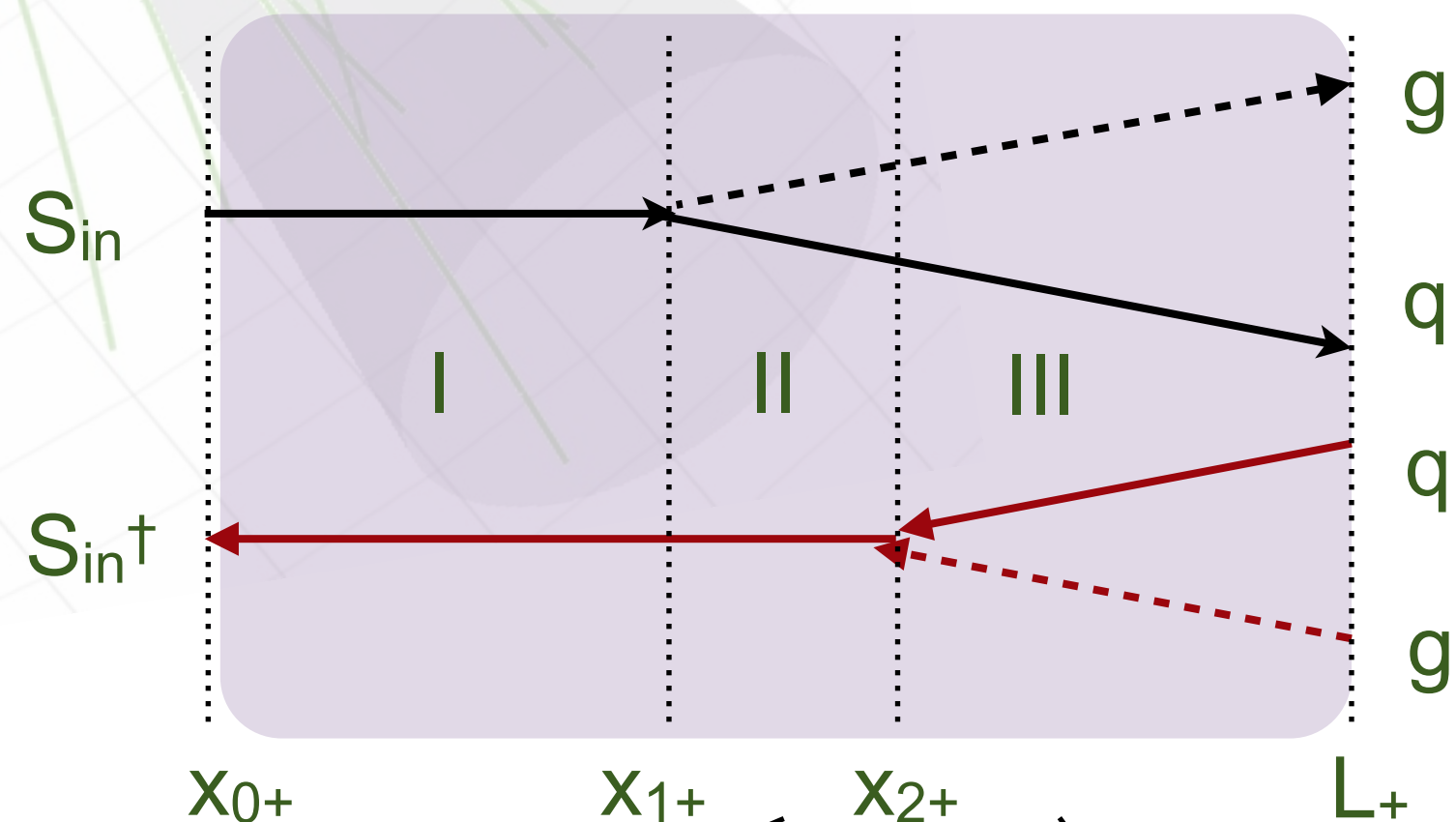
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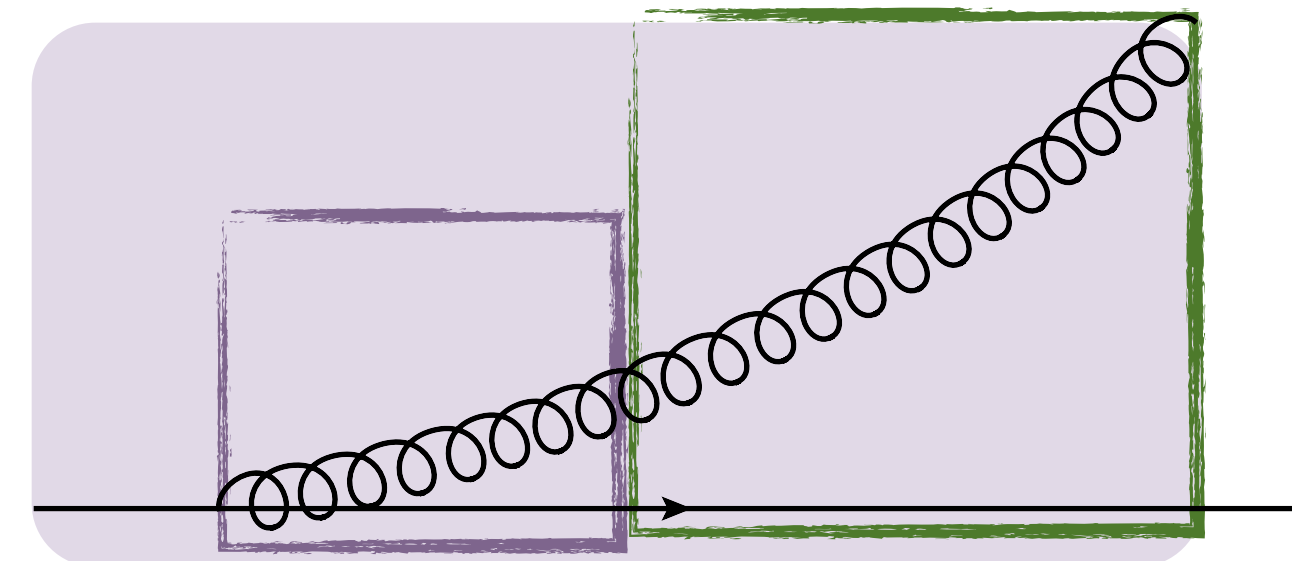
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And, finally, some numerics:

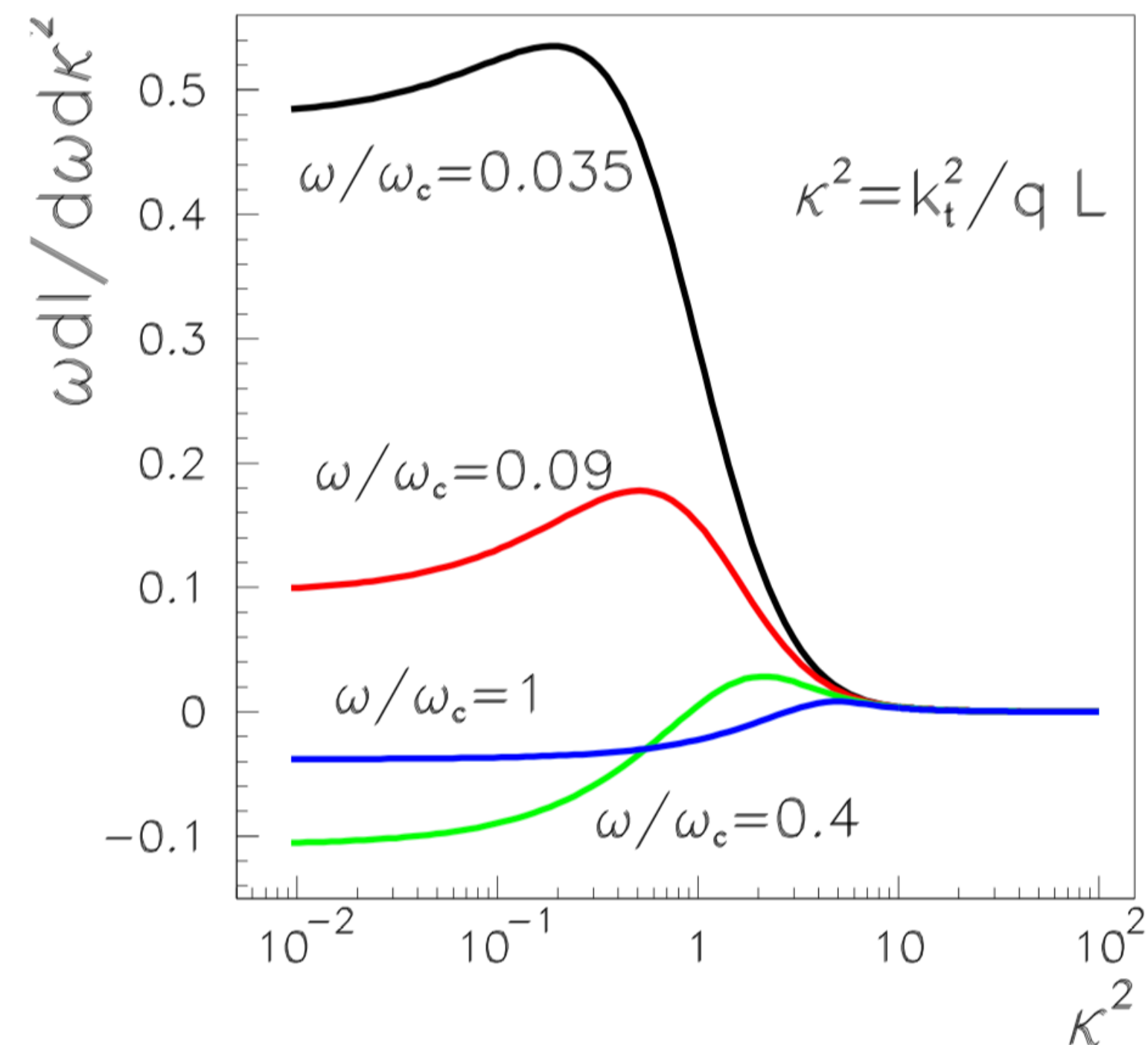
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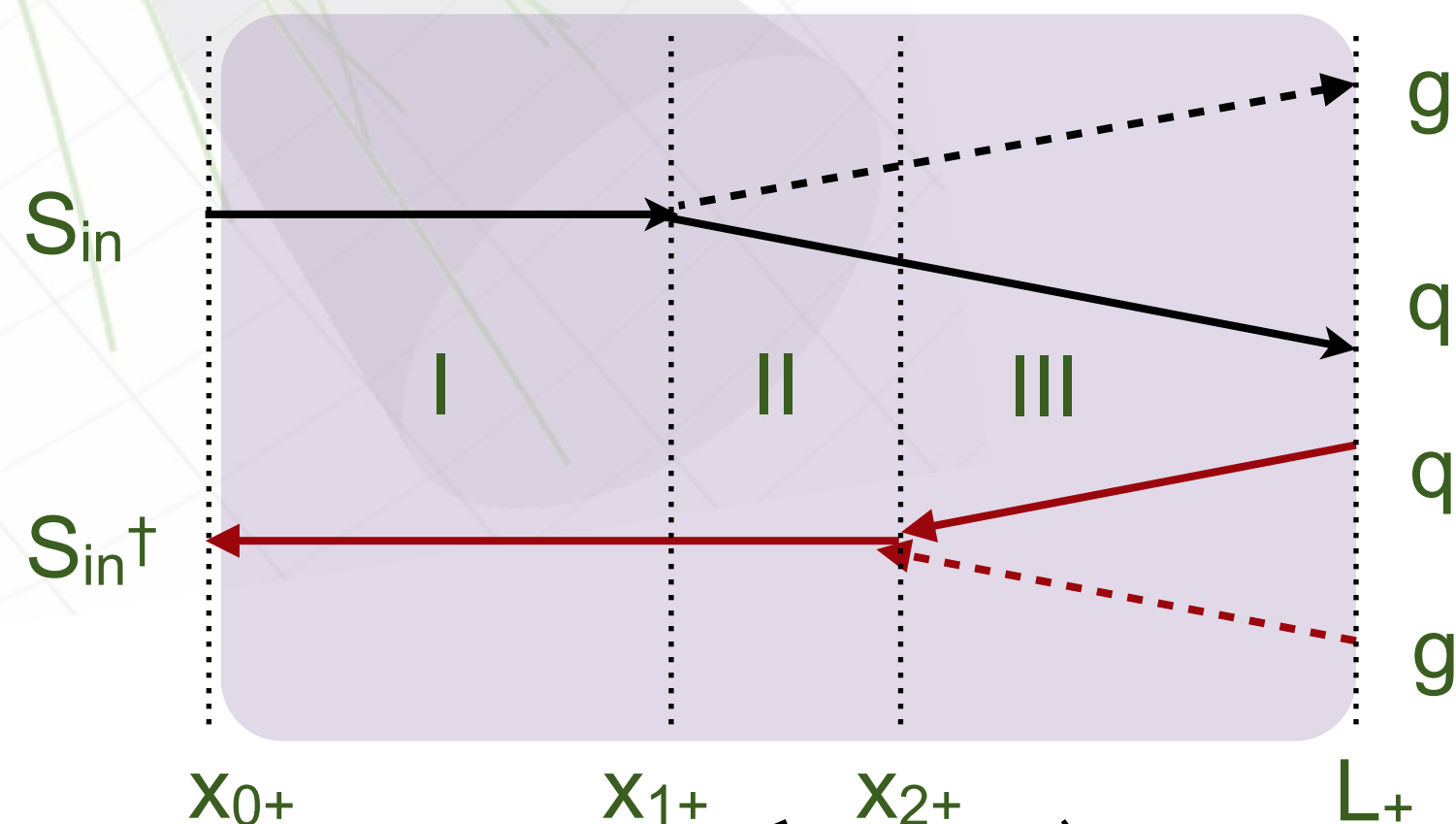
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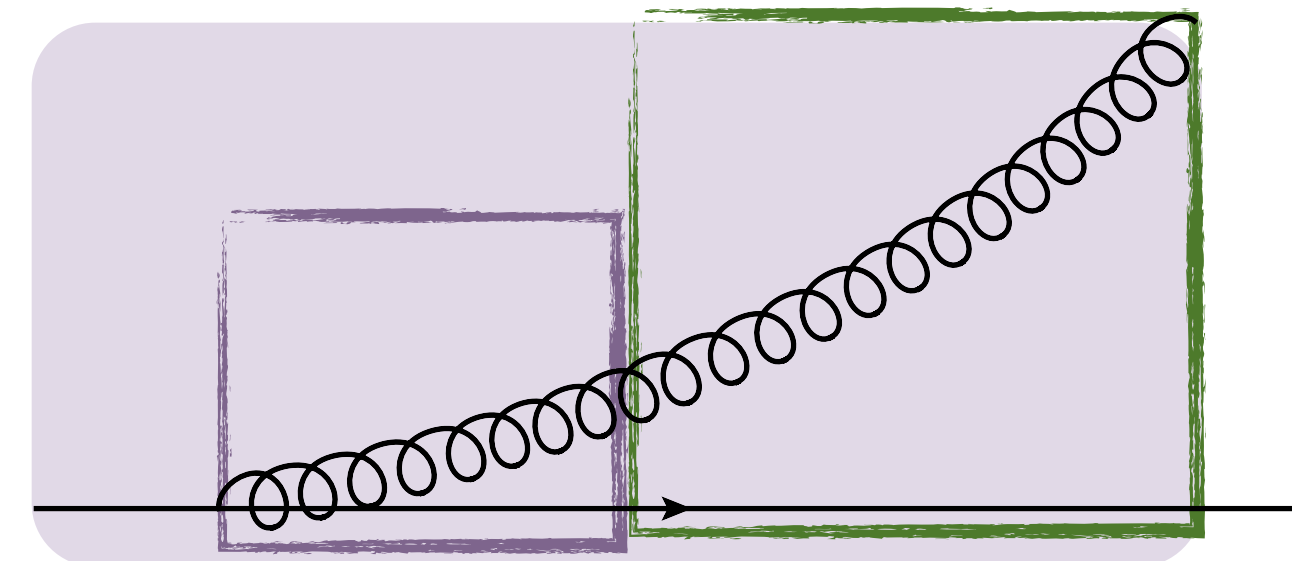
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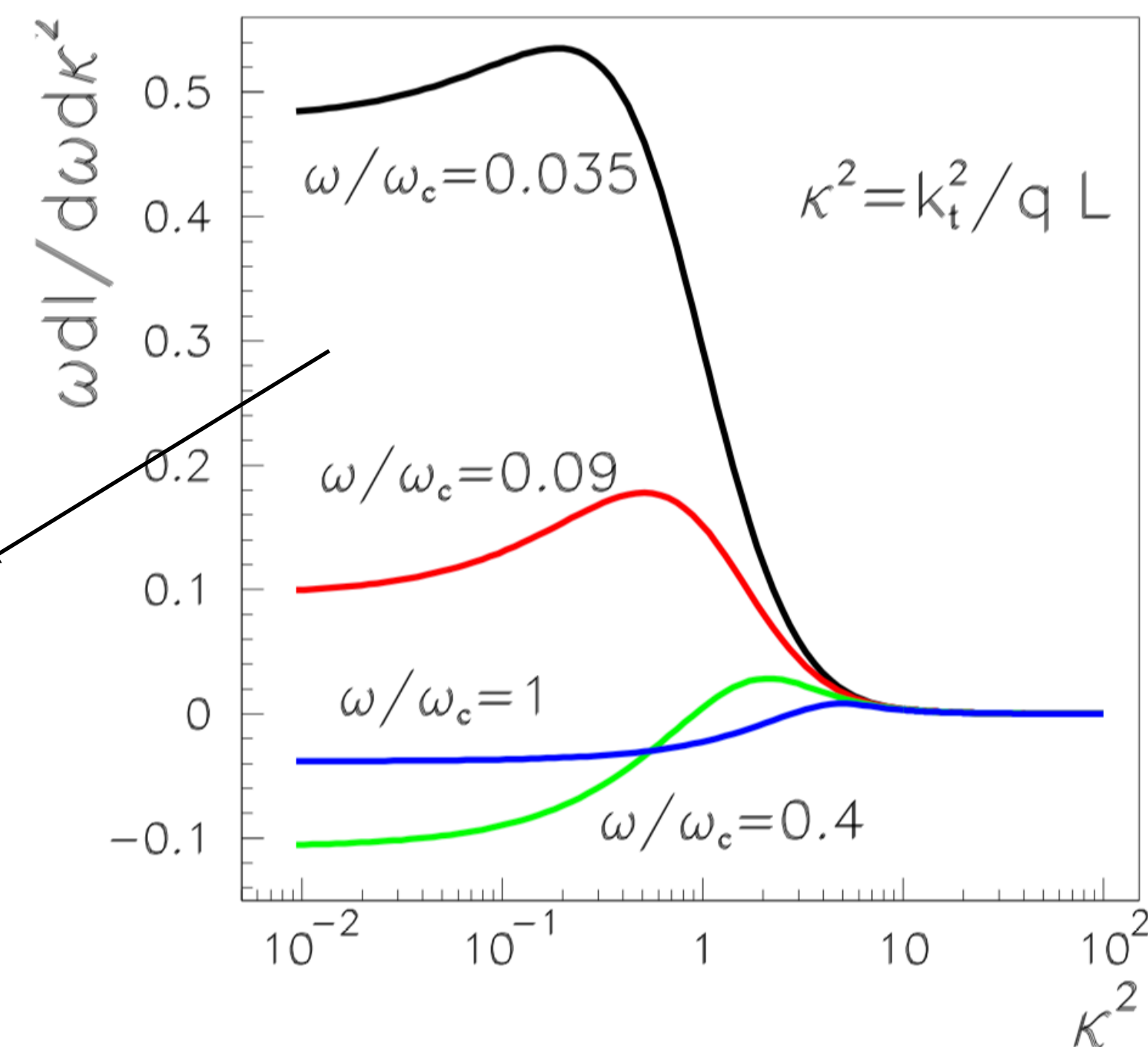
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LPM (QCD) suppression

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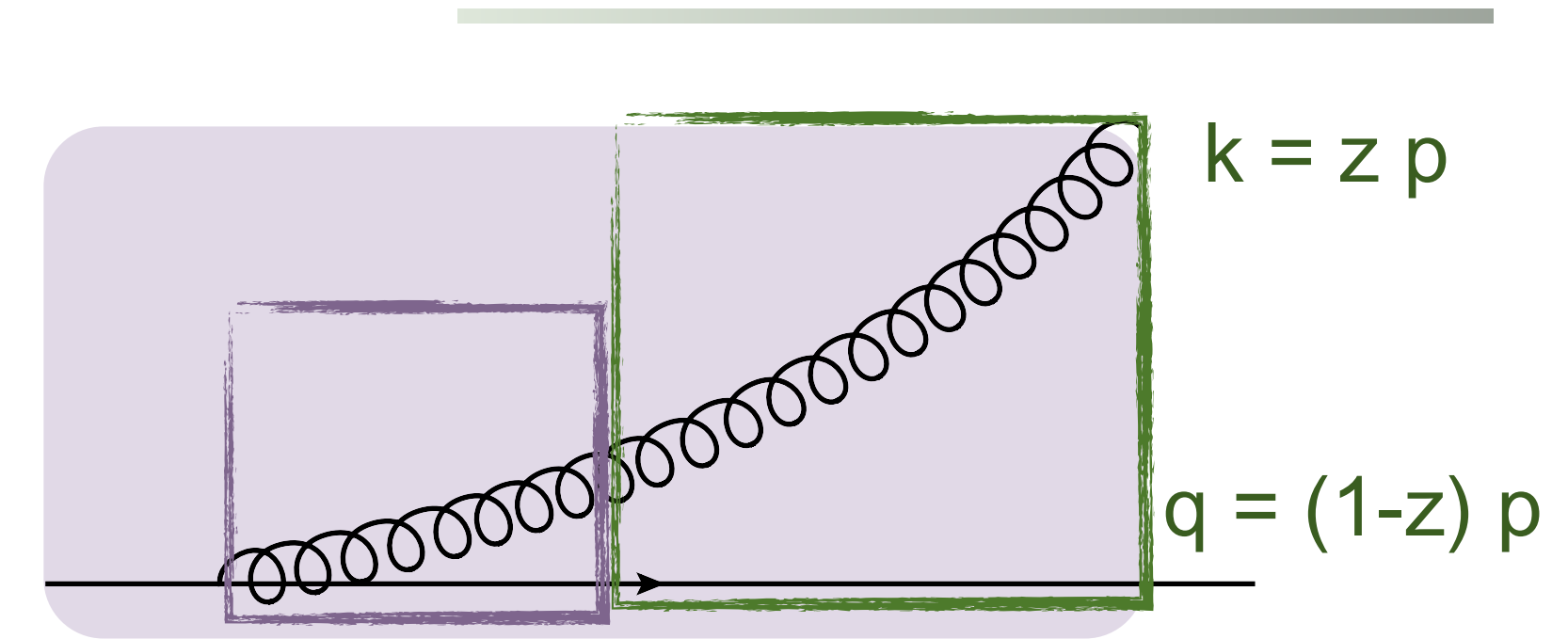


# LPM effect

- ◆ Heuristic discussion on single gluon emission spectrum (BDMPS):

Transport coefficient:

$$\hat{q} = \frac{\langle k_{\perp}^2 \rangle}{\lambda_{mfp}}$$

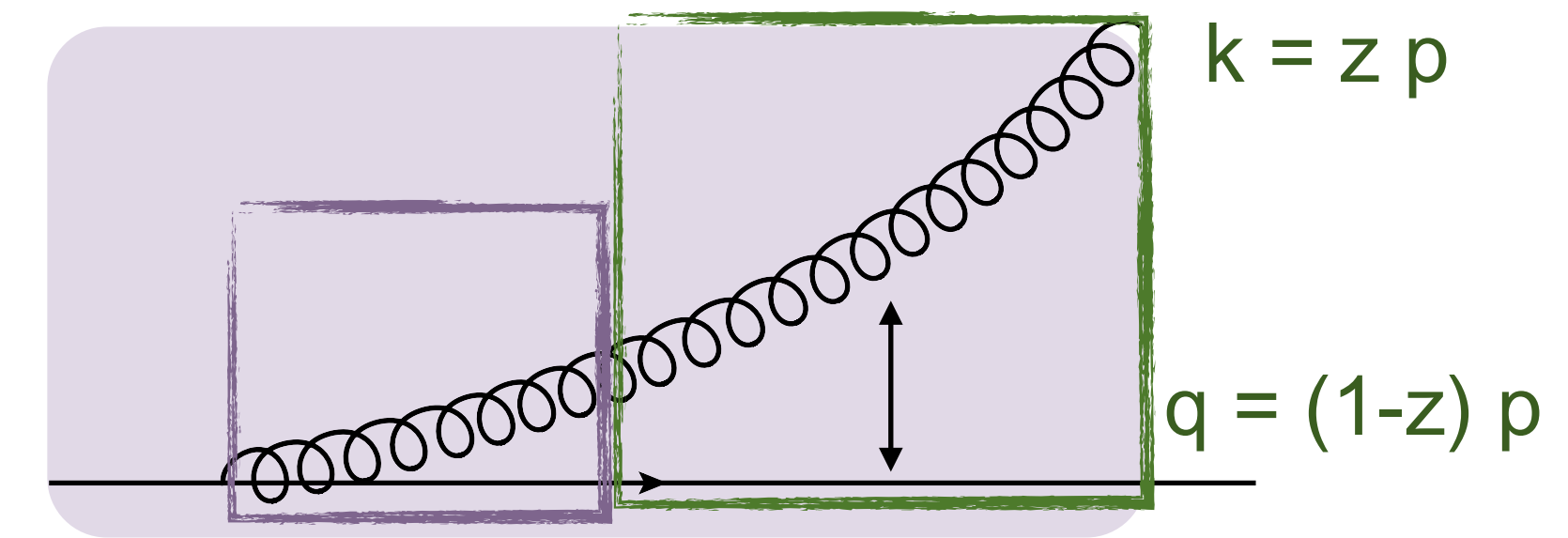


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Soft gluons have shorter formation times

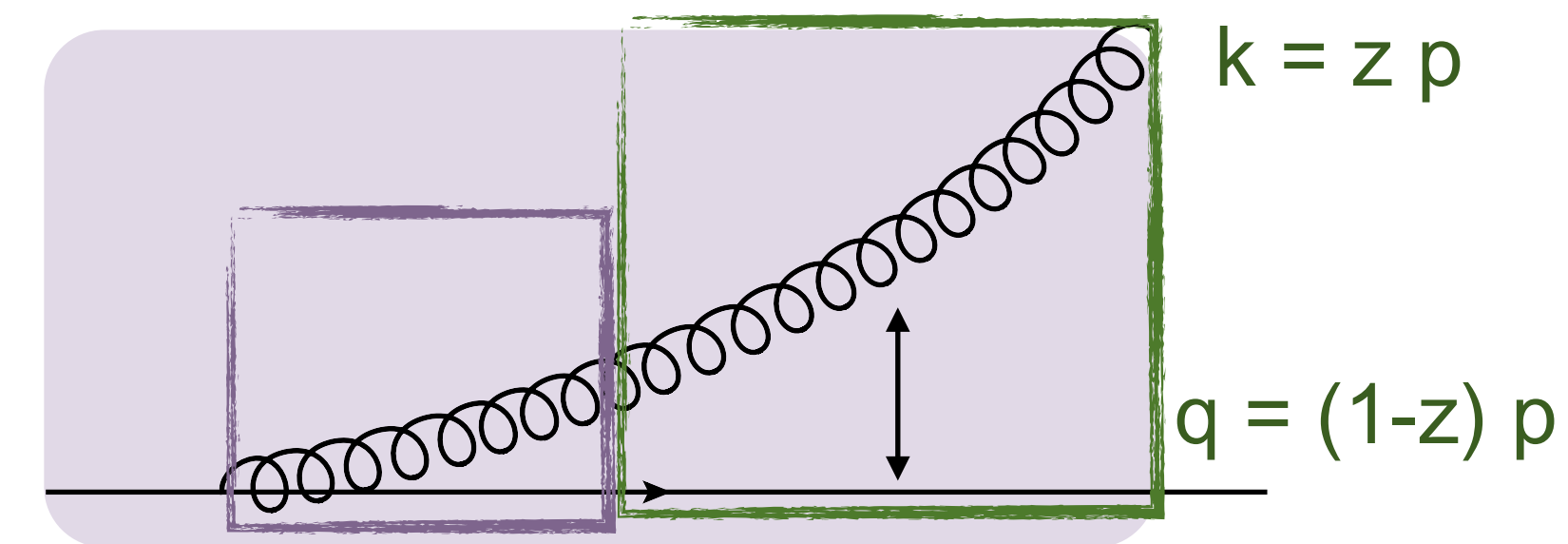


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Soft gluons have shorter formation times

Number of emitted gluons during L:  $N_g \propto \frac{L}{\tau_{form}}$

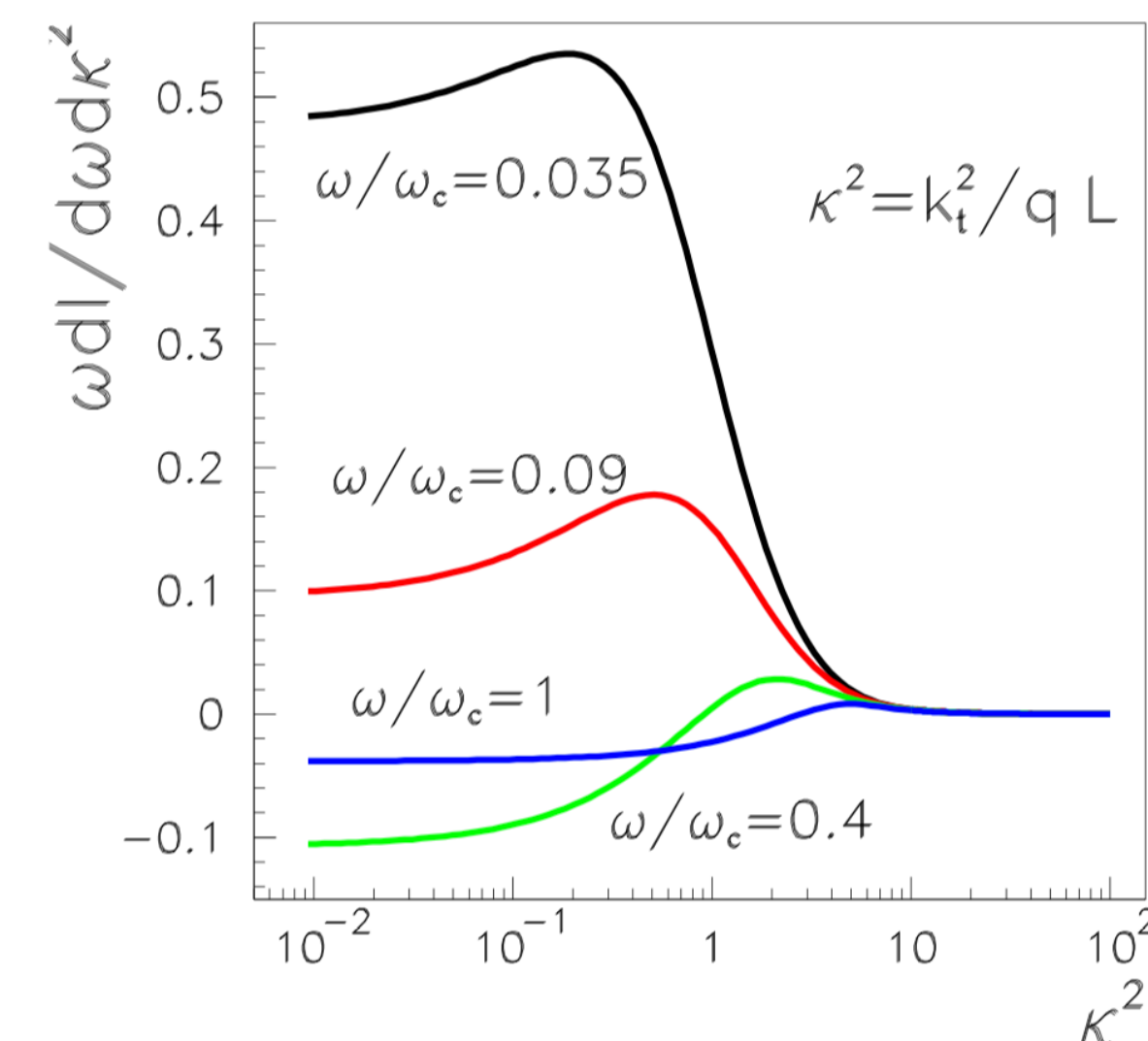
$\tau_{form} \ll L$  Multiple soft emissions (incoherent scatterings)

$\tau_{form} \gg L$  Hard gluon spectrum is suppressed (scattering centres act as a whole)

Critical energy:  $\omega_c(\tau_{form} = L) = \frac{1}{2}\hat{q}L^2$

$$\omega \frac{dI}{d\omega} \Big|_{\omega \leq \omega_c} \sim \alpha_s C_R \sqrt{\frac{\omega_c}{\omega}}$$

$$\omega \frac{dI}{d\omega} \Big|_{\omega > \omega_c} \sim \alpha_s C_R \left(\frac{\omega_c}{\omega}\right)^2$$



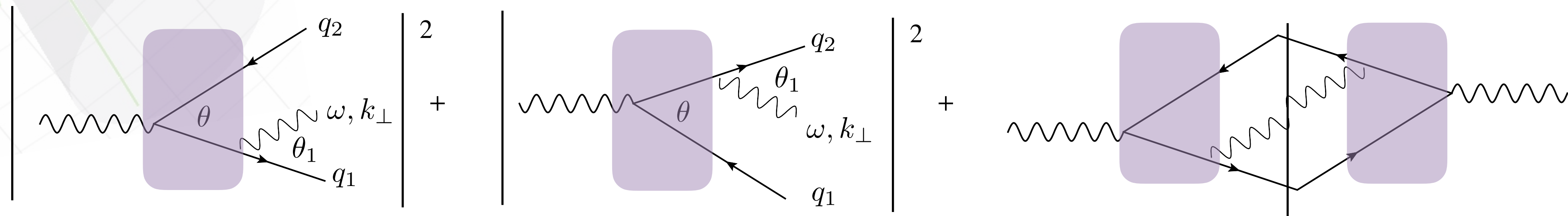
# Intra-jet (de)coherence?

