Colour Evolution

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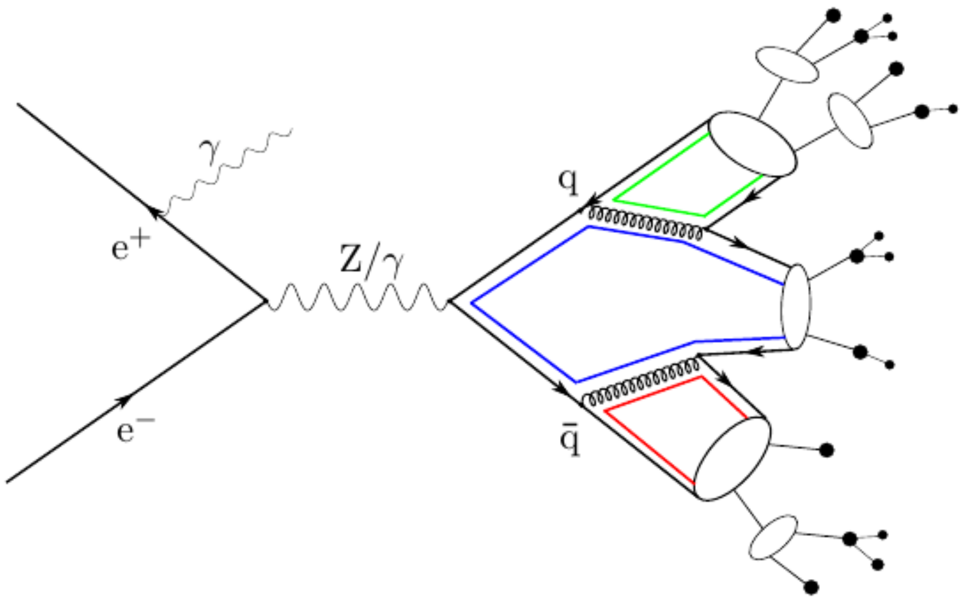
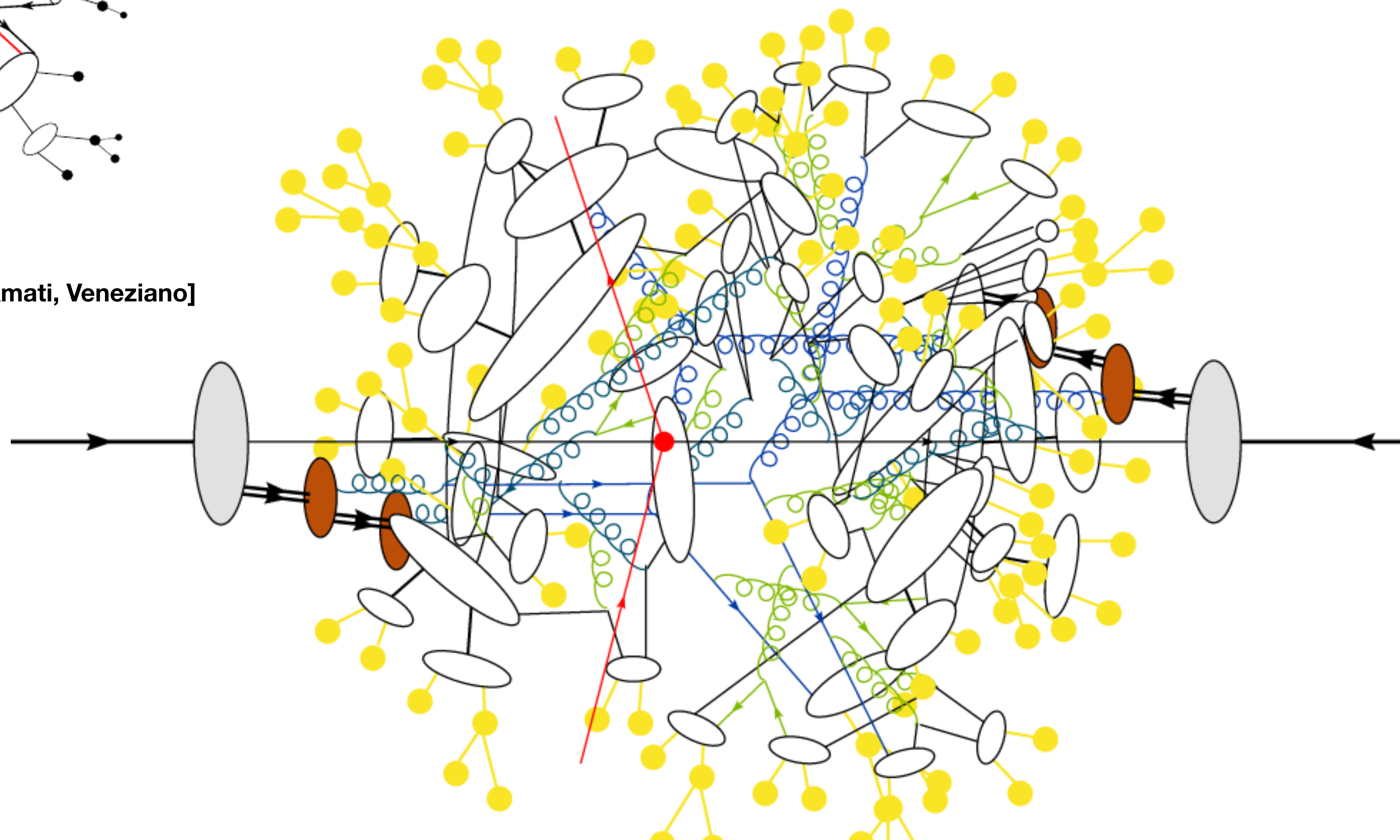
Context/Motivation/Goals



How do we decide which quarks to connect?



[Pre-confinement as a property of pQCD, Amati, Veneziano]

QCD scattering amplitudes are vectors in ~~spin~~ and colour space

$$|\mathcal{M}\rangle = \sum_{\sigma} \mathcal{M}_{\sigma} |\sigma\rangle$$

With the colour flow basis

$$|\sigma\rangle = \left| \begin{array}{ccc} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{array} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}$$

The bare amplitude can be related to the renormalized amplitude as

$$|\mathcal{M}(\{p\},\mu^2)\rangle = \mathbf{Z}^{-1}(\{p\},\mu^2,\epsilon)|\tilde{\mathcal{M}}(\{p\},\epsilon)\rangle$$

and the renormalization constant Z is an operator in the space of colour structures.

The structure of Z governed by RGE

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} |\mathcal{M}(\{p\}, \mu^2)\rangle = \mathbf{\Gamma}(\{p\}, \mu^2) |\mathcal{M}(\{p\}, \mu^2)\rangle$$

Where the pre factor is called soft anomalous dimension

$$\mathbf{\Gamma}(\{p\}, \mu^2) = -\mathbf{Z}^{-1}(\{p\}, \mu^2, \epsilon) \mu^2 \frac{\partial}{\partial \mu^2} \mathbf{Z}(\{p\}, \mu^2, \epsilon)$$

The evolution equation can be solved by

$$|\mathcal{M}(\{p\}, \mu^2)\rangle = \mathbf{U}(\{p\}, \mu^2, \{M_{ij}^2\}) |\mathcal{H}(\{p\}, Q^2, \{M_{ij}^2\})\rangle$$

where

$$U = \exp\left\{ \int_{\mu^2}^{M_{\alpha\beta}^2} \frac{dq^2}{q^2} \Gamma(\{p\}, q^2) \right\}$$

Final colour structure depends on the soft anomalous dimension matrix

Conjecture for soft anomalous dimension matrix

[Becher, Neubert, Phys. Rev. Lett. 102 (2009)]

$$\Gamma = \sum_{\{i,j\}} \frac{T_i \cdot T_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln(\frac{\mu^2}{-s_{ij}}) + \sum_i \gamma^i(\alpha_s)$$

At 1-loop: $\gamma_{\text{cusp}} = \alpha_s/2\pi$

Neglect γ^i **(does not change the color structure)**

$$\Gamma = \sum_{i \leq j} \Omega_{i\bar{j}} T_i \cdot T_j + \sum_{i<j} \Omega_{ij} T_i \cdot T_j + \sum_{i<j} \Omega_{\bar{i}\bar{j}} T_{\bar{i}} \cdot T_{\bar{j}}$$

With $\Omega_{\alpha\beta} = \int_{\mu^2}^{M^2_{\alpha\beta}} \frac{dq^2}{q^2} \frac{\alpha_s}{2\pi} \left(\ln \frac{M^2_{\alpha\beta}}{q^2} - i\pi \right)$

$$= \frac{\alpha_s}{2\pi} \left(\frac{1}{2} \ln^2 \frac{M^2_{\alpha\beta}}{q^2} - i\pi \ln \frac{M^2_{\alpha\beta}}{\mu^2} \right)$$

Putting everything together

$$\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right) = \exp\left(\sum_{i\neq j}\mathbf{T}_i\cdot\mathbf{T}_j\,\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\ln^2\frac{M_{ij}^2}{\mu^2} - i\pi\ln\frac{M_{ij}^2}{\mu^2}\right)\right)$$

