UNIVERSITY OF BERGEN

GENERATING FUNCTIONAL FOR QUENCHED OBSERVABLES

Konrad Tywoniuk

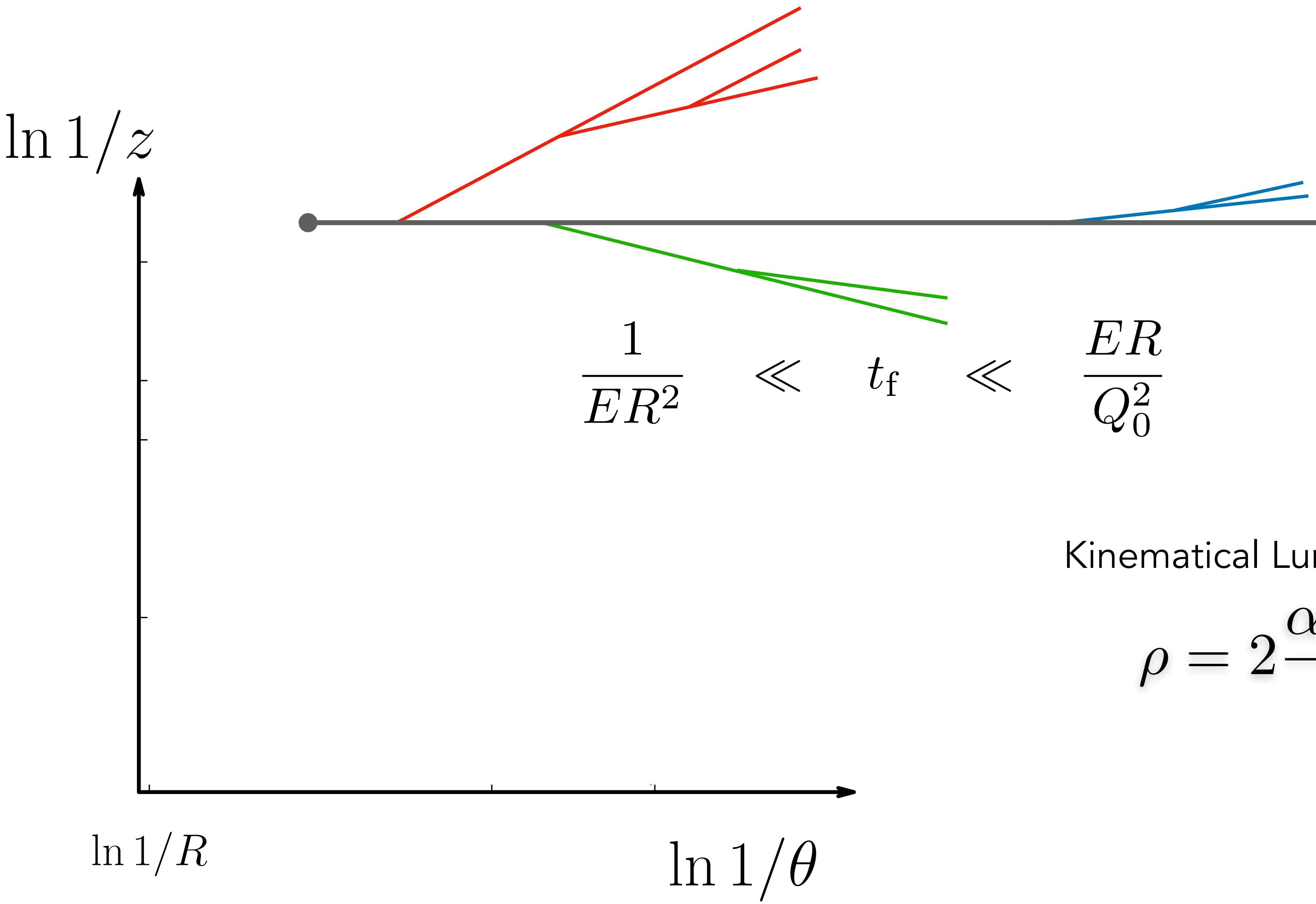
COST Workshop on Interplay of hard and soft QCD probes for collectivity in heavy-ion collisions
25 Feb - 1 Mar 2019, Lund

INTRODUCTION

- jet quenching: suppression & modification of hard probes wrt vacuum (pp) baseline
- unique observables to extract properties of the medium
- motivation: theoretical guidance at high-p_T
  - **what drives quenching & substructure modifications?**

MAPPING THE SPLITTINGS

Andersson, Gustafson, Lönnblad, Pettersson Z.Phys.C (1989)
Dokshitzer, Khoze, Mueller, Troyan "Basics of Perturbative QCD" (1991)



$$\frac{1}{ER^2} \ll t_{\rm f} \ll \frac{ER}{Q_0^2}$$

Kinematical Lund diagram

$$\rho = 2\frac{\alpha_s C_R}{\pi}$$

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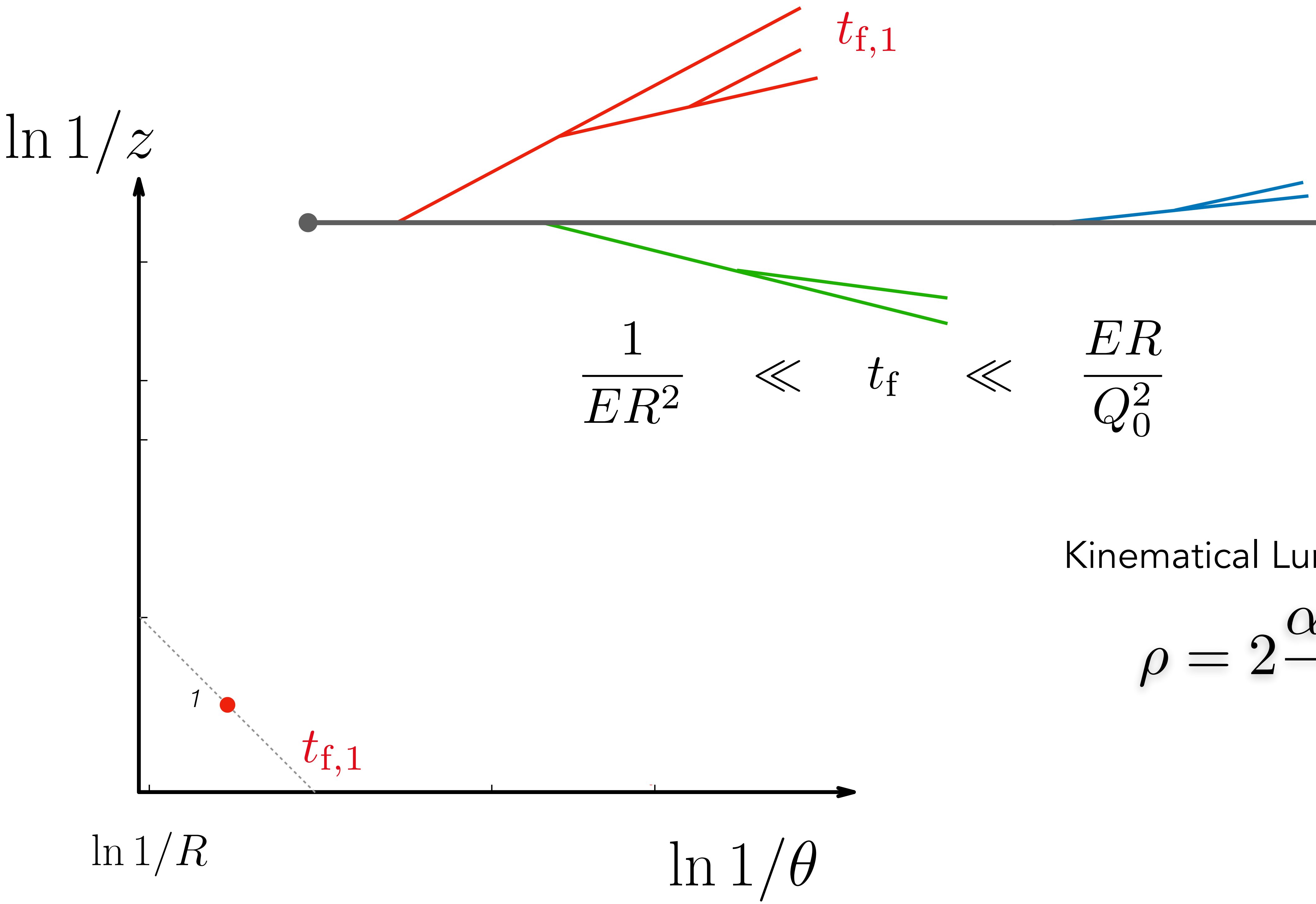