[Mehtar-Tani, Salgado, Tywoniuk (2010-2011)]

[Casalderrey-Solana, Iancu (2011)]

- ◆ Considering the in-medium singlet quark - antiquark antenna in the simplest case:

- ➔ Soft gluon emission outside medium



Eikonal approximation: 
$$\frac{dI}{d\Omega_k} = R_q + R_{\bar{q}} - 2J(1 - \Delta_{med}) = R_{coh} + 2J\Delta_{med}$$

$$R_q \sim \alpha_s C_F \frac{q_{1+}}{(k \cdot q_1)}$$

$$R_{\bar{q}} \sim \alpha_s C_F \frac{q_{2+}}{(k \cdot q_2)}$$

$$2J \sim \alpha_s C_F \left[ \frac{q_{1+}}{(k \cdot q_1)} + \frac{q_{2+}}{(k \cdot q_2)} - \frac{k_+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)} \right]$$



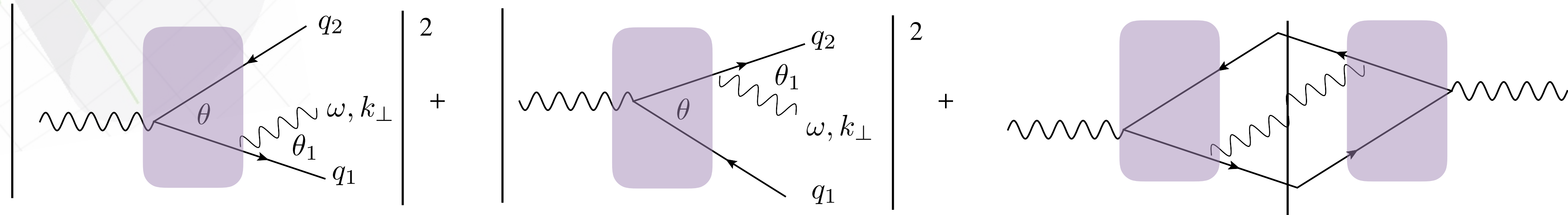
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new in-medium contribution:

$$1 - \Delta_{med} = \frac{1}{N_c^2} \text{Tr} \left\langle W_A(\mathbf{x}_q) W_A^\dagger(\mathbf{x}_{\bar{q}}) \right\rangle$$

as in vacuum:  $R_{coh} \sim \alpha_s C_F \frac{k_+(q_1 \cdot q_2)}{(k \cdot q_1)(k \cdot q_2)}$

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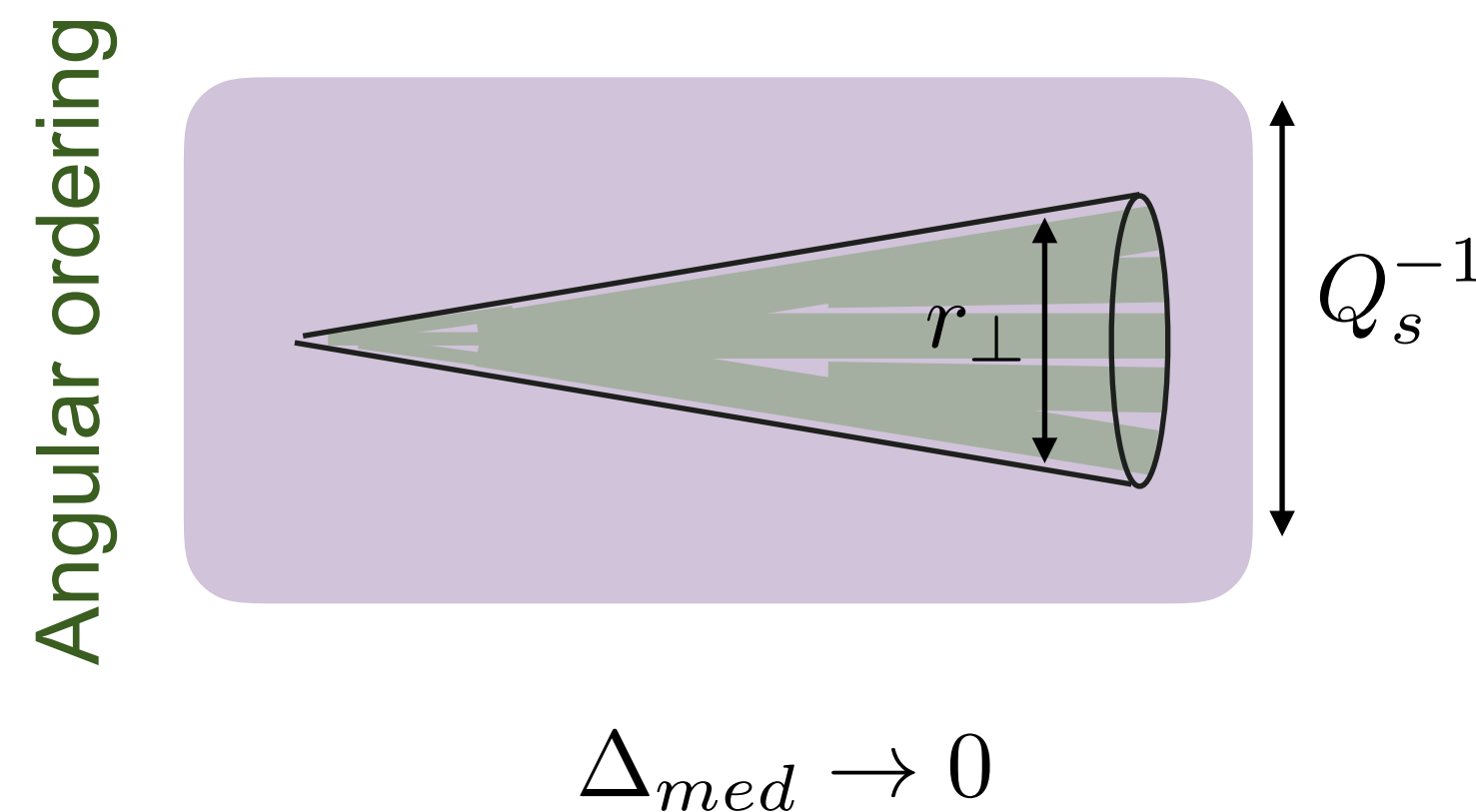
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Antenna Transverse resolution:  $r_\perp = \theta L$   
 Medium Transverse Scale:  $Q_s^{-1} = \sqrt{(\hat{q} L)^{-1}}$



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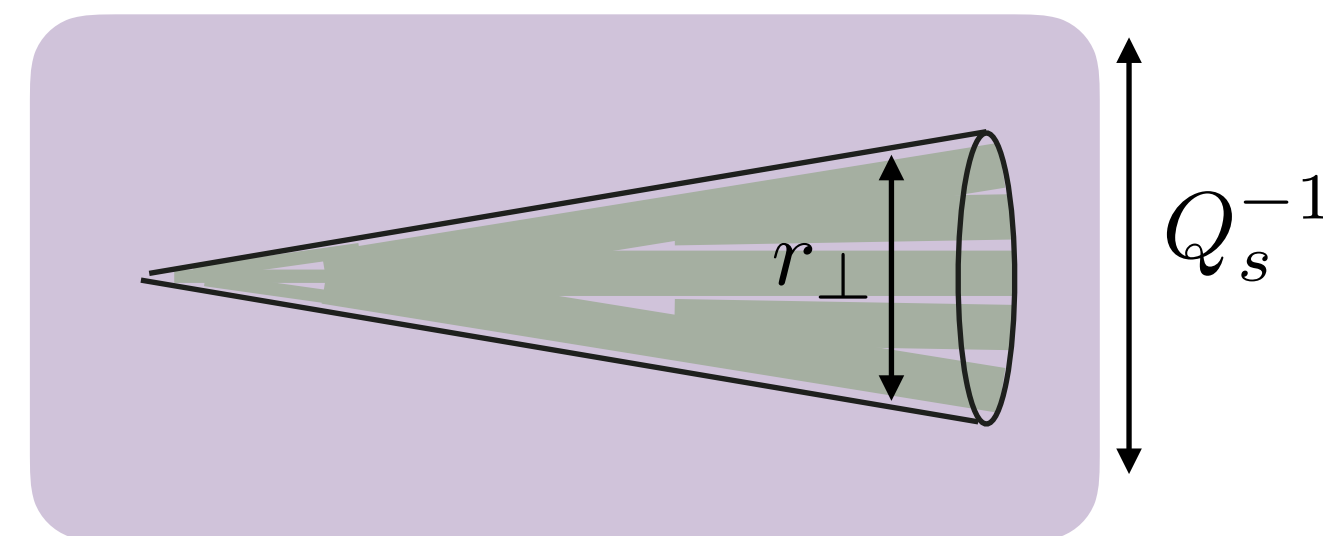
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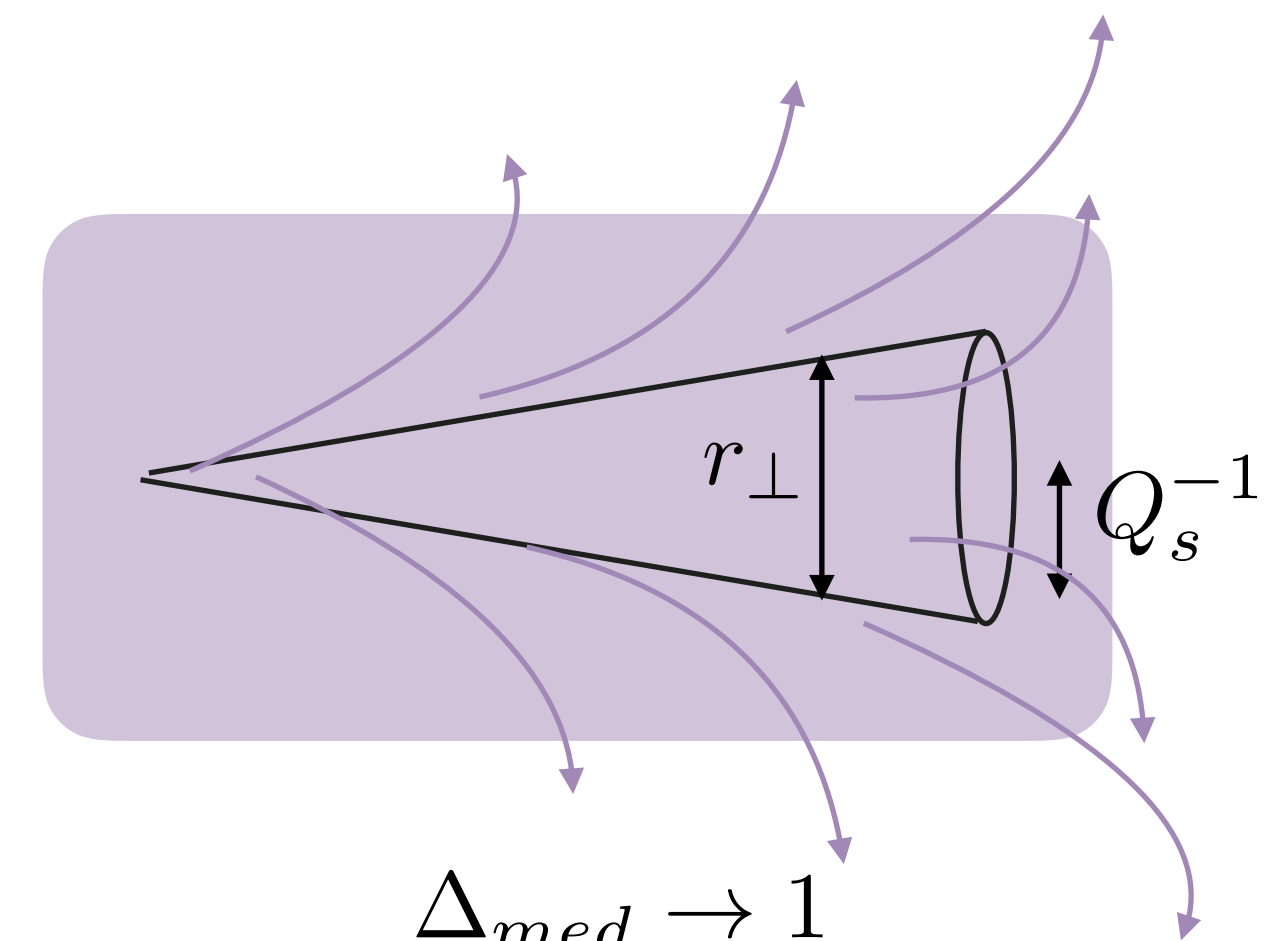
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Angular ordering



$\Delta_{med} \rightarrow 0$

Anti-Angular ordering

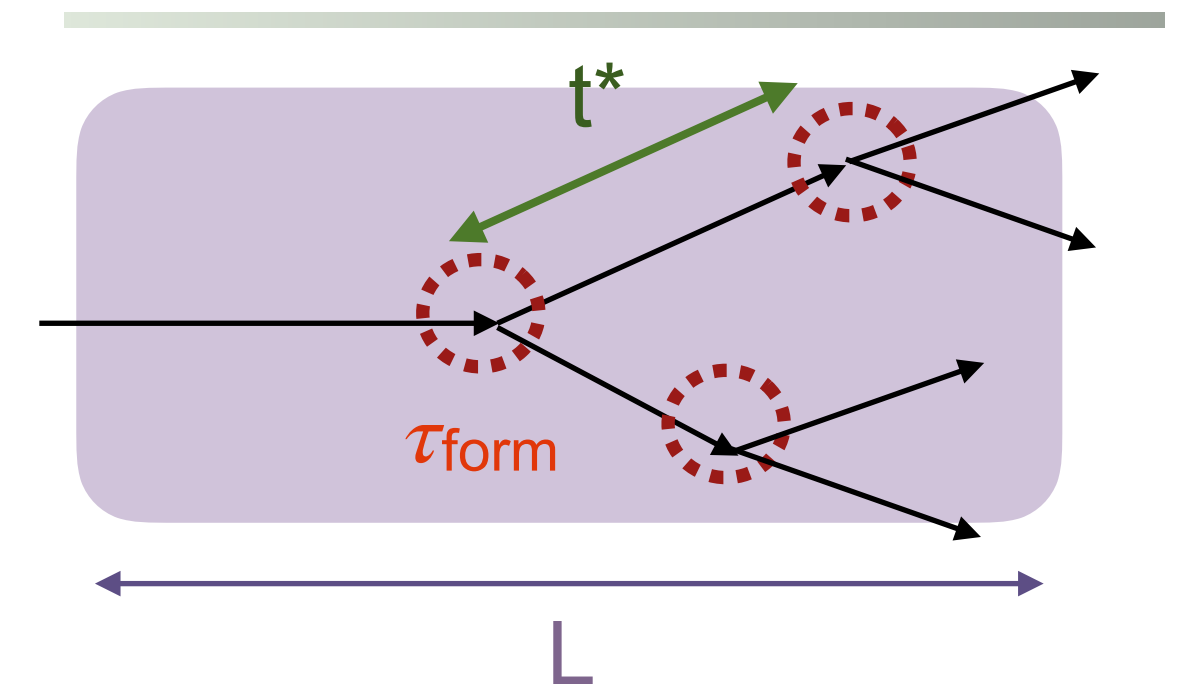


$\Delta_{med} \rightarrow 1$



# More th developments

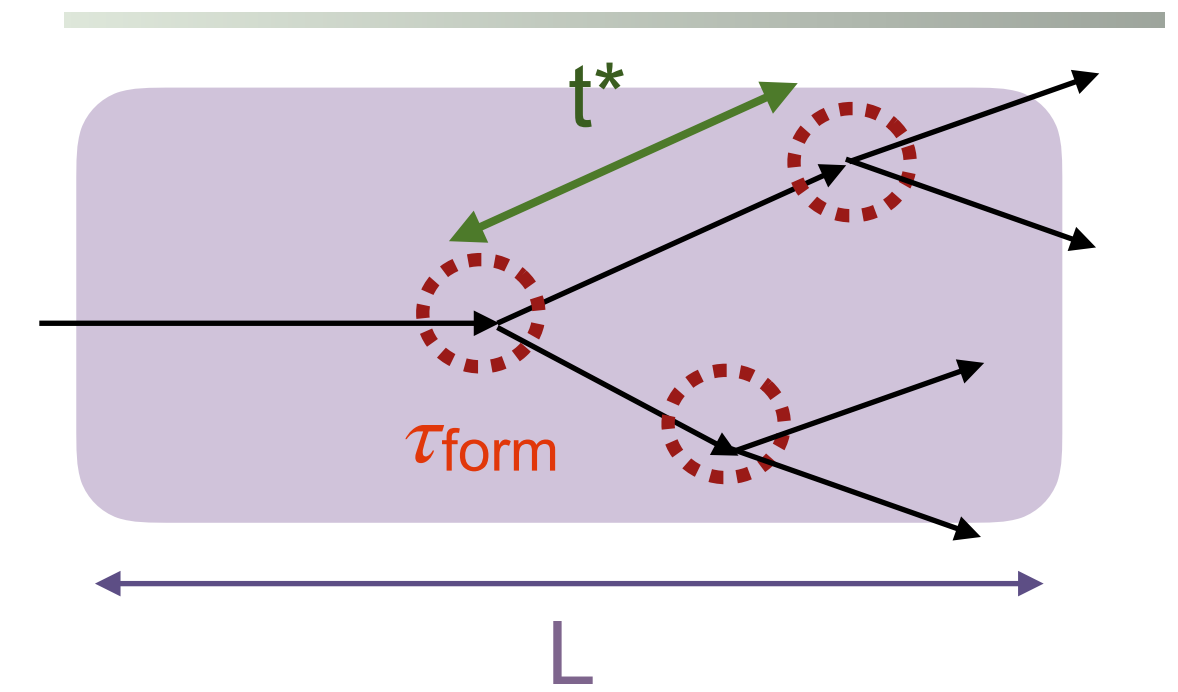
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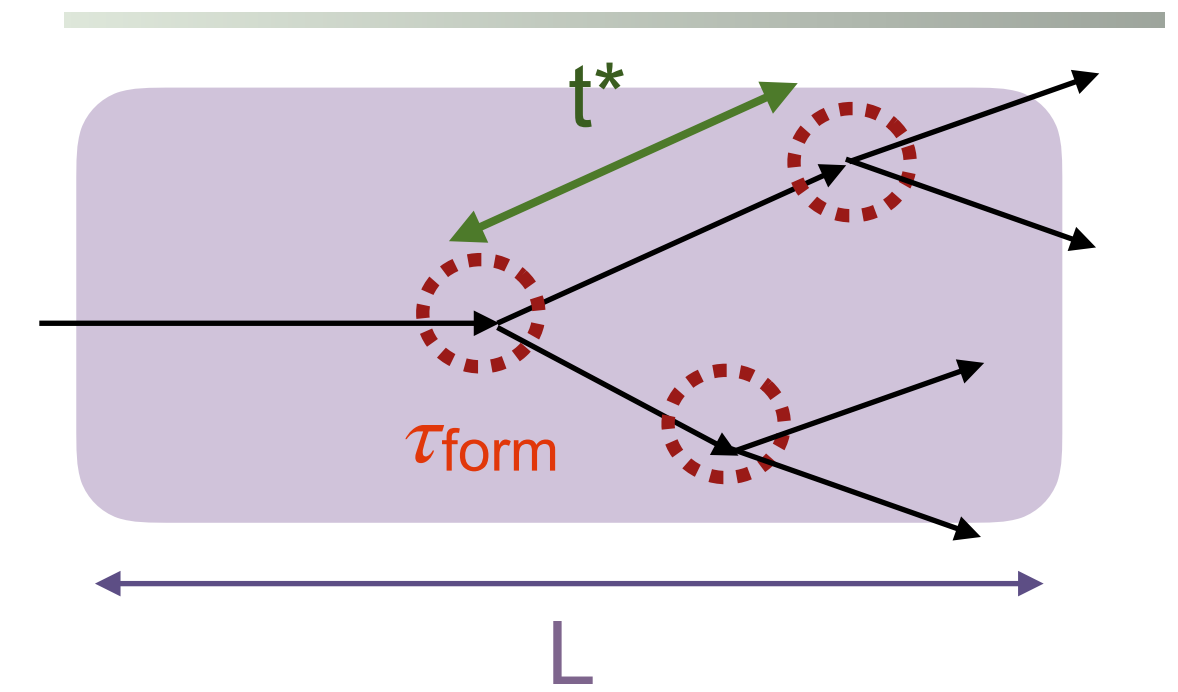


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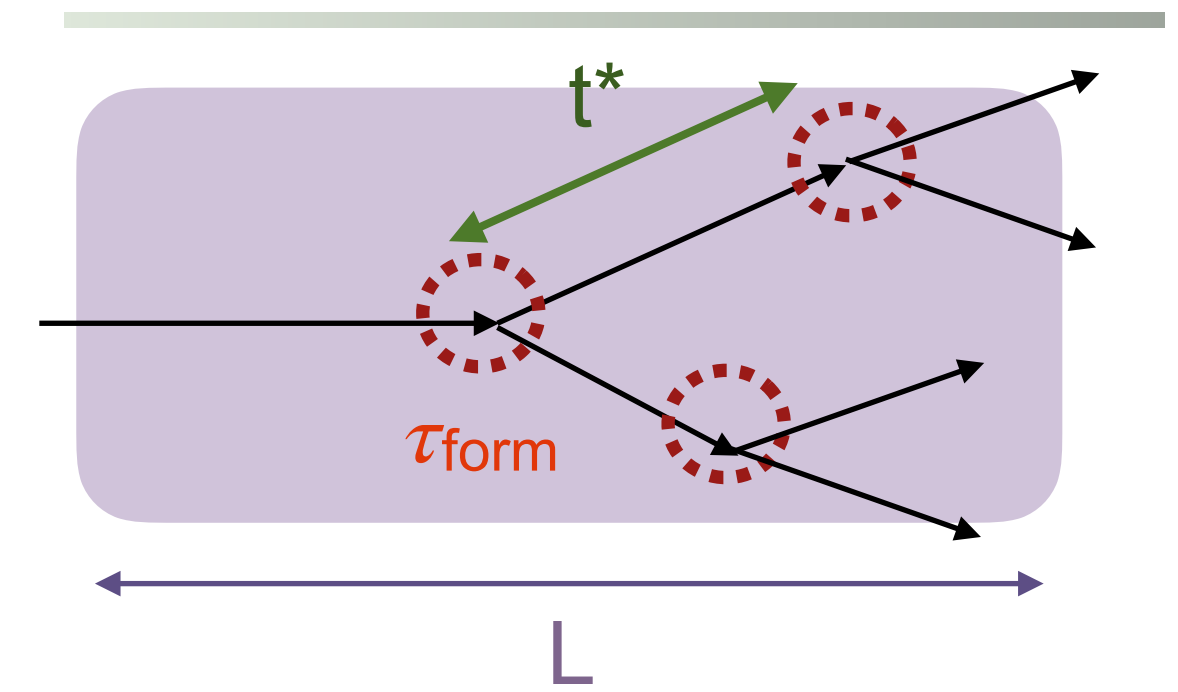
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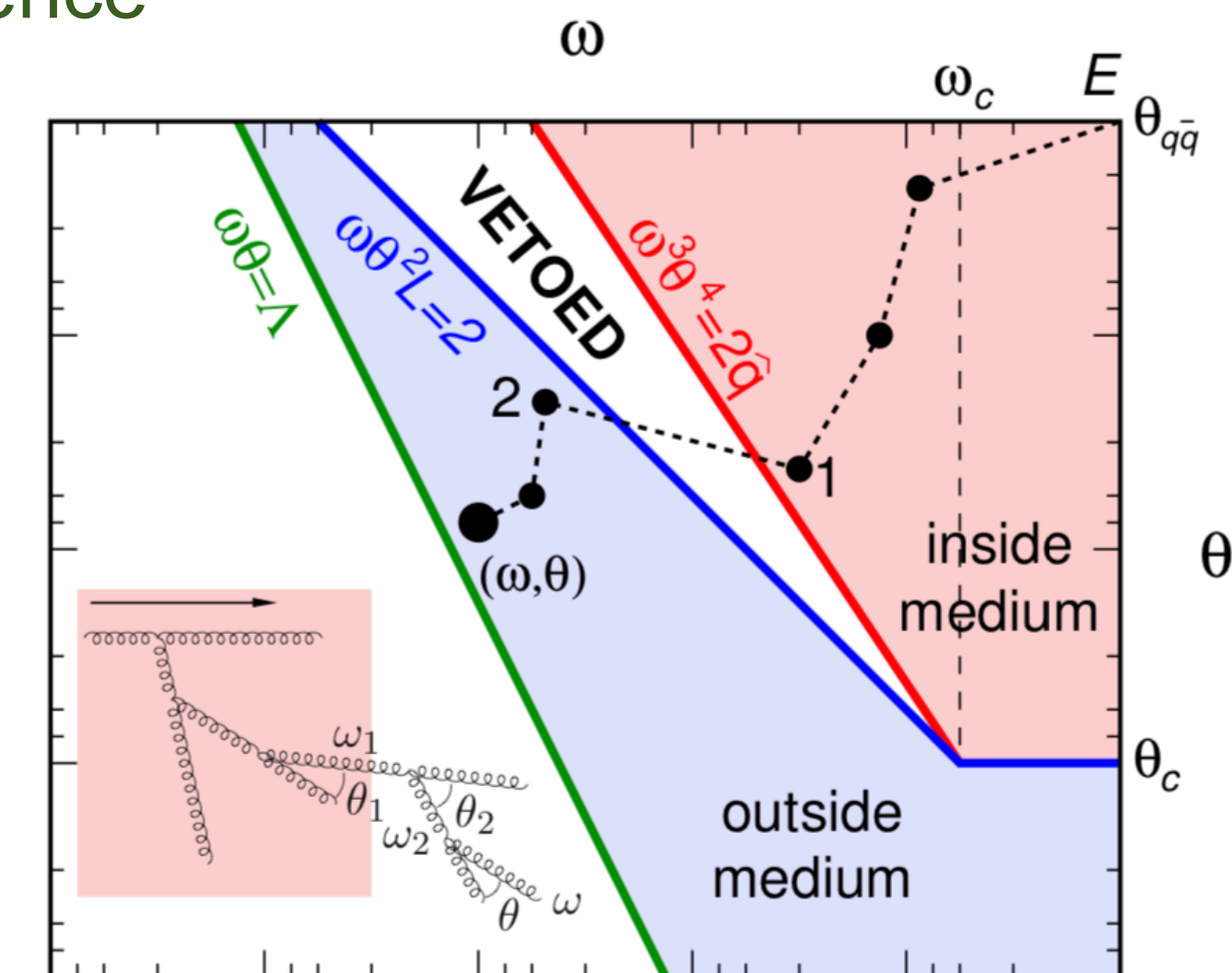
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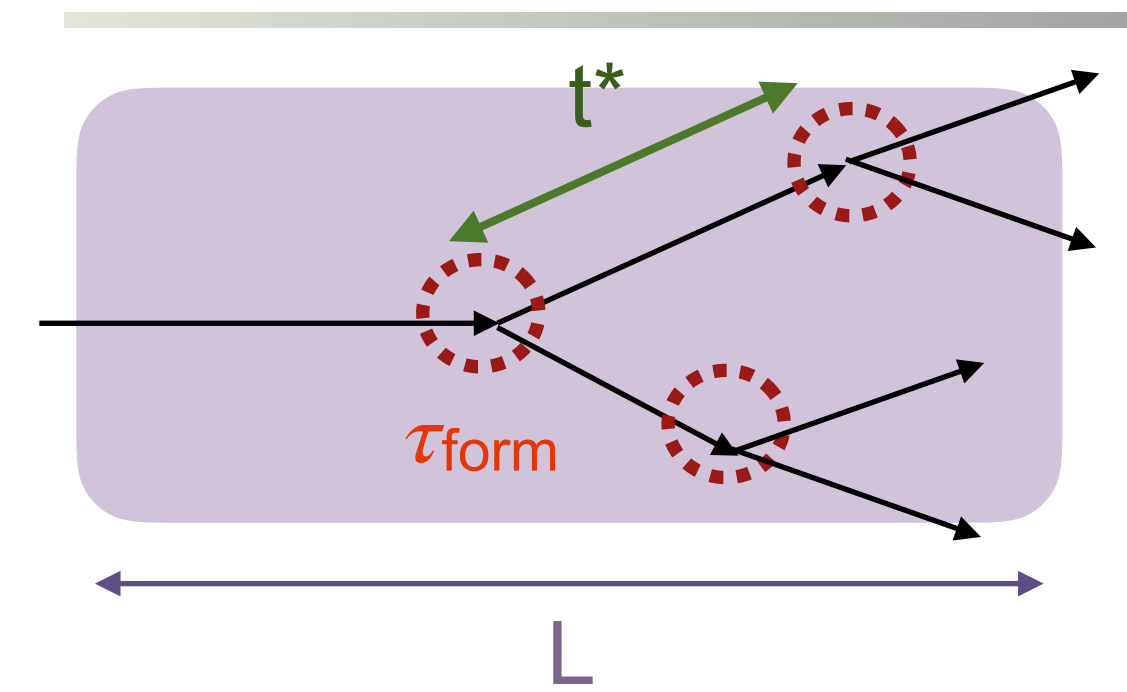




# More th developments

See K. Tywoniuk (Thursday)

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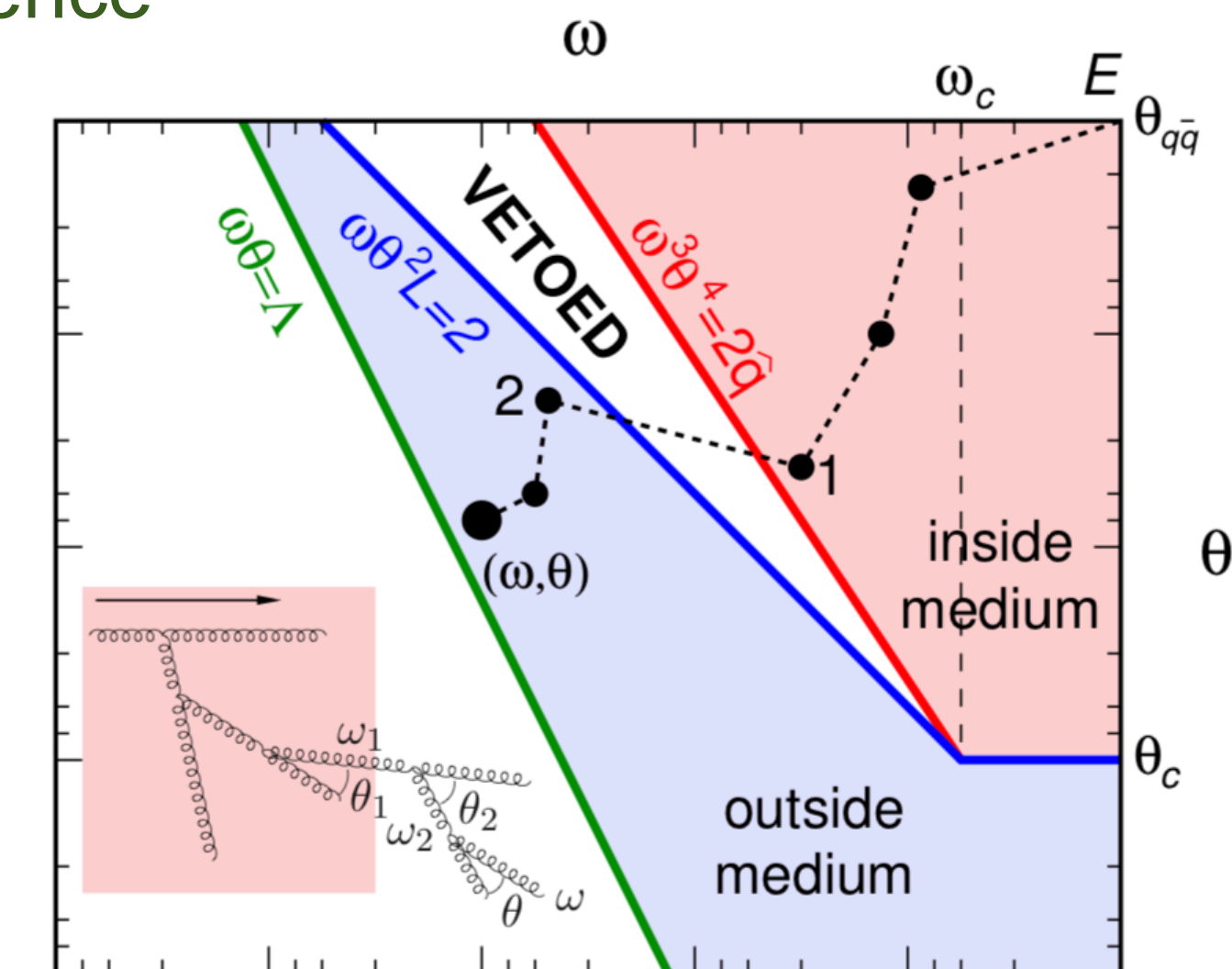
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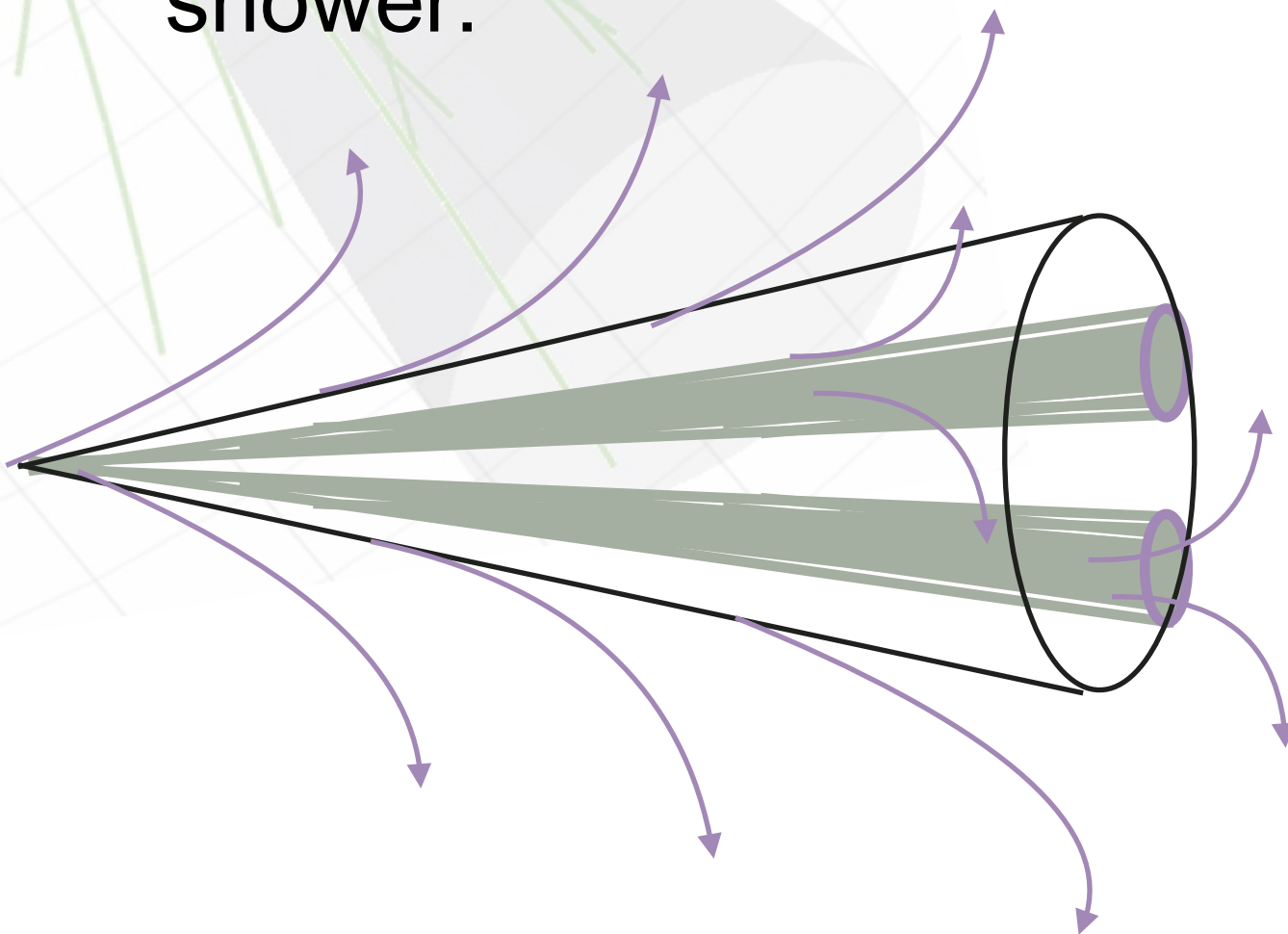


# Resulting picture

[See also: 1801.09703]

[1401.8293]

◆ Resulting picture of a medium-modified parton shower:



Finite size “pp-like” structures  
(Angular Ordering)