Starting point for evolution of a colour flow

$$\mathcal{A}_{\tau\to\sigma} = \langle\sigma|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle\ .$$

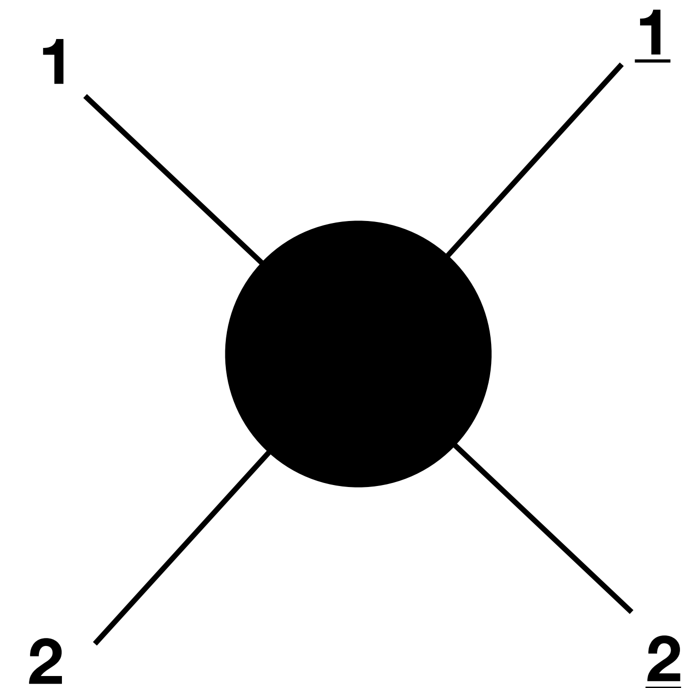
Define reconnection probability

$$P_{\tau\to\sigma} = \frac{|\mathcal{A}_{\tau\to\sigma}|^2}{\sum_\rho |\mathcal{A}_{\tau\to\rho}|^2}$$

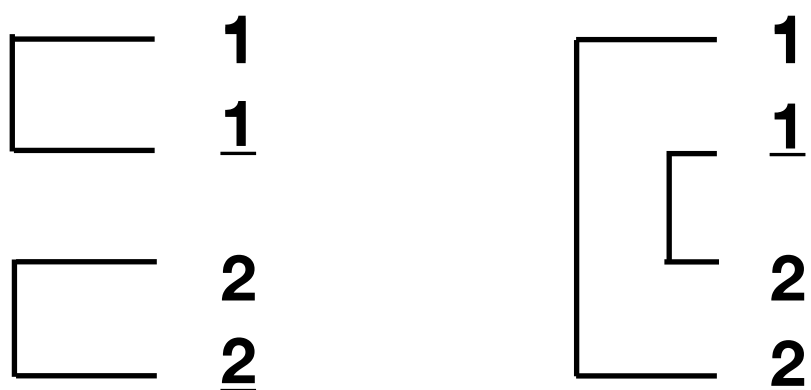
Evolution in colour flow basis (compact notation)

$$|\sigma\rangle = \left|\begin{matrix}\bar{1} & \bar{2} & \dots & \bar{n} \\ 1 & 2 & \dots & n\end{matrix}\right\rangle = \delta_1^{\bar{1}}\ \delta_2^{\bar{2}}\ \dots\ \delta_n^{\bar{n}}$$

Each index runs over N colours



2 cluster system (4 legs) 2 different color flows



$$\left|\begin{matrix}\bar{1} & \bar{2} \\ 1 & 2\end{matrix}\right\rangle = |1\ 2\rangle \qquad |2\ 1\rangle$$

States in color flow notation

$$|12\rangle = \begin{pmatrix}1 \\ 0\end{pmatrix} \qquad |21\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix}$$

[Plätzer, EPJC 74 (2014) 6]

[Martinez, De Angelis, Forshaw, Plätzer, Seymour, JHEP 05 (2018) 044]

Example: Two cluster evolution

Start evolution with initial colour flow $|12\rangle$

$$|\tau\rangle = U|12\rangle = \begin{pmatrix} U_{11} & U_{21} \\ U_{12} & U_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = U_{11}|12\rangle + U_{12}|21\rangle$$

Project out all possible color flows

$$\langle 12|\tau\rangle = U_{11}\langle 12|12\rangle + U_{12}\langle 12|21\rangle$$

$$\langle 21|\tau\rangle = U_{11}\langle 21|12\rangle + U_{12}\langle 21|21\rangle$$

Where [Martinez, De Angelis, Forshaw, Plätzer, Seymour, 1802.08531]

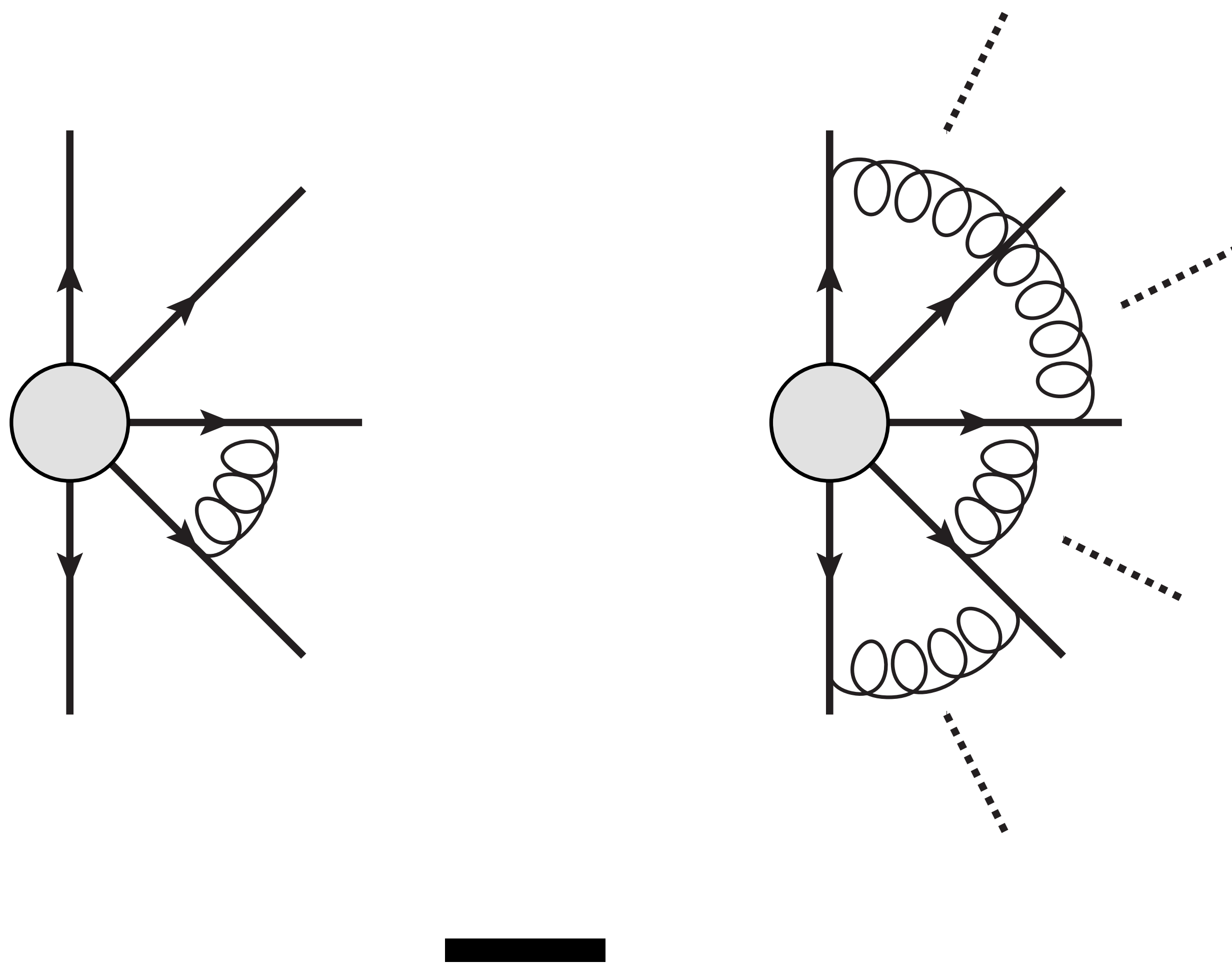
$$\langle \sigma|\tau\rangle = N^{m-\#\mathrm{transpositions}(\sigma,\tau)}$$

Define probability for alternative colour flow (reconnection probability)

$$\mathcal{P} = \frac{|\langle 21|\tau\rangle|^2}{|\langle 12|\tau\rangle|^2 + |\langle 21|\tau\rangle|^2}$$