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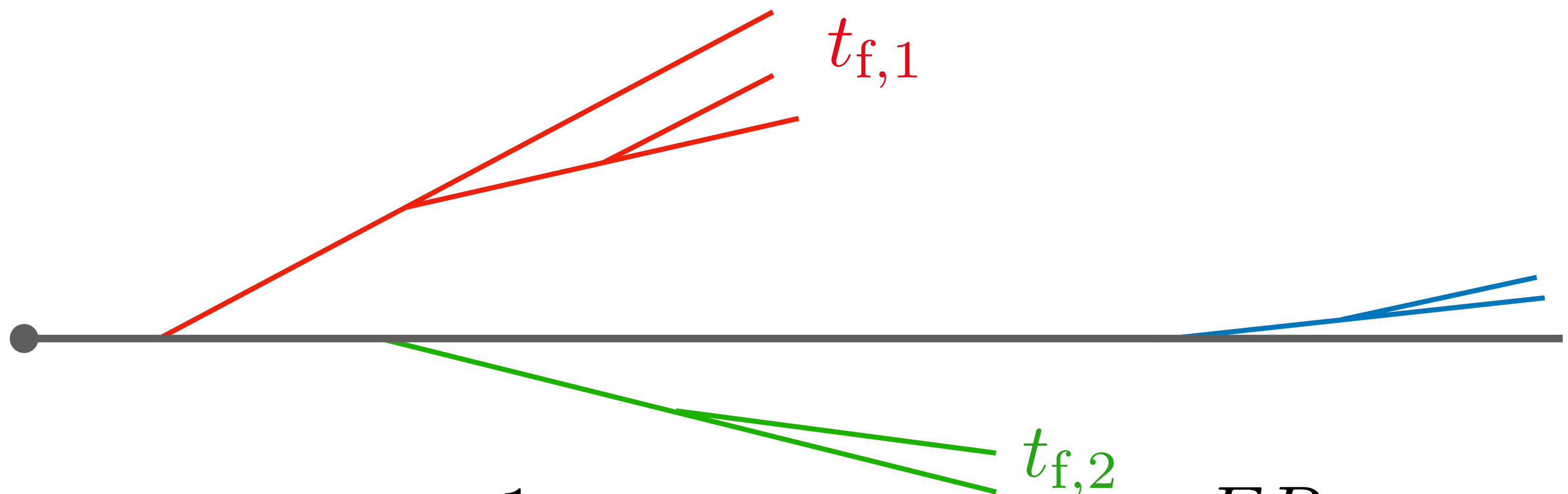

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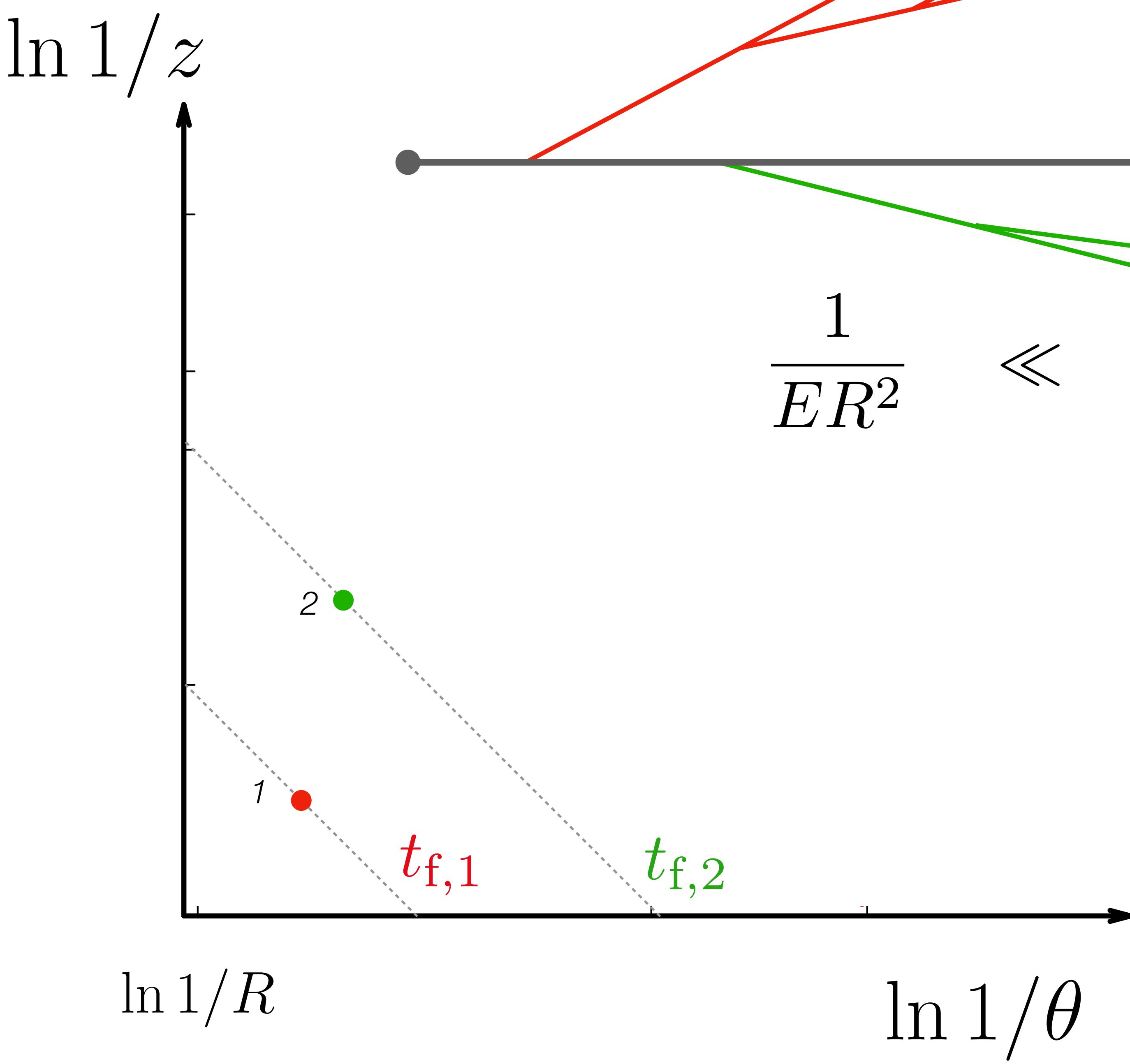

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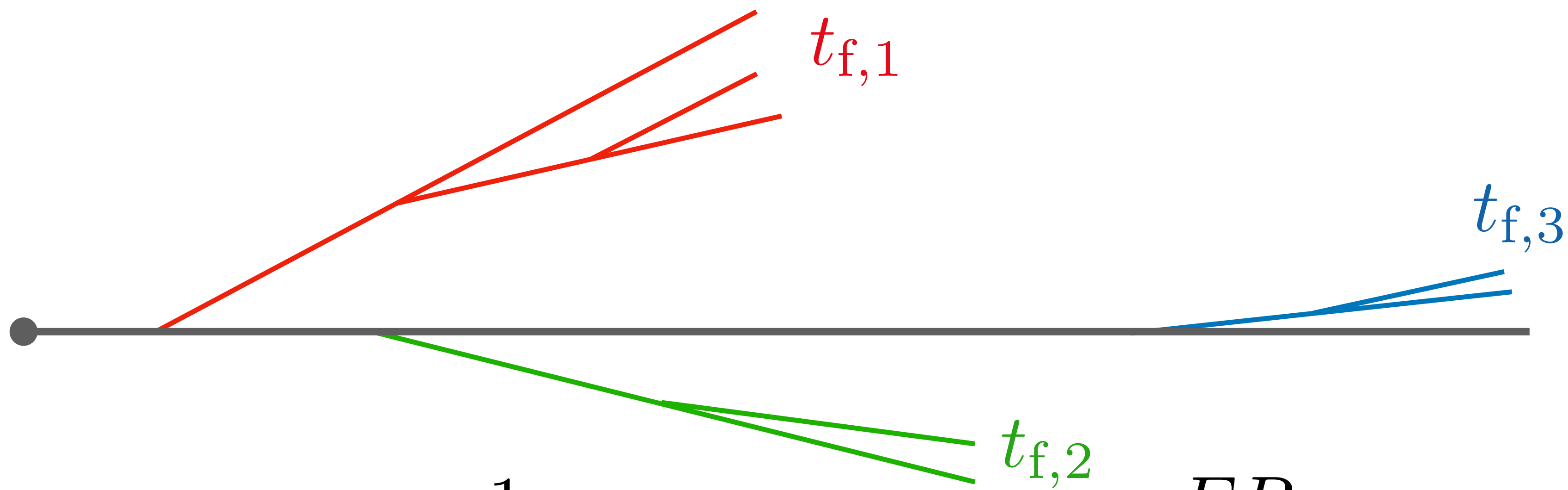