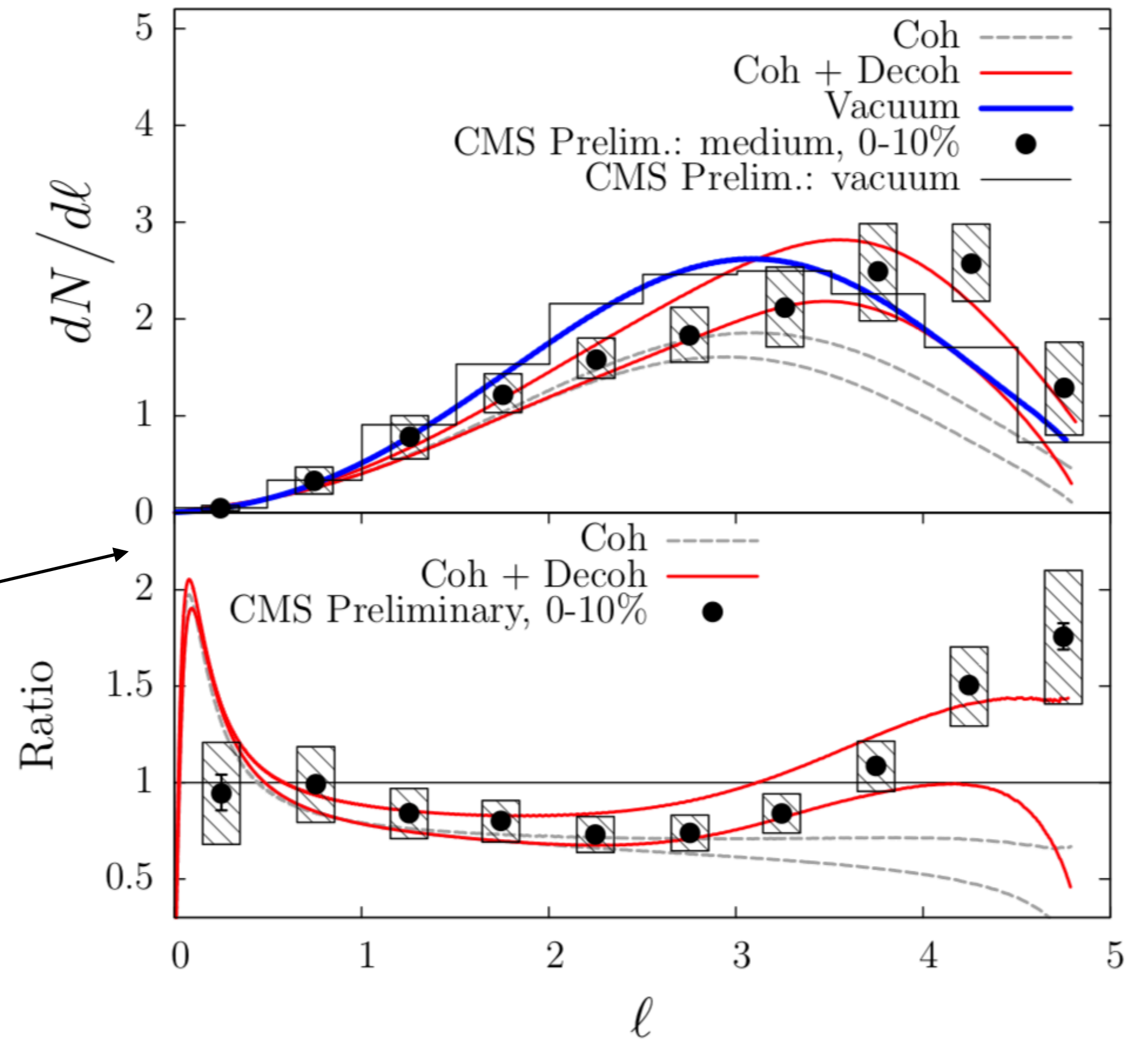
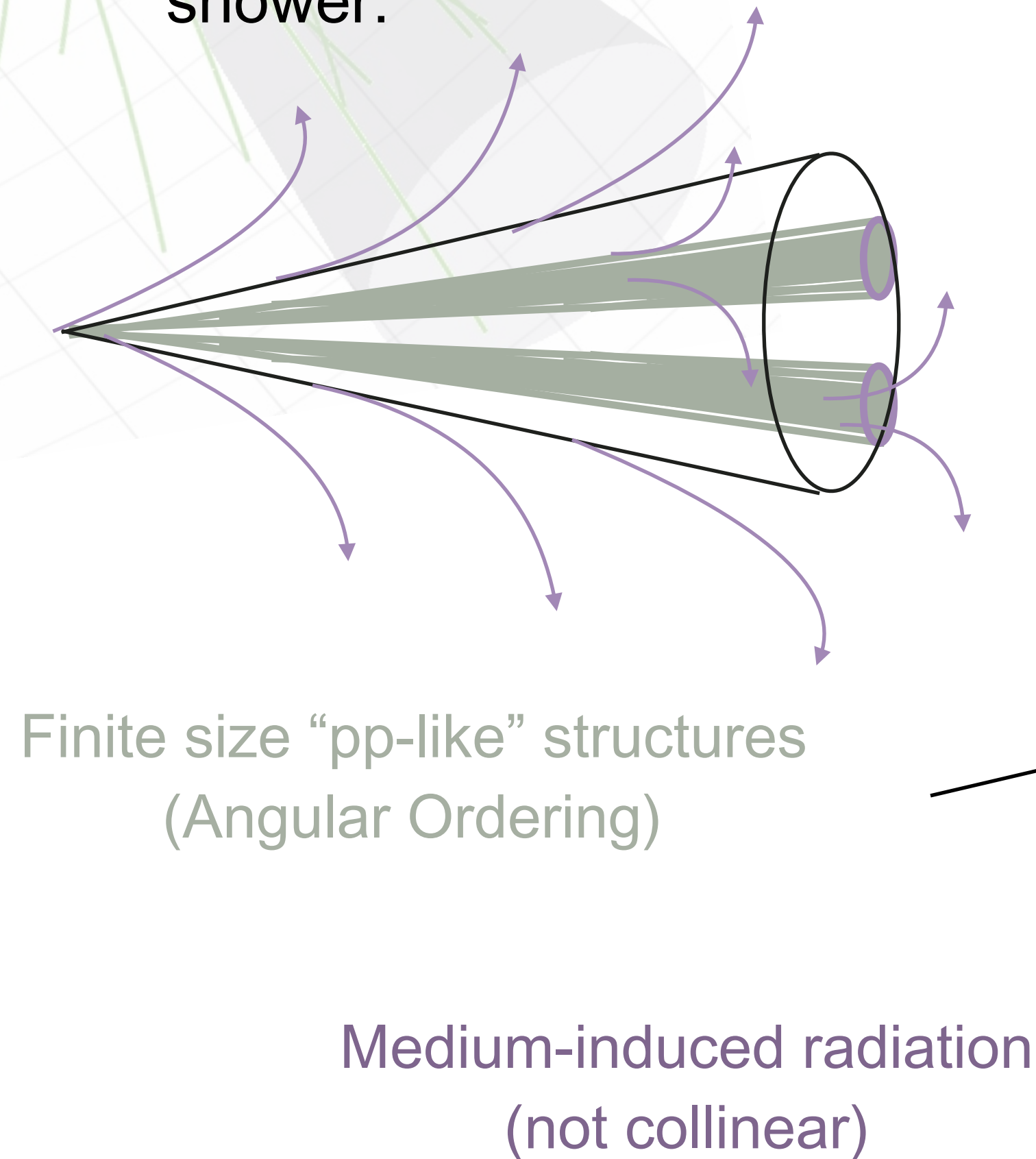
Medium-induced radiation  
(not collinear)

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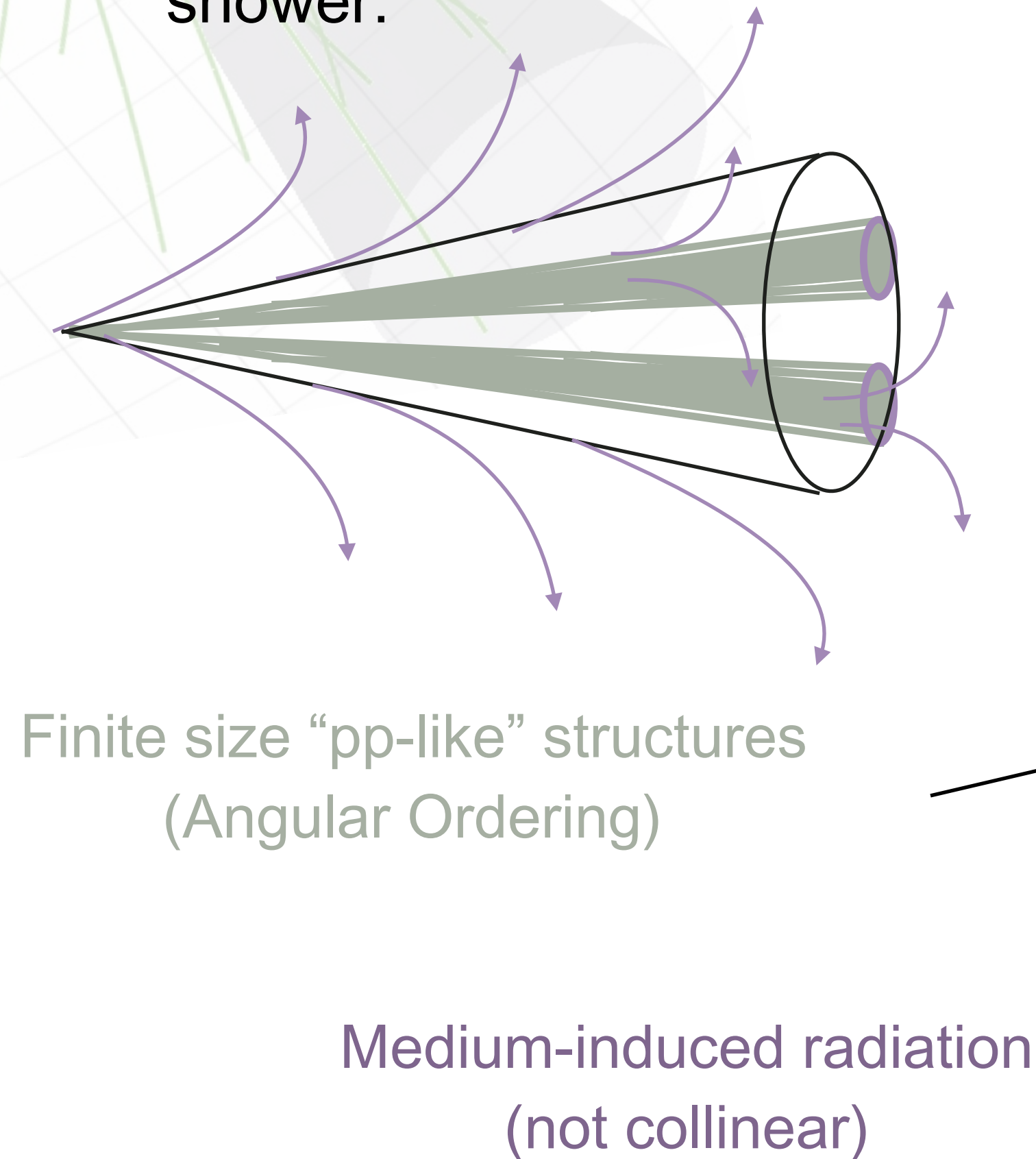


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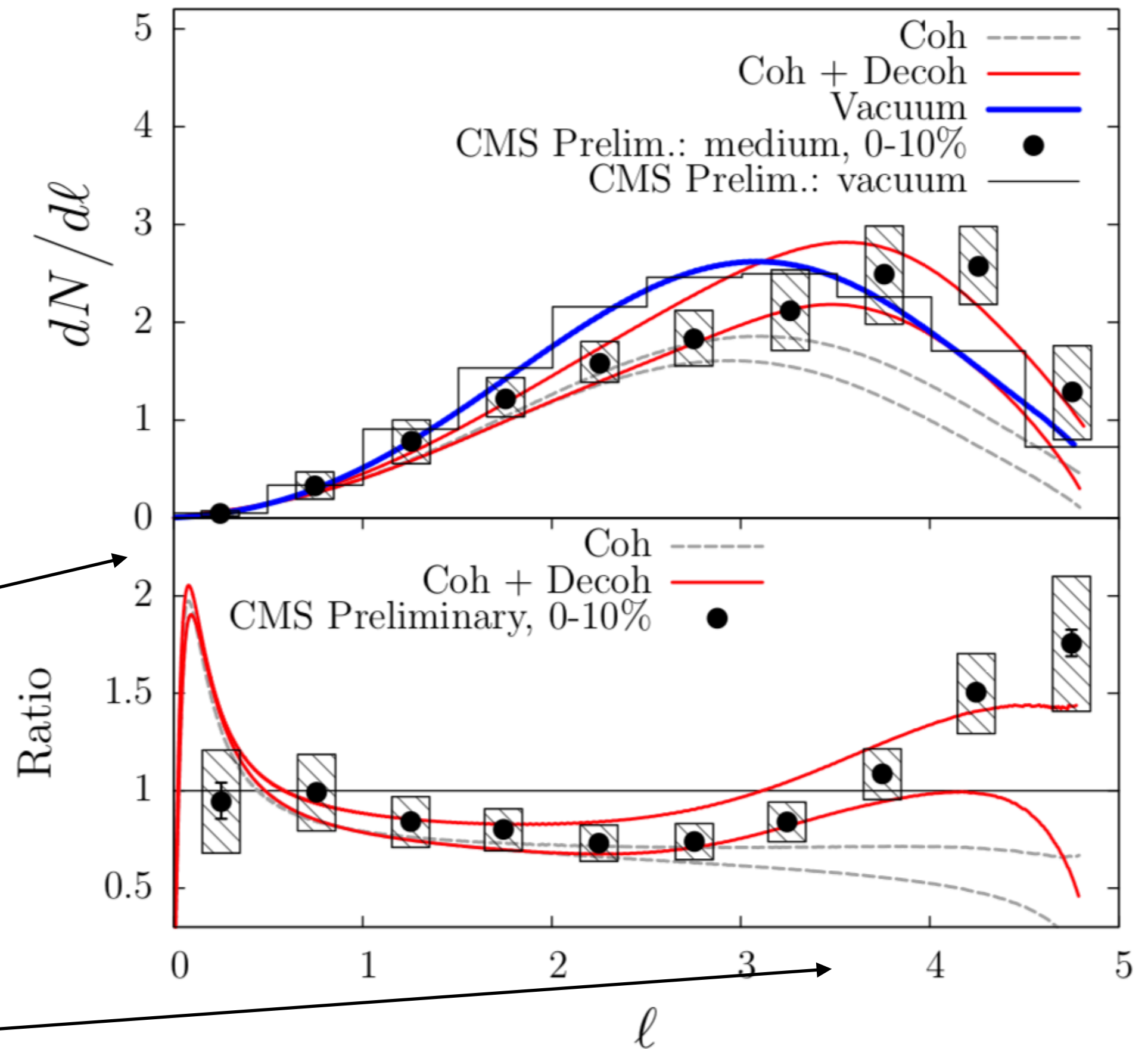
[1401.8293]

◆ Resulting picture of a medium-modified parton shower:

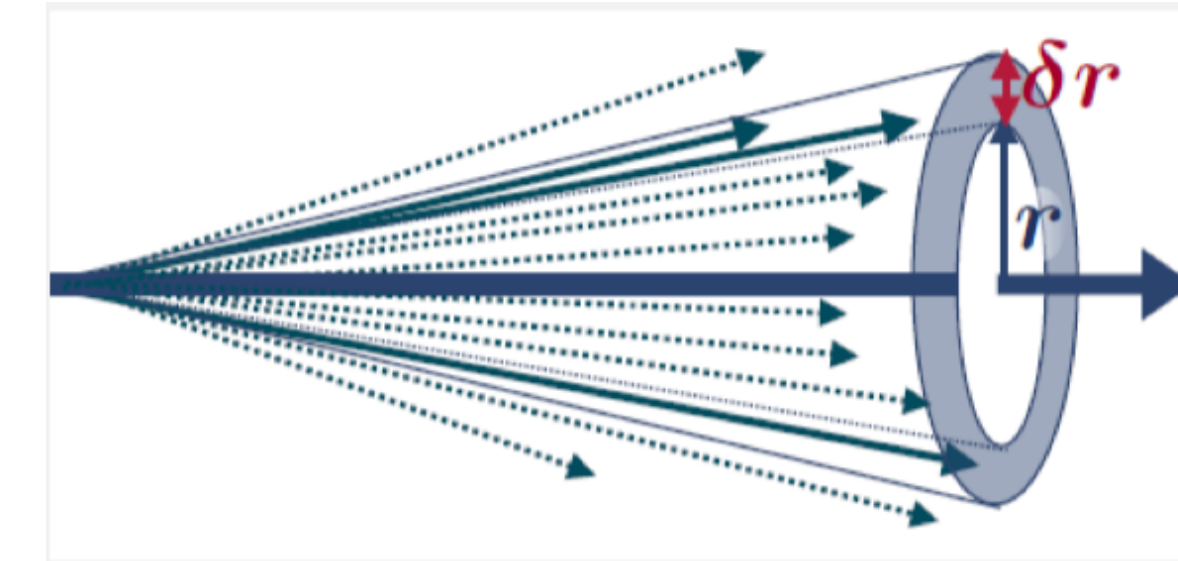


hard structure:  
unmodified

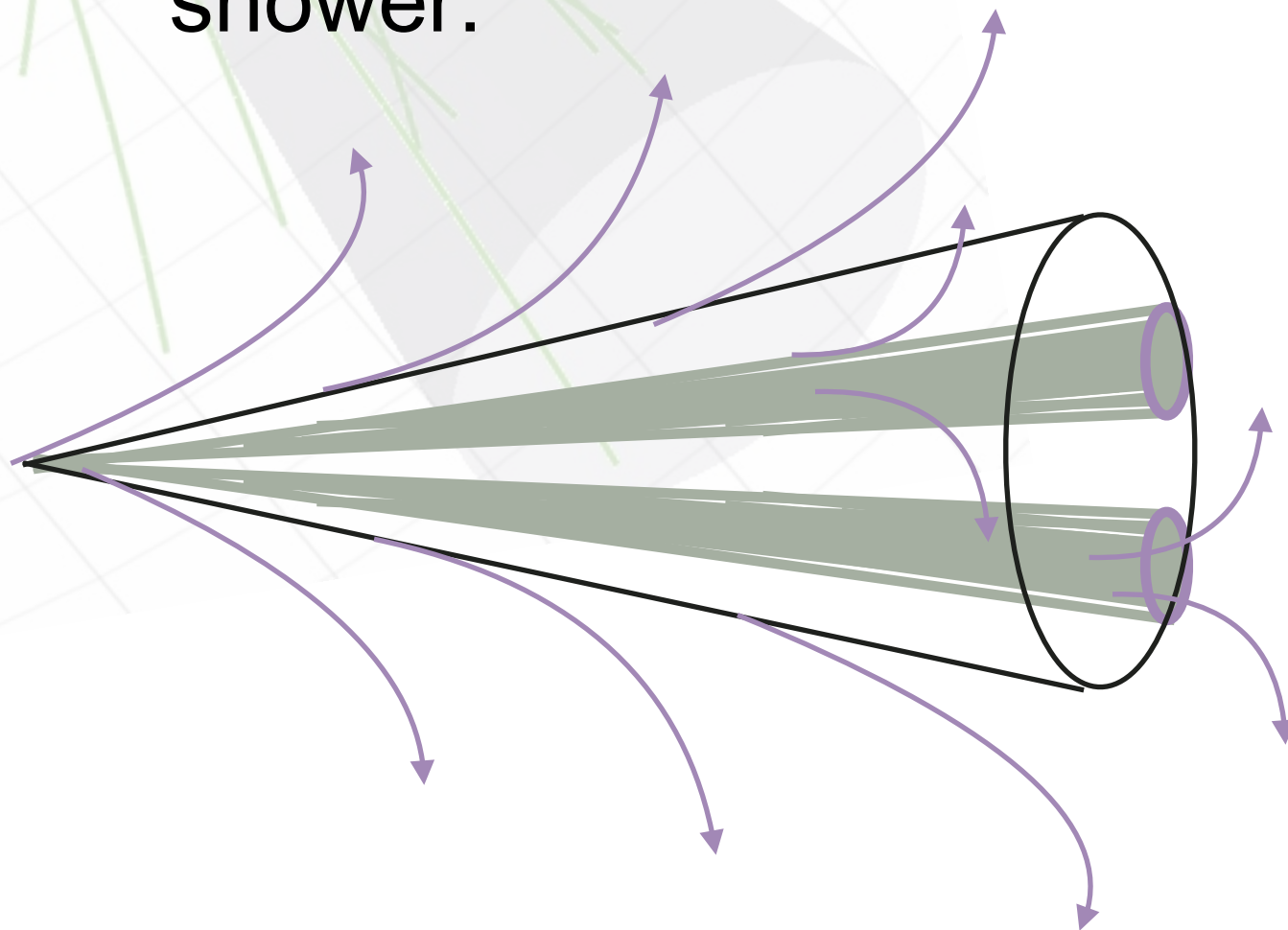
increase of soft  
fragments



# Resulting picture



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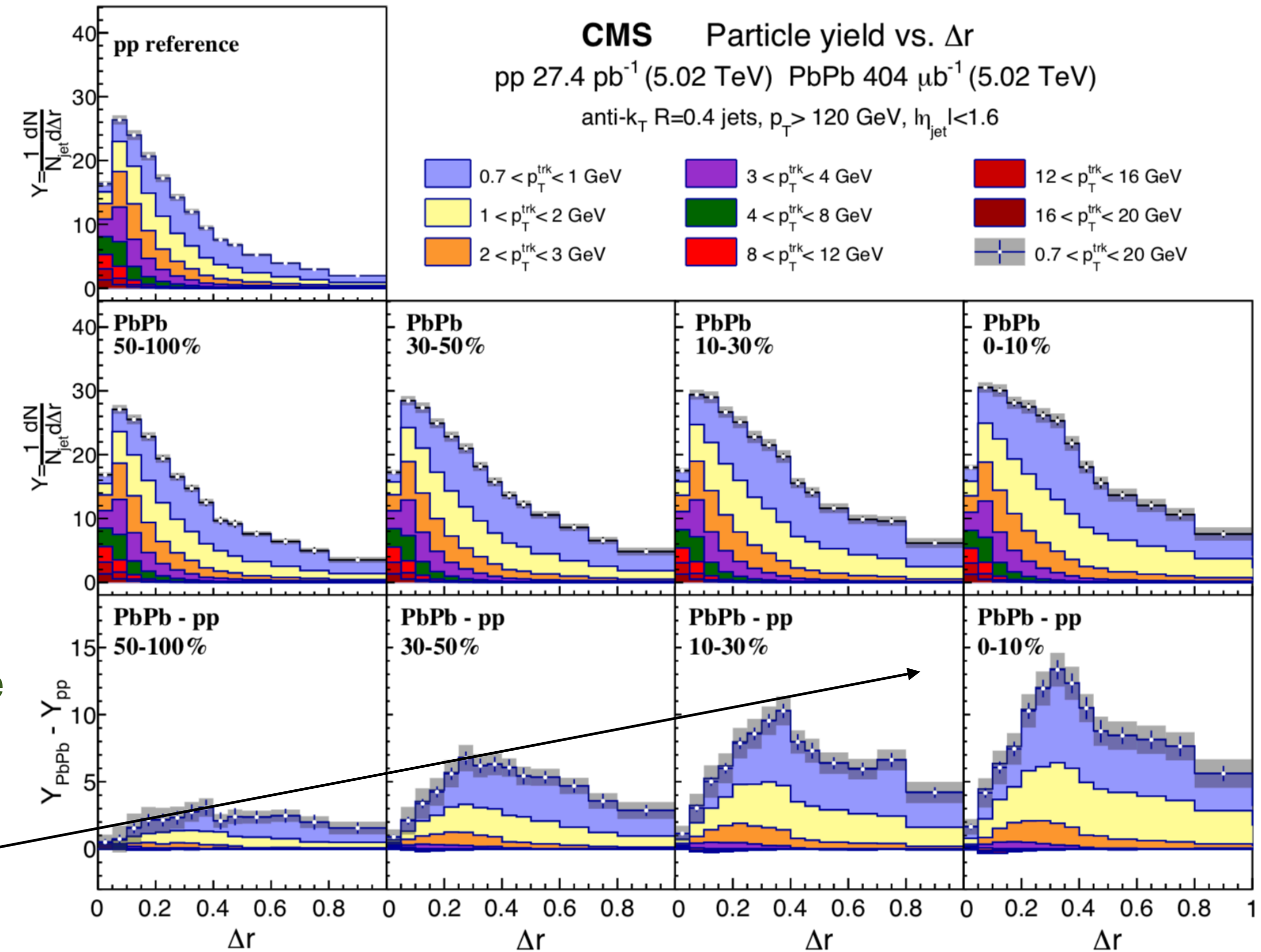


Finite size "pp-like" structures  
(Angular Ordering)

Soft fragments  
radiated up to large  
angles

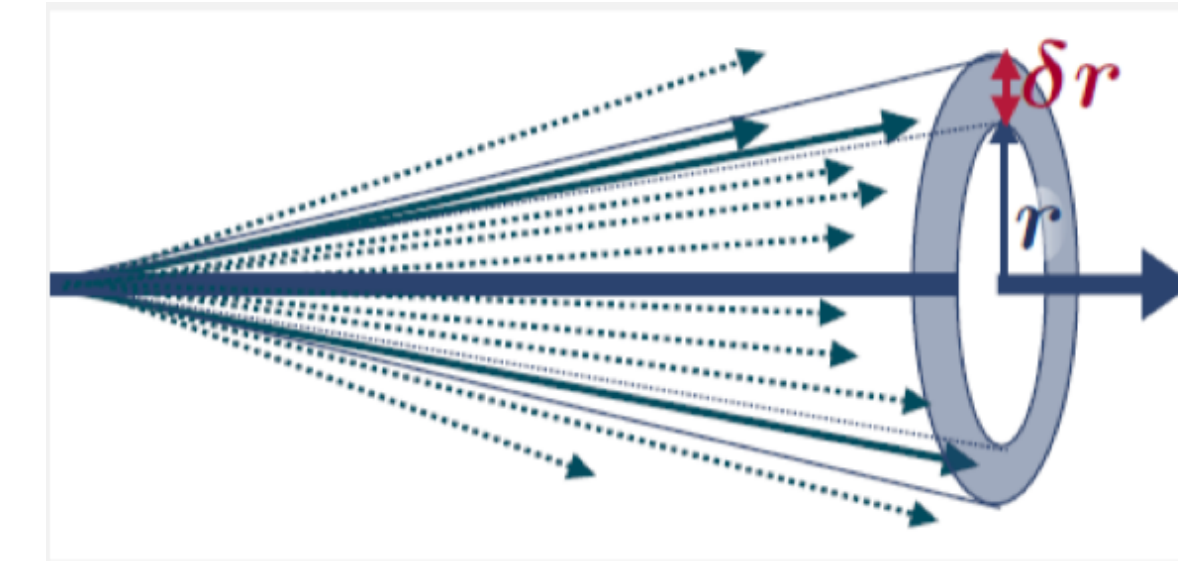
Medium-induced radiation  
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arXiv:1803.00042

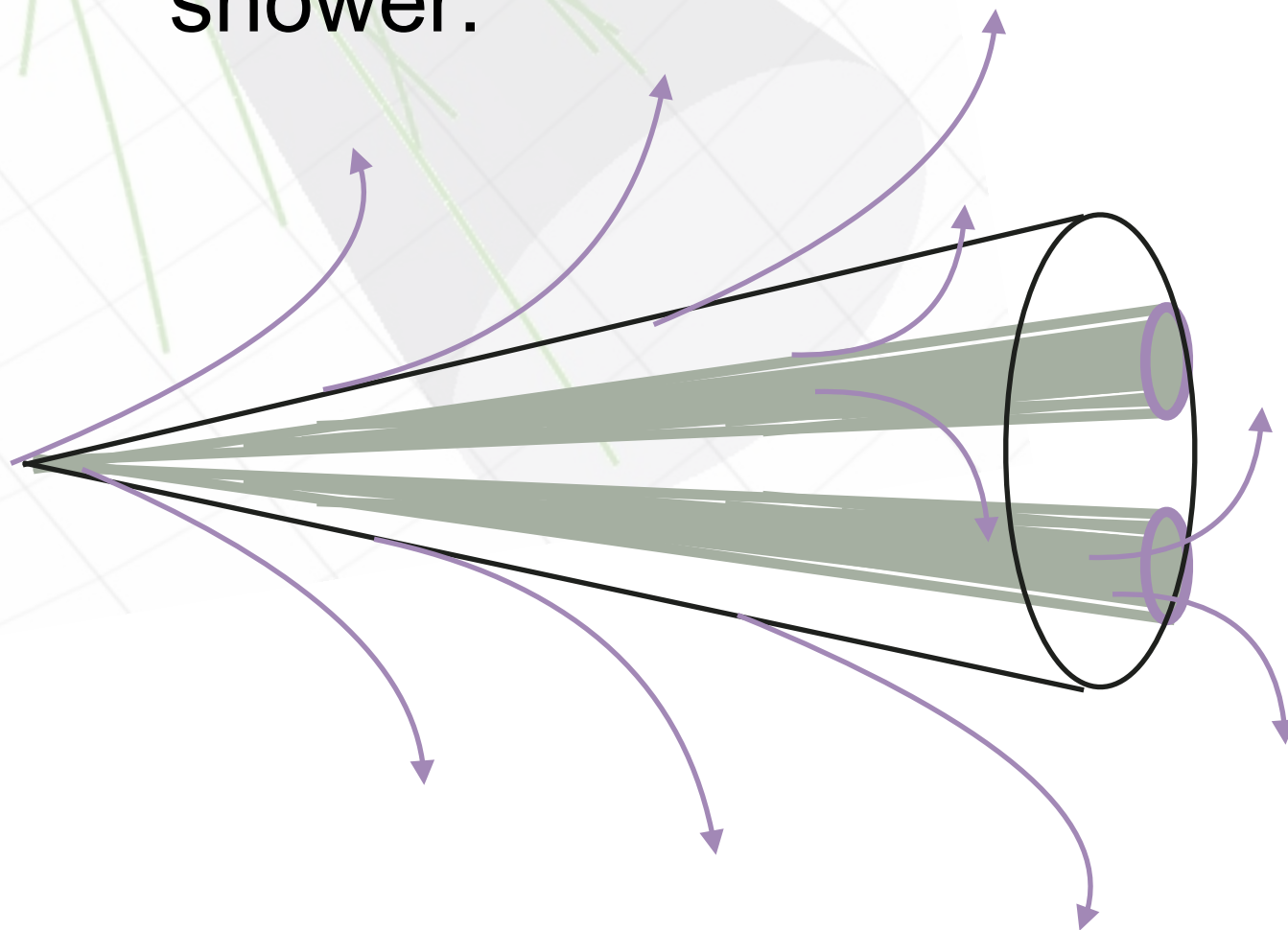




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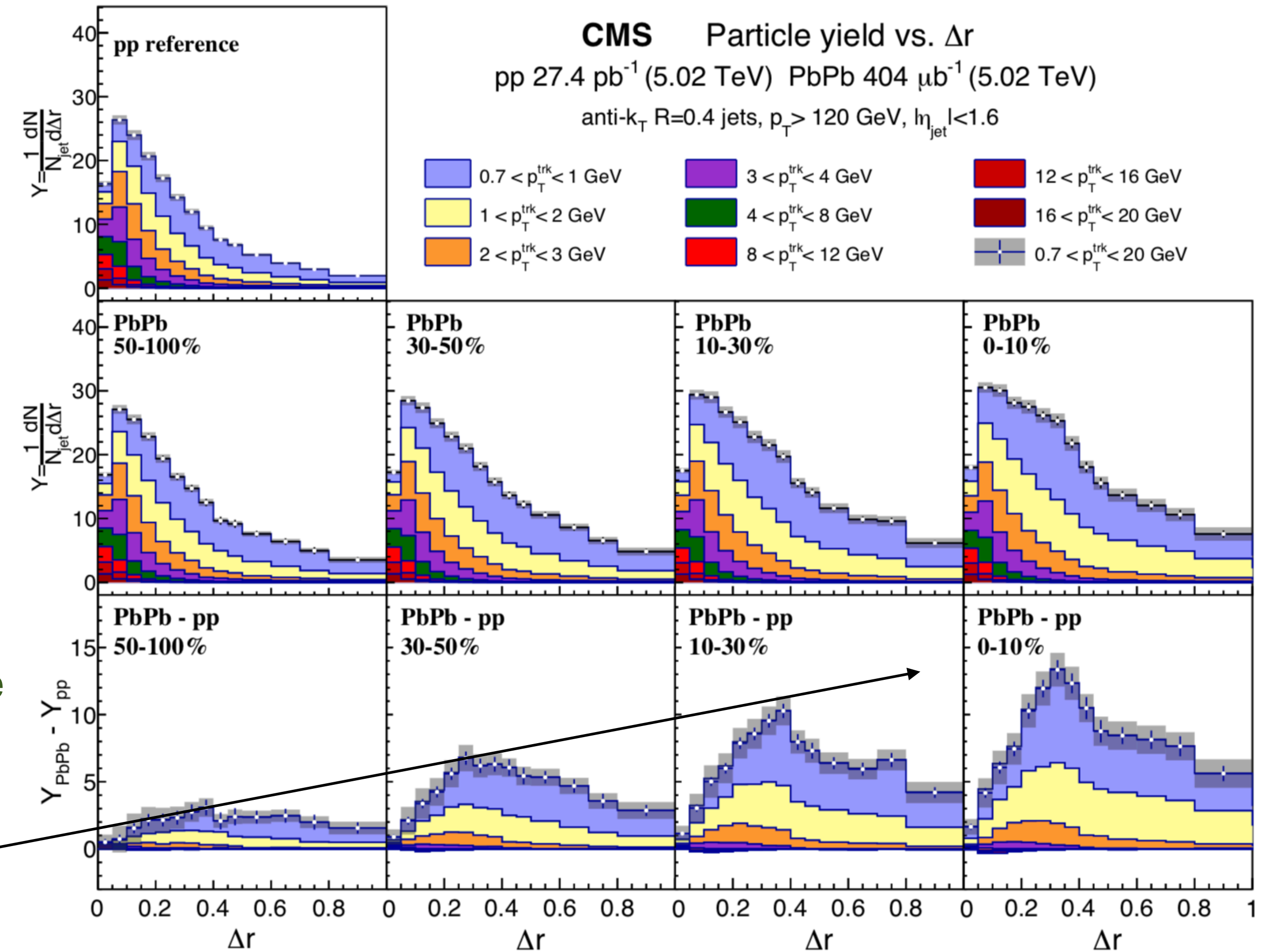


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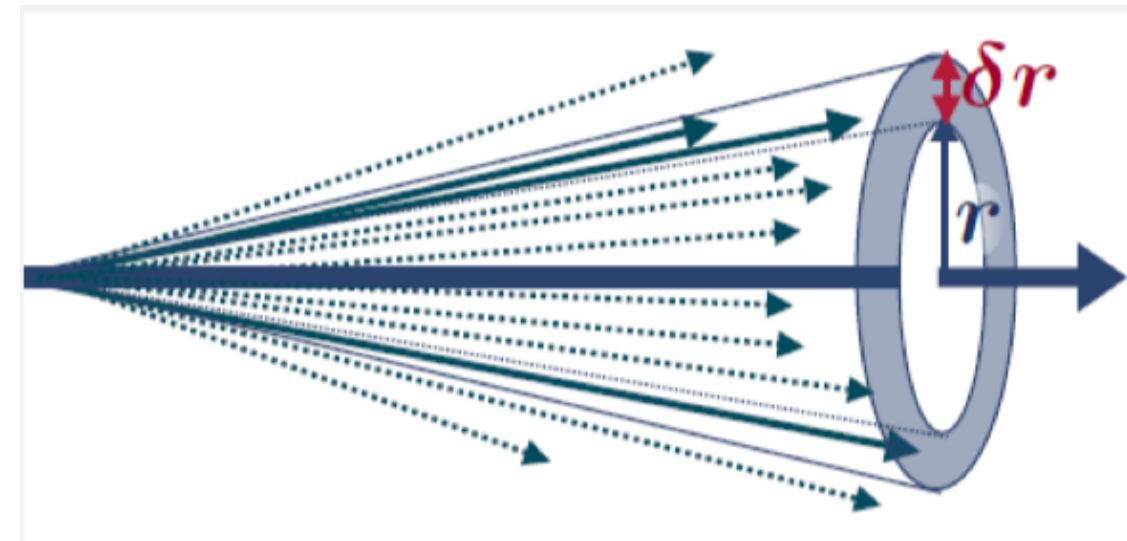


Everything seems to work, we can go home!