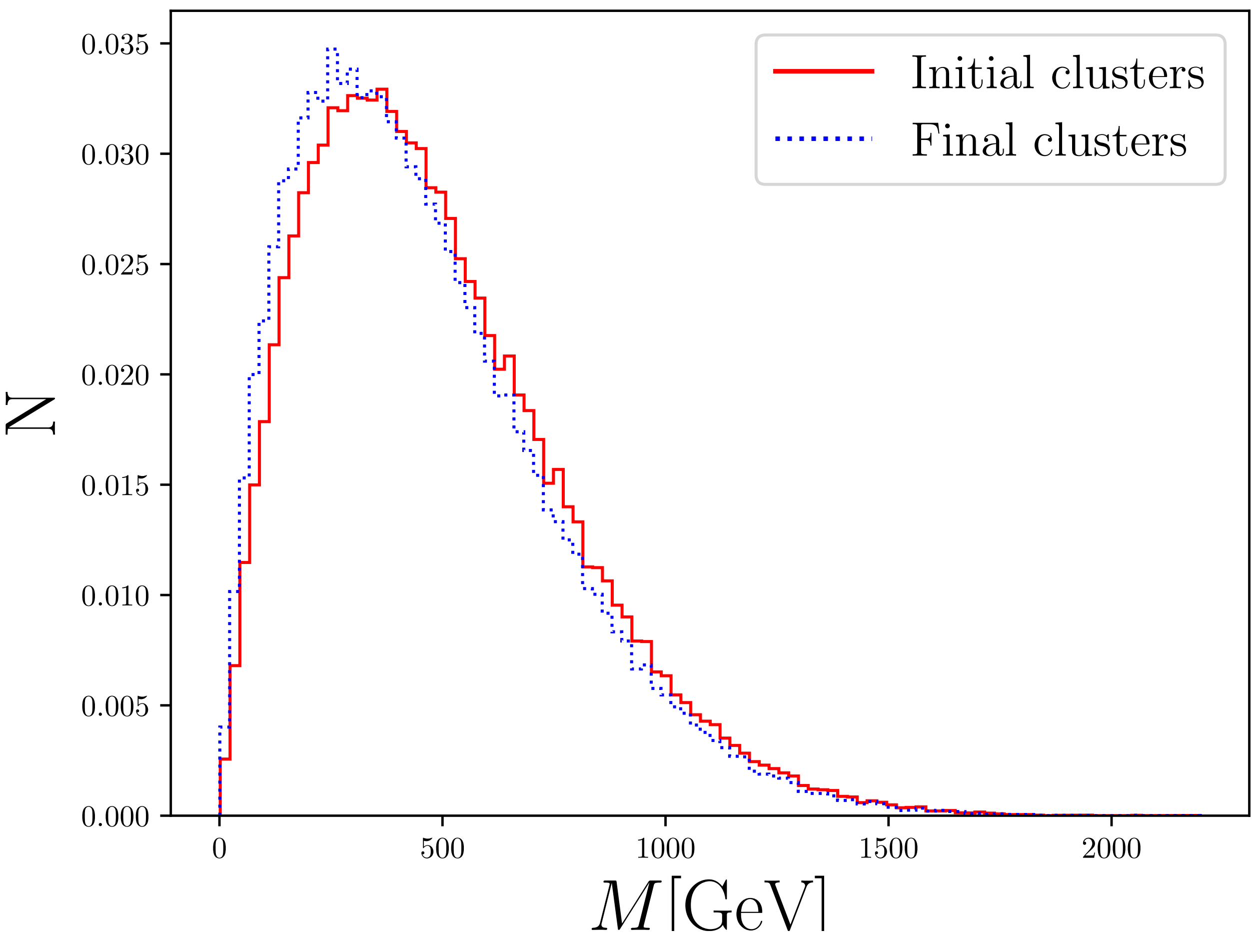
In short

- Study small systems (2-5 clusters = 4-10 coloured legs) (toy mc)
- Consider iterated soft gluon exchange between any two legs to all orders
- Evolve colour structure of legs to decide which quarks to connect
- Different input for hadronization (cluster model)

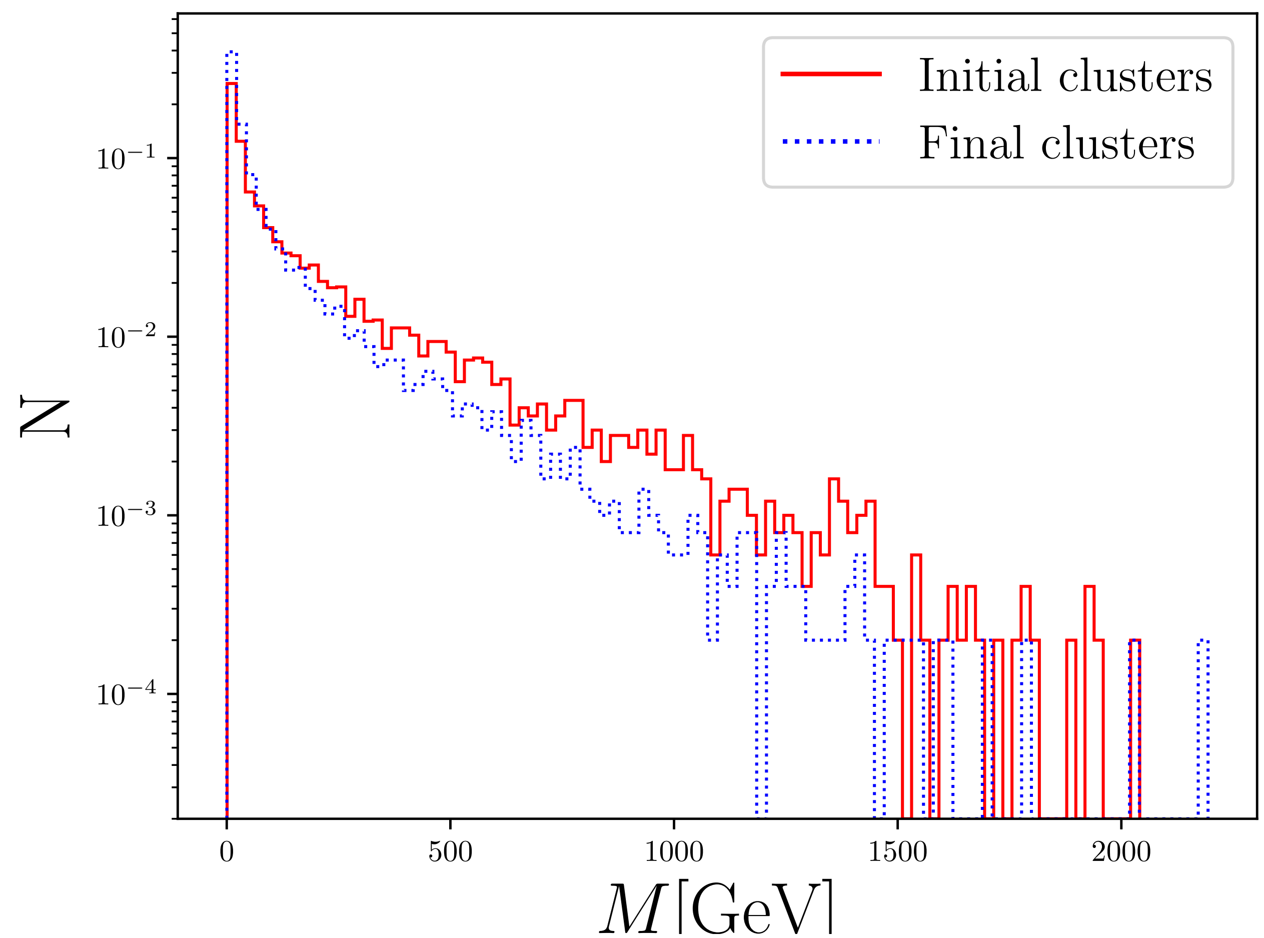


Numerical results

- **Toy Monte Carlo for up to 5 clusters**
- **2 phase space algorithms (RAMBO,UA5 type model) to create quark kinematics**