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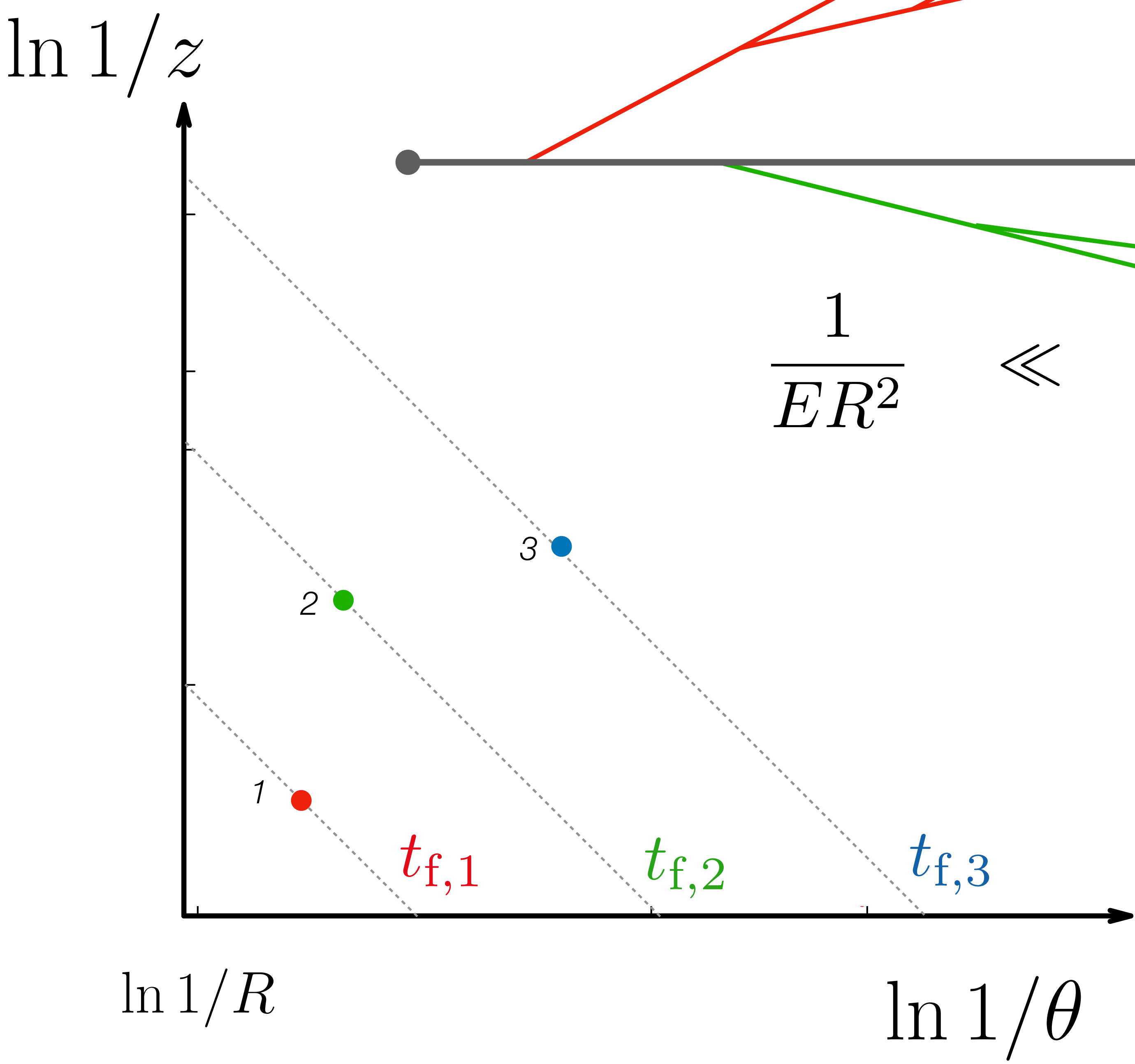

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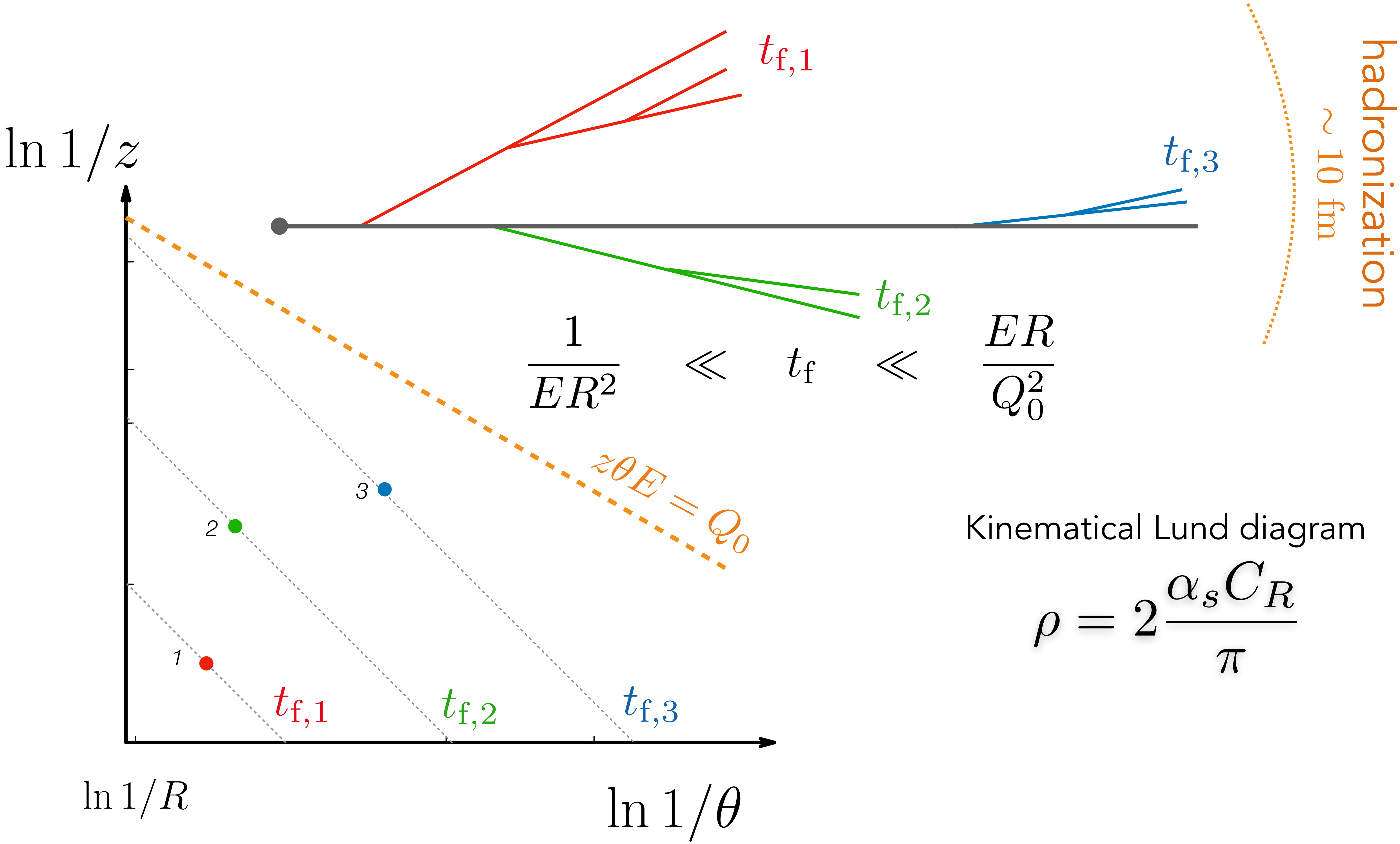

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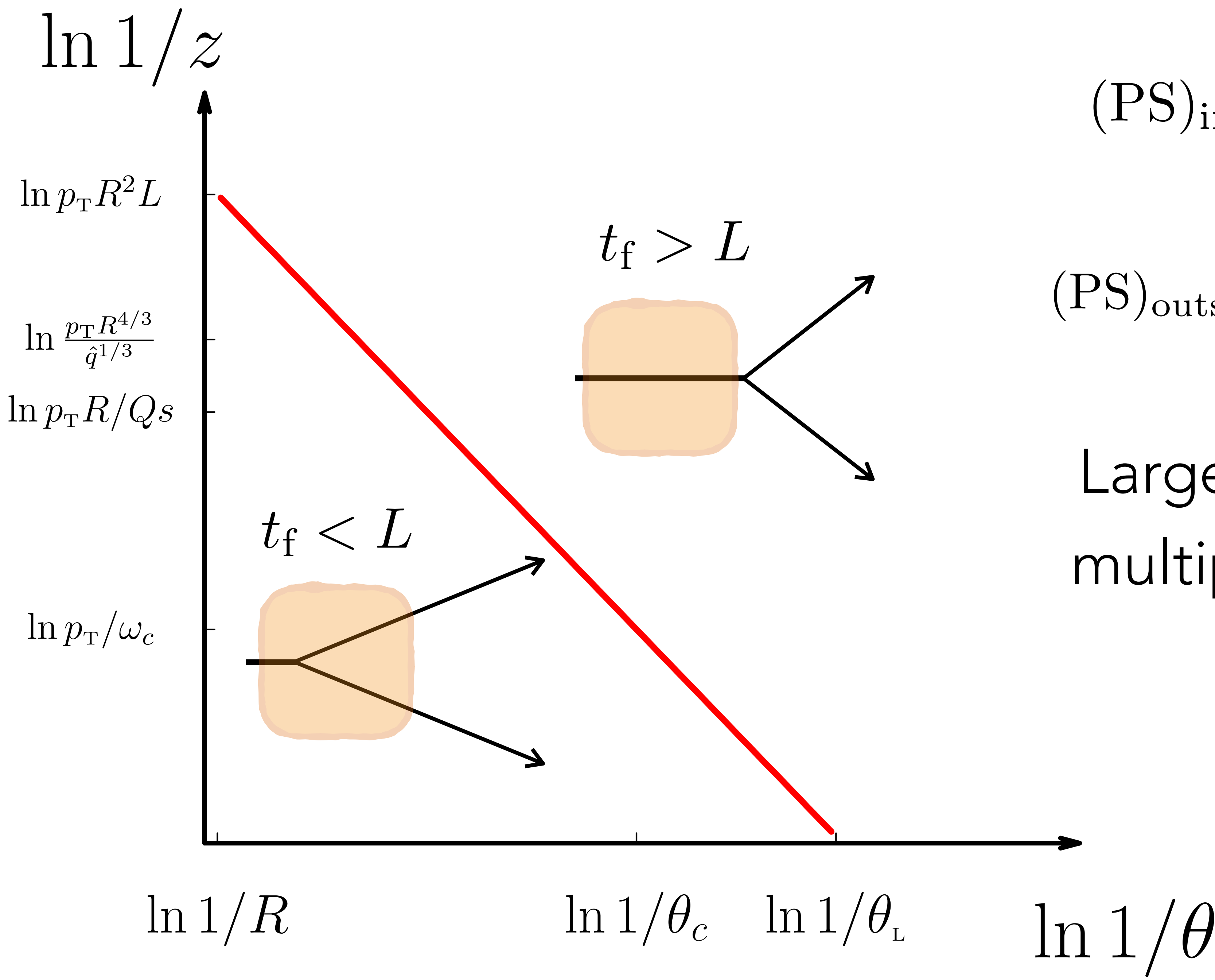
Medium length as perturbative scale

Medium interactions will impose new scales one the plane.



$$(\mathrm{PS})_{\mathrm{inside}} = \frac{\bar{\alpha}}{4} \ln^2 \left(p_{\mathrm{T}} R^2 L \right)$$

$$(\mathrm{PS})_{\mathrm{outside}} = \frac{\bar{\alpha}}{2} \ln^2 \left(\frac{p_{\mathrm{T}} R}{Q_0} \right) - (\mathrm{PS})_{\mathrm{inside}}$$

Large $\sim\mathcal{O}(1)$ probability of multiple splitting inside the medium.