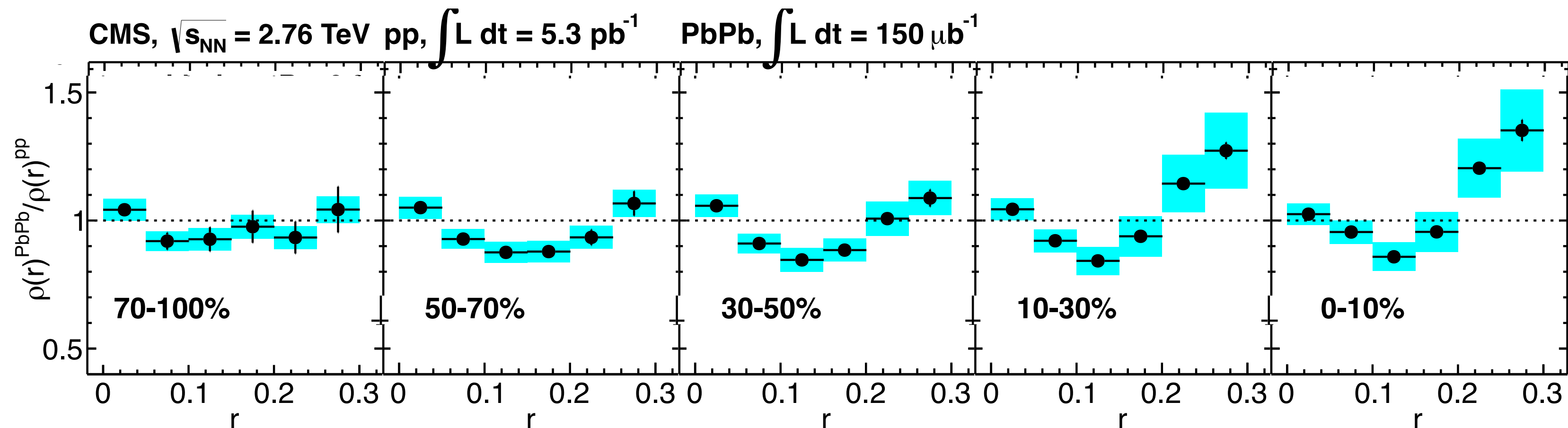
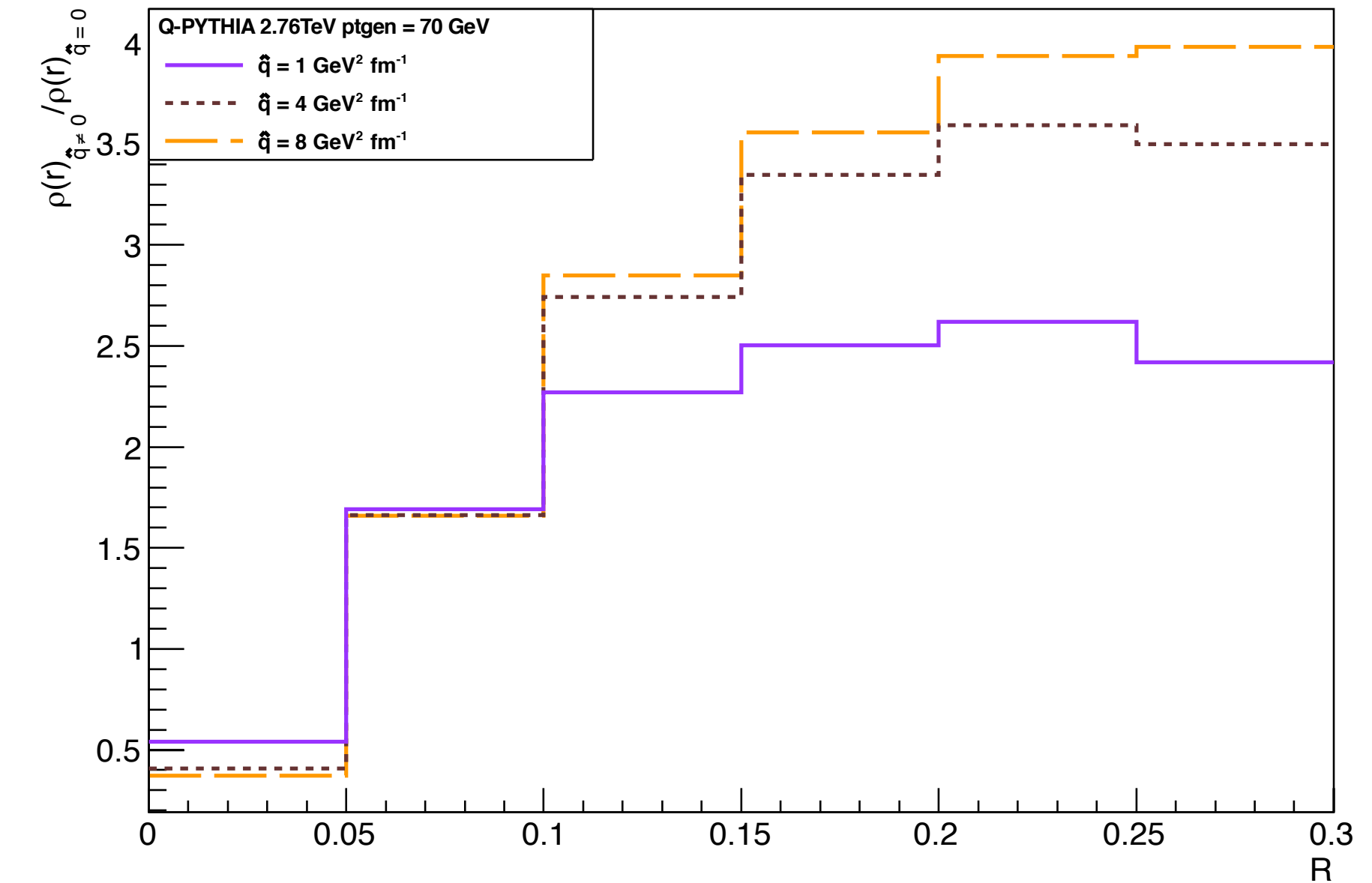
# Not so fast...

- Quantitative comparisons show some disagreement...
- Monte Carlo approaches based on in-medium single gluon radiation fail to describe some of the intra-jet features!

$$\rho(r) = \frac{1}{\delta r} \frac{1}{N_{\text{jet}}} \sum_{\text{jets}} \frac{\sum_{\text{tracks} \in [r_a, r_b]} p_{\text{T}}^{\text{track}}}{p_{\text{T}}^{\text{jet}}}$$



[L.Apolinário, QCD Forward Physics 2014]

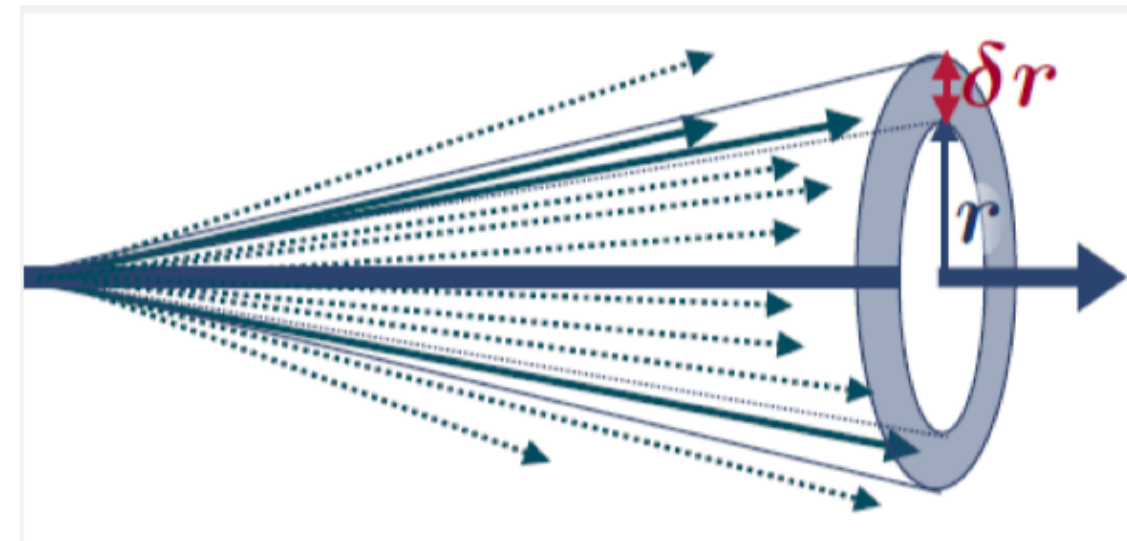




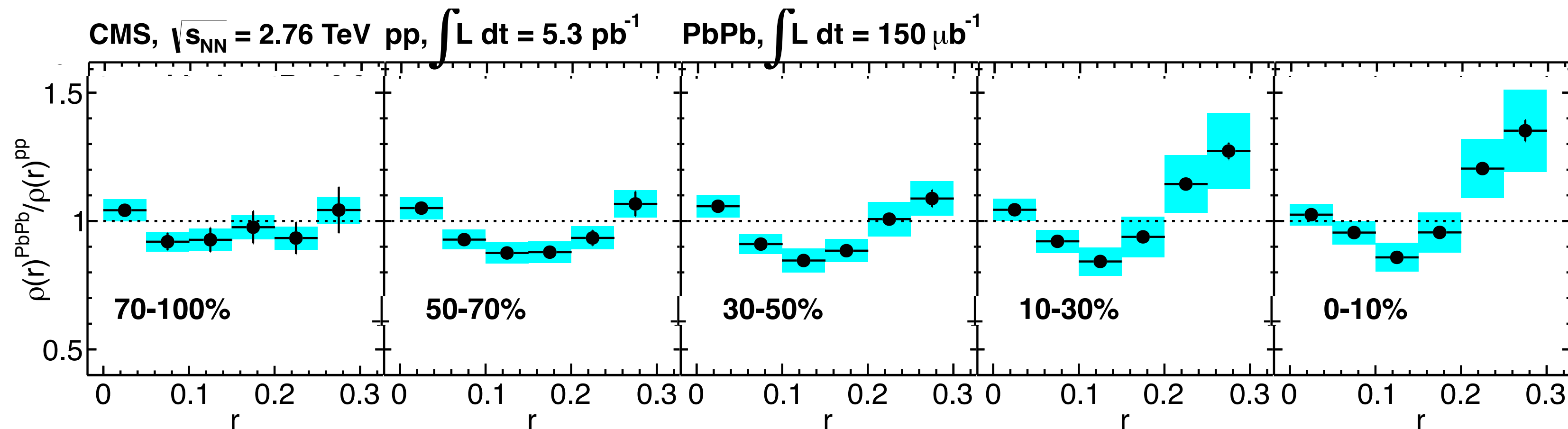
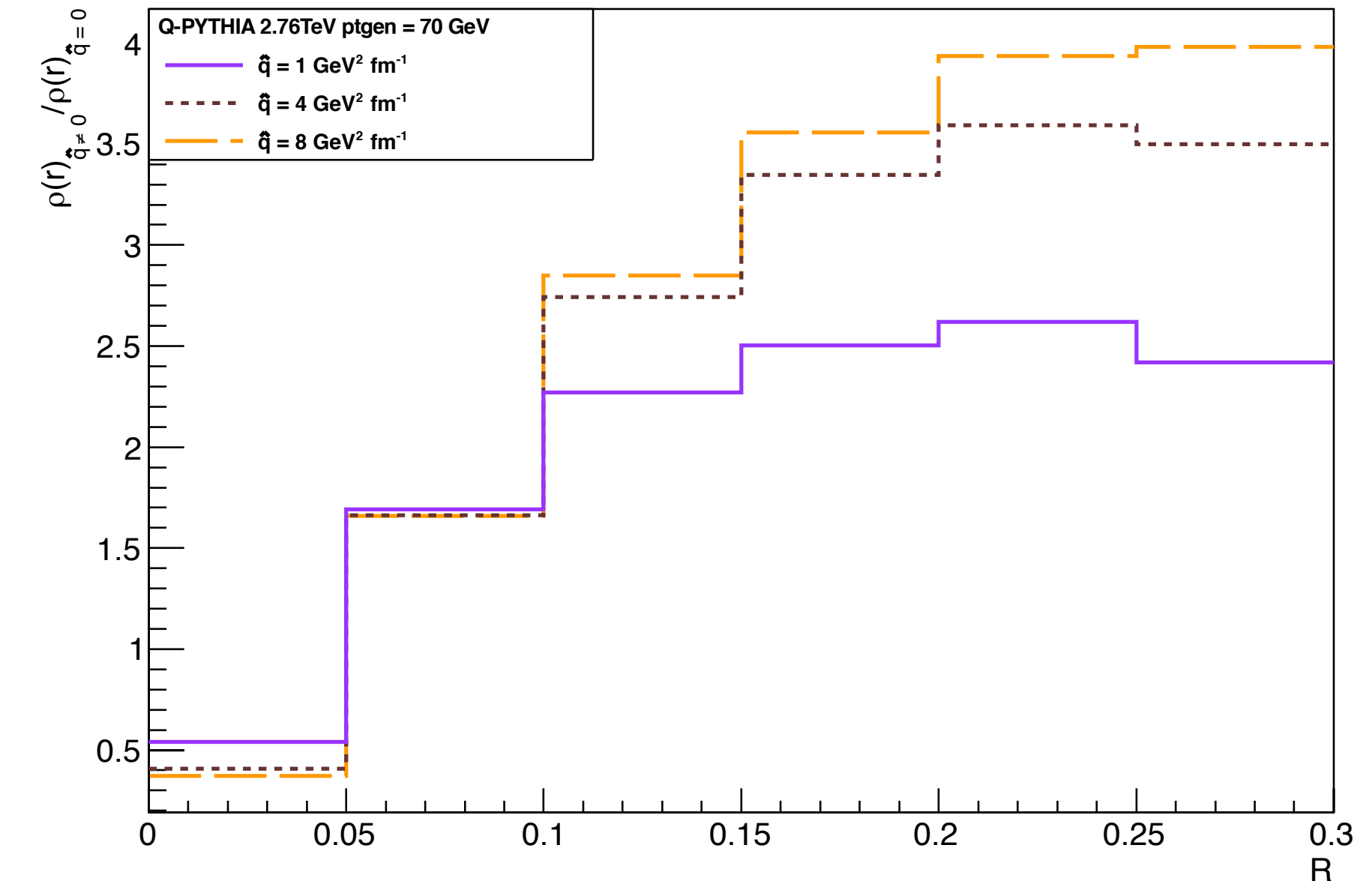
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Qualitative disagreement...

Let's go back again...

# What is a jet?

---

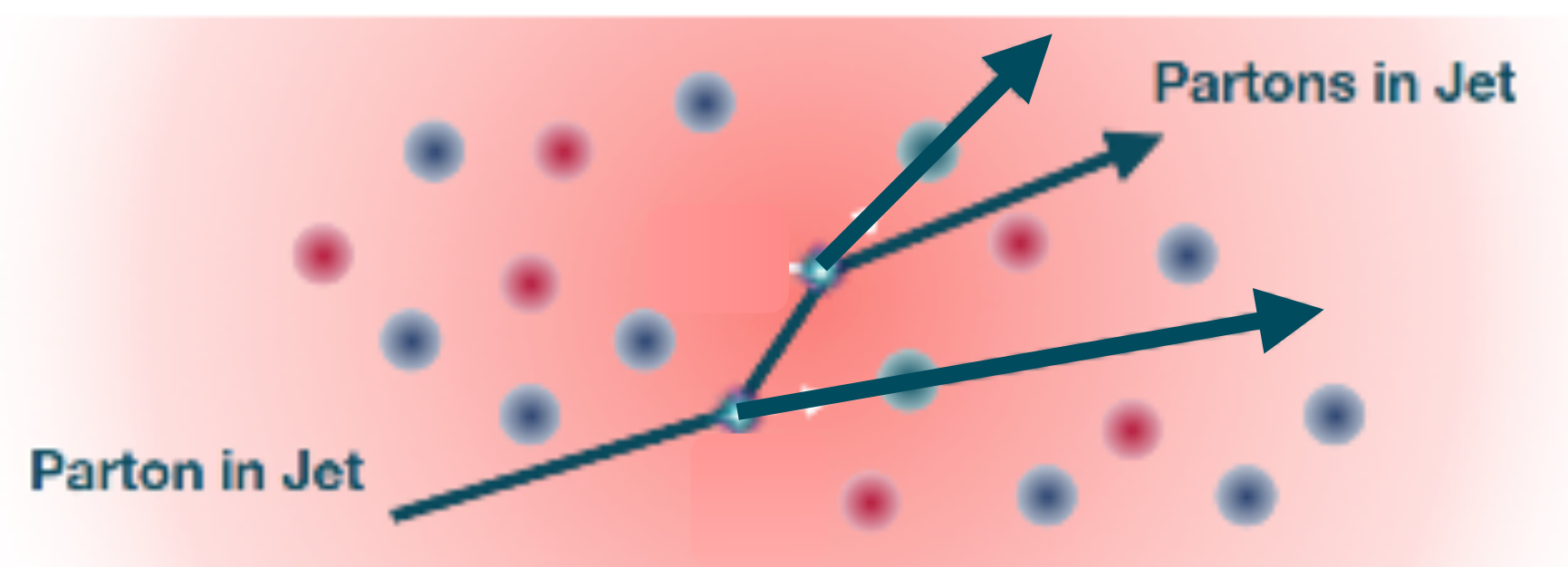
- ◆ A jet in pp:
- ◆ Defined with a jet clustering algorithm based (or not) in QCD principles: anti- $k_T$ ,  $k_T$ , C/A, ...
  - ➔ Have an object that can be related to the parton shower
  - ➔ Have an object that can be equally treated at parton, particle or calorimetric level



# What is a jet?

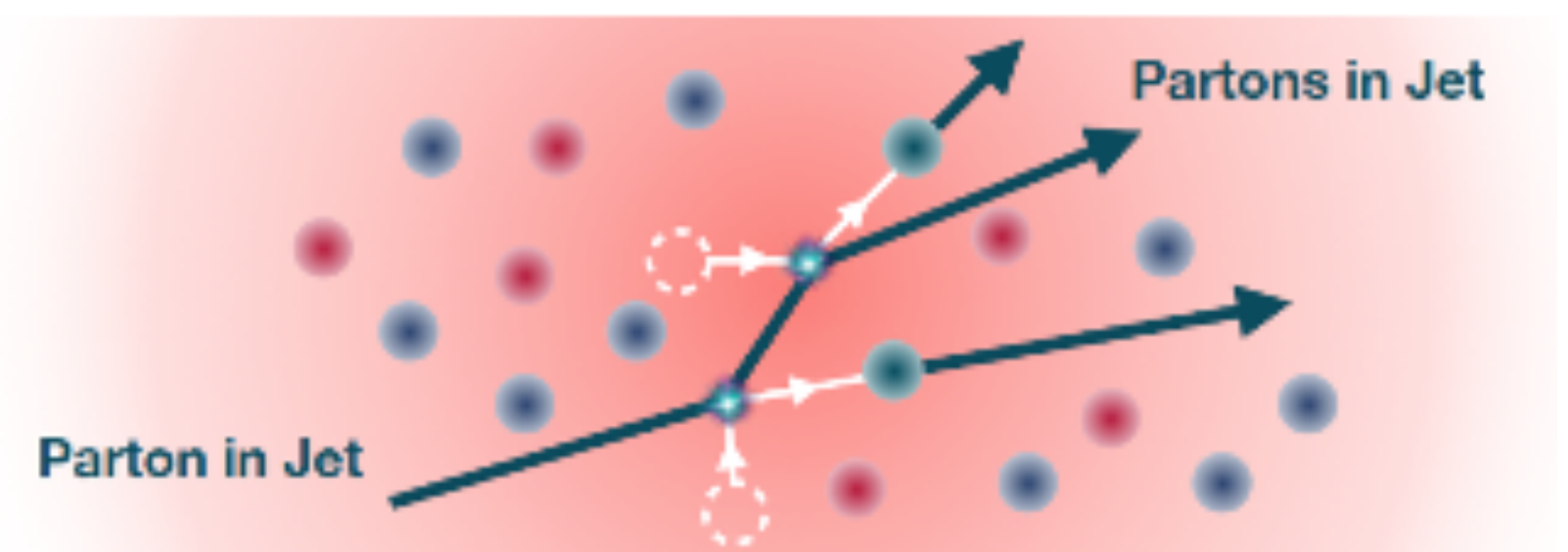
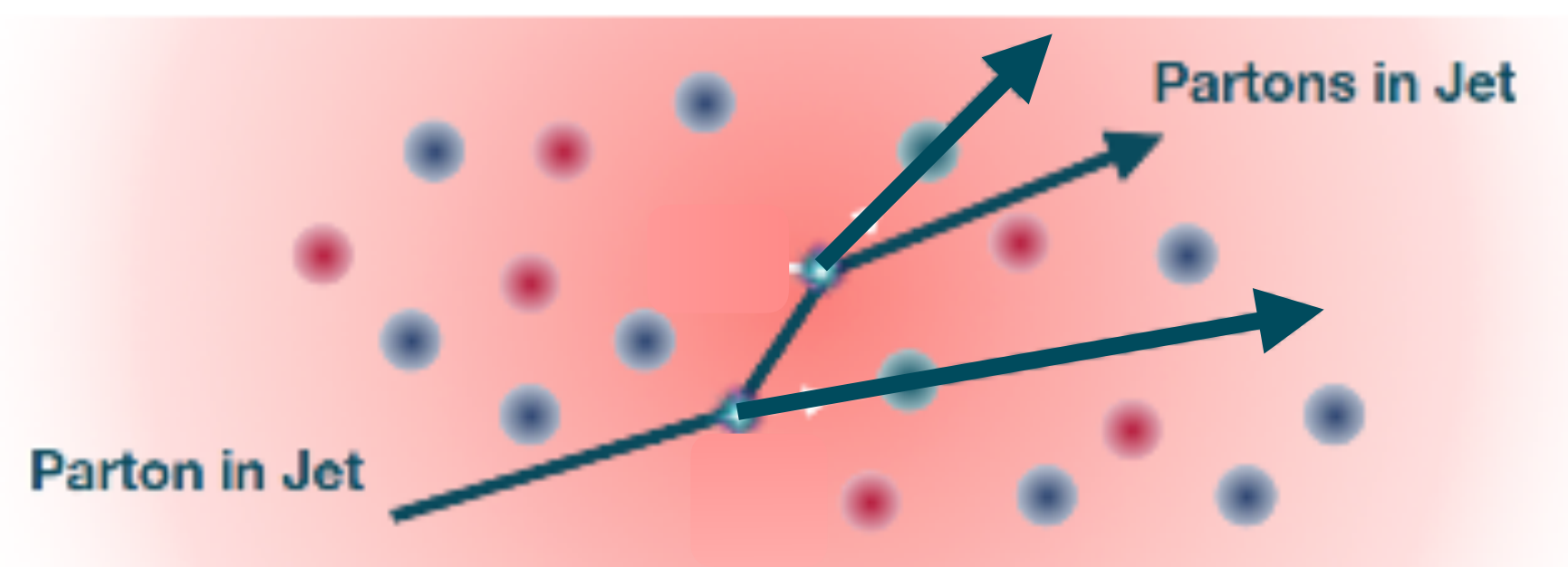
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- ◆ A jet in heavy-ions:
  - ◆ Defined with a jet clustering algorithm based (or not) in QCD principles: anti- $k_T$ ,  $k_T$ , C/A,...
  - ➔ Have an object that can be related to the parton shower
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# What is a jet?

- ◆ A jet in heavy-ions:
  - ◆ Defined with a jet clustering algorithm based (or not) in QCD principles: anti- $k_T$ ,  $k_T$ , C/A,...
  - ➔ Have an object that can be related to the parton shower ???
  - ➔ Have an object that can be equally treated at parton, particle or calorimetric level





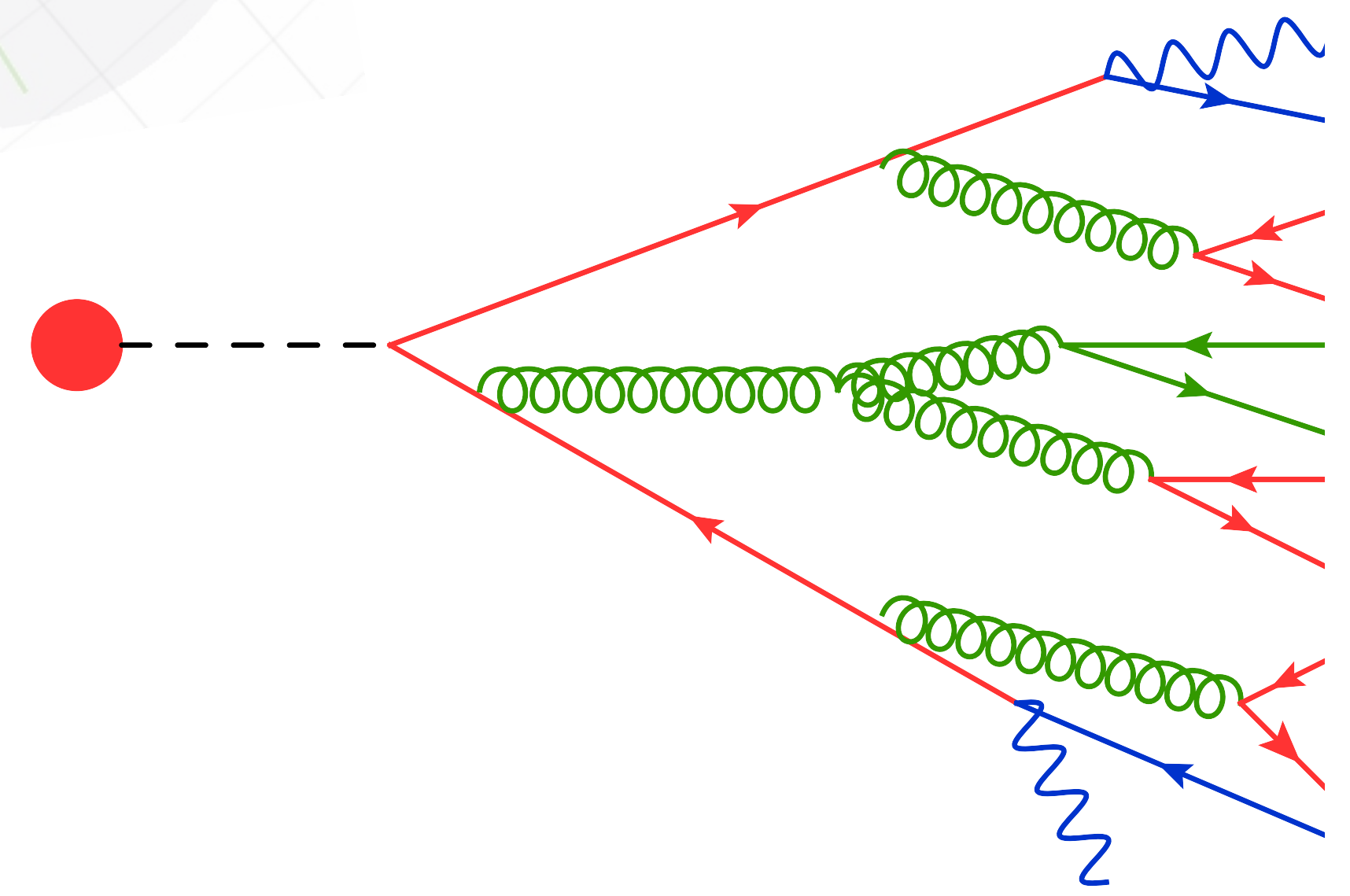
The background features a light gray grid. Overlaid on this are several purple lines of varying thickness and style, including solid lines, dotted lines, and lines with a dashed-dotted pattern. Some lines are straight, while others are curved. In the bottom-left corner, there are several overlapping, semi-transparent geometric shapes in shades of yellow, green, and gray, resembling a stylized cityscape or abstract architecture. A single, solid purple line starts from the top-left and curves towards the right, crossing several other lines.

**How to define a jet  
in a heavy-ion?**



# Jet Quenching v2.0

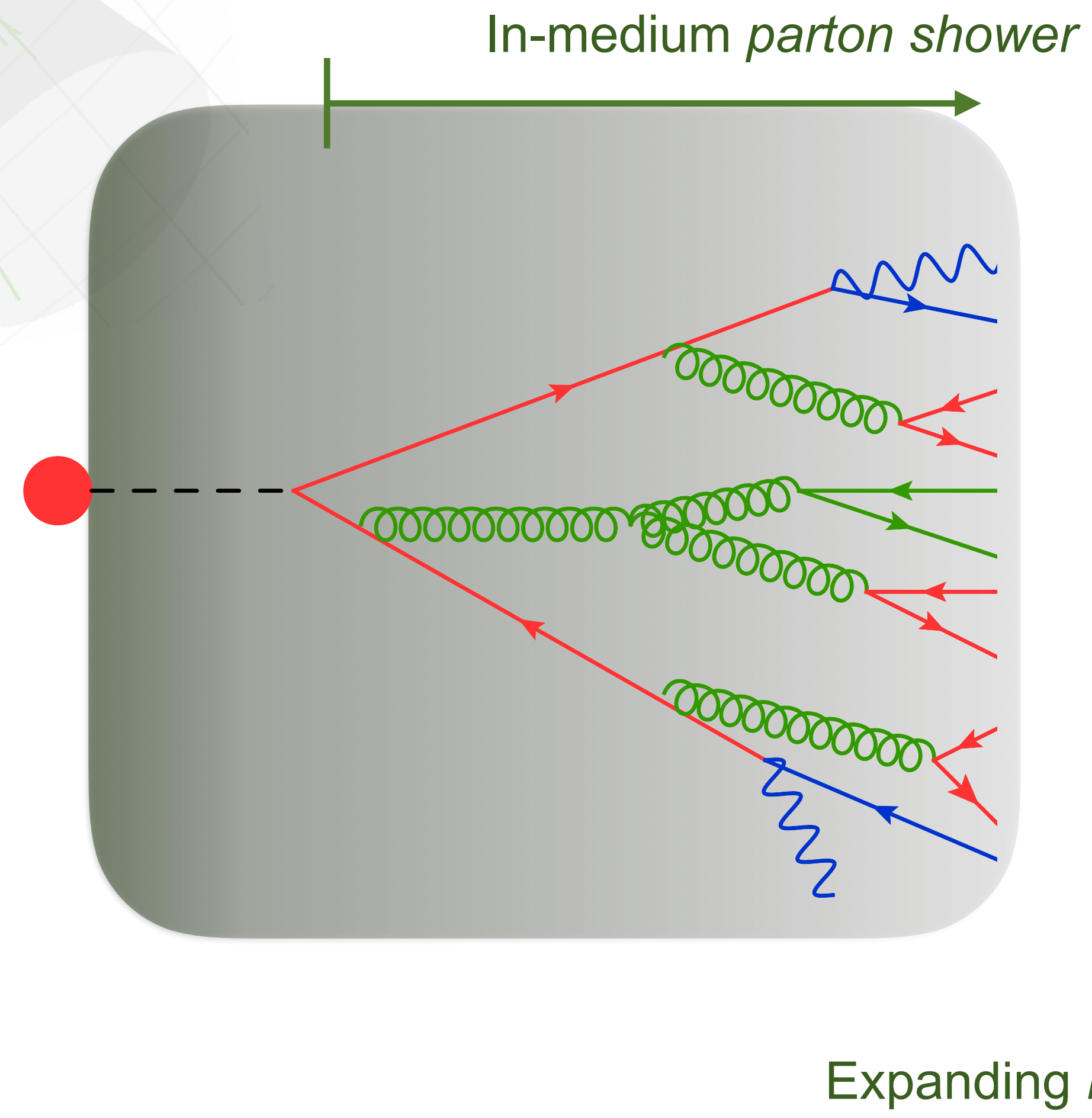
- ◆ The way we see and define a jet should include all momentum scales:





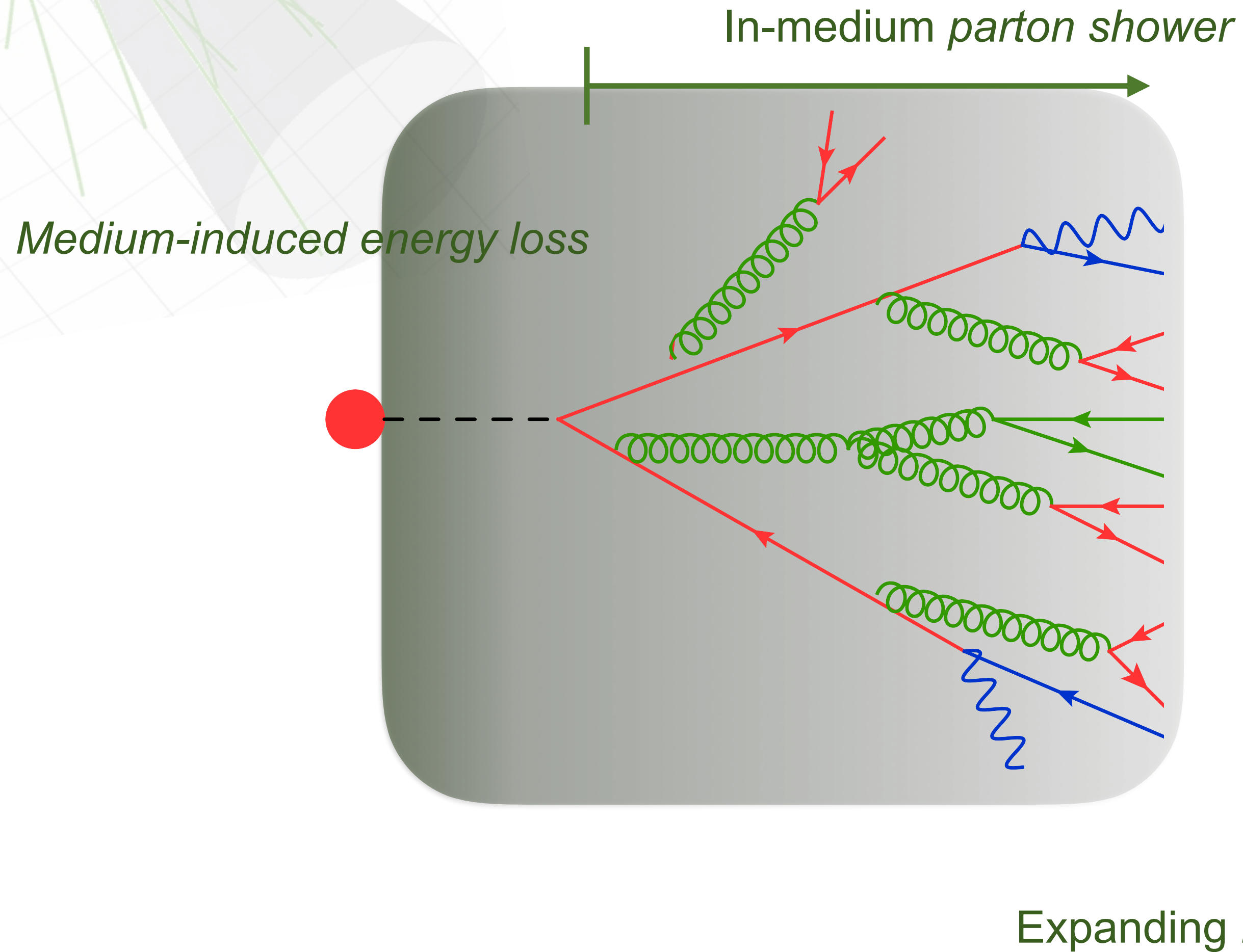
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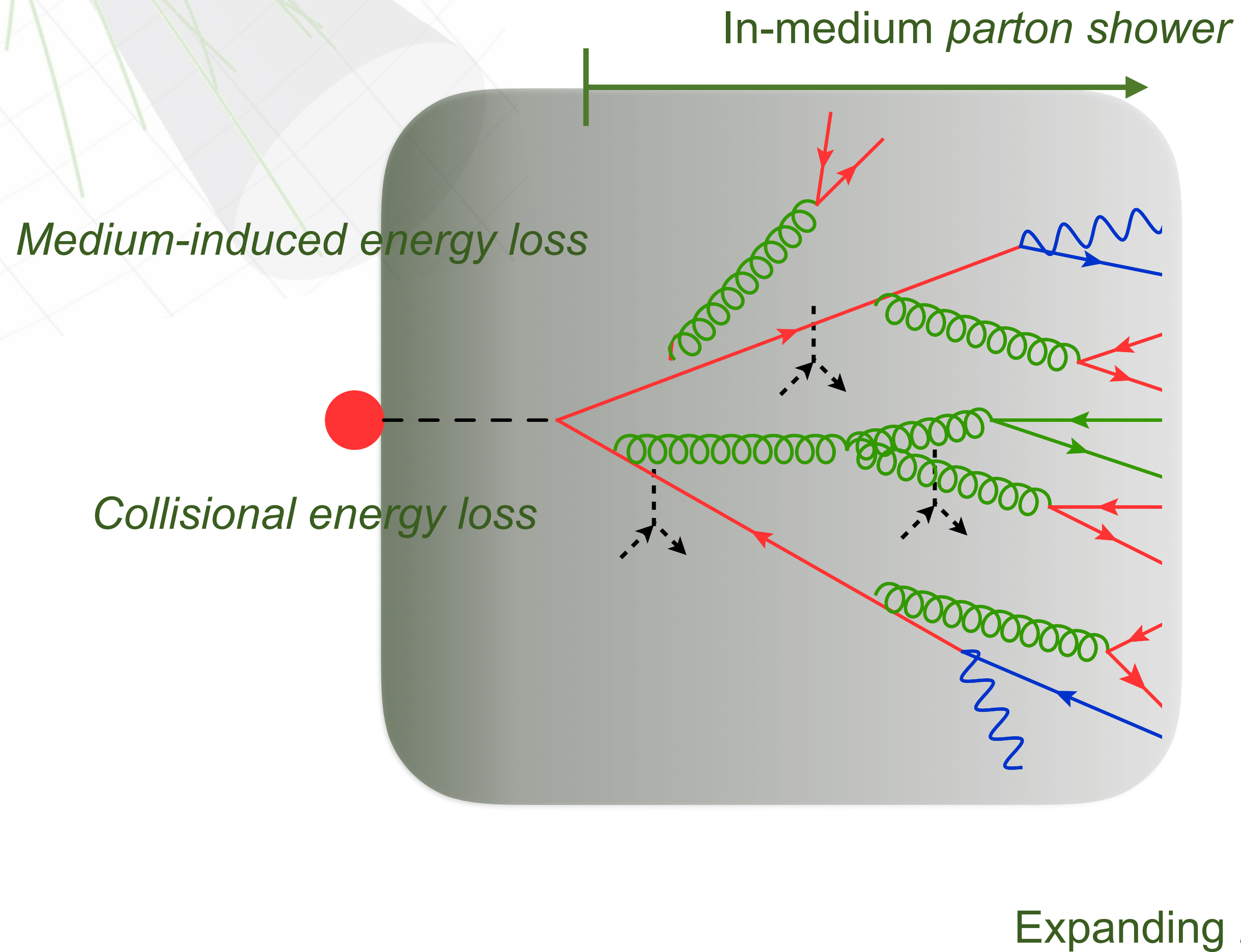
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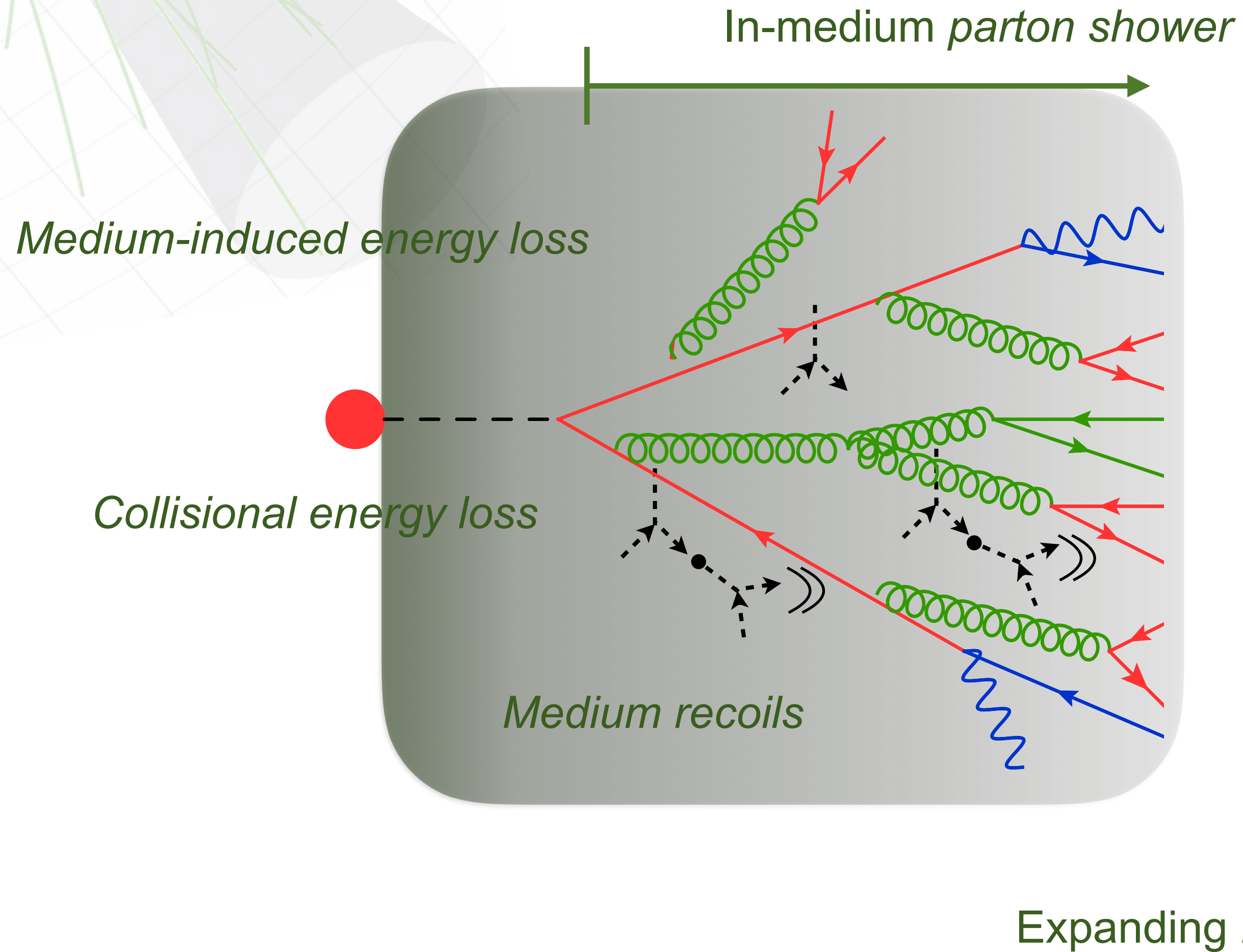
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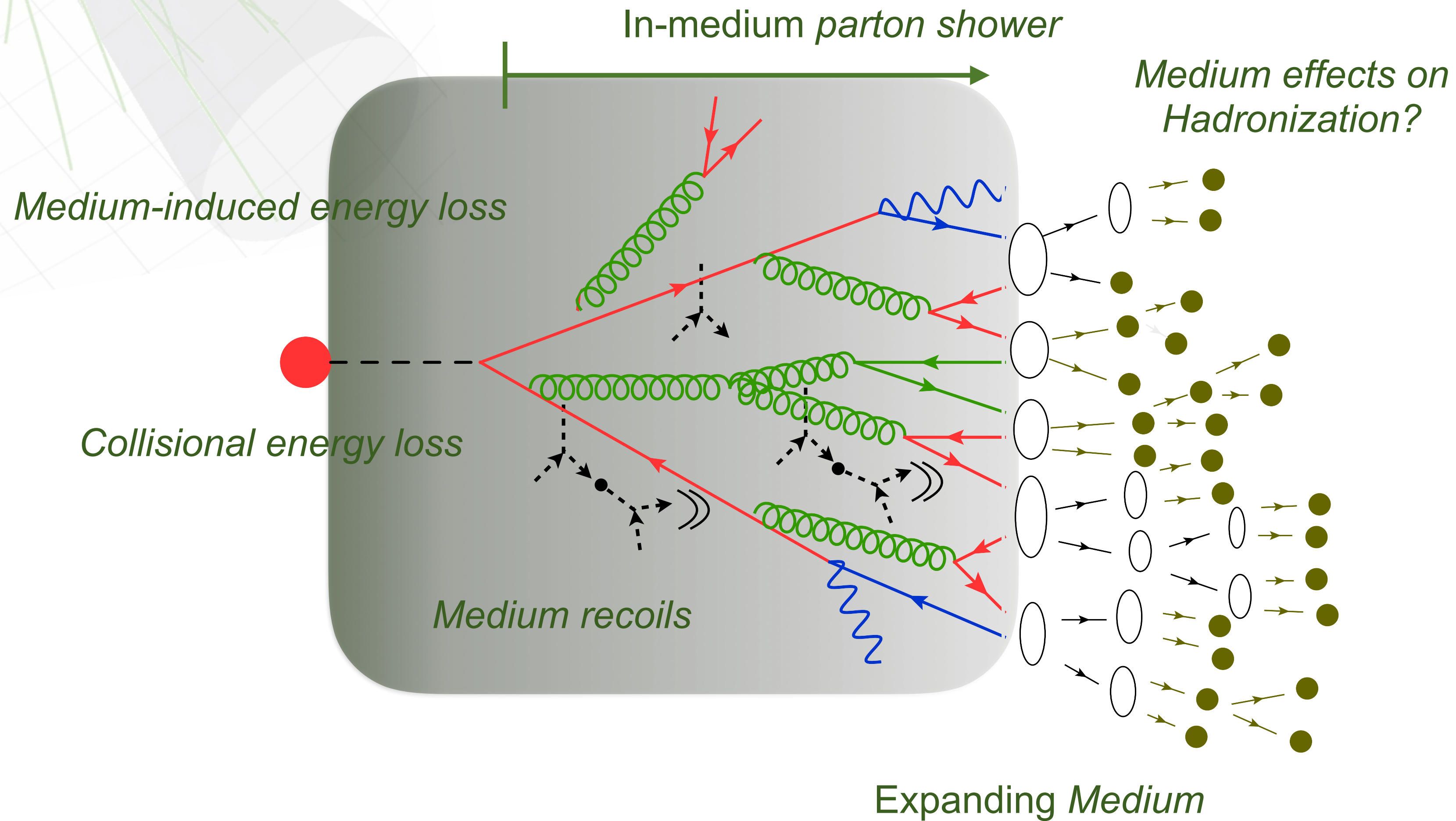
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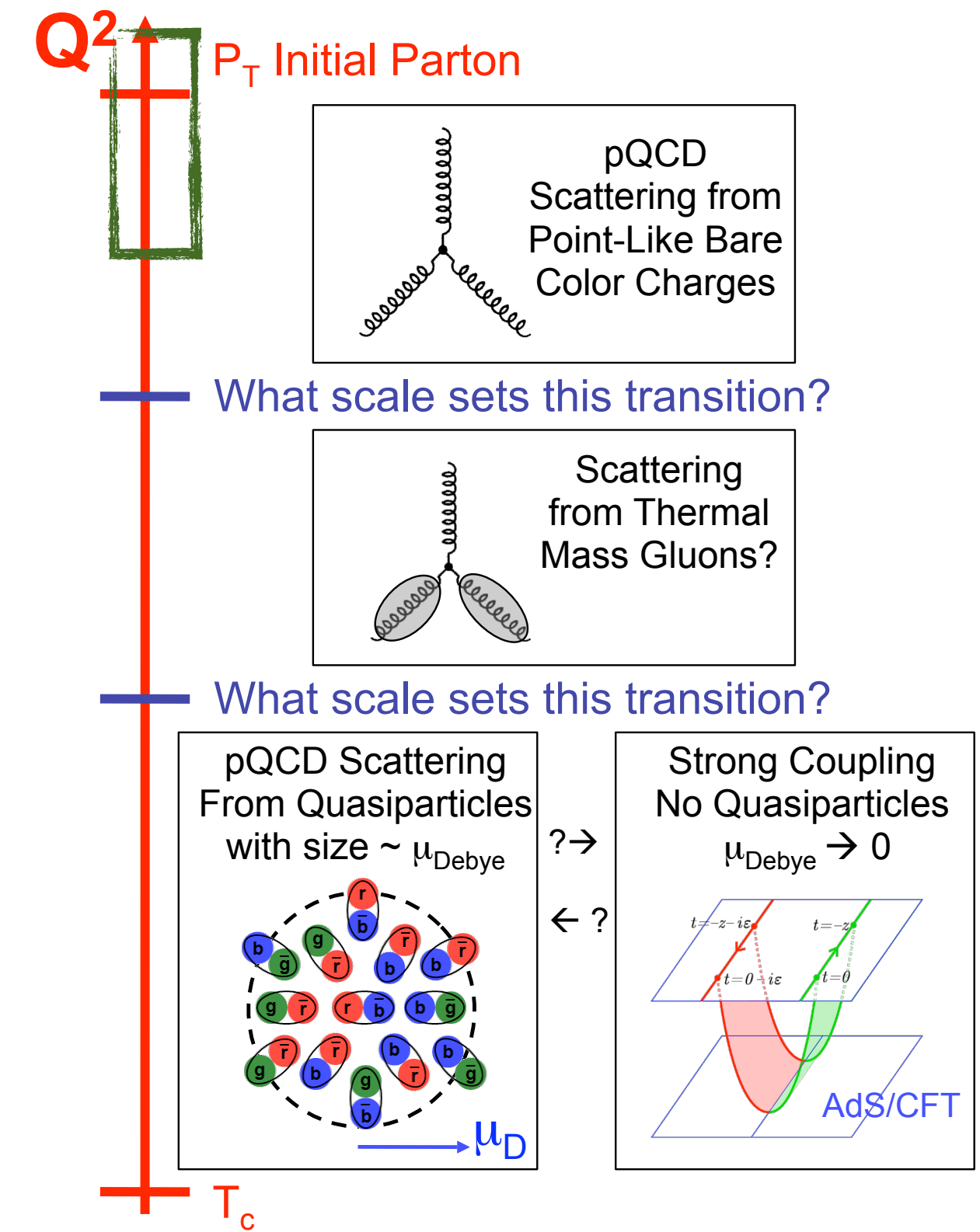
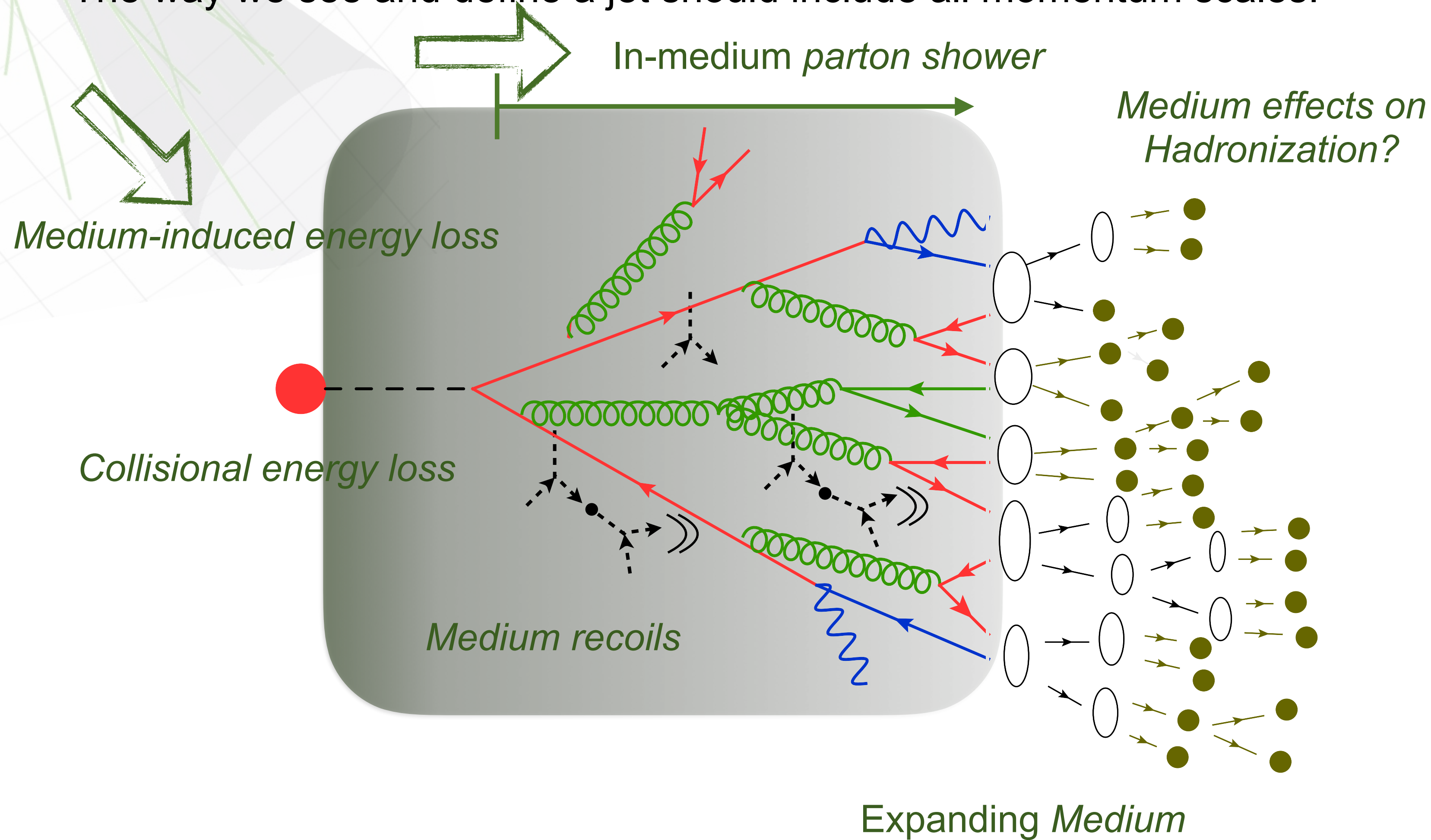
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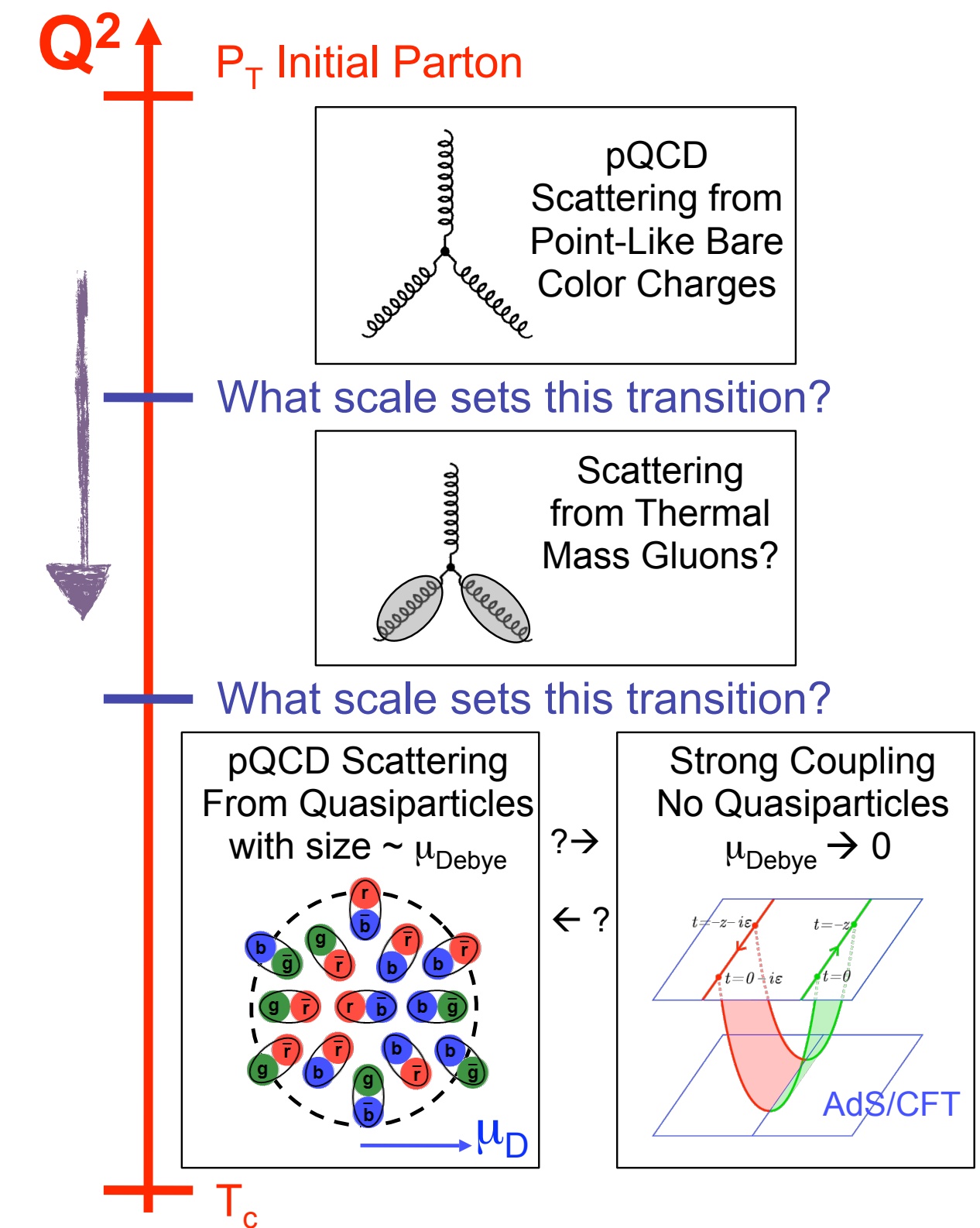
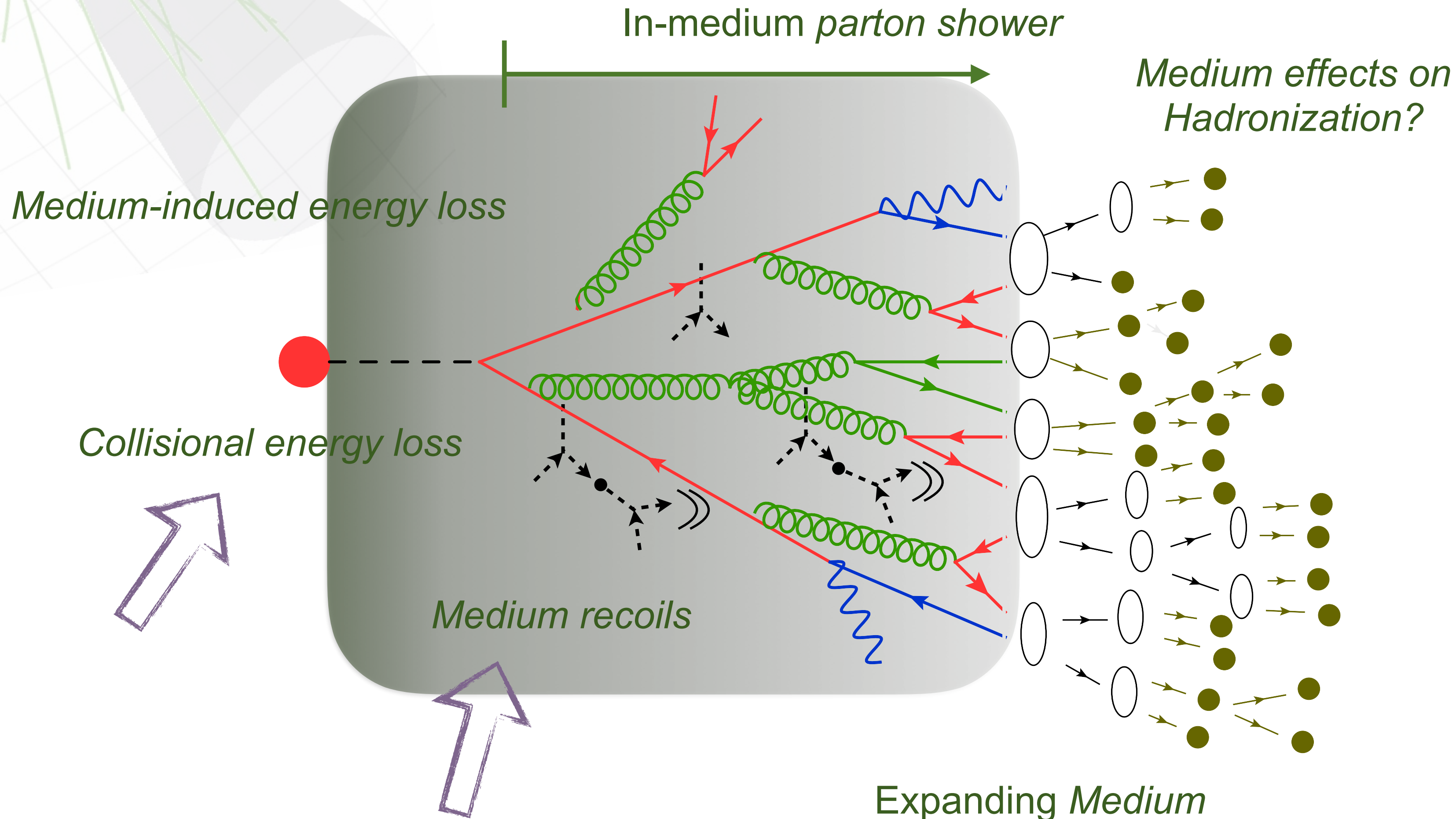
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# Jet Quenching v2.0

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# Medium response

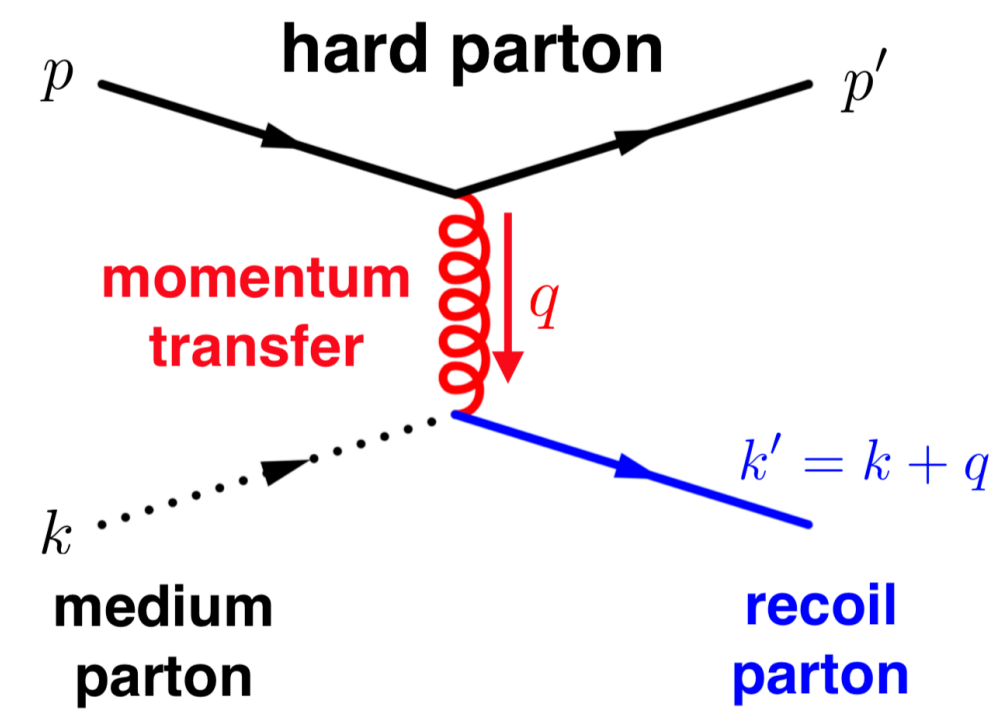
LBT: [Cao, Luo, Qin, Wang (16) He, Luo, Wang, Zhu (17)]

MARTINI: [Schenke, Gale, Jeon (09)]

JEWEL: [Elayavalli, Zapp (17)]

- ◆ QGP part that become correlated with the jet (not to be subtracted!)
- ◆ Seen as (pQCD approach):
  - ◆ Recoils from jet-medium interactions with a thermal/3D hydro particle distribution
    - ➔ Recoiled particle makes part of the jet: JEWEL

E.g: JEWEL  $\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, |\hat{t}|) \simeq \frac{C_R 2\pi\alpha_s^2}{(|\hat{t}| + \mu_D^2)^2}$





# Medium response

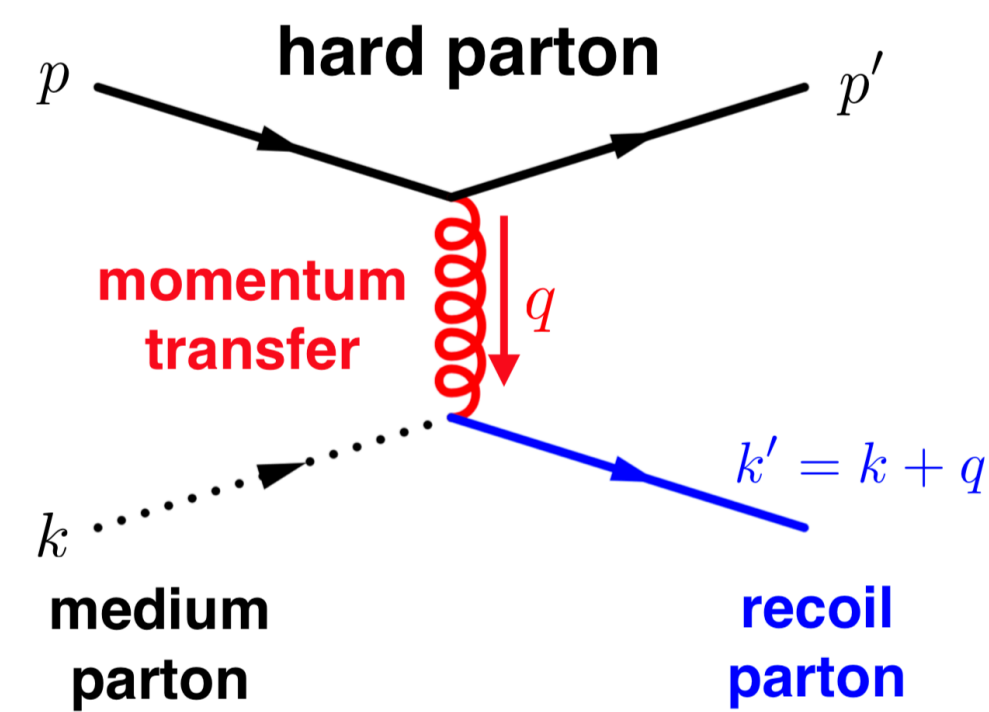
LBT: [Cao, Luo, Qin, Wang (16) He, Luo, Wang, Zhu (17)]

MARTINI: [Schenke, Gale, Jeon (09)]

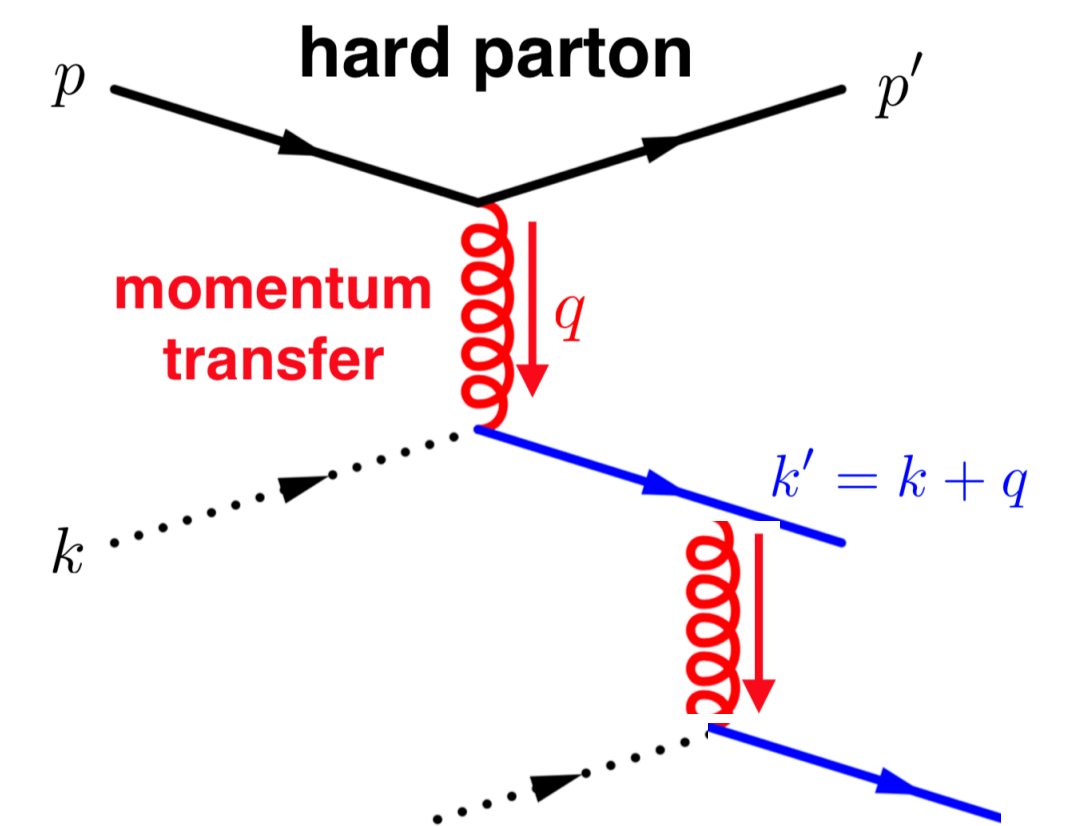
JEWEL: [Elayavalli, Zapp (17)]

- ◆ QGP part that become correlated with the jet (not to be subtracted!)
- ◆ Seen as (pQCD approach):
  - ◆ Recoils from jet-medium interactions with a thermal/3D hydro particle distribution
    - ➔ Recoiled particle makes part of the jet: JEWEL
    - ➔ Recoiled particle can further interact with medium constituents: MARTINI, LBT

E.g: JEWEL  $\frac{d\hat{\sigma}}{d\hat{t}}(\hat{s}, |\hat{t}|) \simeq \frac{C_R 2\pi\alpha_s^2}{(|\hat{t}| + \mu_D^2)^2}$