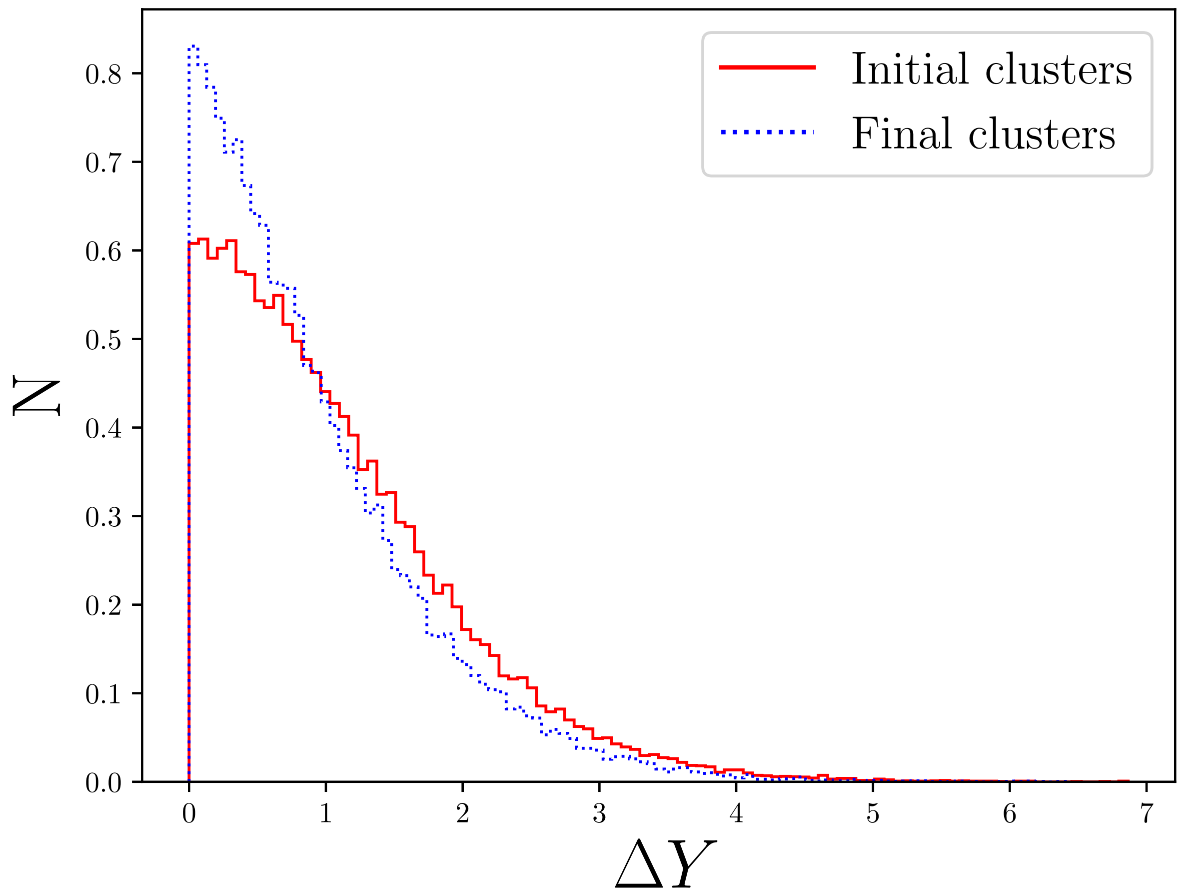


RAMBO high mass clusters (unphysical)