Andrews et al. arXiv:1808.03689
Dreyer, Salam, Soyez 1807.04758
Chien, Elayavalli arXiv:1803.03589
Cunqueiro, Płoskoń arXiv:1812.00102

Computing quenched observables

quenching weight: probability distribution of losing energy

$$\frac{d\sigma_{\rm med}}{dp_T^2 dy} = \int_0^\infty d\epsilon\, \mathcal{P}(\epsilon) \frac{d\sigma_{\rm vac}(p_T+\epsilon)}{dp_T^2 dy}$$

quenching factor = nuclear modification factor

$$R_{\rm jet} = \left(\frac{d\sigma_{\rm med}}{dp_T^2 dy}\right) \Big/ \left(\frac{d\sigma_{\rm vac}}{dp_T^2 dy}\right)$$

For $\epsilon/p_T \ll 1$ and large n:

$$\frac{1}{(p_T+\epsilon)^n} \approx \frac{1}{p_T^n} e^{-\epsilon n/p_T}$$

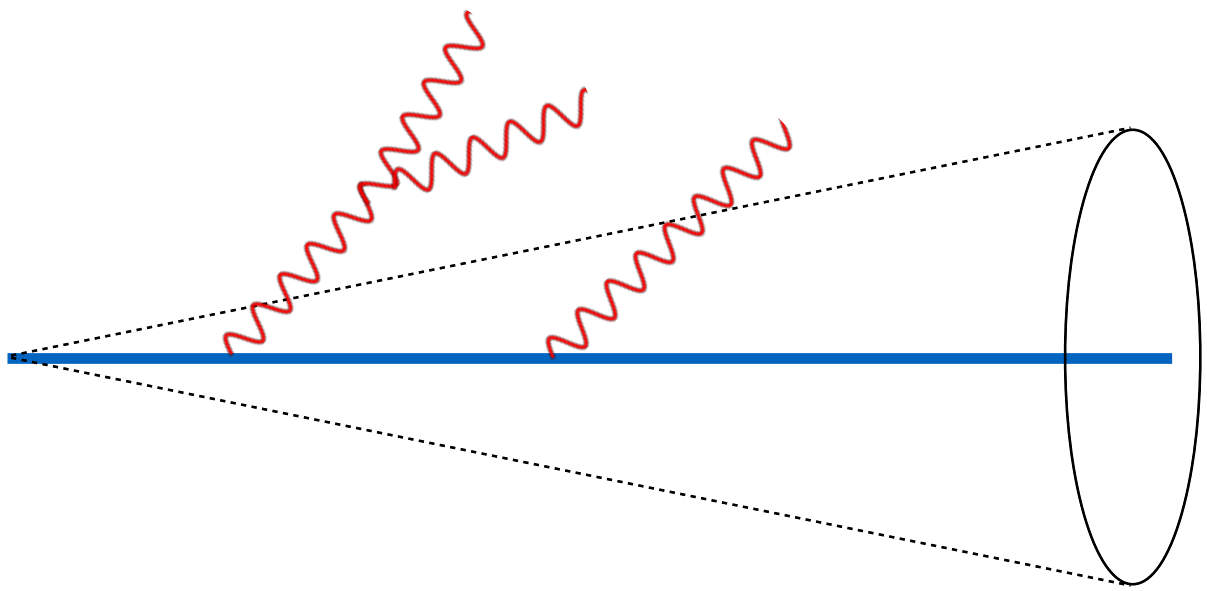
$$\tilde{\mathcal{P}}(\nu) = \int_0^\infty d\epsilon\, e^{-\nu\epsilon} \mathcal{P}(\epsilon)$$

$$R_{\rm jet} \sim \tilde{\mathcal{P}}\left(n/p_T\right) \equiv \mathcal{Q}(p_T)$$

quenching factor is Laplace transform of energy loss probability

QUENCHING OF SINGLE PARTON

Baier, Dokshitzer, Mueller, Schiff JHEP 0109 (2001) 033



Wavy lines: medium induced gluons generated via the rate in *time*

$$\frac{\mathrm{d}N}{\mathrm{d}\omega\,\mathrm{d}t} = \bar{\alpha}\sqrt{\frac{\hat{q}}{\omega^3}}$$

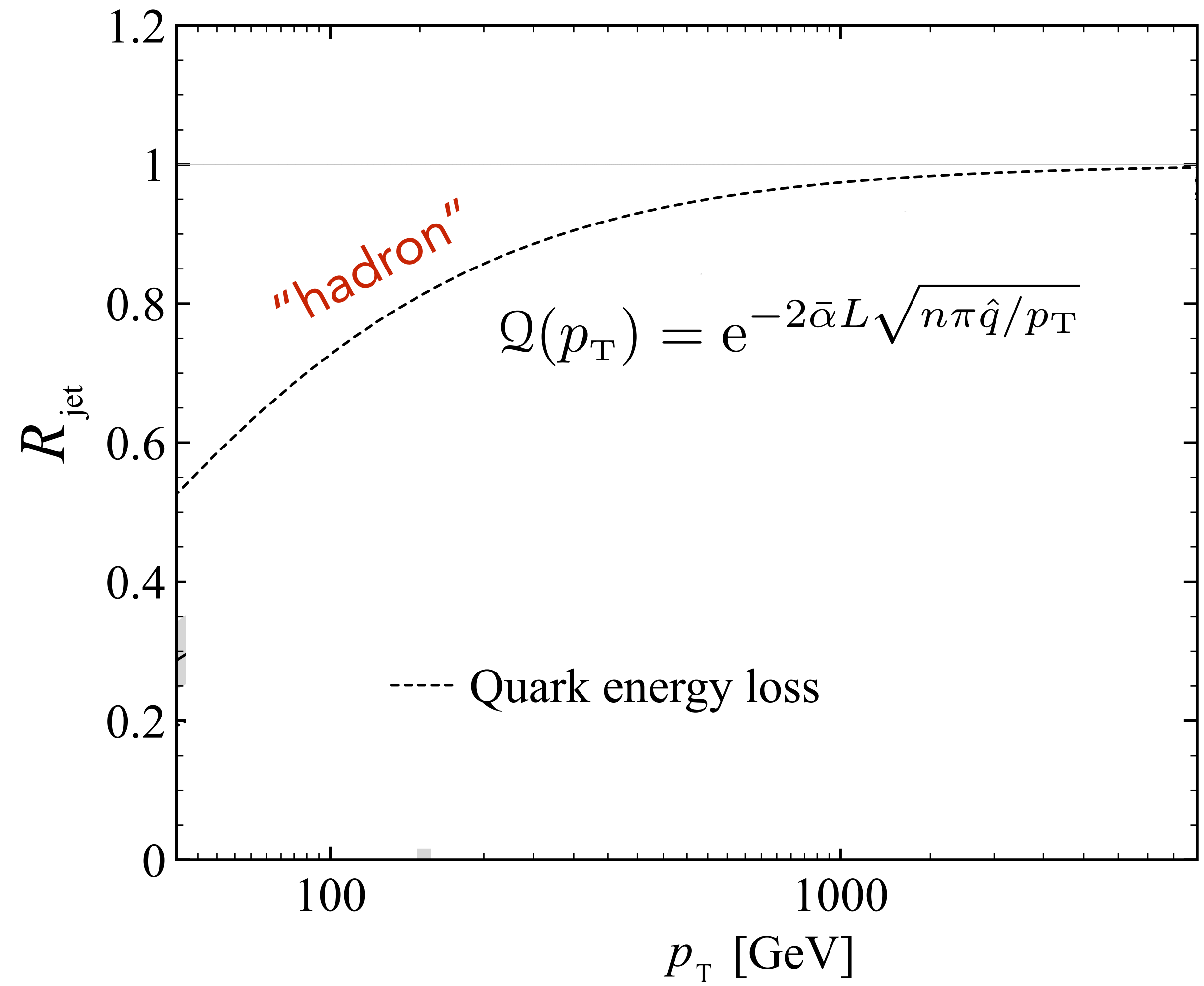
- form factor suppression related to multiplicity of virtual gluons