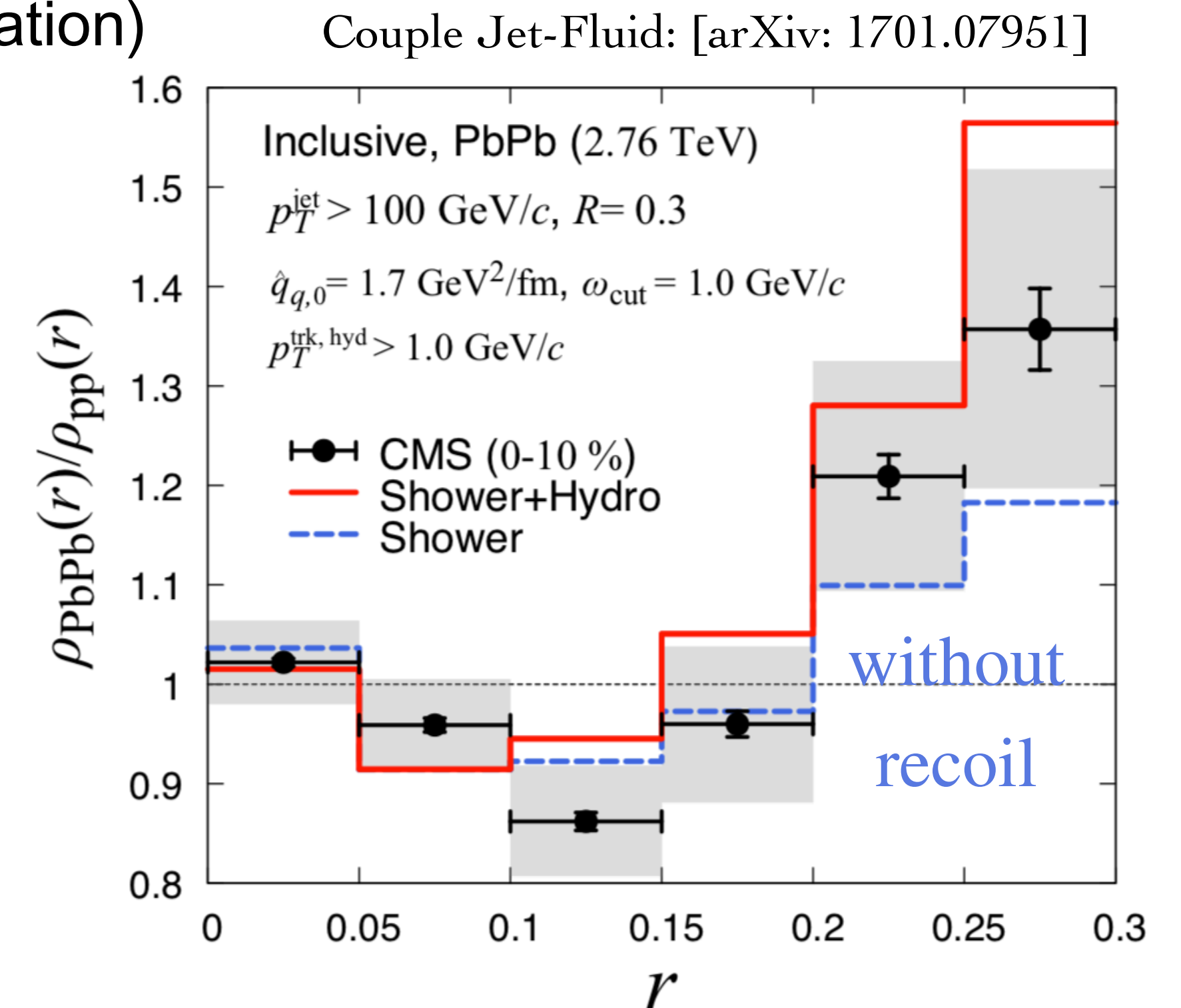
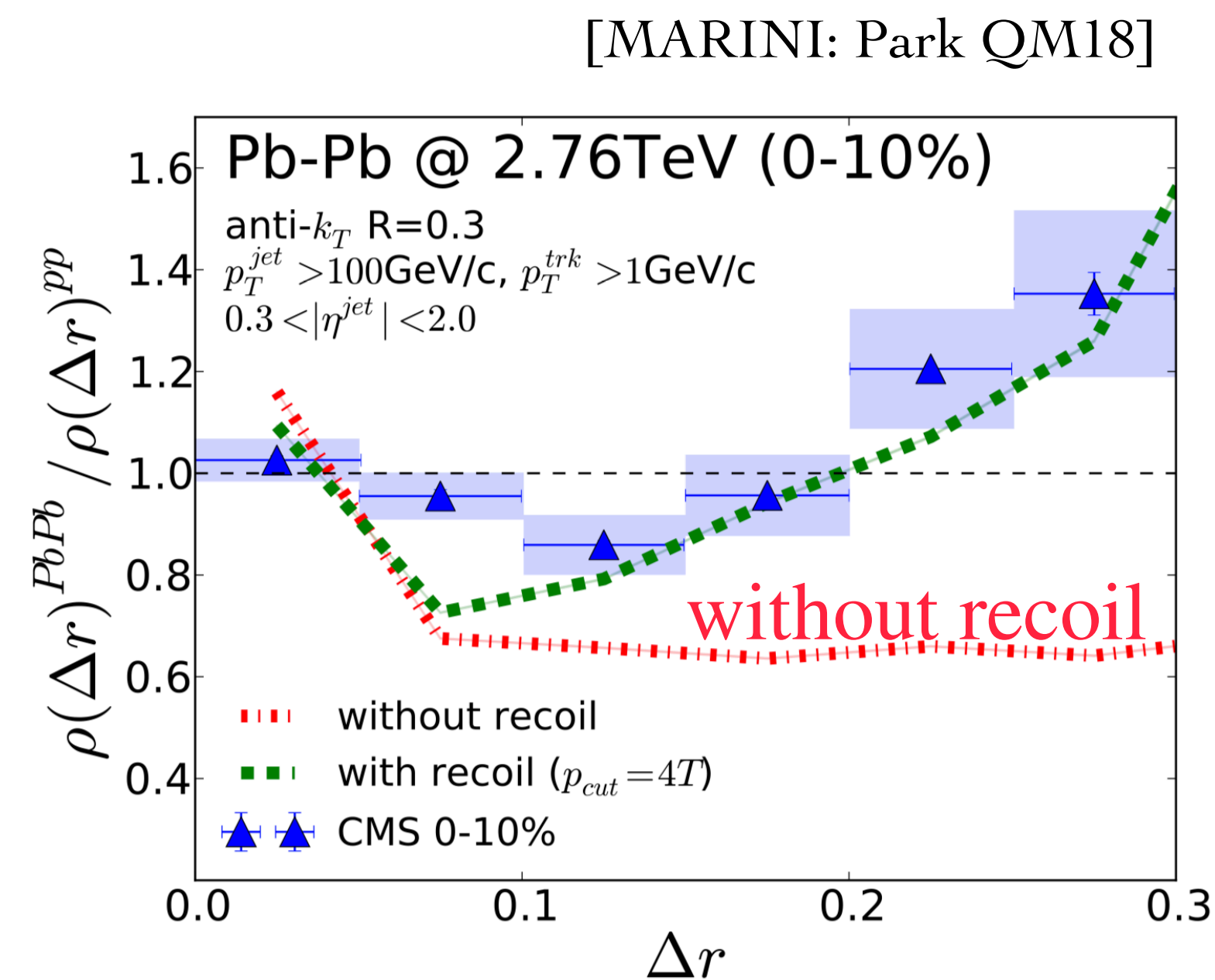
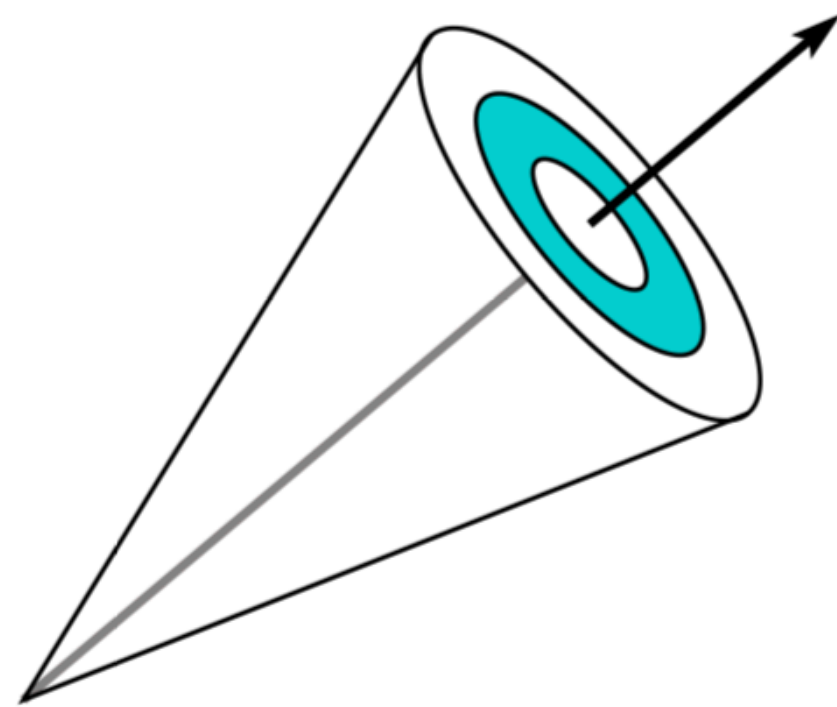
E.g: MARTINI



# There are still problems...

- ◆ Magnitude of the medium recoil component to the jet varies from model to model...
- ◆ Coupled Jet-Fluid (Analytical approach)
- ◆ MARTINI (Monte Carlo approach based on in-medium gluon radiation)

Recoil component  
Best observed in the  
“jet radial profile”

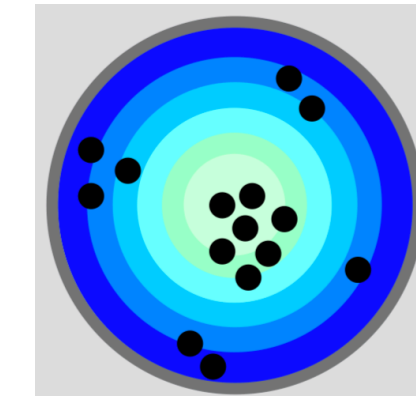
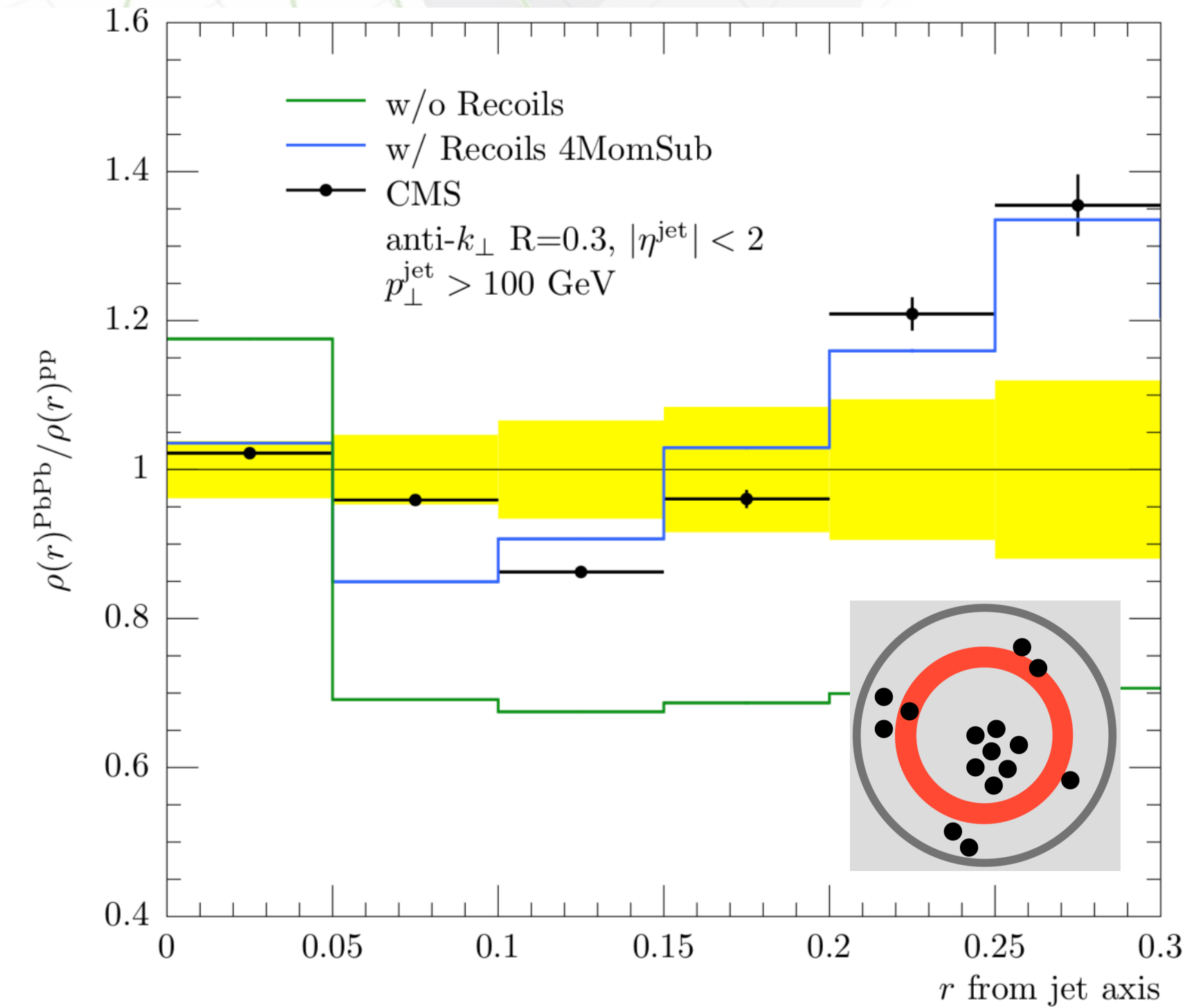




# And more problems...

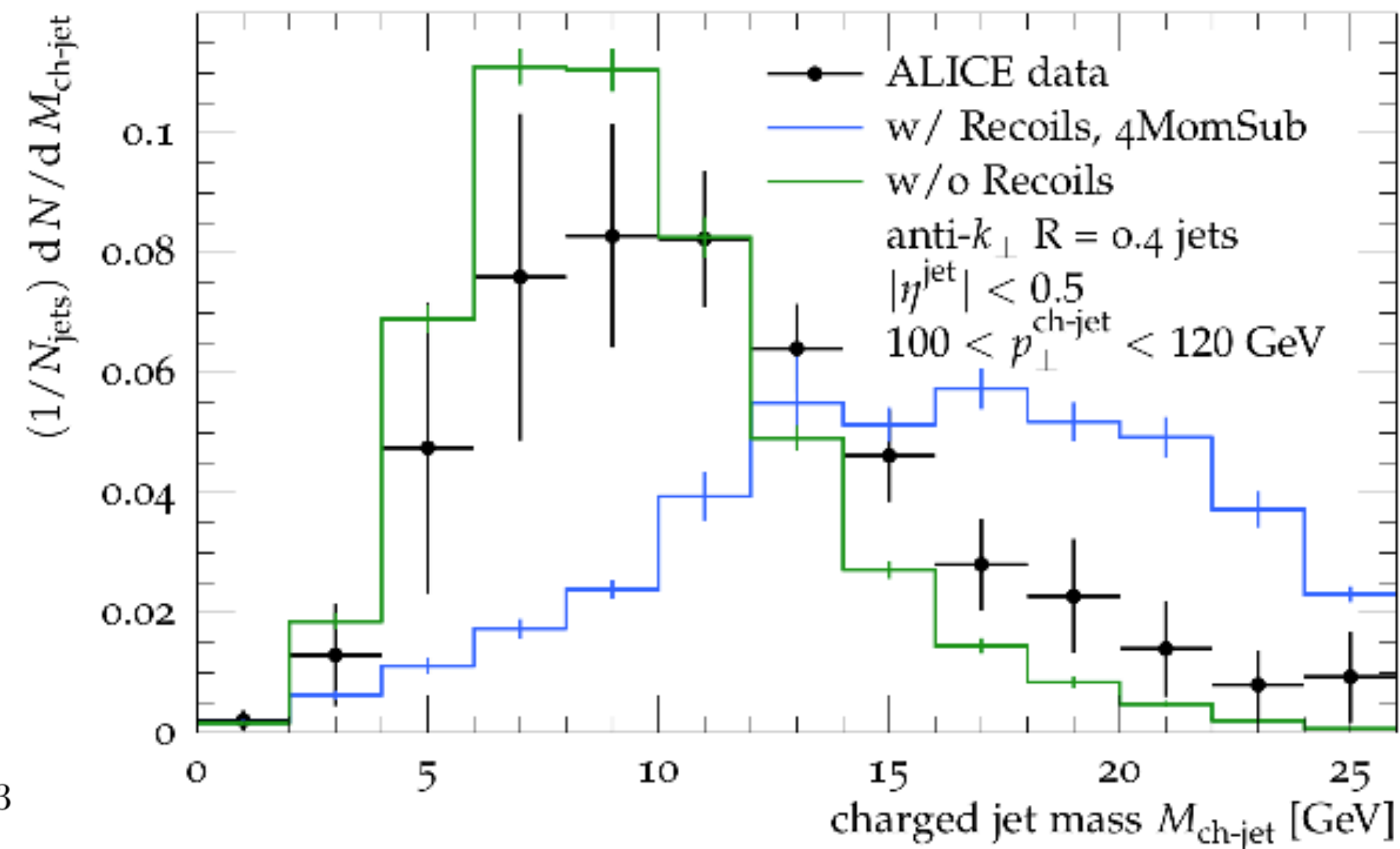
- ◆ Within the same model, not so easy to simultaneously describe jet radial profile and jet mass:

[JEWEL:1707.01539]



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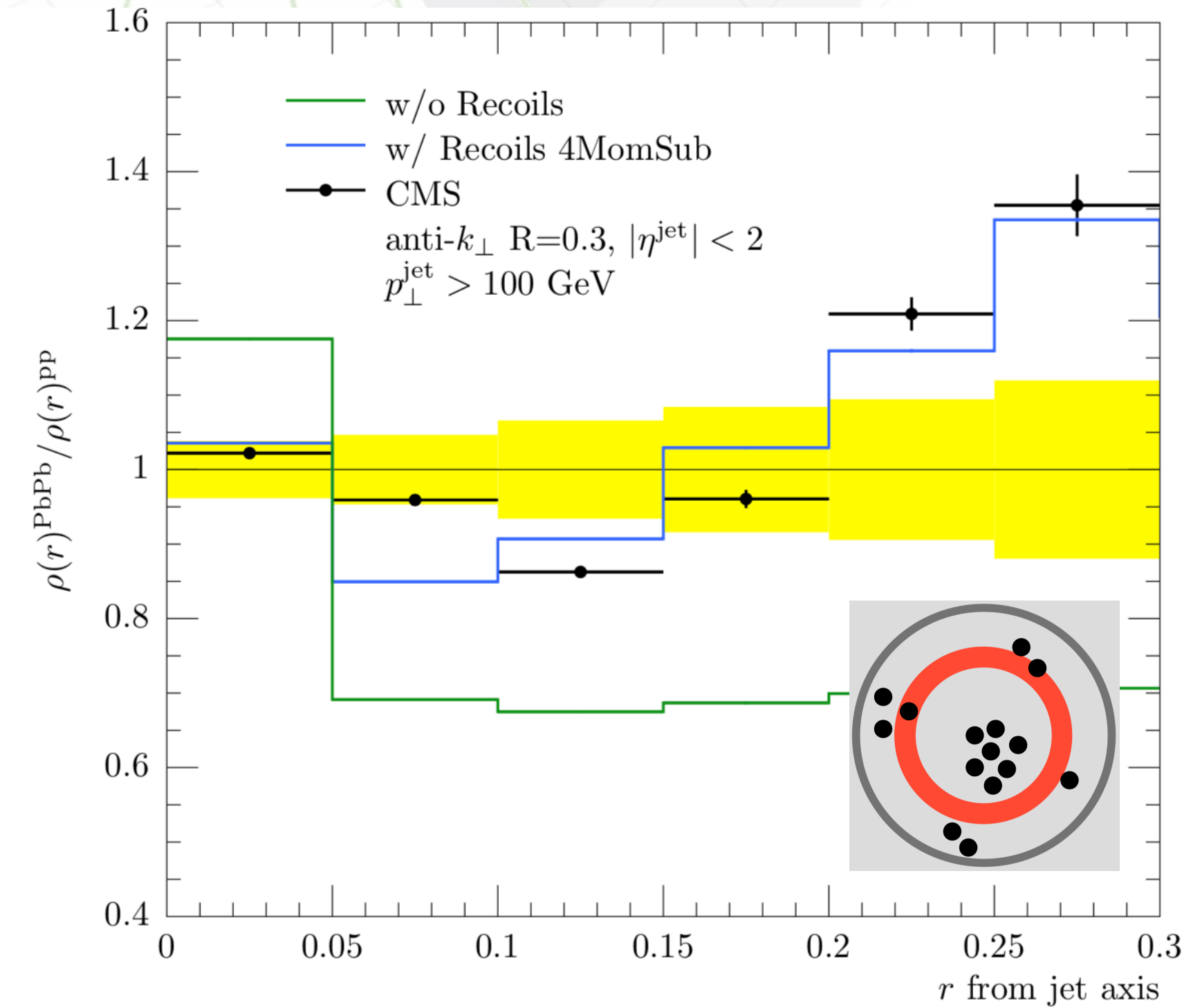
JEWEL+PYTHIA Pb+Pb (0 – 10%) (2.76 TeV)



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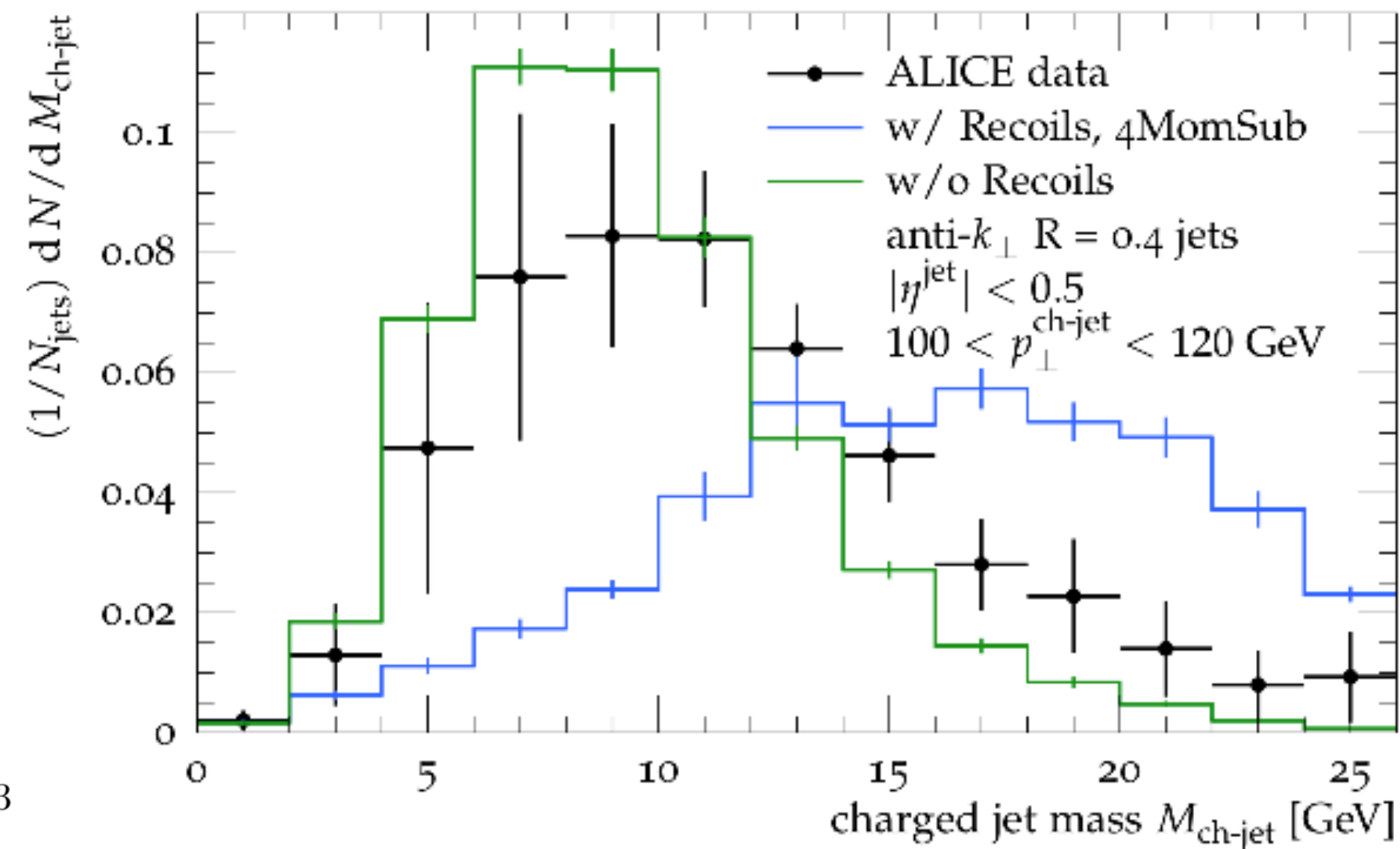
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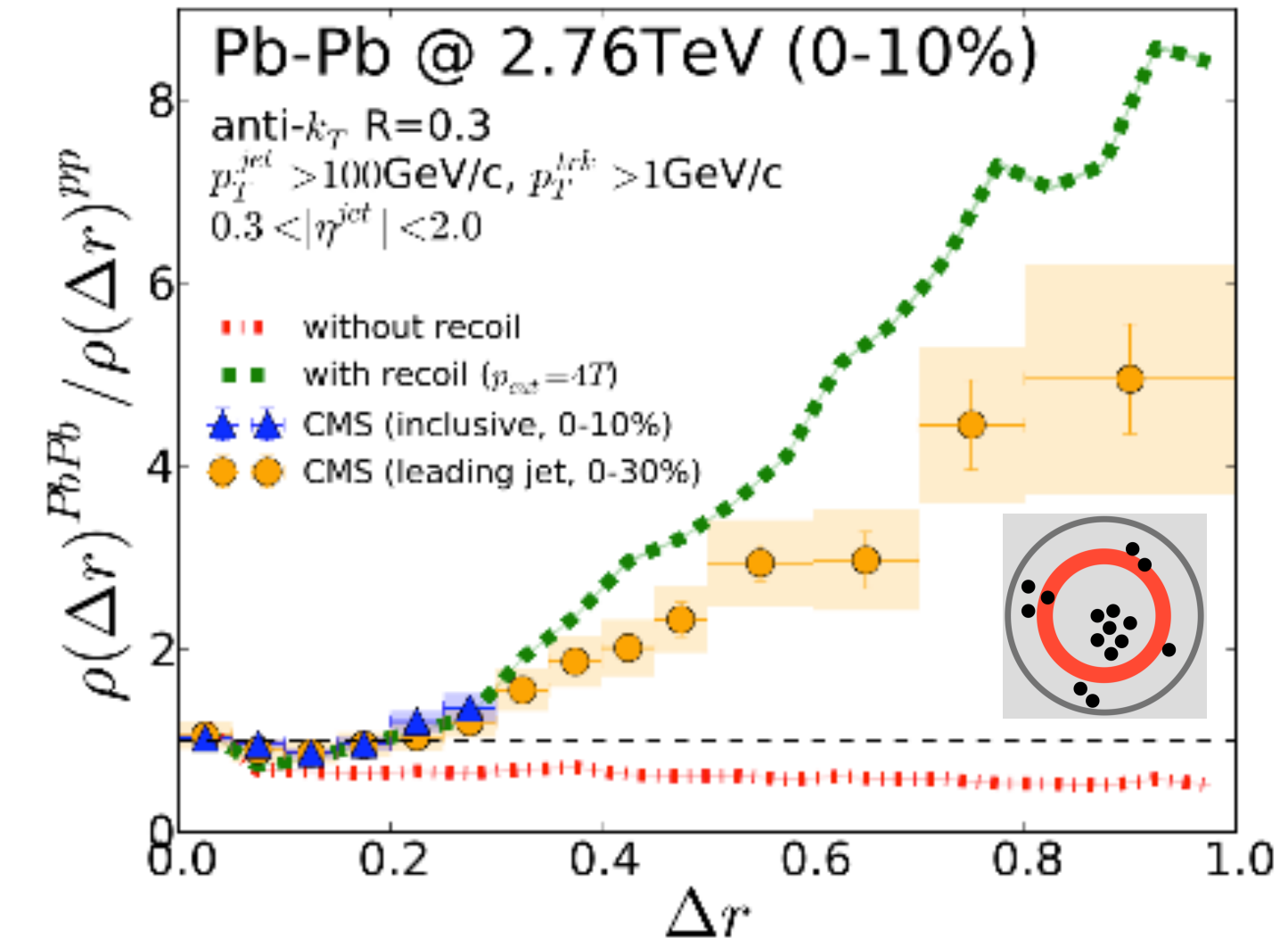


[JEWEL:1707.01539]

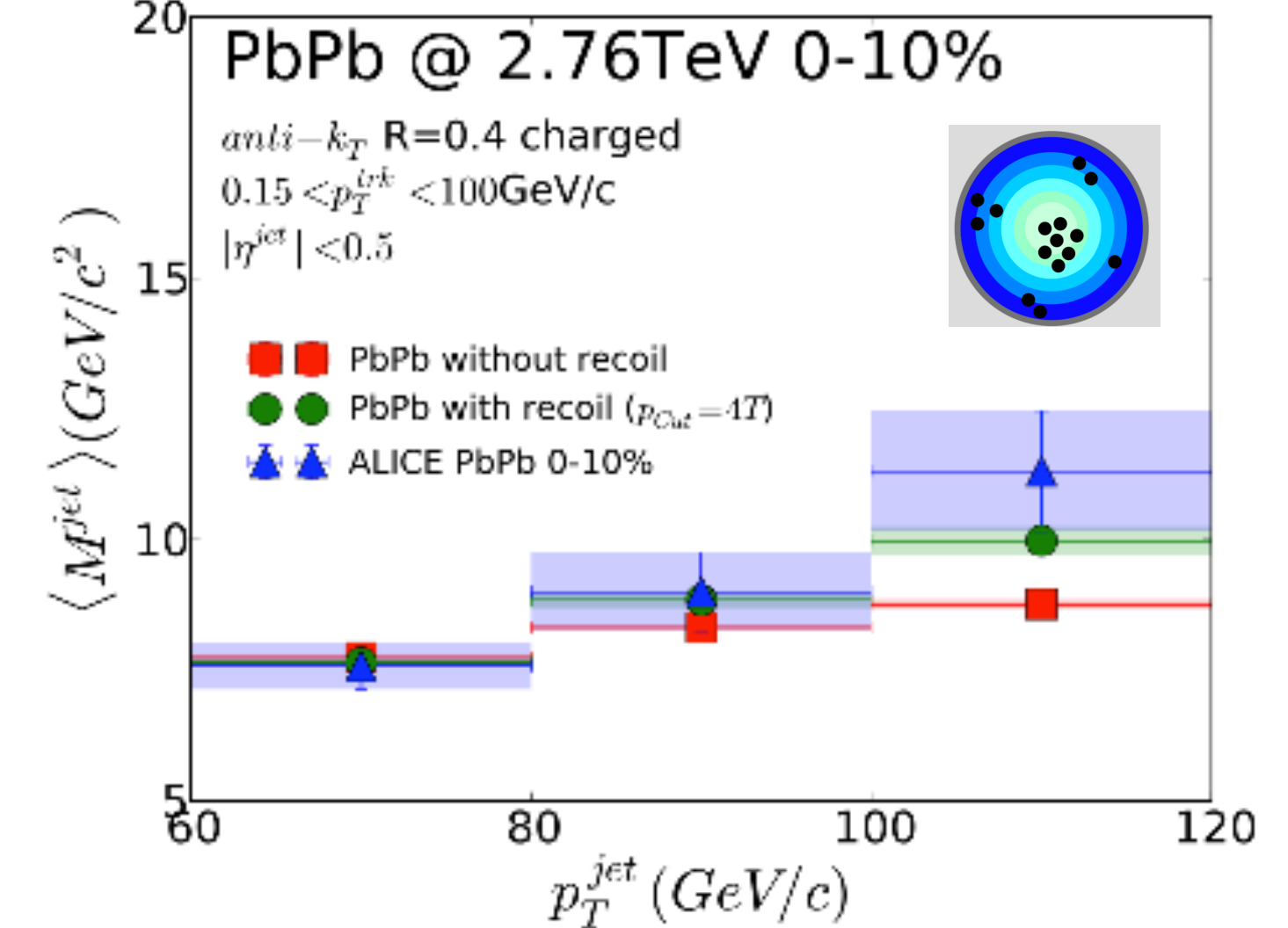
JEWEL+PYTHIA Pb+Pb (0 – 10%) (2.76 TeV)



[MARTINI:1807.06550]



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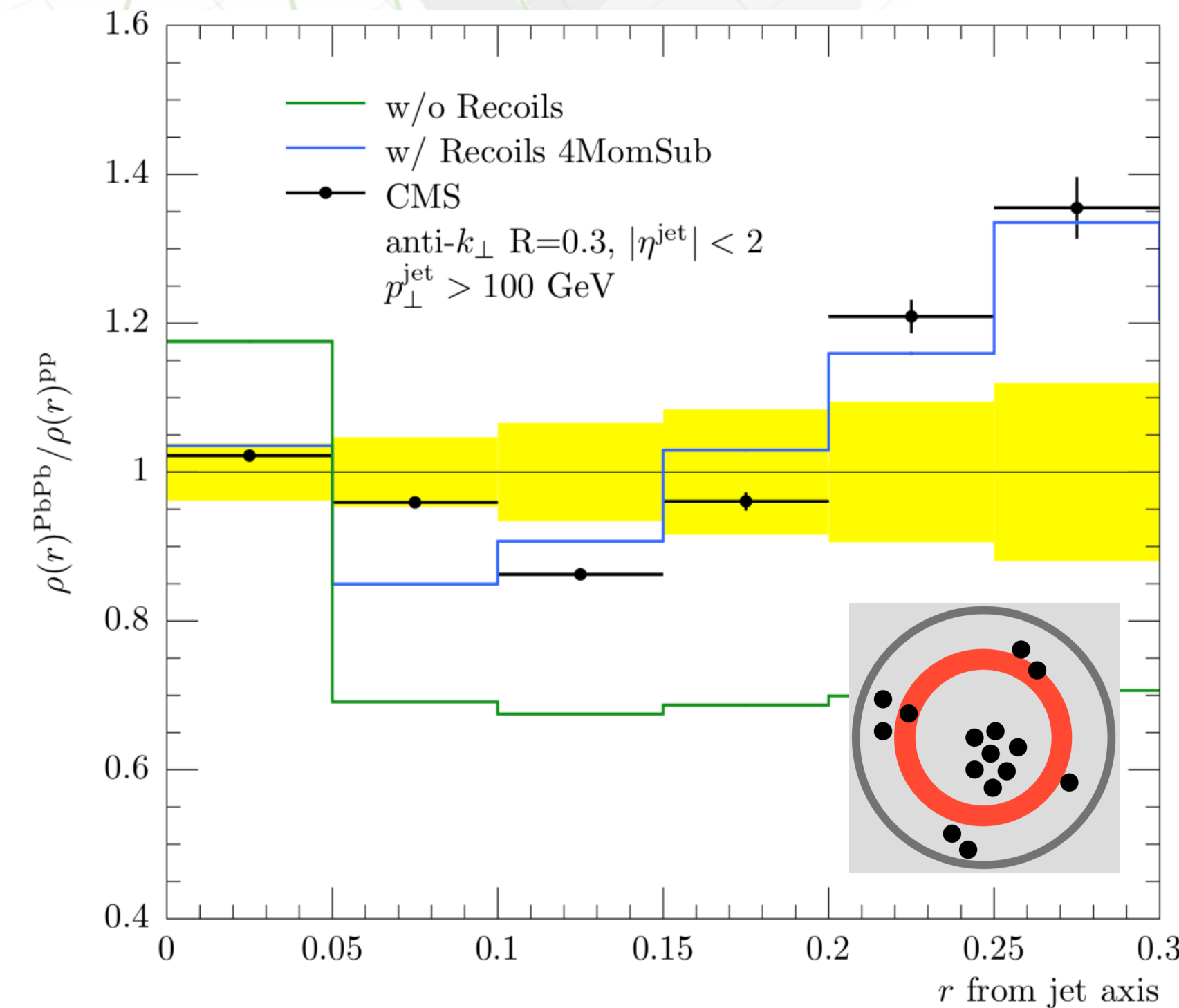