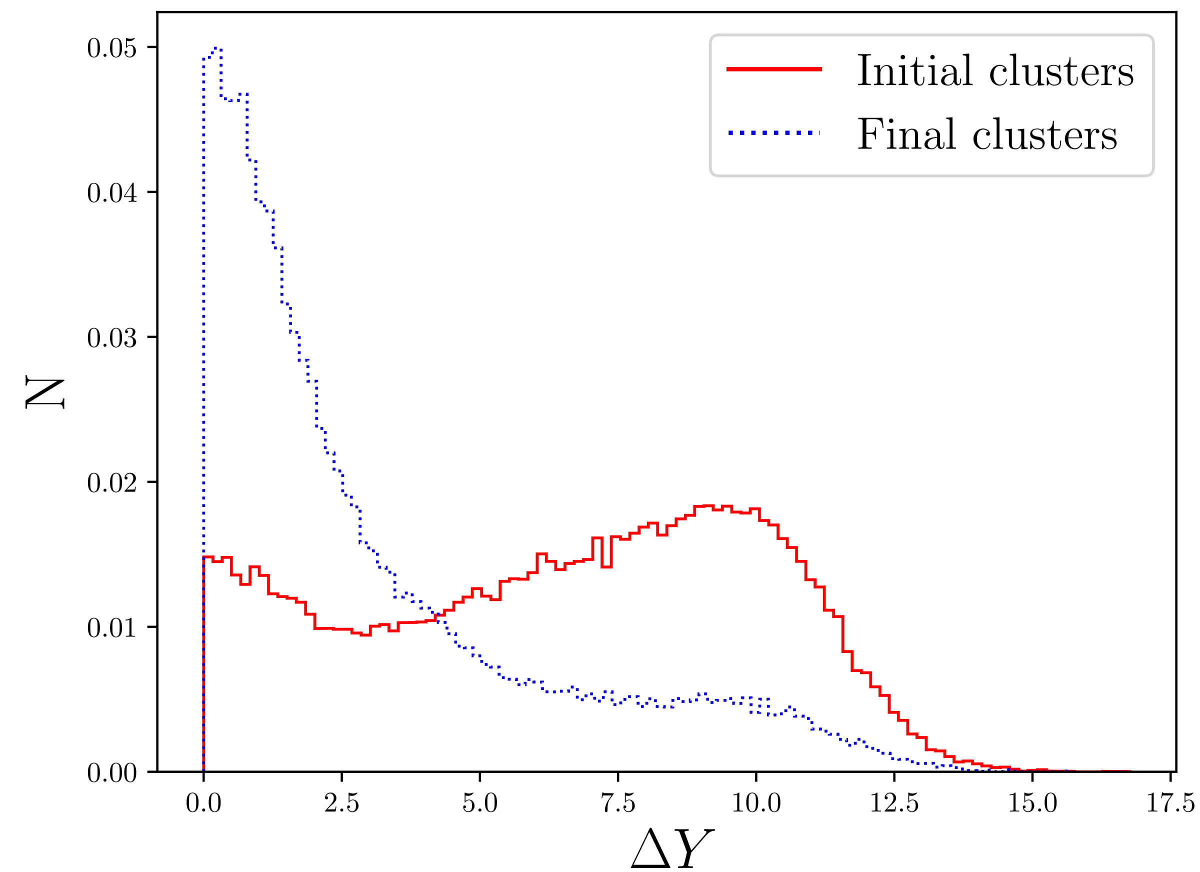


UA5 with random initial connections

- **Change in Delta Y between the constituent quarks**



RAMBO

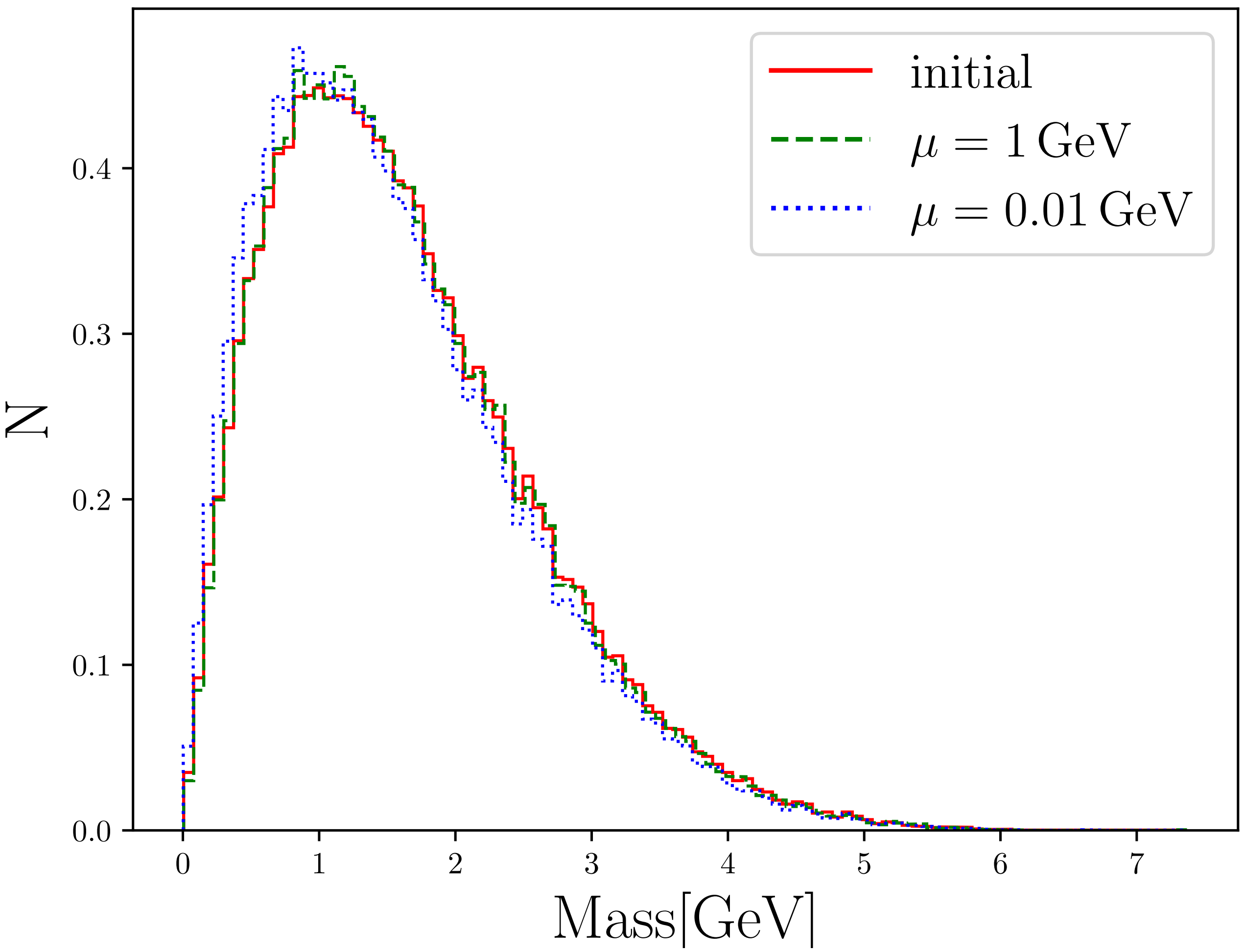


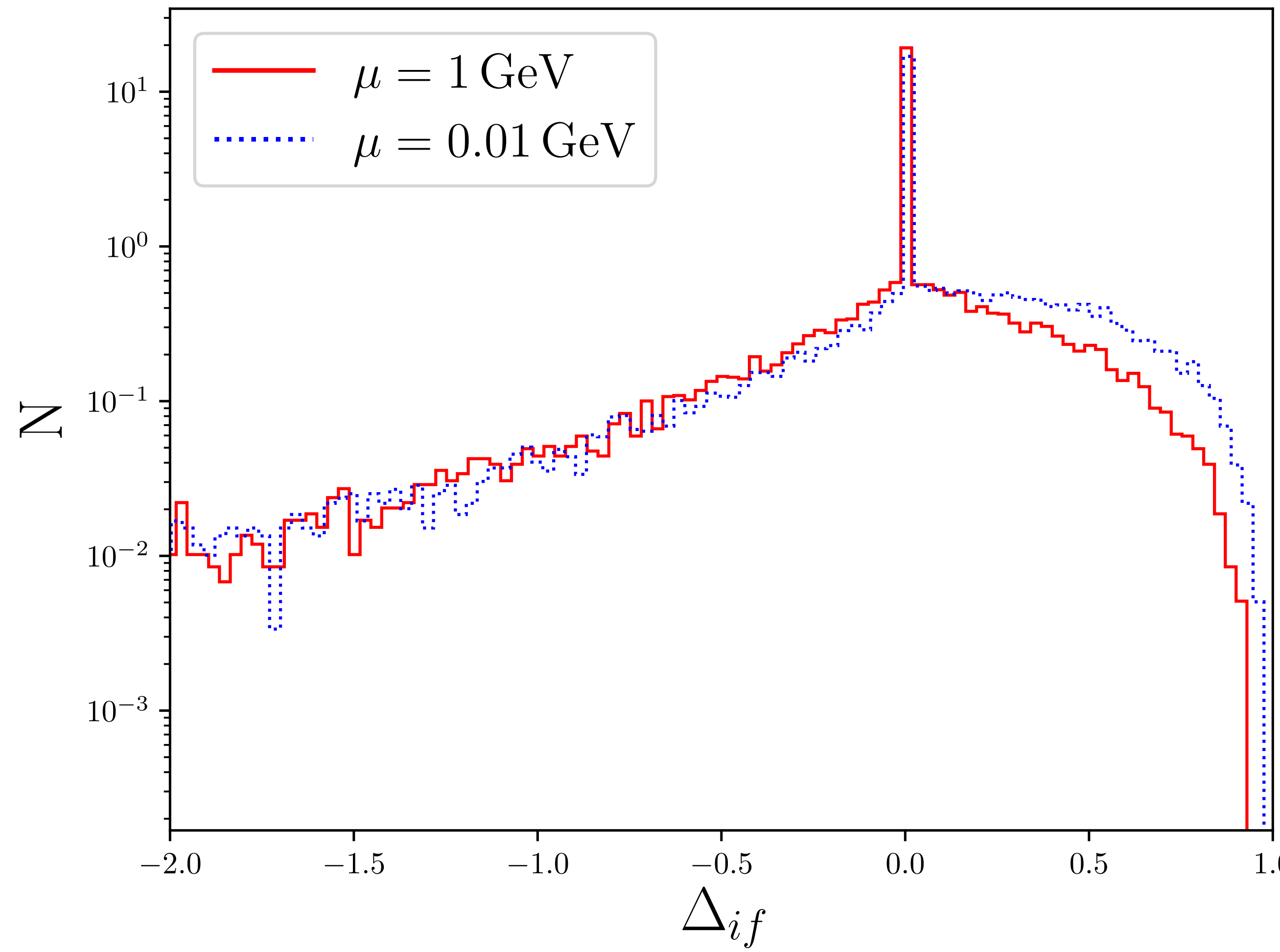
UA5 with random initial connections

Algorithm produces results attributed to properties of CR (e.g reduction of invariant cluster masses, connects quarks which are closer in spacetime)

Numerical results

- **Physical cluster masses O(GeV)**





$$\Omega_{\alpha\beta} = \frac{\alpha}{2\pi}\left(\frac{1}{2}\ln^2\frac{M_{\alpha\beta}^2}{\mu^2} - i\pi\ln\frac{M_{\alpha\beta}^2}{\mu^2}\right)$$

Colour length drop

$$\Delta_{if} = 1 - \frac{\sum M_f^2}{\sum M_i^2}$$

Bottleneck for full colour flow evolution of a LHC event

Explicit form of soft anomalous dimension for the two cluster evolution

$$\Gamma = \begin{bmatrix} \frac{-3}{2}(\Omega_{23} + \Omega_{14}) & \frac{1}{2}(\Omega_{12} - \Omega_{23} - \Omega_{14} + \Omega_{34}) \\ \frac{1}{2}(\Omega_{12} - \Omega_{13} - \Omega_{24} + \Omega_{34}) & \frac{-3}{2}(\Omega_{13} + \Omega_{24}) \end{bmatrix}$$

Needs to be exponentiated

Even for a general 2×2 real matrix, however, the matrix exponential can be quite complicated

$$\exp\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \frac{1}{\Delta}\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$$

where

$$m_{11} = e^{(a+d)/2}\left[\Delta \cosh\left(\tfrac{1}{2}\Delta\right) + (a-d)\sinh\left(\tfrac{1}{2}\Delta\right)\right]$$
$$m_{12} = 2\,b\,e^{(a+d)/2}\sinh\left(\tfrac{1}{2}\Delta\right)$$
$$m_{21} = 2\,c\,e^{(a+d)/2}\sinh\left(\tfrac{1}{2}\Delta\right)$$
$$m_{22} = e^{(a+d)/2}\left[\Delta \cosh\left(\tfrac{1}{2}\Delta\right) + (d-a)\sinh\left(\tfrac{1}{2}\Delta\right)\right],$$

and

$$\Delta \equiv \sqrt{(a-d)^2 + 4\,b\,c}\,.$$

For 5 clusters 120x120 matrix

n! colourflows

Rowland, Todd and Weisstein, Eric W. "Matrix Exponential." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/MatrixExponential.html

Baryonic configurations

- **Severe underestimation of produced baryons (strangeness also) at LHC (ALICE, ATLAS, CMS)**
- **Improved description with new production mechanisms possible**

[Pythia, (Christiansen, Skands) JHEP08 (2015) 003]
[Herwig, (Gieseke, PK, Plätzer) Eur.Phys.J. C78 (2018) 99]

- **Herwig: Baryonic clusters**

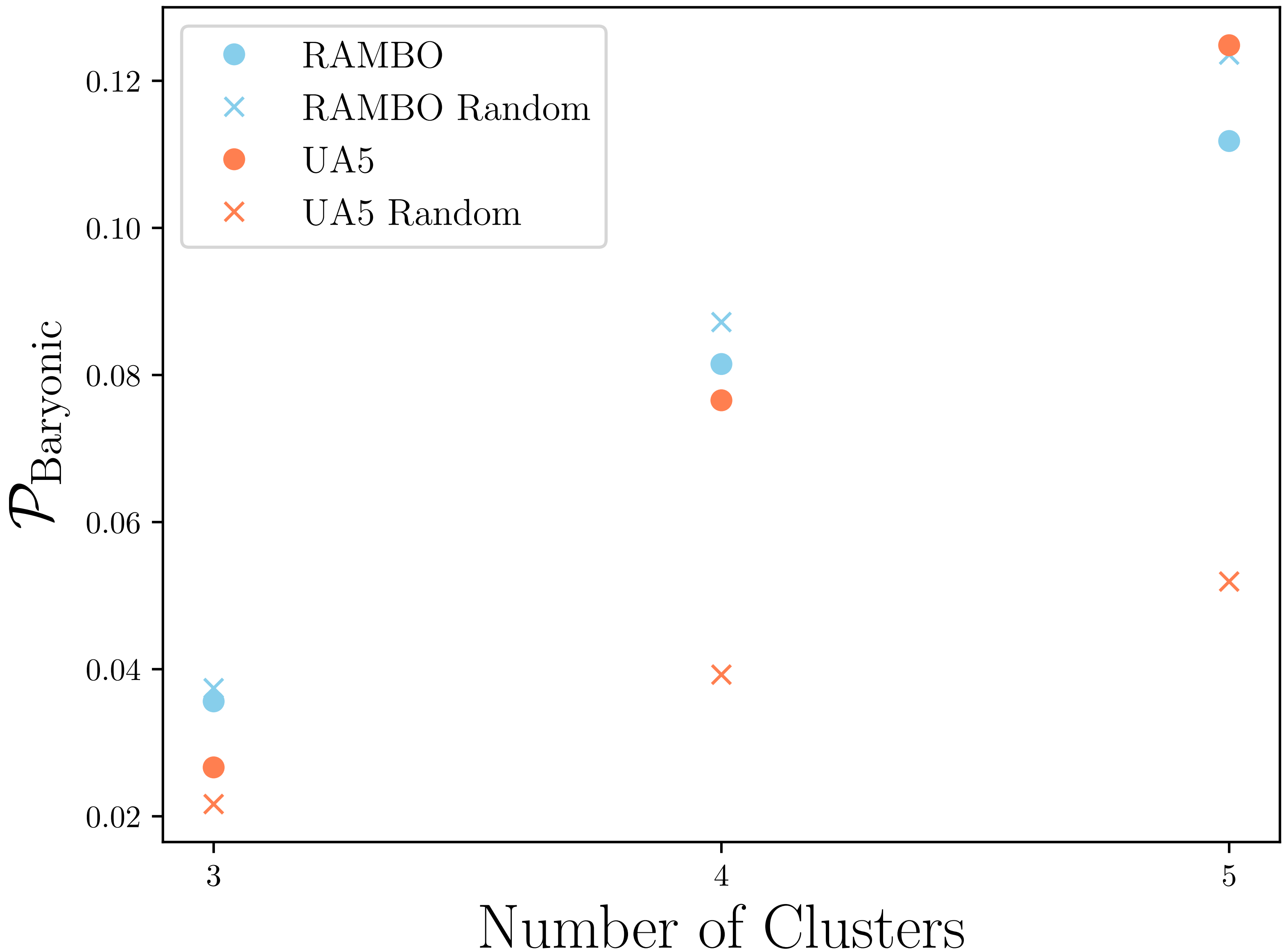
Can construct a baryonic state (3 quarks, 3 antiquarks)