$$\mathcal{Q}(p_\mathrm{T}) \sim \mathrm{e}^{-N(\omega > p_\mathrm{T}/n)}$$

- strong quenching for

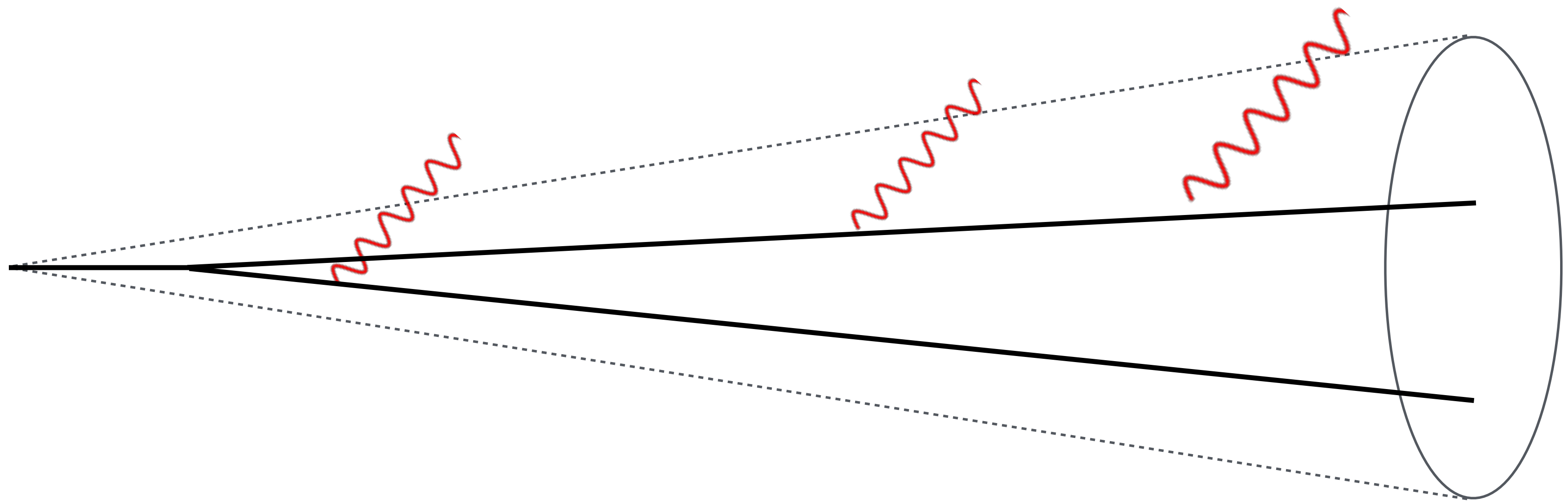
$$p_\mathrm{T} \ll n\bar{\alpha}^2\hat{q}L^2$$

- quark ≈ meson (rescaling)



NEIGHBORING JET ENERGY LOSS

Y. Mehtar-Tani, KT arXiv:1706.06047 [hep-ph]



$$\mathcal{P}_2(\nu) = \mathcal{P}(\nu) \times \mathcal{P}_{\rm sing}(\nu)$$

total color charge

contributions from interferences

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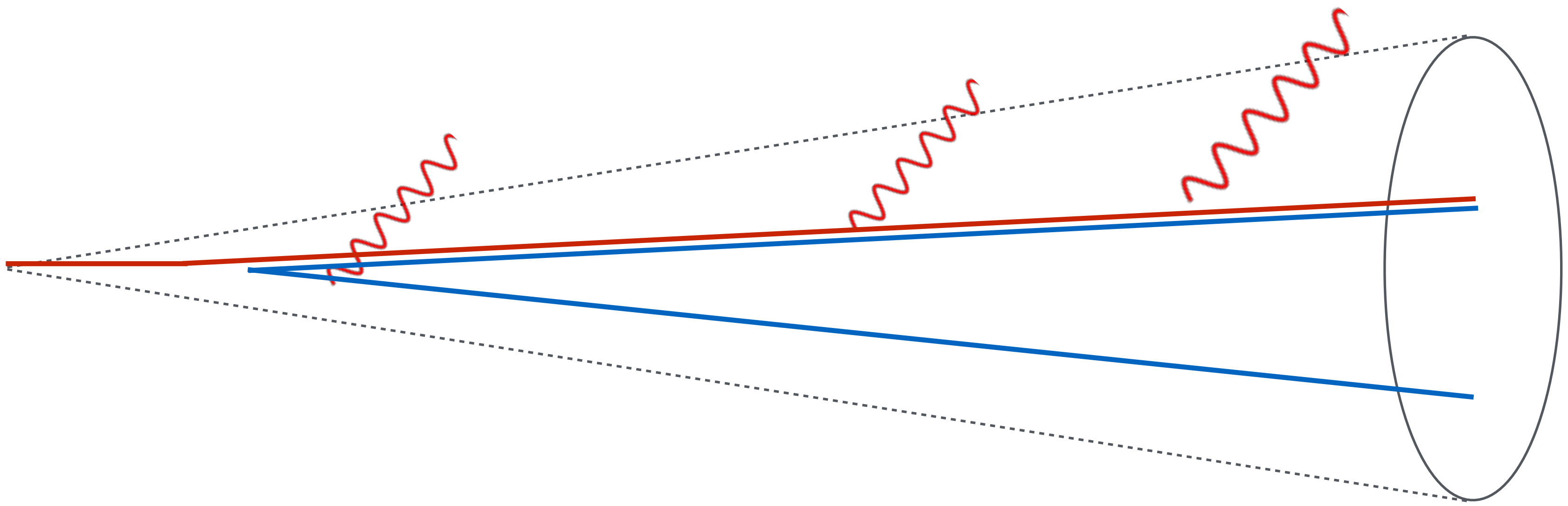

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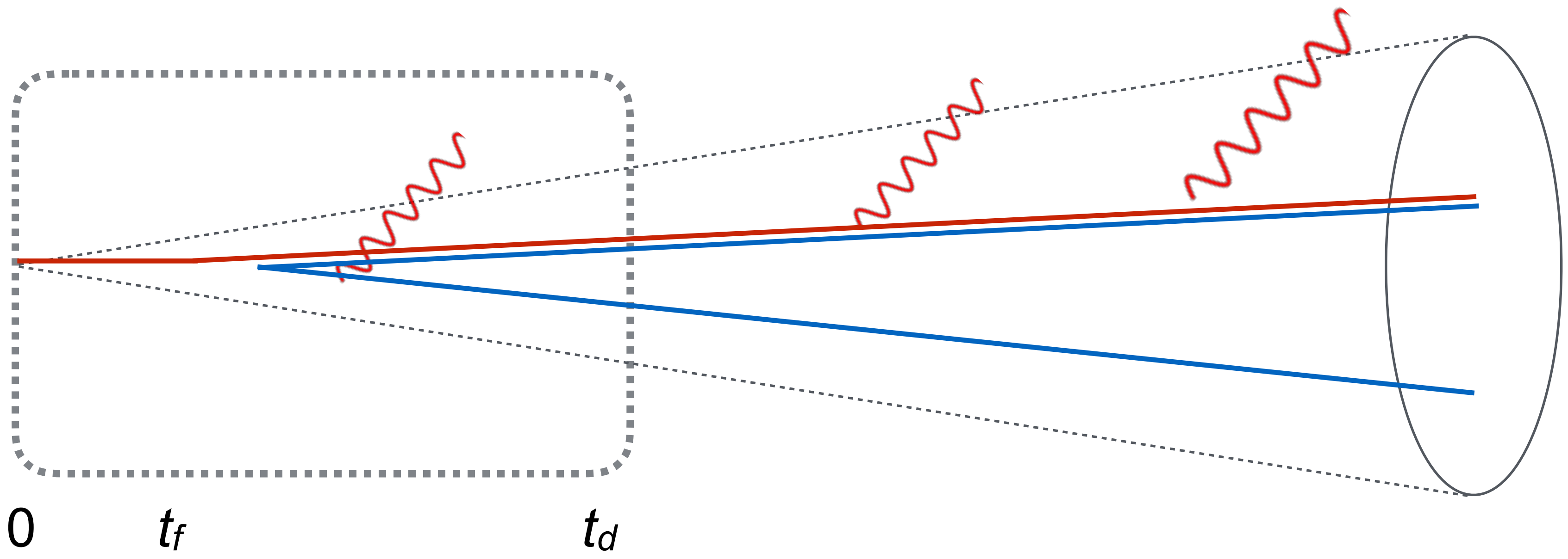


$$t_{\mathrm{d}} \sim (\hat{q}\theta_{12})^{-1/3}$$

$$\theta_c \sim \sqrt{\frac{1}{\hat{q}L^3}}$$

$$\mathcal{P}_2(\nu) = \mathcal{P}(\nu) \times \mathcal{P}_{\mathrm{sing}}(\nu)$$

total color charge

contributions from interferences

$$\mathcal{S}_2(t) = \exp\left[-\frac{1}{4}\int_0^t \mathrm{d}s\, \hat{q}(\boldsymbol{x}_{12}, t)\boldsymbol{x}_{12}^2(s)\right]$$