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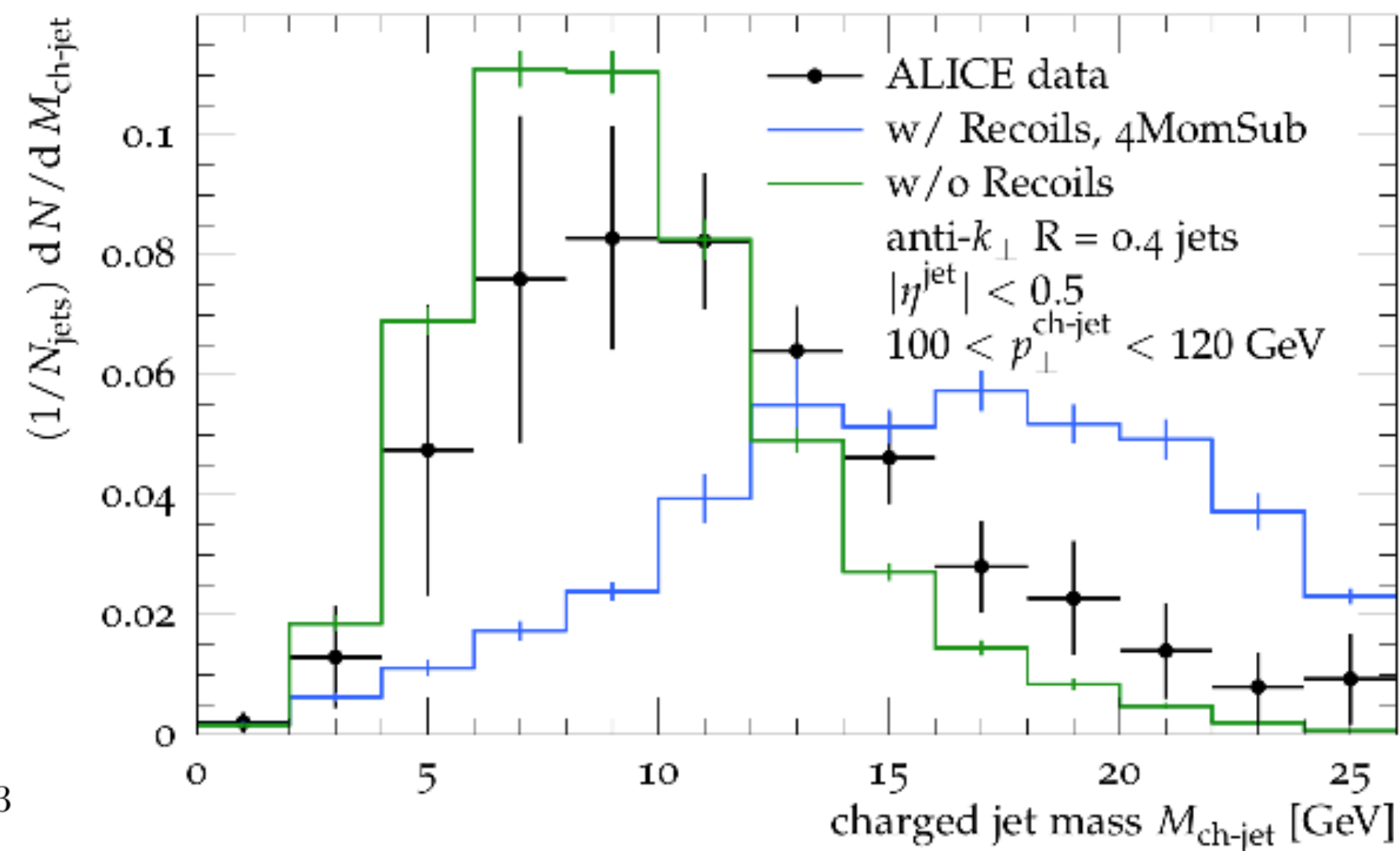
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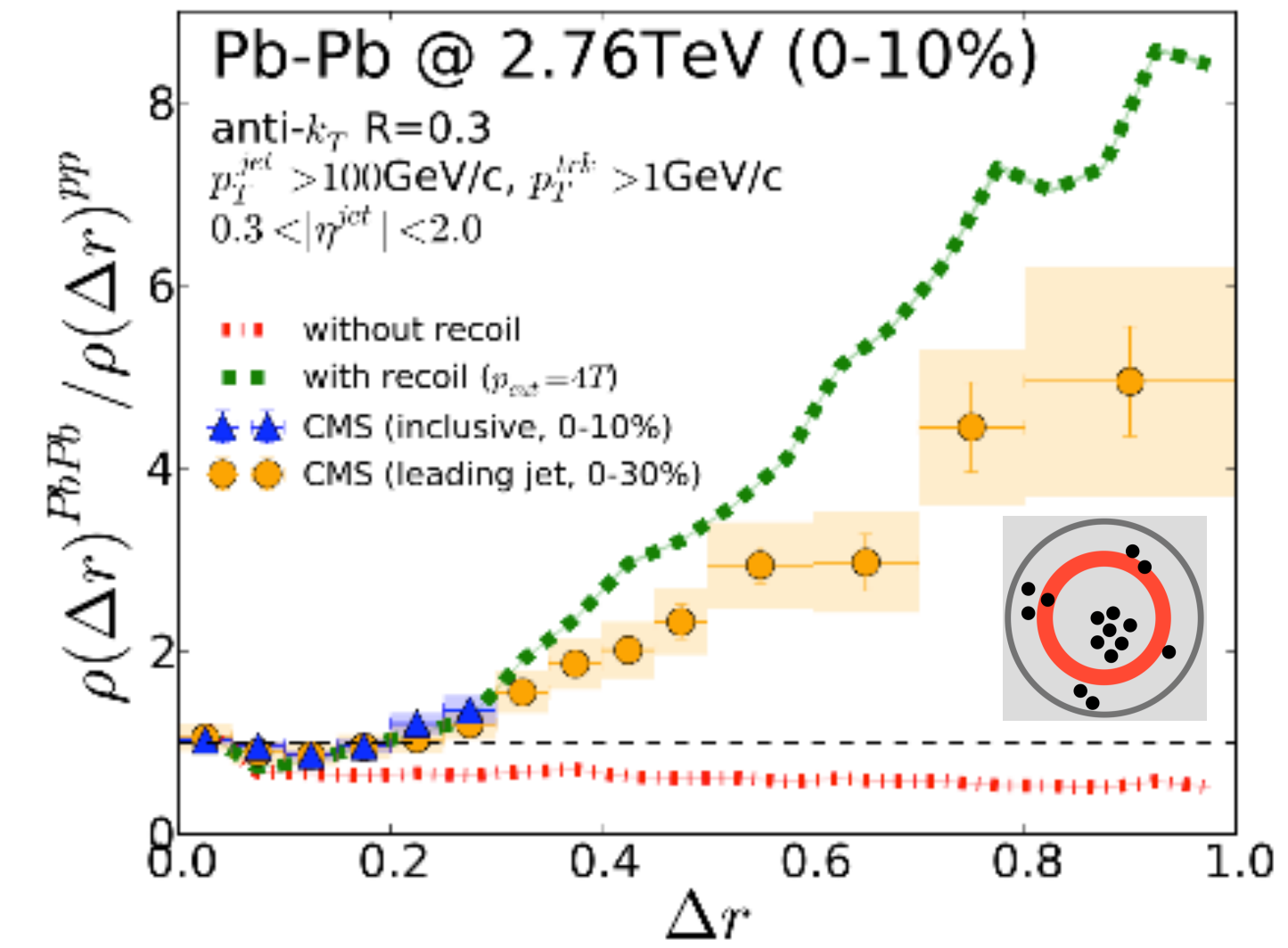
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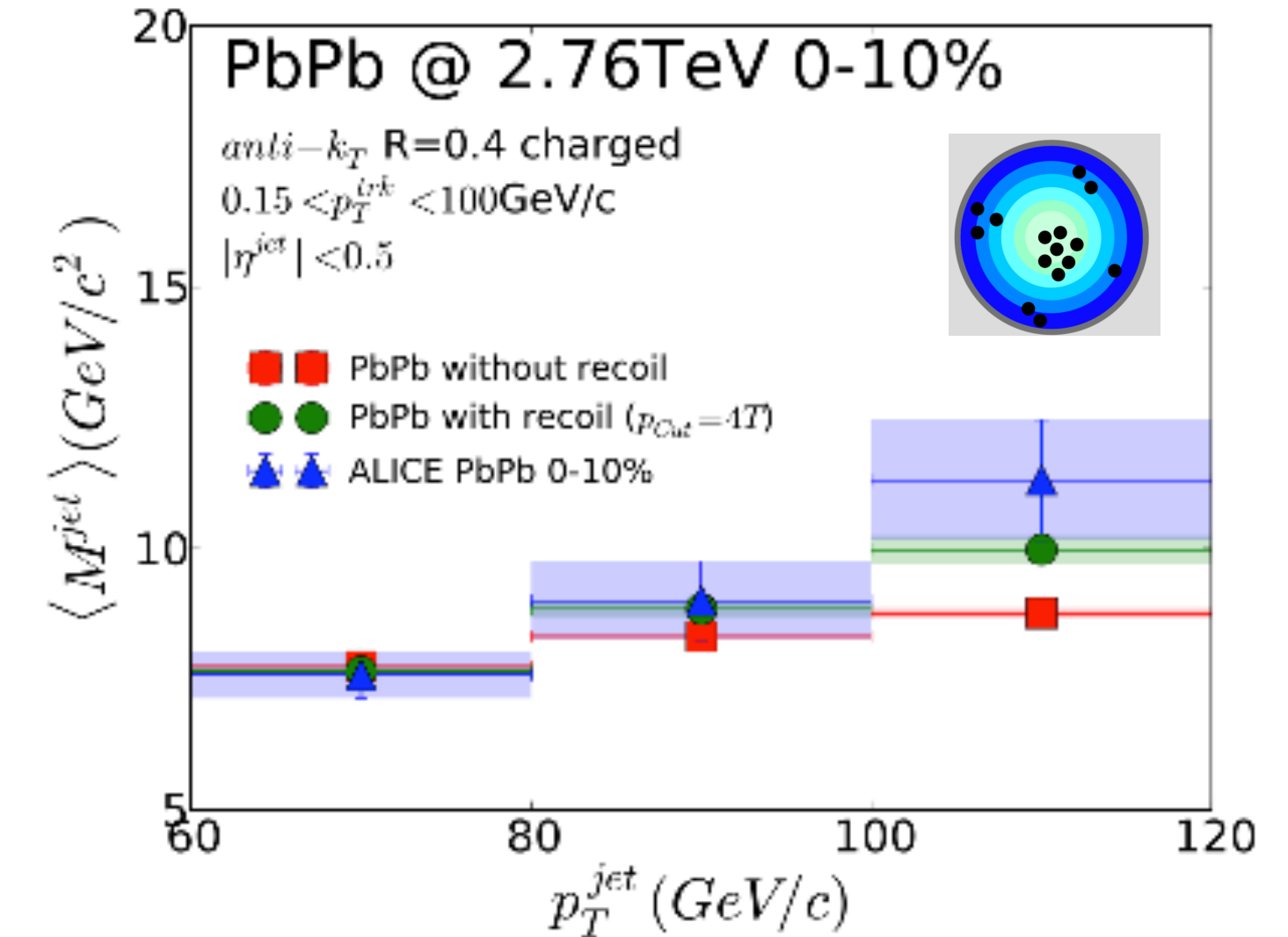


See K. Zapp (Thursday) for news about JEWEL

[MARTINI:1807.06550]



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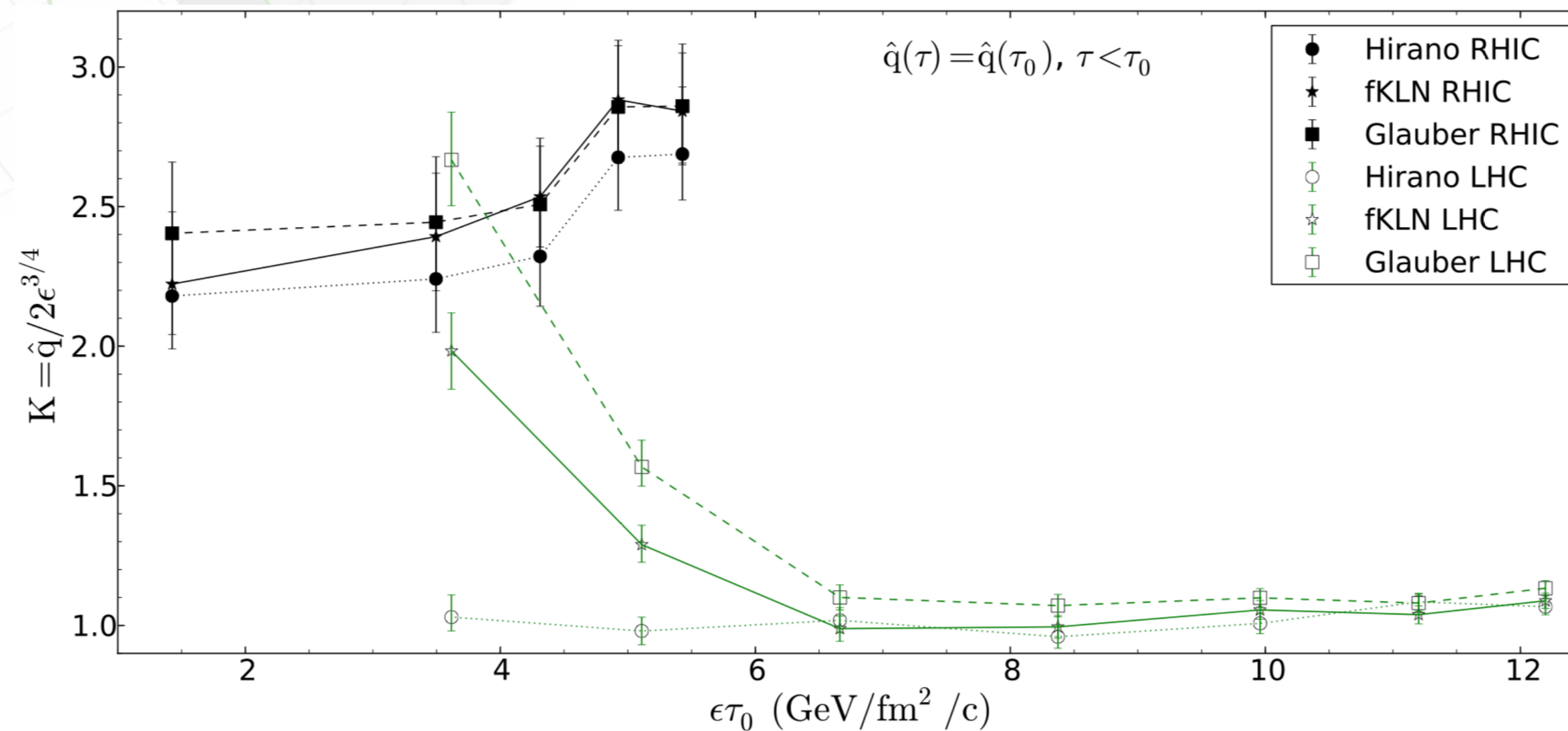
# And even more problems...

[Jet collaboration (13)]

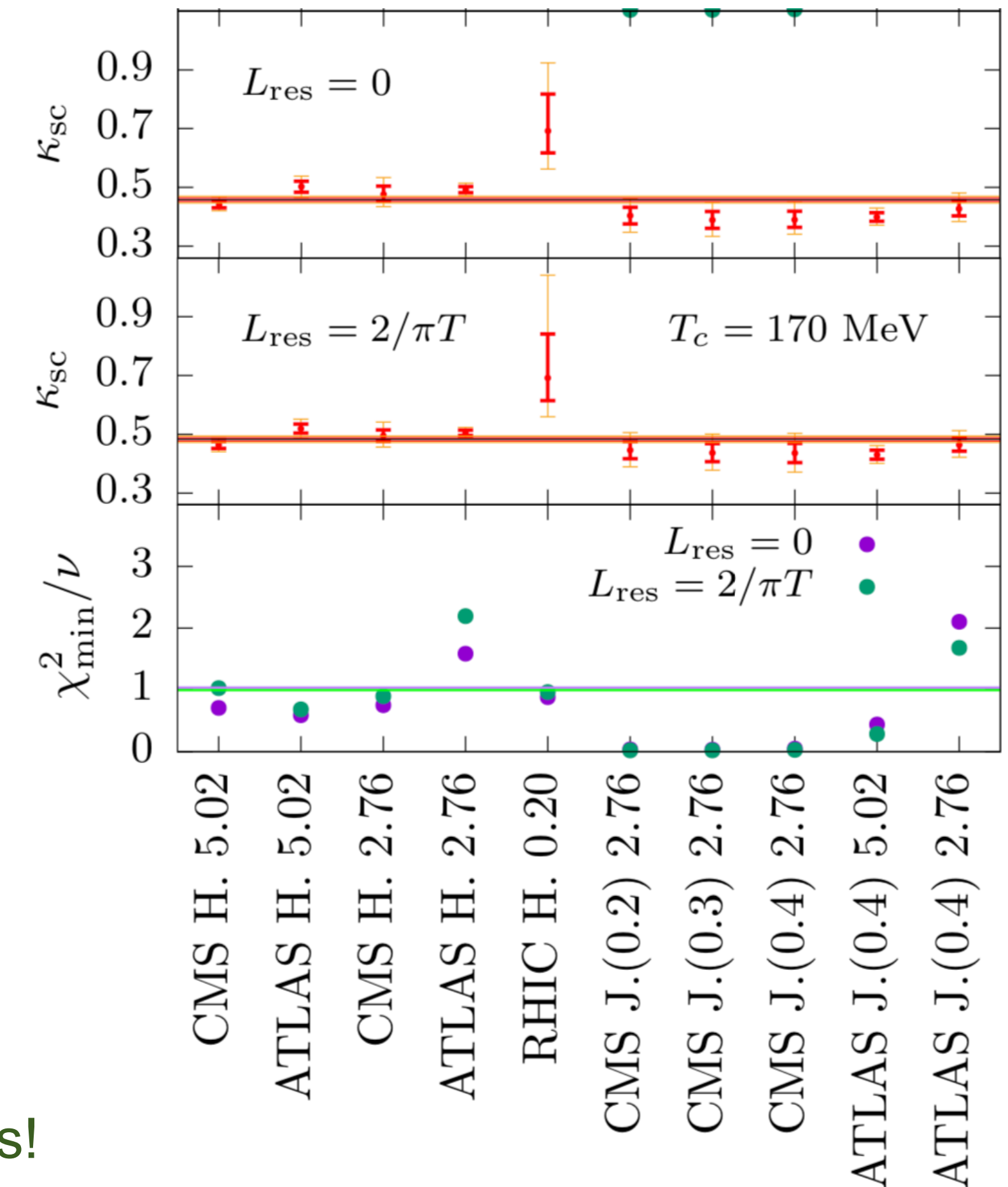
[Andrés, Armesto, Luzum, Salgado, Zurita (17)]

[Casalderrey-Solana, Hulcher, Milhano, Pablos, Rajagopal (18)]

- ◆ Quenching parameter ( $\hat{q}$ ) at RHIC larger is than at LHC
- ◆ Not clearly understood so far...



both pQCD and non-pQCD approaches!





# JQ Phenomenology

---

- ◆ Theoretical developments that address elementary jet processes:
  - ✓ Able to build up a clear qualitative picture
  - ✗ Not suited for describing medium recoil component (essential to withdraw QGP properties)

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But Jet Quenching must feed (and be fed) with jet phenomenology!

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See L. Apolinário (Thursday)



The background features a light gray grid. Overlaid on this are several purple lines of varying thickness and style, including solid, dotted, and dashed lines. Some lines are straight, while others are curved. In the bottom right corner, the text 'Wrapping-up' is written in a bold, black, sans-serif font. The overall aesthetic is modern and technical.

**Wrapping-up**



# Summary

---

- ◆ This was a brief lecture/overview of “jet quenching”:
- ◆ Things I didn’t cover: Heavy-quarks, AdS/CFT approaches, Monte Carlo approaches,...
- ◆ Several developments towards the understanding of what is a jet in a heavy-ion environment!
- ◆ But no consistent picture has emerged yet...
  - ◆ Not clear the role of medium recoil effects, missing in-medium evolution equation (coupled to vacuum radiation), ...
  - ➔ Several developments in building new observables (particular QGP effect, a particular type of probe (q/g-jets) and/or different QGP timescales ⇒ Jet substructure!)

See J. Barata (Thursday)

# Summary

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## Questions?

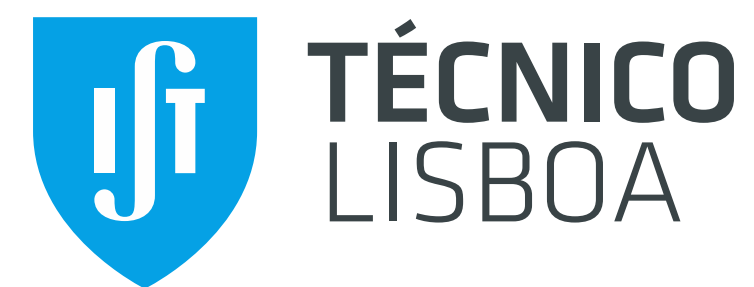
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See J. Barata (Thursday)



# Acknowledgements

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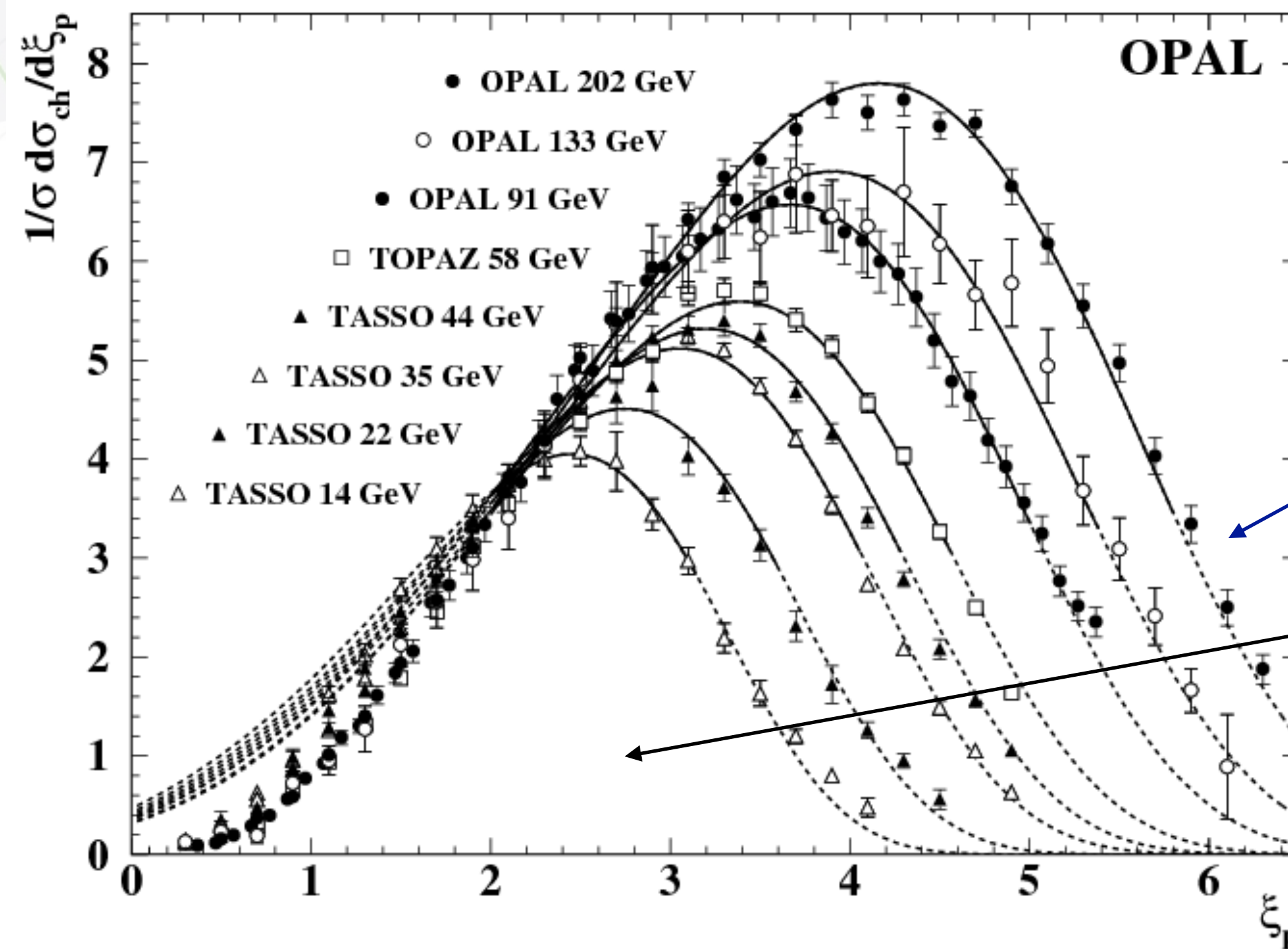
The background features a light gray grid pattern. Overlaid on this are several abstract elements: a cluster of purple lines, some solid and some dotted, that curve and converge towards the right side of the frame; a 3D-style light green hexagonal shape in the upper left; and a series of overlapping, semi-transparent geometric shapes in shades of yellow and green along the left edge.

**Backup Slides**



# Experimental Evidences

- ◆ MLLA: only the leading behaviour,
- ◆ Good description in the hard region (two different energies)



$$\xi_p = \log\left(\frac{1}{z}\right) = \log\left(\frac{p_{jet}}{p_{had}}\right)$$

Soft jet fragments

Hard jet fragments

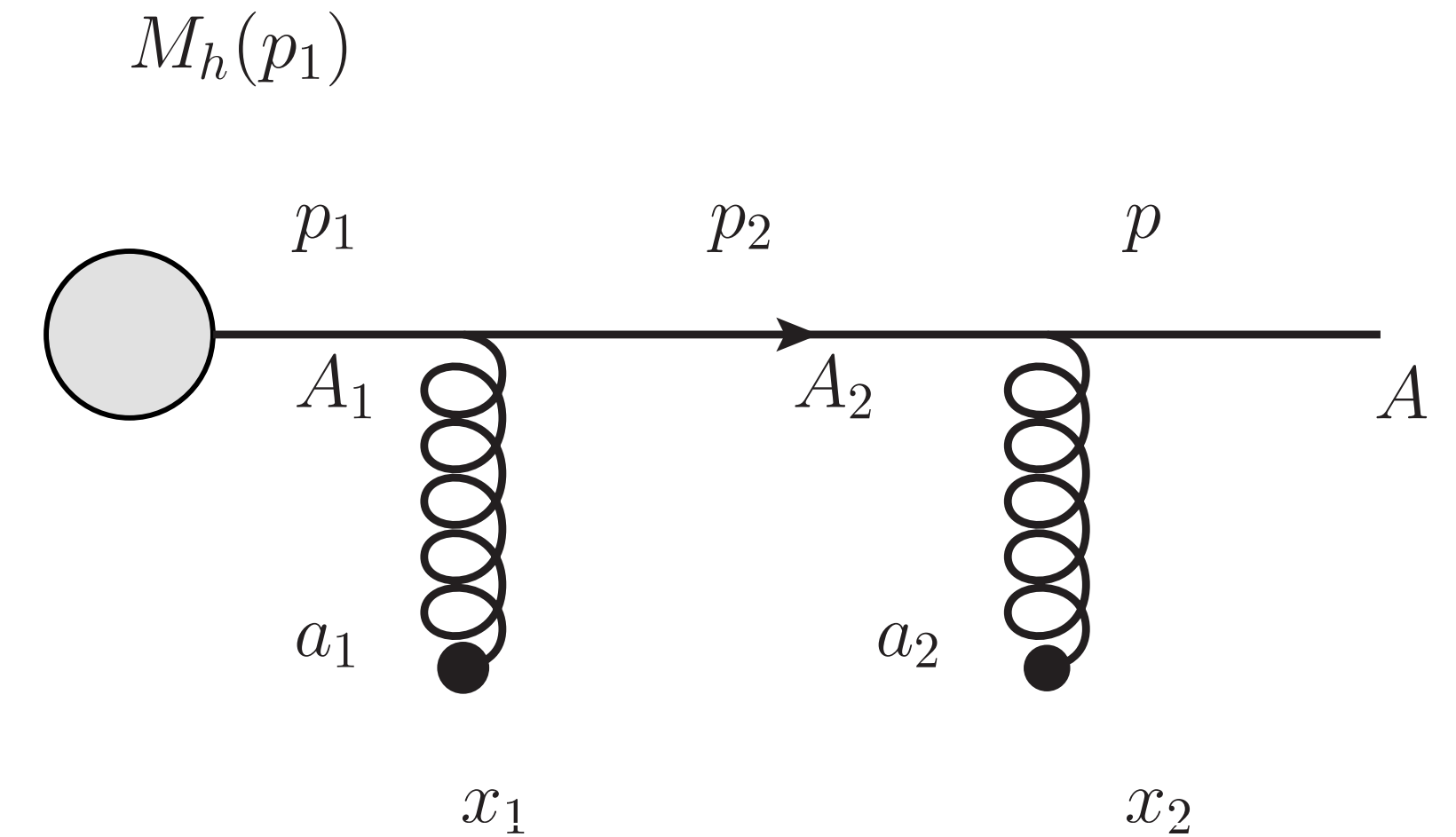
# Eikonal Approximation

See: arXiv:0712.3443

- Consider a high energetic particle propagating interacting twice with the medium:

$$S_2 = \int d^4x_1 d^4x_2 \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \bar{u}(p) e^{ix_2(p-p_2)+ix_1(p_2-p_1)}$$

$$\mathcal{A}_{AA_2}(x_{2+}, x_{2\perp}) \frac{ip_2}{p_2^2 + i\epsilon} ig \mathcal{A}_{A_2A_1}(x_{1+}, x_{1\perp}) \frac{ip_1}{p_1^2 + i\epsilon} M_h(p_1)$$



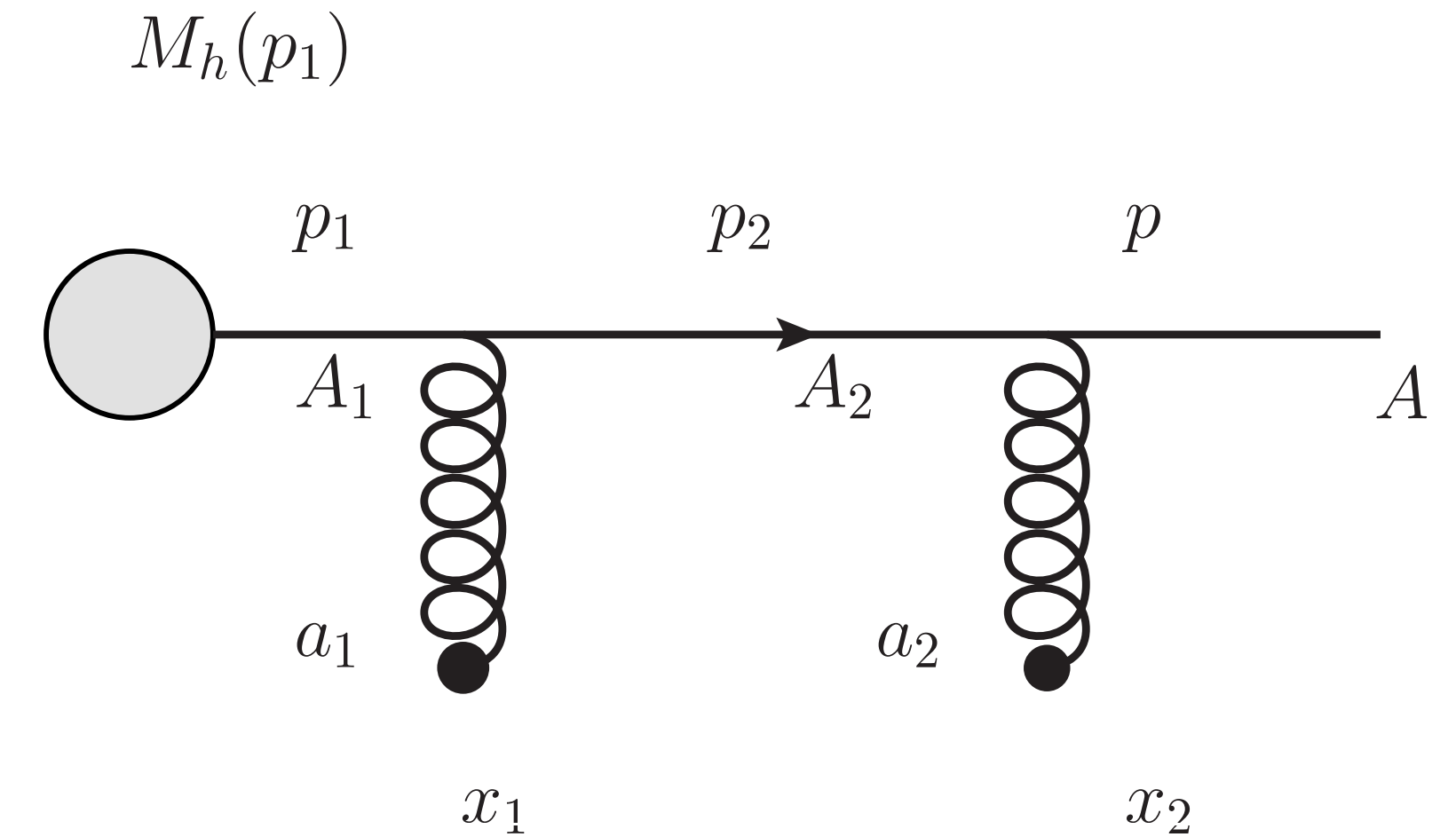


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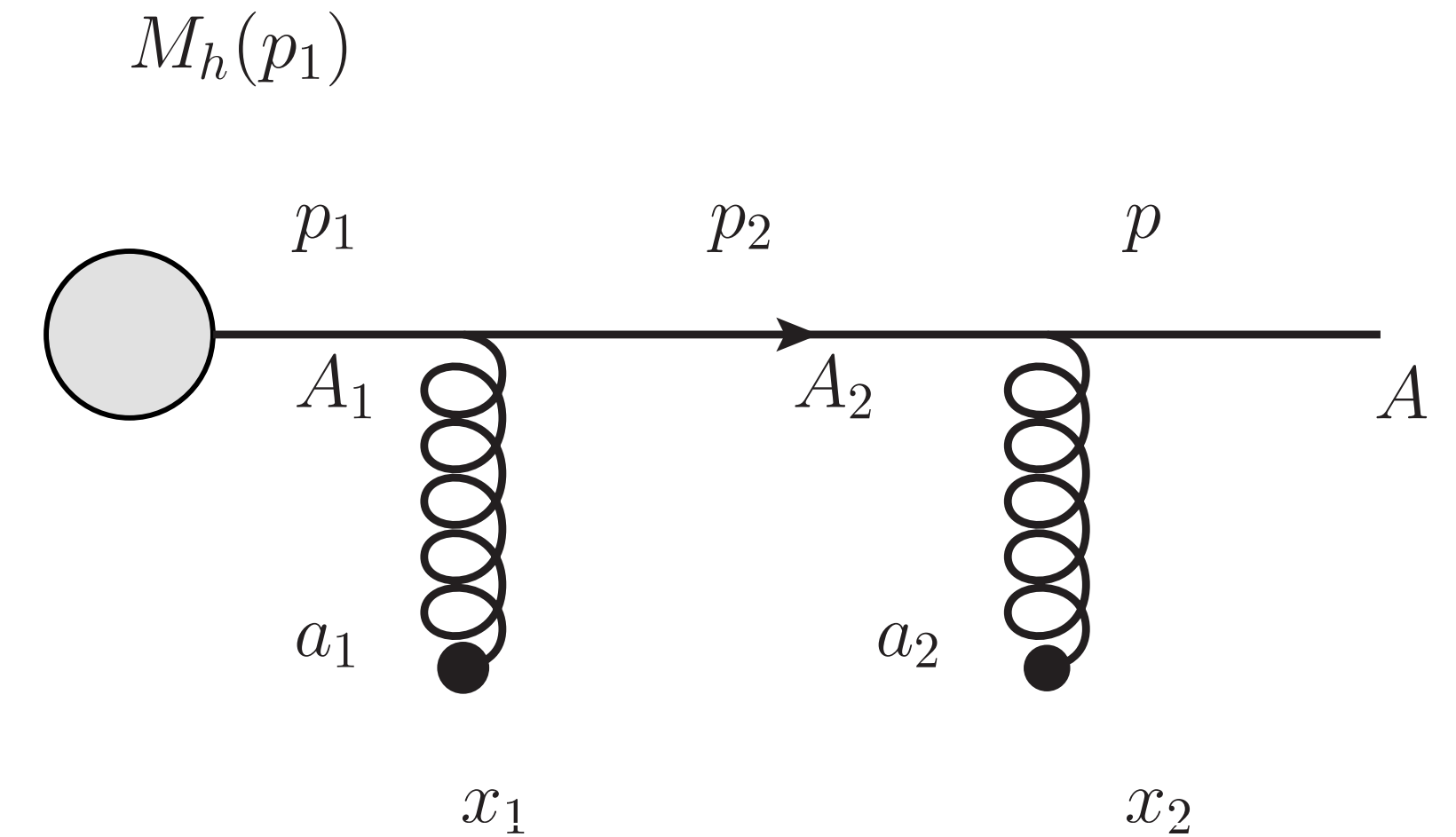
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No dependence on  $x_i$ :  $\int dx_{1-} dx_{2-} e^{ix_{2-}(p-p_2)+ix_{1-}(p_2-p_1)} = (2\pi)^2 \delta(p-p_2)_+ \delta(p_2-p_1)_+ \Rightarrow p_+ = p_{1+} = p_{2+}$