$$|[ijk]\rangle = \frac{1}{K}\epsilon^{ijk}\epsilon_{\bar{i}\bar{j}\bar{k}} = \frac{1}{K}\left(|ijk\rangle - |ikj\rangle - |jik\rangle + |jki\rangle + |kij\rangle - |kji\rangle\right)$$

Calculate baryonic reconnection probability as before where the amplitude is

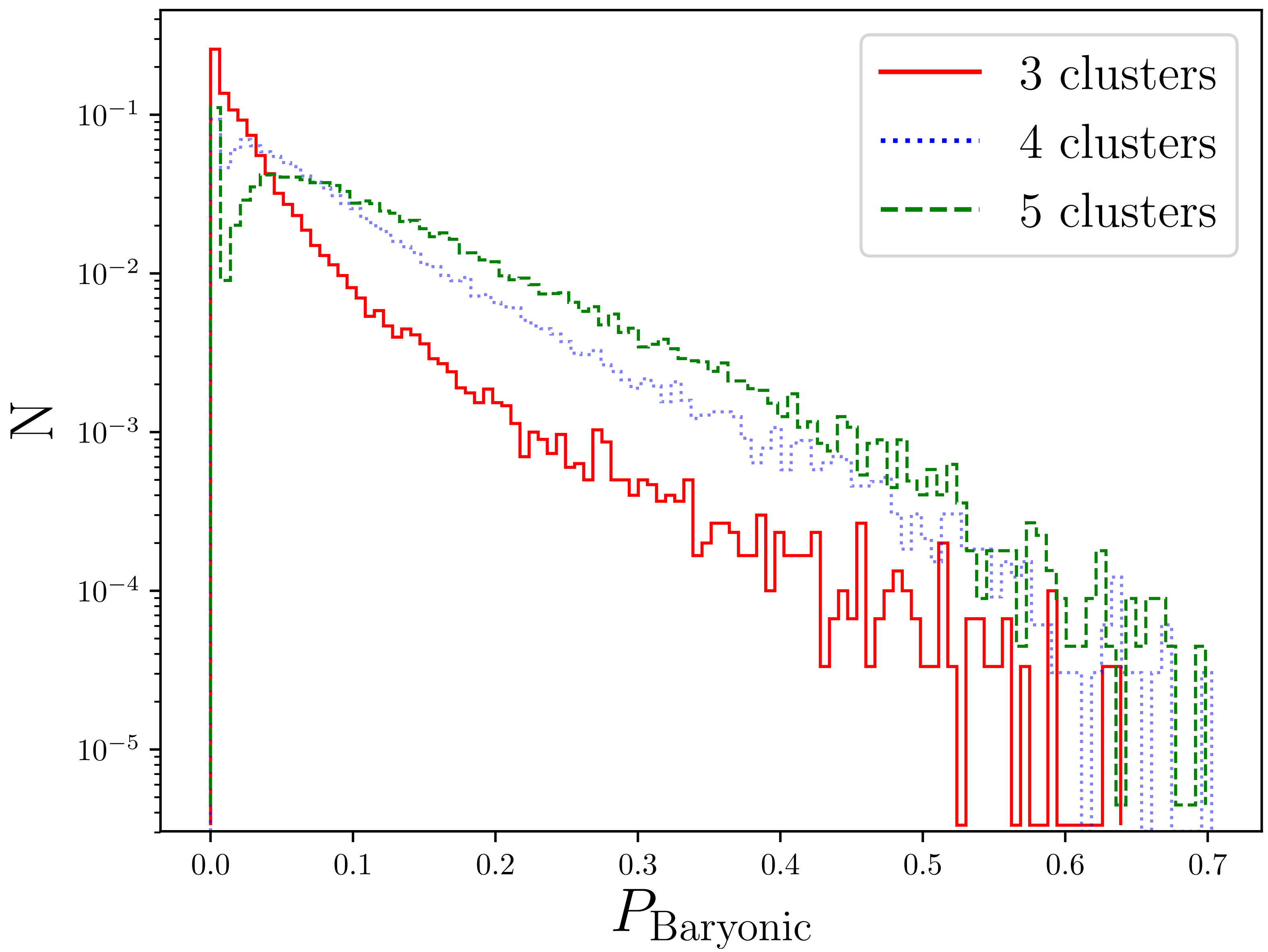
$$\mathcal{A}_{\tau\rightarrow B_{ijk}\otimes\tilde{\sigma}_{ijk}} = \langle B_{ijk}| \otimes \langle\tilde{\sigma}_{ijk}|\mathbf{U}\left(\{p\},\mu^2,\{M_{ij}^2\}\right)|\tau\rangle$$

Probabilities



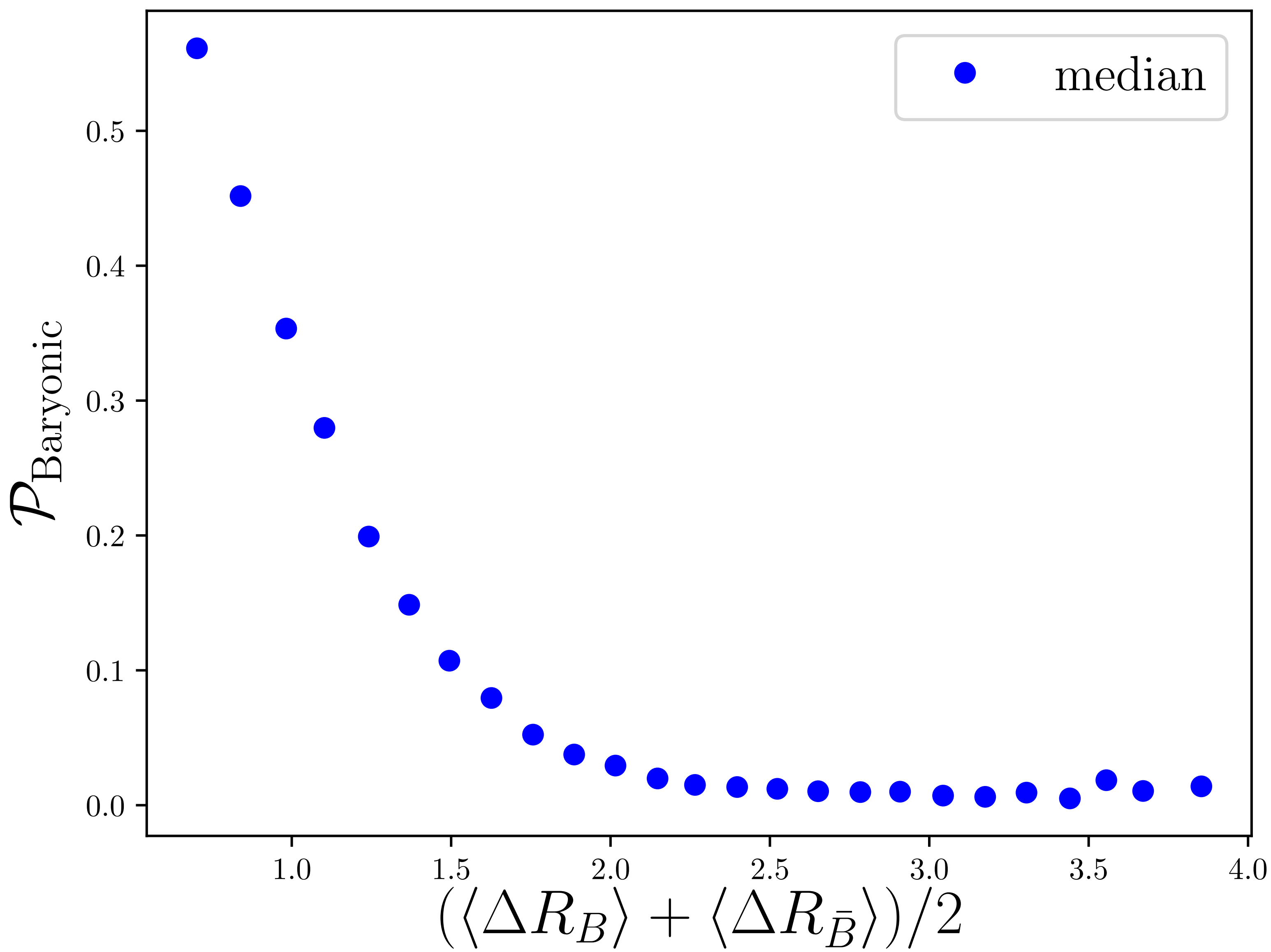
Reasonable values with clear dependence on number of clusters