decoherence parameter

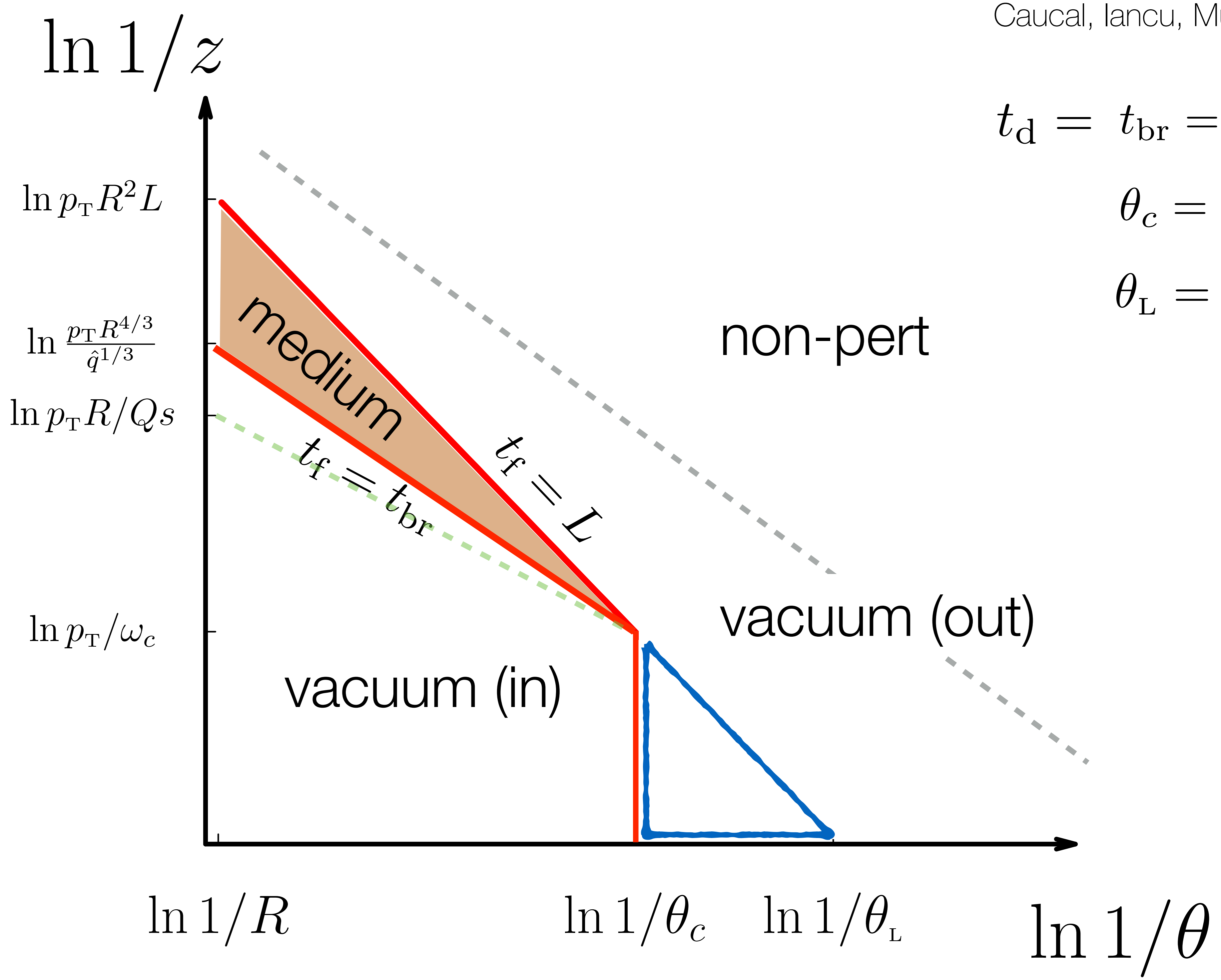
color randomization of a $q\bar{q}$ pair

Mehtar-Tani, Salgado, KT PLB (2012), JHEP (20132; Casalderrey, Iancu JHEP (2011)

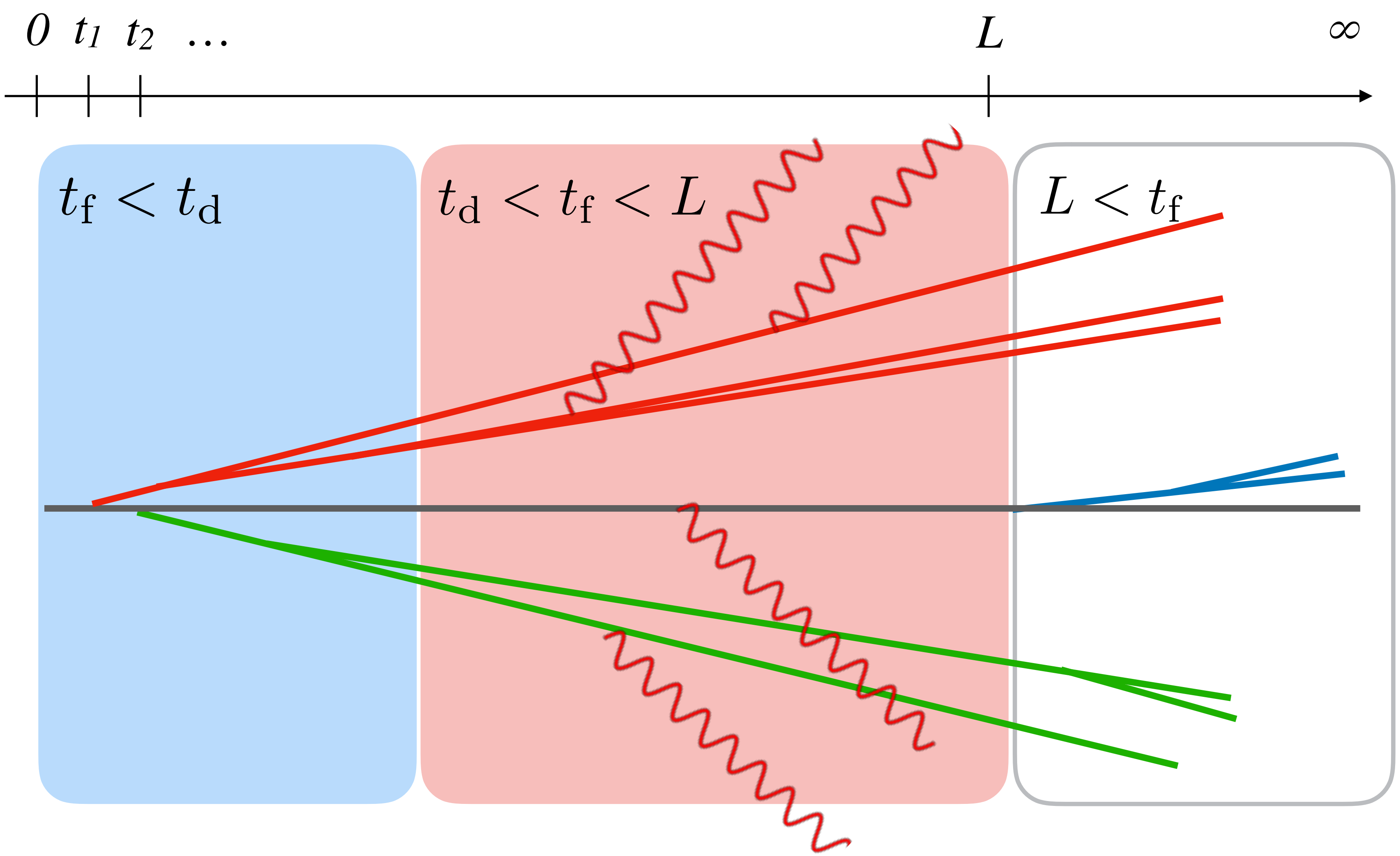
LUND PLANE IN MEDIUM

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]
Andrews et al. arXiv:1808.03689
Caucal, Iancu, Mueller, Sozey PRL (2018)

$$t_{\rm d} = t_{\rm br} = (\hat{q}\theta^2)^{-1/3}$$
$$\theta_c = (\hat{q}L^3)^{-1/2}$$
$$\theta_{\rm L} = (p_{\rm T}L)^{-1/2}$$



SUMMARY: EMERGING PICTURE (DLA)



	hard, in-medium radiation	energy-loss, broadening	fragmentation, hadronization
Regimes:	*"logarithmic" time*	*"linear" time*	*"logarithmic", non-perturbative*

GENERATING FUNCTIONAL METHOD

Konishi, Ukawa, Veneziano Nucl. Phys. B1567 (1979);
Bassetto, Ciafaloni, Marchesini Phys. Rept. 100 (1983)
Dokshitzer, Khoze, Mueller, Troyan "Basics of Perturbative QCD" (1991)

Generating function: $$G(u) = \sum_n P_n u^n$$

Normalization (conservation of probability) $$G(u=1) = 1$$

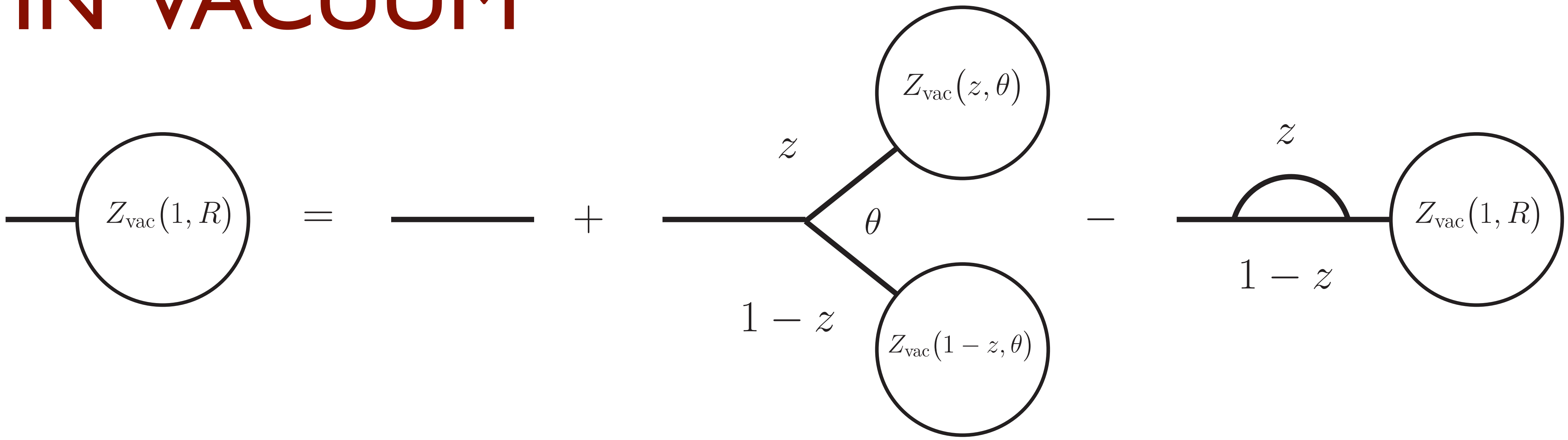
Applications to jets

$$P_n \to P(k_1, \ldots, k_n) \qquad u_n \to u(k_1) \ldots u(k_n)$$

Fragmentation function

$$D(x|p,R) \equiv x \frac{\mathrm{d}N}{\mathrm{d}x} \equiv x \frac{\delta Z(p,R;u)/\delta u(x)\Big|_{u=1}}{Z(p,R;u)\Big|_{u=1}}$$