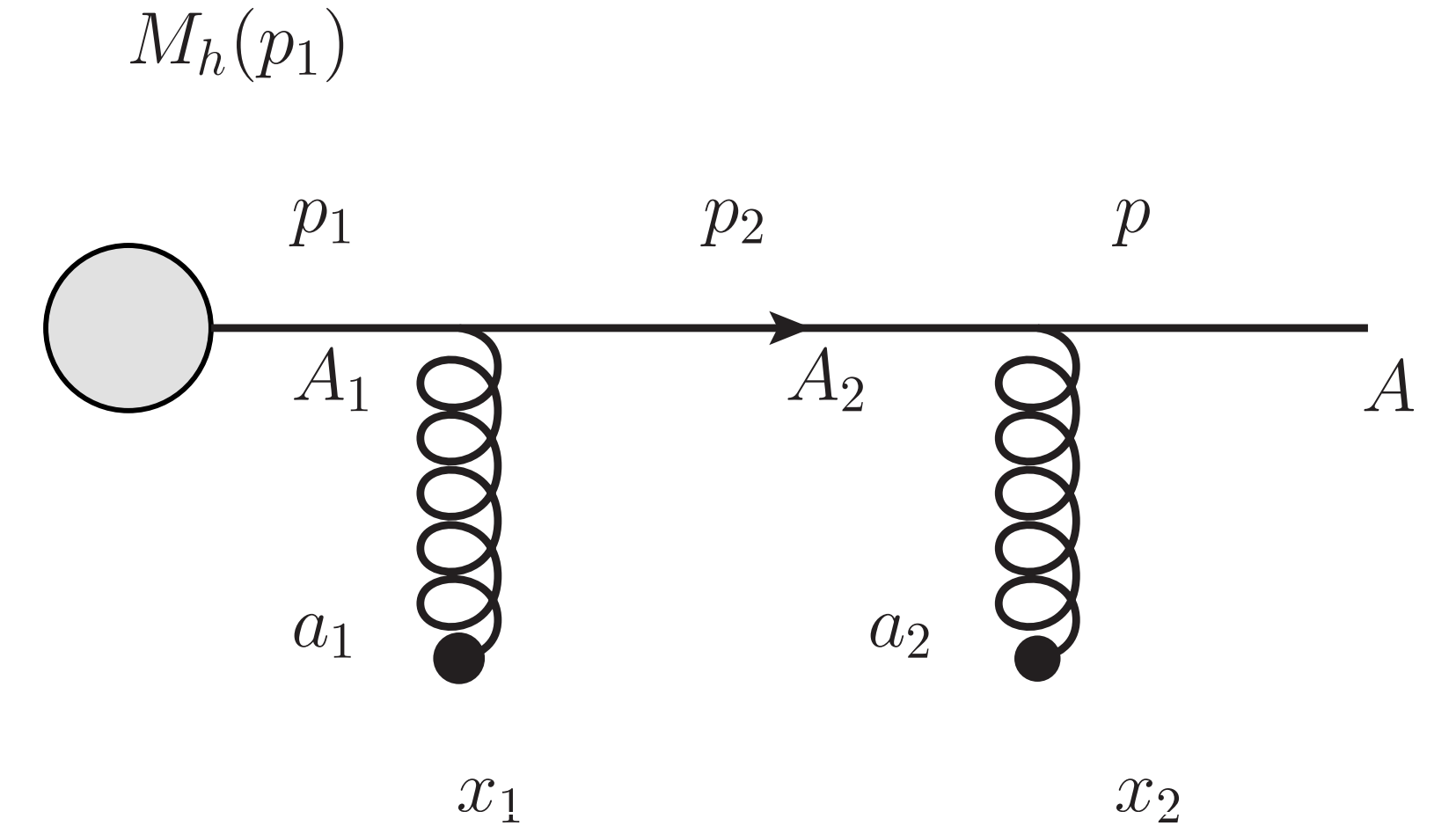


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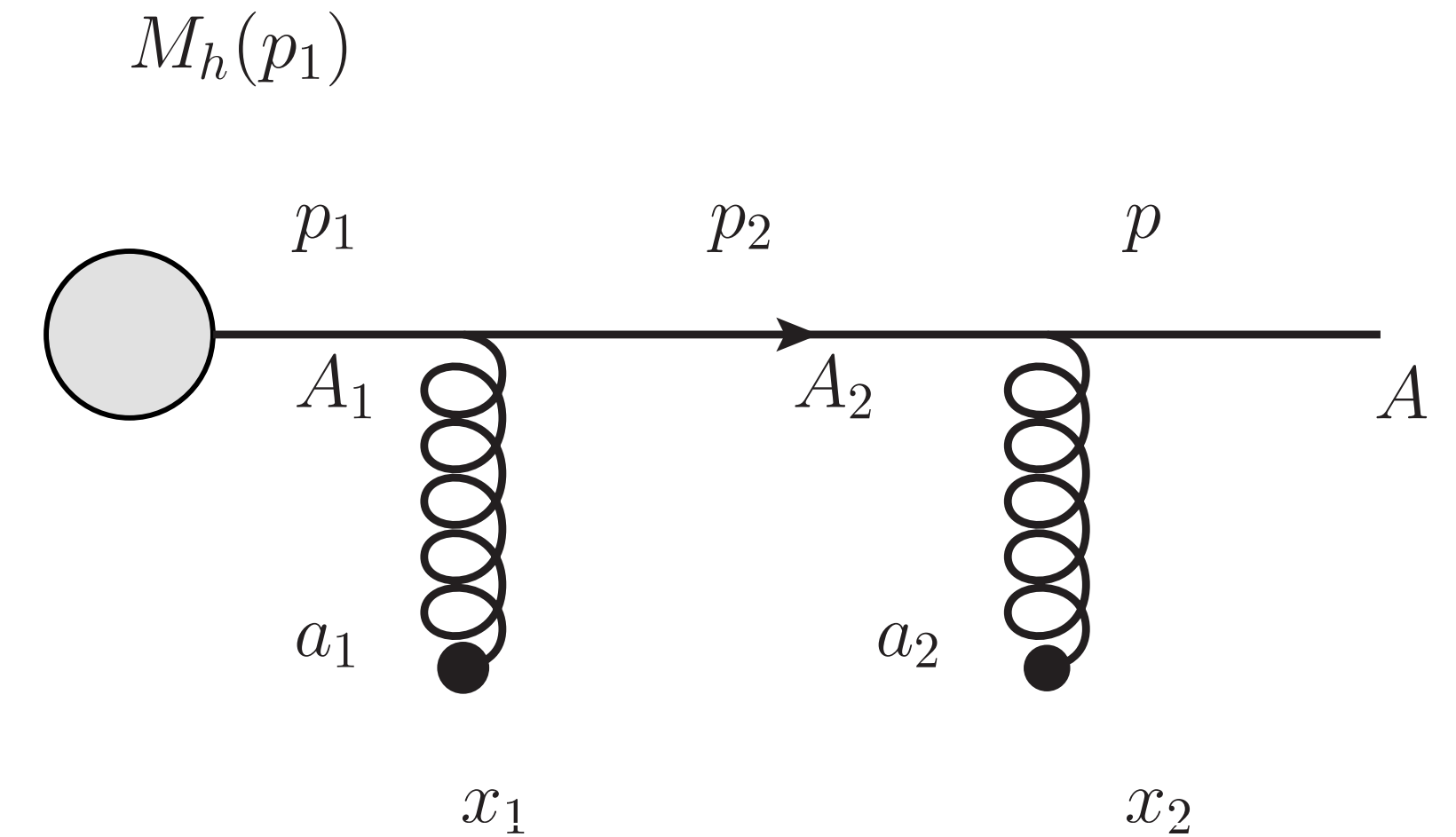
$$\bar{u}(p) \not{A}_{AA_2} \not{p}_2 = 2p_2 \cdot A_{AA_2} \bar{u}(p) - \bar{u}(p) \not{p}_2 \not{A}_{AA_2}$$

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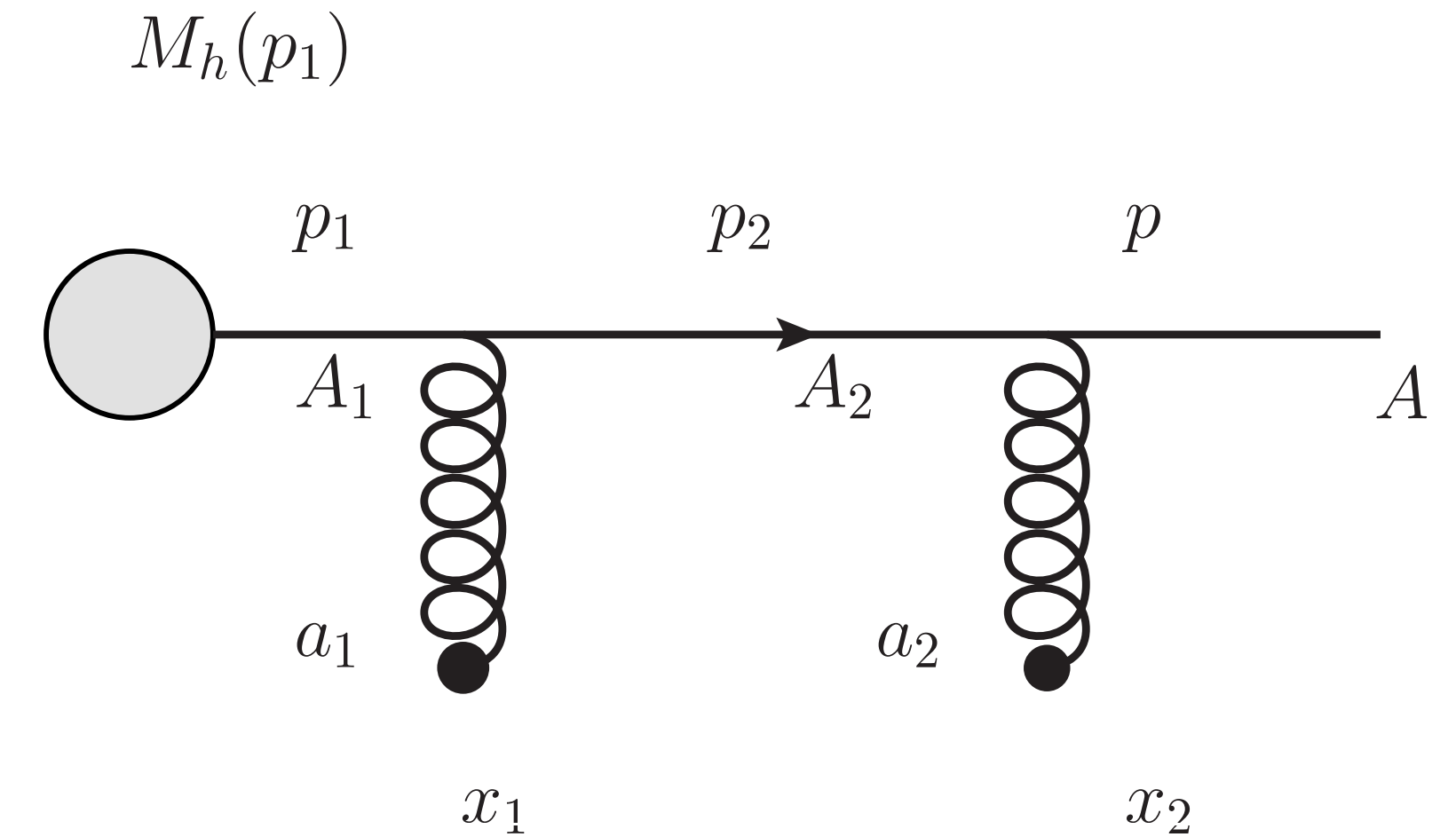
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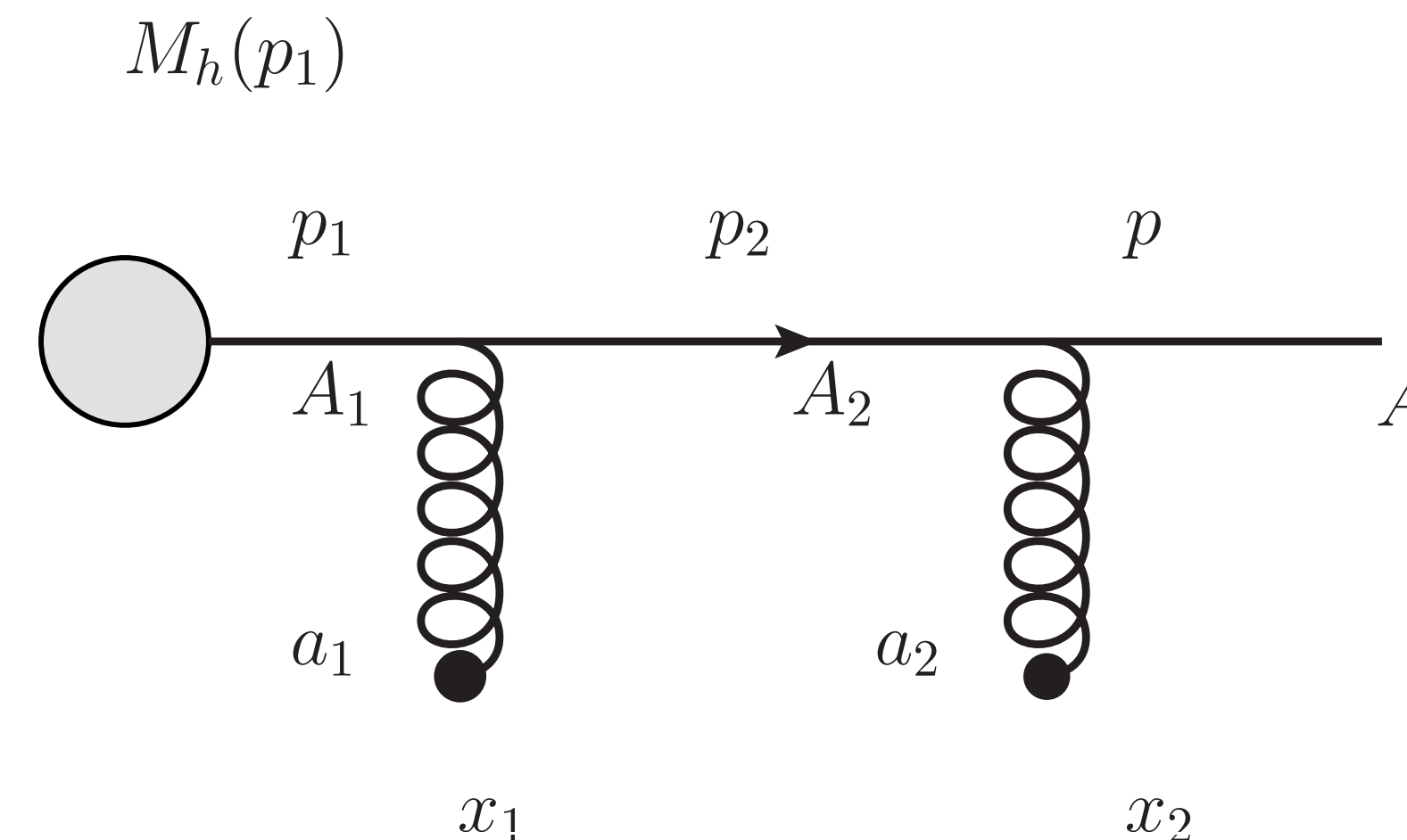
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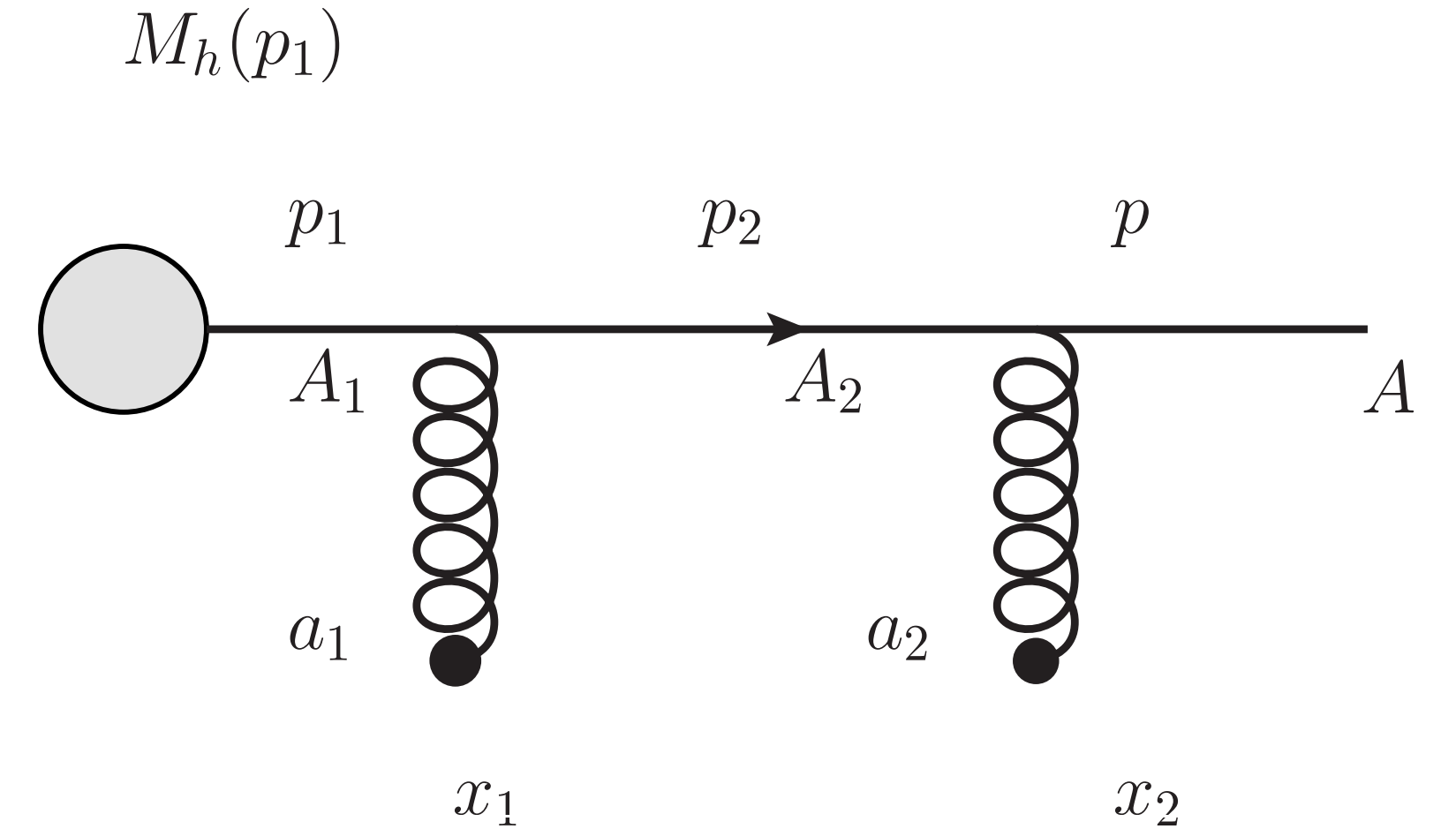
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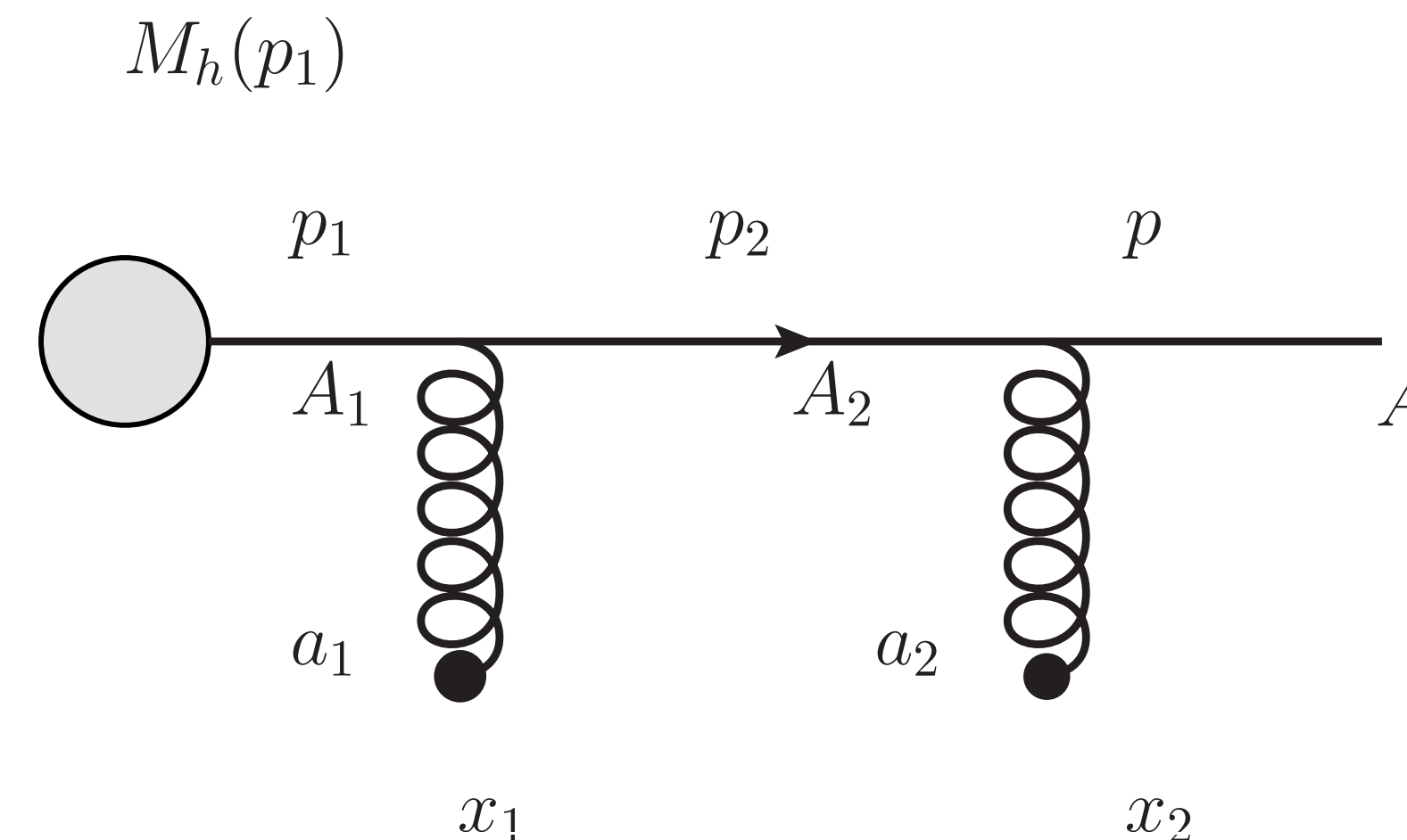
Integral in  $p_{T}$ :  $\int \frac{d^2p_{2\perp}}{(2\pi)^2} \frac{d^2p_{1\perp}}{(2\pi)^2} e^{-i(x_1-x_2)_\perp - i(0-x_1)_\perp} = \delta^2(x_1 - x_2)_\perp \delta^2(x_1) \Rightarrow x_{1\perp} = x_{2\perp} = 0$

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$$\begin{aligned}
 S_2 &= \int d^4x_1 d^4x_2 \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \bar{u}(p) e^{ix_2(p-p_2)+ix_1(p_2-p_2)} \\
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 &\simeq \int dx_{1+} dx_{2+} ig(A_-)_{AA_2}(x_{2+}, 0_\perp) \theta(x_2 - x_1)_+ ig(A_-)_{A_2A_1}(x_{1+}, 0_\perp) \theta(x_1)_+ \\
 &\quad \cdot \bar{u}(p) M_h(p)
 \end{aligned}$$





# Eikonal Approximation

See: arXiv:0712.3443

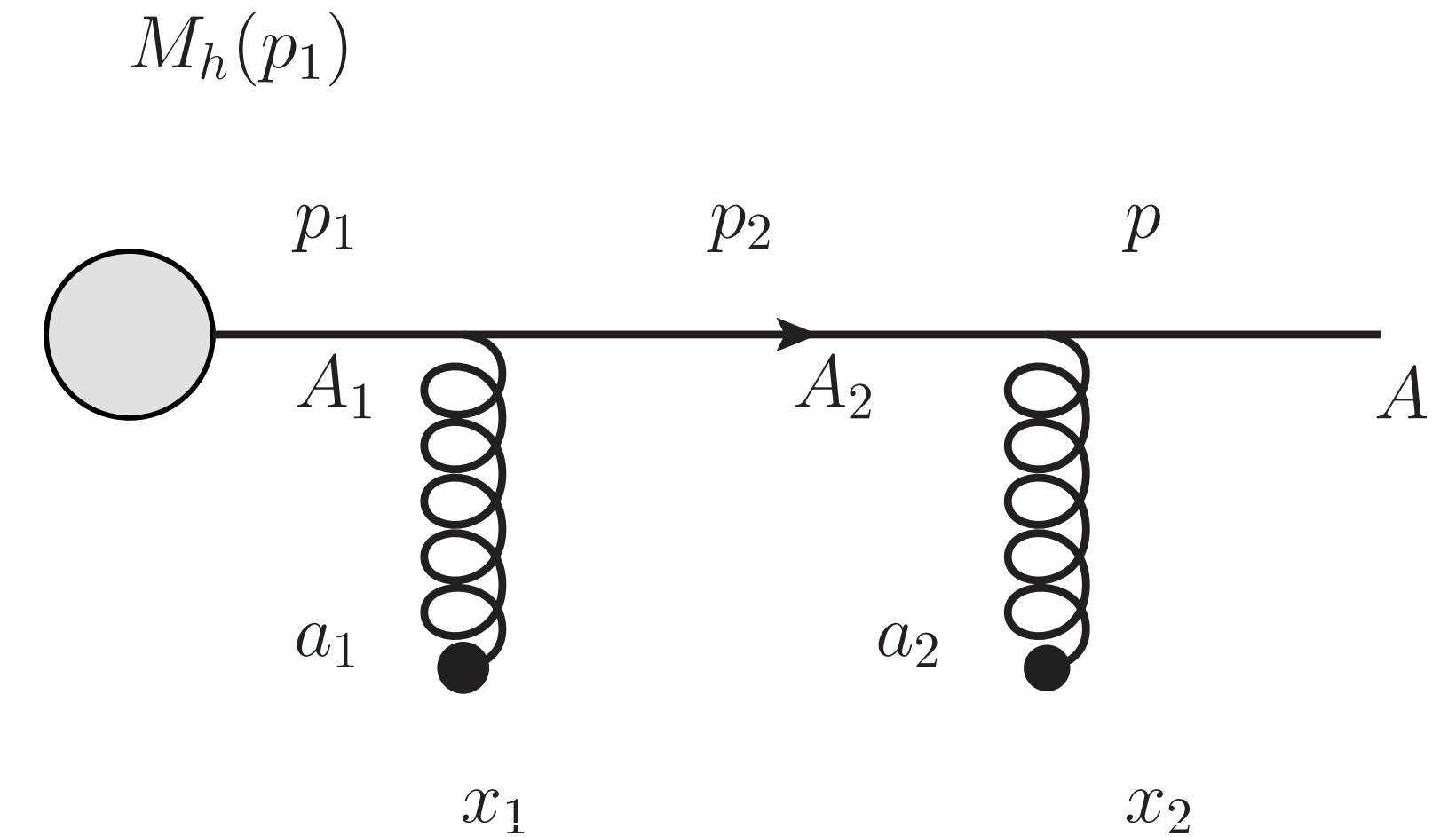
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$$A_{AA_2}(x_{2+}, x_{2\perp}) \frac{ip_2}{p_2^2 + i\epsilon} ig A_{A_2A_1}(x_{1+}, x_{1\perp}) \frac{ip_1}{p_1^2 + i\epsilon} M_h(p_1)$$

$$\simeq \int dx_{1+} dx_{2+} ig(A_-)_{AA_2}(x_{2+}, 0_\perp) \theta(x_2 - x_1)_+ ig(A_-)_{A_2A_1}(x_{1+}, 0_\perp) \theta(x_1)_+$$

$$\bar{u}(p) M_h(p)$$



No interaction term

# Eikonal Approximation

See: arXiv:0712.3443

- Consider a high energetic particle propagating interacting twice with the medium:

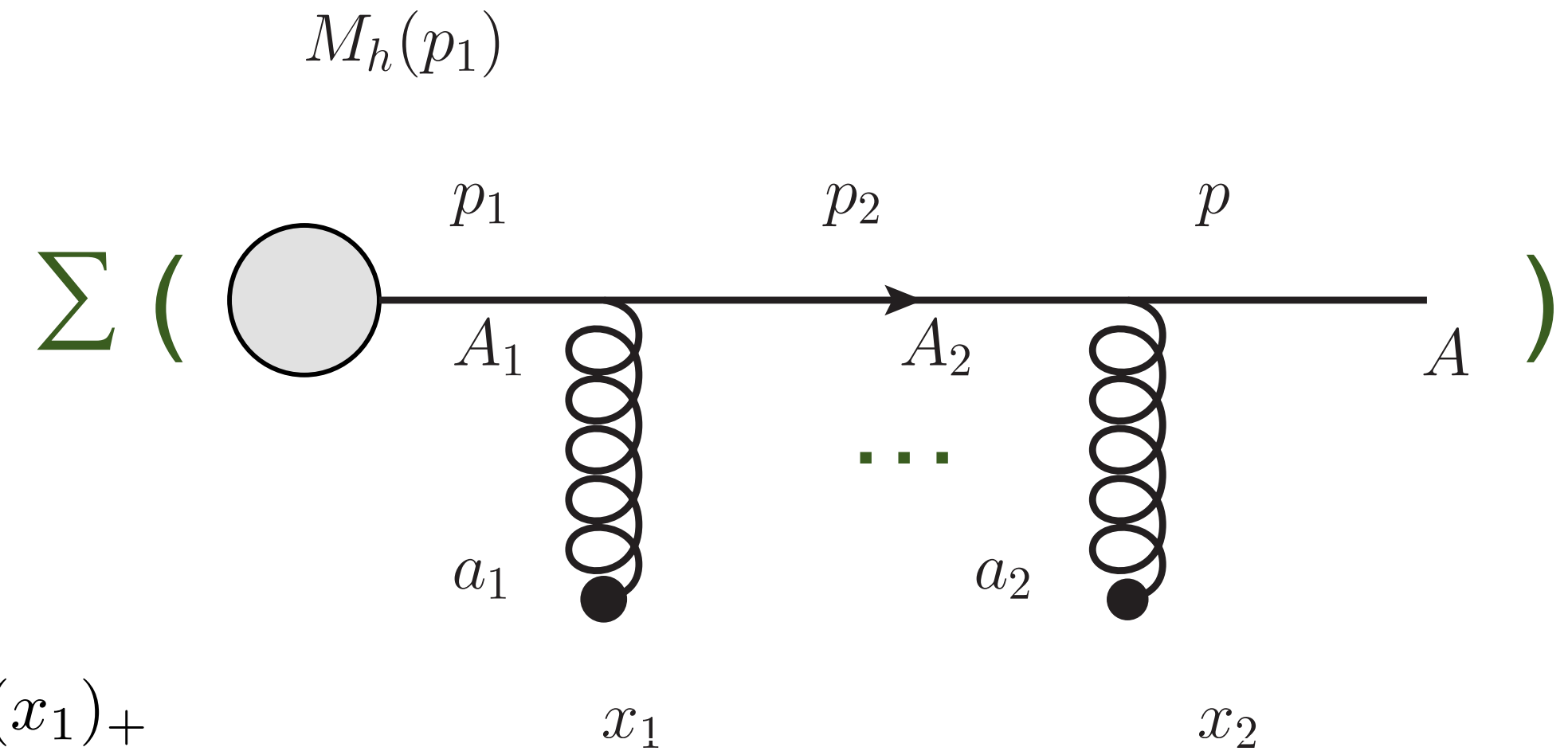
$$S_2 = \int d^4x_1 d^4x_2 \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \bar{u}(p) e^{ix_2(p-p_2)+ix_1(p_2-p_1)}$$

$$A_{AA_2}(x_{2+}, x_{2\perp}) \frac{ip_2}{p_2^2 + i\epsilon} ig A_{A_2A_1}(x_{1+}, x_{1\perp}) \frac{ip_1}{p_1^2 + i\epsilon} M_h(p_1)$$

$$\simeq \int dx_{1+} dx_{2+} ig(A_-)_{AA_2}(x_{2+}, 0_\perp) \theta(x_2 - x_1)_+ ig(A_-)_{A_2A_1}(x_{1+}, 0_\perp) \theta(x_1)_+$$

summing over all 'n' interaction terms

$$\mathcal{P} \exp \left\{ ig \int_{x_{1+}}^{x_{n+}} dx_+ A_-(x_+, x_\perp = 0) \right\} = W(x_{n+}, x_{1+}; x_\perp = 0)$$



$\bar{u}(p) M_h(p)$

No interaction term



# MC Bibliography

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- ◆ Monte Carlo models for jet quenching:
  - ◆ Hybrid Strong/Weak coupling: [Casalderrey-Solana, Gulhan, Milhano, Pablos, Rajagopal (14;17); Helcher, Pablos, Rajagopal (18)]
  - ◆ JETSCAPE: [JETSCAPE Collab. (17)]
  - ◆ JEWEL: [Krauss, Wiedemann, Zapp(13); Zapp (14); Elayavalli, Zapp (16;17)]
  - ◆ LBT/Co-LBT: [Wang and Y. Zhu (16); Cao, Luo, Qin, Wang (15); He, Luo, Wang, Zhu (17);]
  - ◆ MARTINI: [Schenke, Gale, Jeon (09); Park, Jeon, Gale (18)]
  - ◆ MATTER: [Majumder (13); Kordell, Majumder (17); Cao, Majumder (18)]
  - ◆ PYQUEN: [Lokhtin, Snigirev (06)]
  - ◆ Q-PYTHIA: [Armesto, Cunqueiro, Salgado (09)]

# MC Bibliography

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- ◆ Monte Carlo models for heavy-ions:
  - ◆ **AMPT**: [Ko, Li, Lin, Pal, Zhang (00; 01)]
  - ◆ **BAMPS**: [Xu, Greiner (03; 07)]
  - ◆ **CUJET**: [Buzzatti and Gyulassy (11; 12)]
  - ◆ **HiJING/HiJING++**: [Gyulassy, Wang (91; 94); Barnaföldi et al (17)]
  - ◆ **HYDJET/HYDJET++**: [Lokhtin, Malinina, Petrushanko, Snigirev, Arsene, Tywoniuk (09)]
  
- ◆ Analytical approaches:
  - ◆ **Coupled Jet-Fluid**: [Tachibana, Chang, Qin (17)]