Baryonic reconnection probabilities in detail



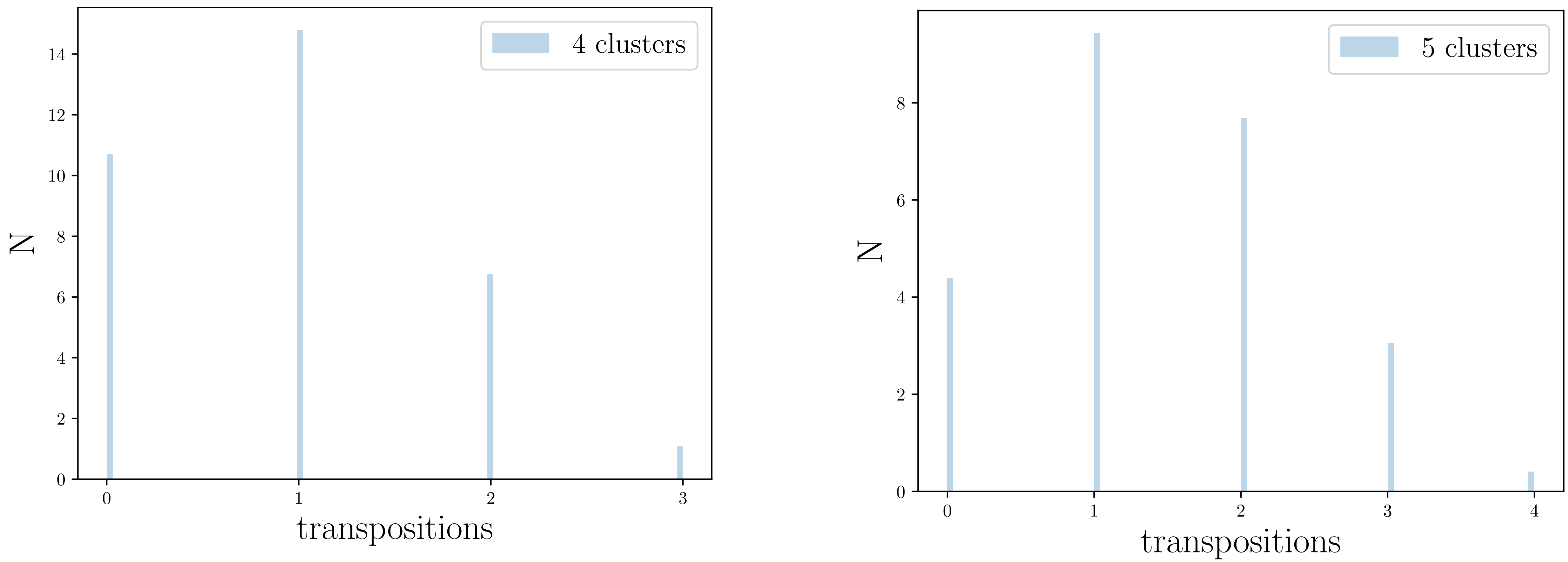
Tail towards higher values -> preferred kinematic configuration?

Baryonic reconnection probability for 3 cluster evolution



Clear kinematic dependence

- **No 1/N dependence -> independent subsystems**



Effects of perturbative colour flow evolution may be factorizable
-> allows to implement in MCEG

- **Toy Monte Carlo for full Colour Flow Evolution for up to 5 clusters**
- **Evolution into baryonic states possible**
- **Strong support for geometric/kinematic models**
- **Future: Study evolution of independent subsystems in Herwig and see where this approach is applicable**

Backup

Backup

Backup

Ba

up

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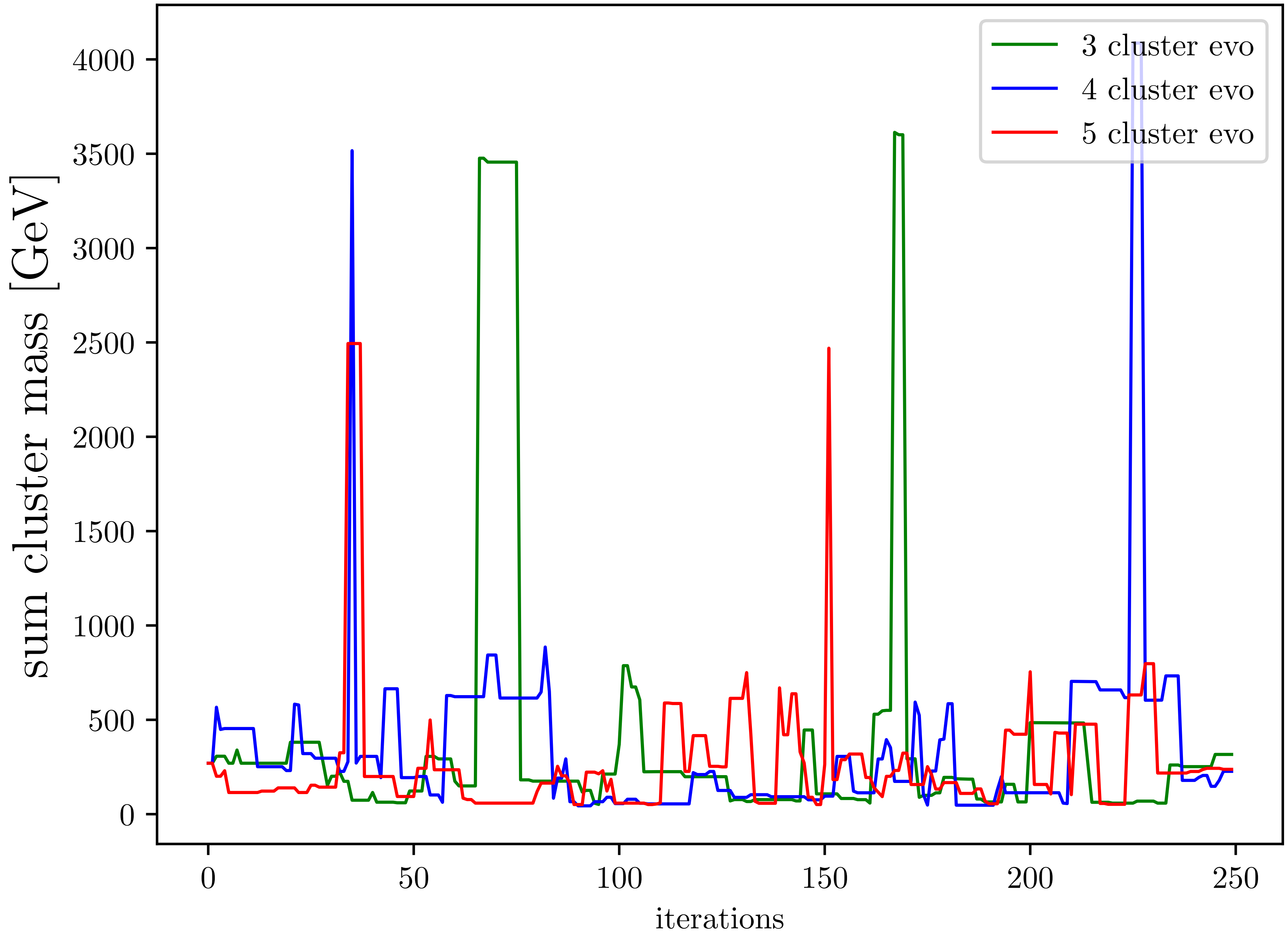
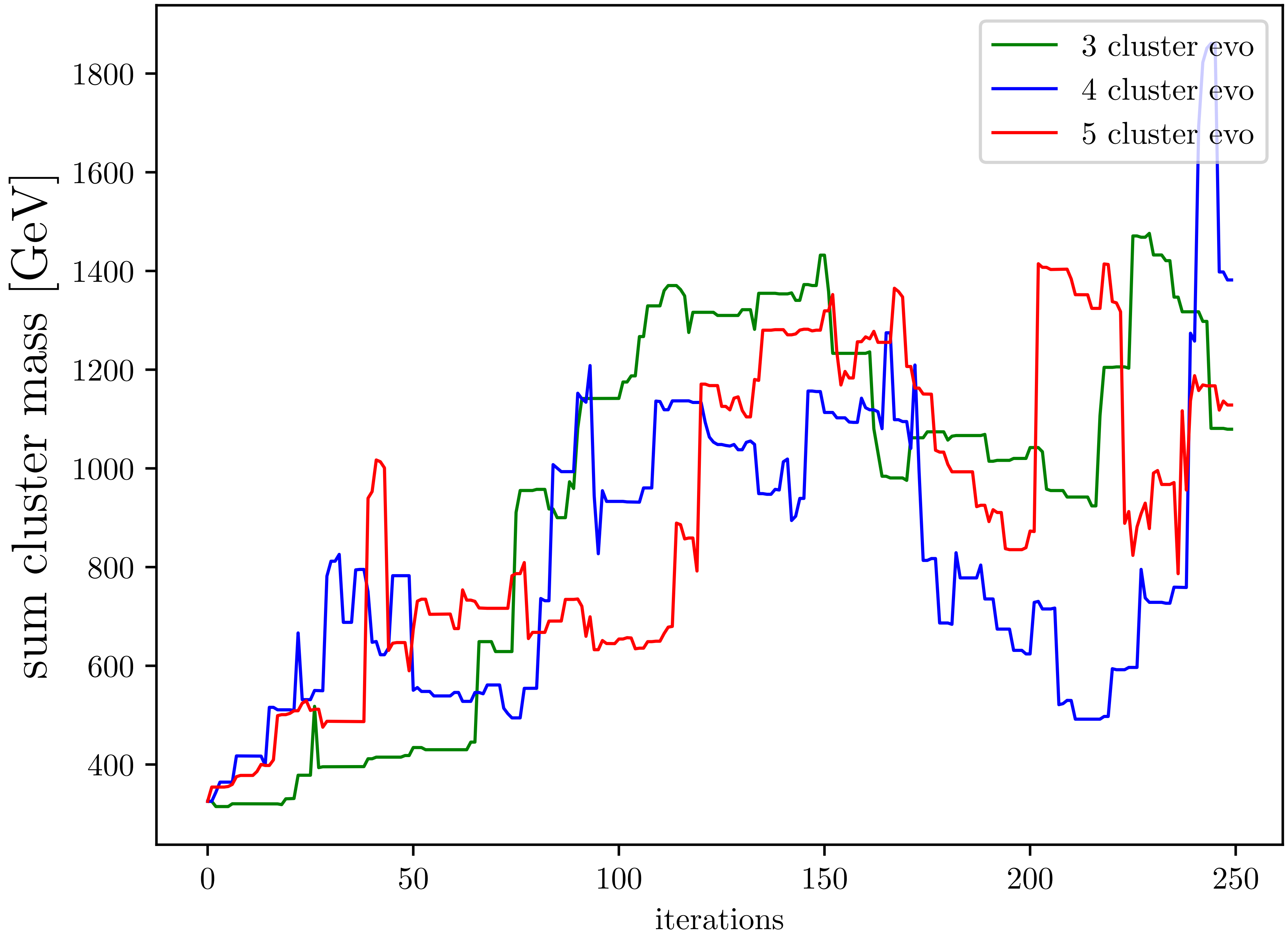
Semi-hard MPI event only, random evolution of 10 clusters, B(10,5) = 252