GF IN VACUUM



$$
Z_{\rm vac}(p,R;u) = u(p) + \int_0^R \frac{\mathrm{d}\theta}{\theta} \int_0^1 \mathrm{d}z \, \frac{\alpha_s}{\pi} P(z) \times \left[Z_{\rm vac}(zp,\theta) Z_{\rm vac}\big((1-z)p,\theta\big) - Z_{\rm vac}(p,\theta) \right]
$$

- angular ordering: $R > \theta > \ldots > Q_0/p$ (but can build in any)
- $Z_{\rm vac}(u{=}1){=}1$ from probability conservation

$$
\frac{\partial}{\partial \ln Q} D(x,\theta) = \int_0^1 \mathrm{d}z \frac{\alpha_s}{\pi} P(z) \left[D(x/z, zQ) - z D(x,Q) \right]
$$

MLLA evolution equation

Implementing Quenching

Assume all particles are produced and, subsequently, quenched ***independently*** in the medium

$$\frac{\mathrm{d}\sigma_{\mathrm{excl}}}{\mathrm{d}k_1\mathrm{d}k_2...\mathrm{d}k_N} = \int \mathrm{d}p \left\{ \int \prod_{i=1}^{N} \mathrm{d}\epsilon_i P(\epsilon_i) \right\} f(k_1+\epsilon_1, k_2+\epsilon_2, ..., k_N+\epsilon_N|p) \times \delta(p-\epsilon-k_1-k_2-...-k_N)\frac{\mathrm{d}\sigma}{\mathrm{d}p}$$

-neglect energy-loss compared to typical energies…

$$\frac{\mathrm{d}\sigma_{\mathrm{excl}}}{\mathrm{d}k_1\mathrm{d}k_2...\mathrm{d}k_N} \simeq \left(\int_0^\infty \mathrm{d}\epsilon P(\epsilon)\exp\left(-\frac{n\epsilon}{p_T}\right)\right)^N \frac{\mathrm{d}\sigma_{0,\mathrm{excl}}}{\mathrm{d}k_1\mathrm{d}k_2...\mathrm{d}k_N} = \mathcal{Q}(p)^N \frac{\mathrm{d}\sigma_{0,\mathrm{excl}}}{\mathrm{d}k_1\mathrm{d}k_2...\mathrm{d}k_N}.$$

Any in-medium, resolved parton is re-weighted by a quenching factor!

GF FOR IN-MEDIUM JET EVOLUTION

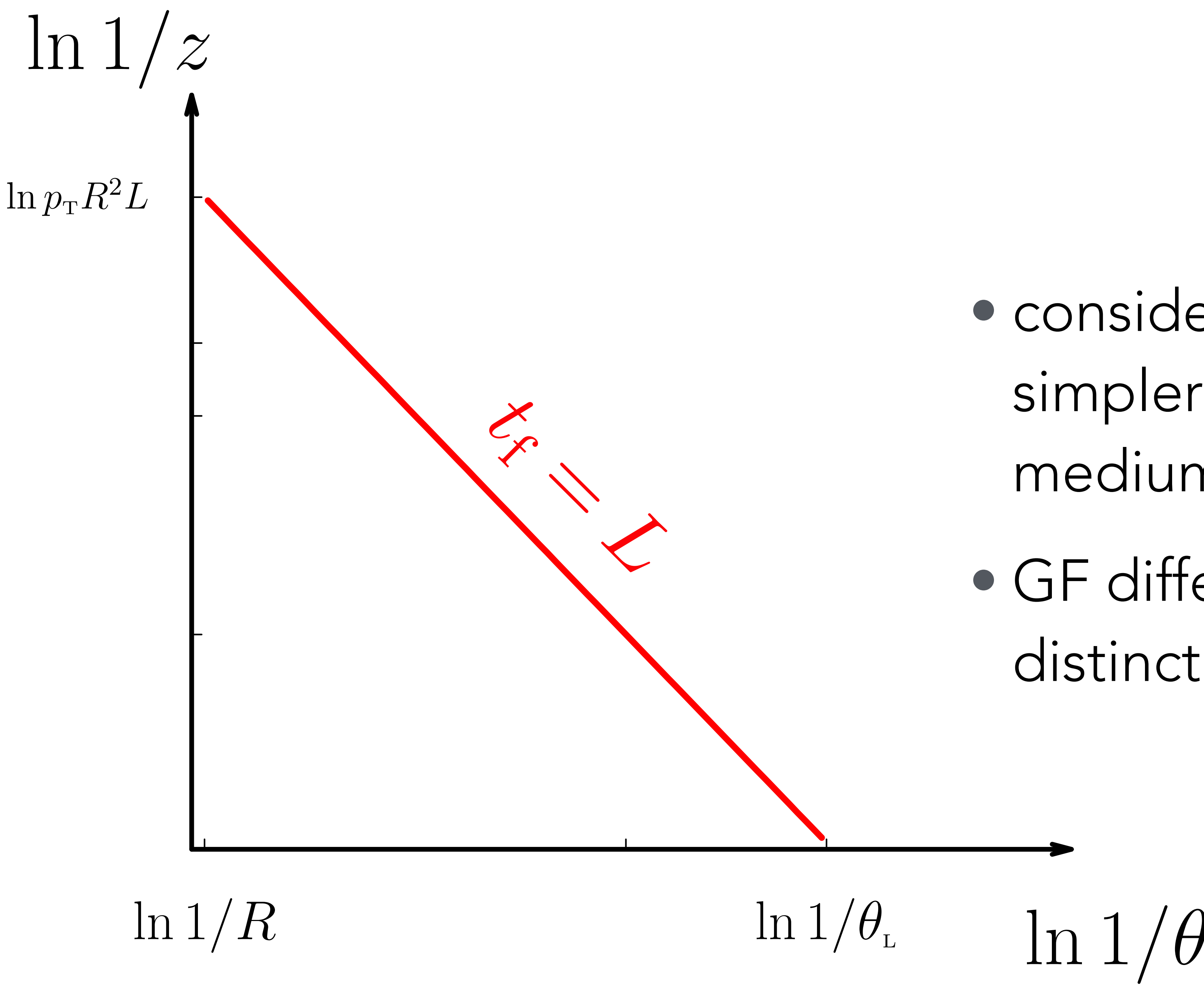
Y. Mehtar-Tani, KT (in preparation…)

$$\mathcal{Z}(p,R;u) = u(p) + \int^R \mathrm{d}\Omega\; \Theta(t_\mathrm{f} < t_\mathrm{d} < L)$$
$$\left[\left(\mathcal{Z}(zp,\theta) + \mathcal{Z}_\mathrm{out}(zp,R',\theta)\right)\left(\mathcal{Z}((1-z)p,\theta) + \mathcal{Z}_\mathrm{out}((1-z)p,R',\theta)\right) \mathcal{Q}^2(p_\mathrm{T}) - \mathcal{Z}(p,\theta)\right]$$
$$+ \int^R \mathrm{d}\Omega\, \left(\Theta(t_\mathrm{d} > L) + \Theta(t_\mathrm{d} < L < t_\mathrm{f})\right) \left[\mathcal{Z}_\mathrm{vac}(zp,\theta)\, \mathcal{Z}_\mathrm{vac}((1-z)p,\theta) - \mathcal{Z}_\mathrm{vac}(p,\theta)\right]$$

$$\int^R \mathrm{d}\Omega \equiv \int_0^R \frac{\mathrm{d}\theta}{\theta} \int_0^1 \mathrm{d}z\, \frac{\alpha_s}{\pi} P(z)$$

- decomposes phase space according to scale analysis
- incorporates correctly (small) out-of-cone energy loss
- incorporates loss of coherence (angular ordering) for particles coming out of the medium (Z_out)