No veto of colour flows

Extreme LHC event, iterated random evolution of 82 (very light) clusters, B(82,5) = 27285336



No veto of colour flows

Perturbative colour flow evolution

Colour charge operators T_i : [Catani, Seymour, Nucl. Phys. B485 (1997) 291-419]

Colour charge products given by

$$T_i \cdot T_j = \frac{1}{2}(\delta_j^{i'}\delta_i^{j'} - \frac{1}{N}\delta_i^{i'}\delta_j^{j'})$$

In the large N limit corresponds to exchange of colour between legs i,j

In color flow basis: Matrices in colour space which change the colour structure

$$\mathbf{T}_i \cdot \mathbf{T}_{\bar{i}} = \mathbf{T}_j \cdot \mathbf{T}_{\bar{j}} \equiv -\begin{pmatrix} N & 1 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{T}_i \cdot \mathbf{T}_{\bar{j}} = \mathbf{T}_j \cdot \mathbf{T}_{\bar{i}} \equiv -\begin{pmatrix} 0 & 0 \\ 1 & N \end{pmatrix} ,$$

$$\mathbf{T}_i \cdot \mathbf{T}_j = \mathbf{T}_{\bar{i}} \cdot \mathbf{T}_{\bar{j}} \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$

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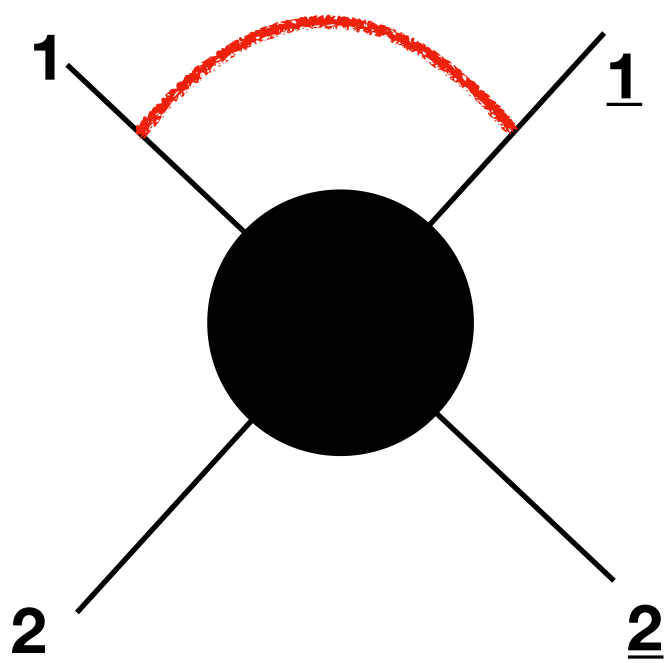
Colour Evolution

Consider evolution of an amplitude in colour space

$$\mathcal{M} = U \mathcal{M}_0$$

$$U = \exp\left(\int_{\mu^2}^{M_{\alpha\beta}^2} \frac{dq^2}{q^2} \Gamma(\{p\}, q^2) \right)$$

Swaps/crosses colour lines between legs



Colour structure of an event determined by the 1-loop soft anomalous dimension

$$\Gamma = \sum_{\{i,j\}} \frac{T_i \cdot T_j}{2} \frac{\alpha_s}{2\pi} \left(\ln \frac{M_{\alpha\beta}^2}{q^2} - i\pi \right)$$

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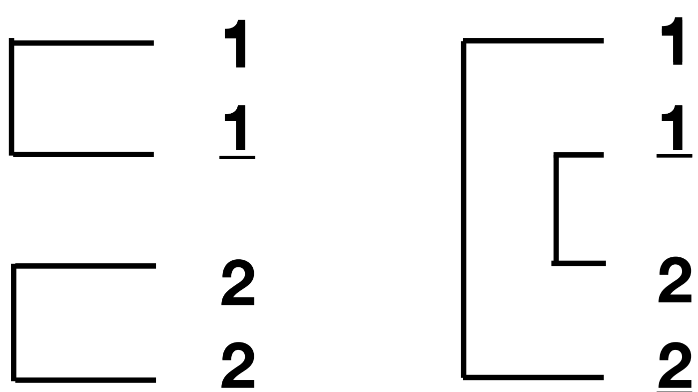
Colour Evolution for a 2 cluster system

Colour state can be represented as product of kronecker deltas

$$|\sigma\rangle = \delta^{i}_{\bar{i}} ... \delta^{j}_{\bar{j}}$$

Possible Colour flows of a 2 cluster system ⟶ **Radiating dipole**



$$M^2_{i\bar{i}} = (p_i + p_{\bar{i}})^2$$

Calculate and exponentiate soft anomalous dimension matrix (2x2) for 2 clusters

kinematics -> matrix elements of U

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Perturbative colour flow evolution

$$-\mathbf{T}_i\cdot\mathbf{T}_{\bar{i}} \equiv \begin{pmatrix} N & 0 & 1 & 0 & 0 & 1 \\ 0 & N & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -\mathbf{T}_i\cdot\mathbf{T}_{\bar{j}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & N & 0 & 1 & 0 \\ 0 & 1 & 0 & N & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -\mathbf{T}_i\cdot\mathbf{T}_{\bar{k}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & N & 0 \\ 1 & 0 & 0 & 1 & 0 & N \end{pmatrix}$$

$$-\mathbf{T}_j\cdot\mathbf{T}_{\bar{i}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & N & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & N & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -\mathbf{T}_j\cdot\mathbf{T}_{\bar{j}} \equiv \begin{pmatrix} N & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & N \end{pmatrix} \quad -\mathbf{T}_j\cdot\mathbf{T}_{\bar{k}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & N & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & N & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-\mathbf{T}_k\cdot\mathbf{T}_{\bar{i}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & N & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & N \end{pmatrix} \quad -\mathbf{T}_k\cdot\mathbf{T}_{\bar{j}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & N & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & N & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad -\mathbf{T}_k\cdot\mathbf{T}_{\bar{k}} \equiv \begin{pmatrix} N & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$+\mathbf{T}_i\cdot\mathbf{T}_j \equiv \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad +\mathbf{T}_i\cdot\mathbf{T}_k \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad +\mathbf{T}_j\cdot\mathbf{T}_k \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$+\mathbf{T}_{\bar{i}}\cdot\mathbf{T}_{\bar{j}} \equiv \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad +\mathbf{T}_{\bar{i}}\cdot\mathbf{T}_{\bar{k}} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad +\mathbf{T}_{\bar{j}}\cdot\mathbf{T}_{\bar{k}} \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$