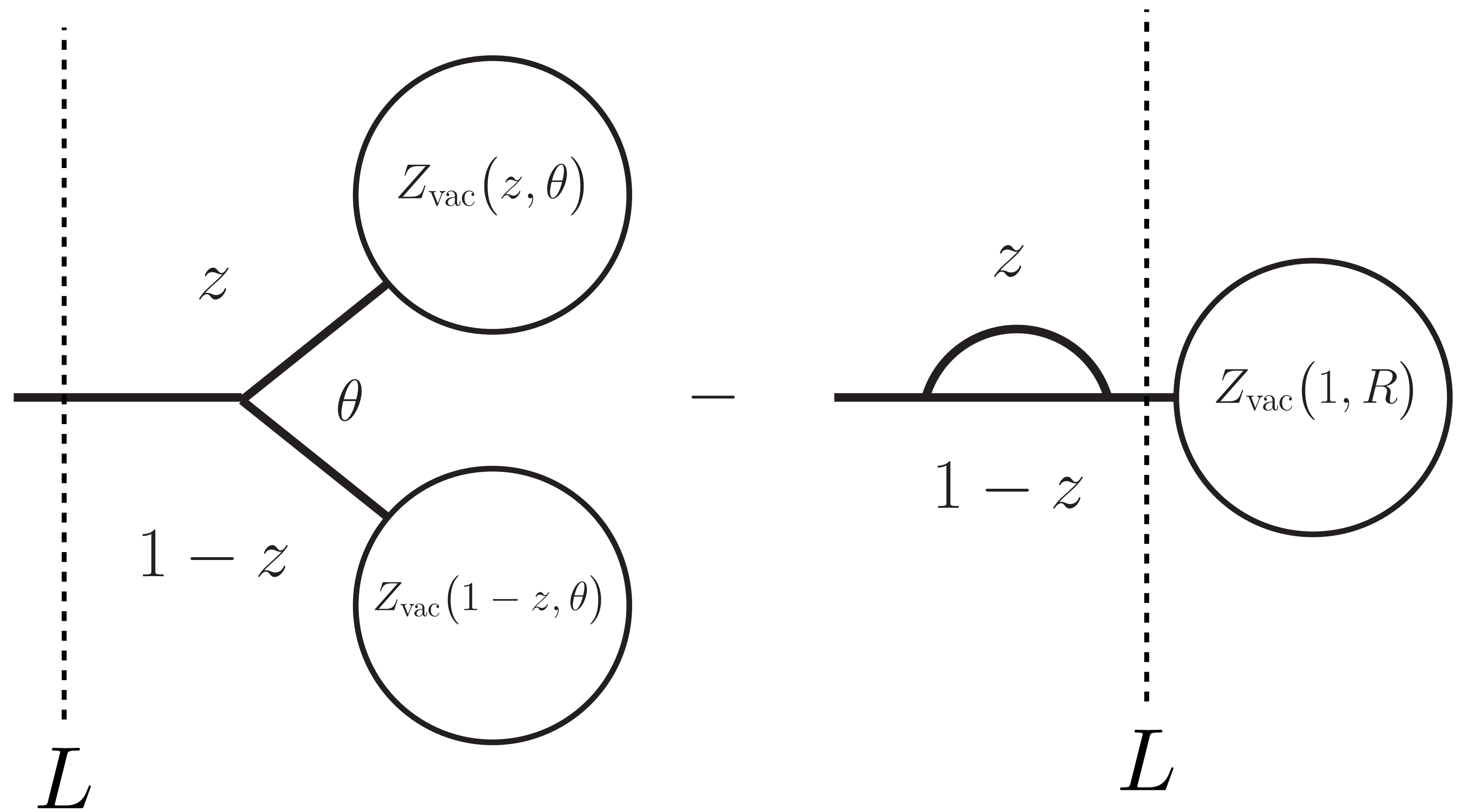
SIMPLIFYING THE DISCUSSION

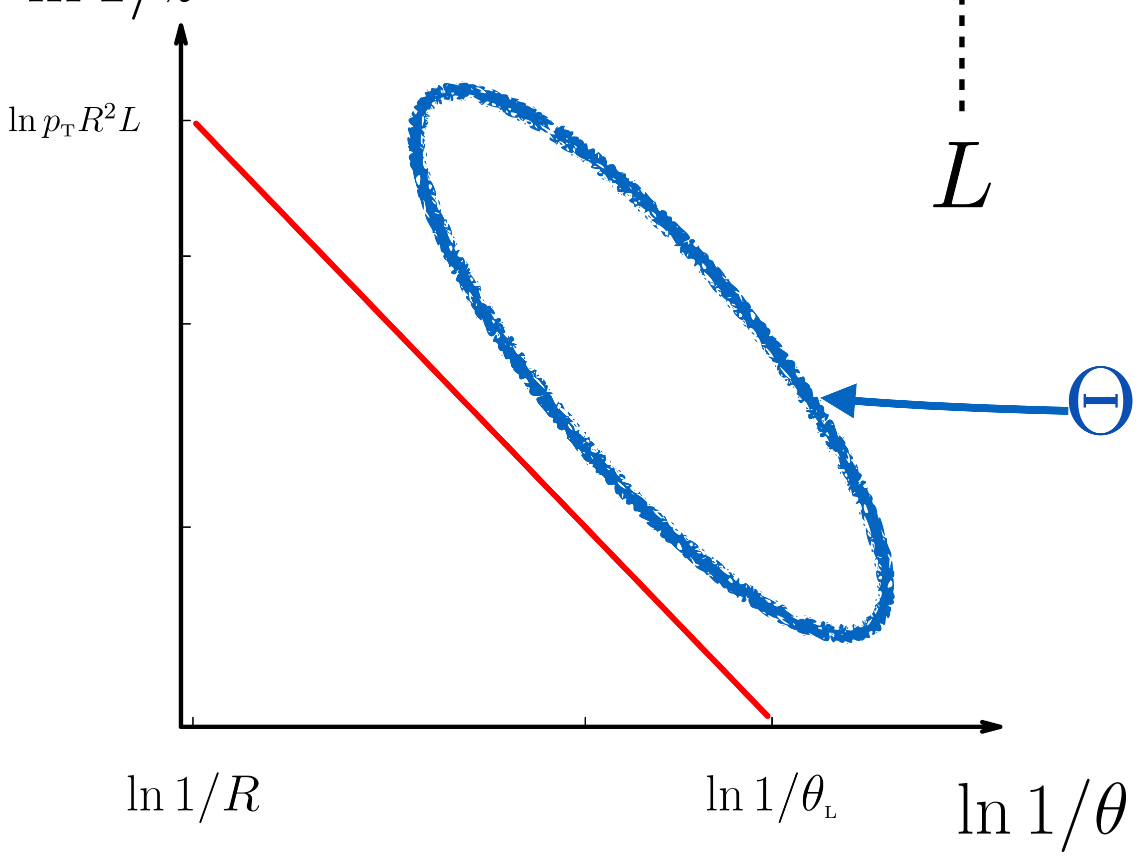


- consider for the moment the simpler situation: **in** or **out** of the medium
- GF differentiates between two distinct cases

GF CONTRIBUTIONS

Fragmentation **outside** the medium



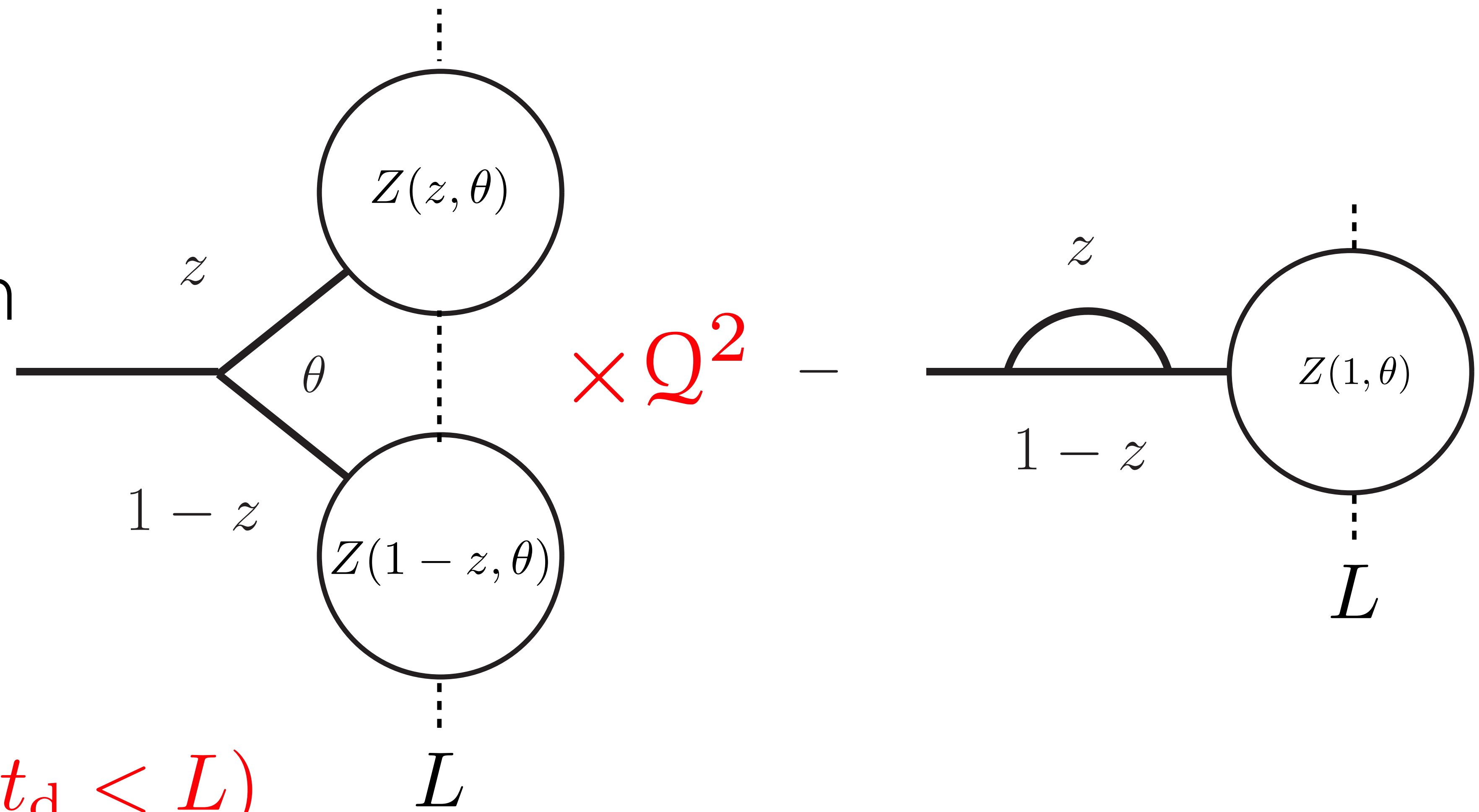


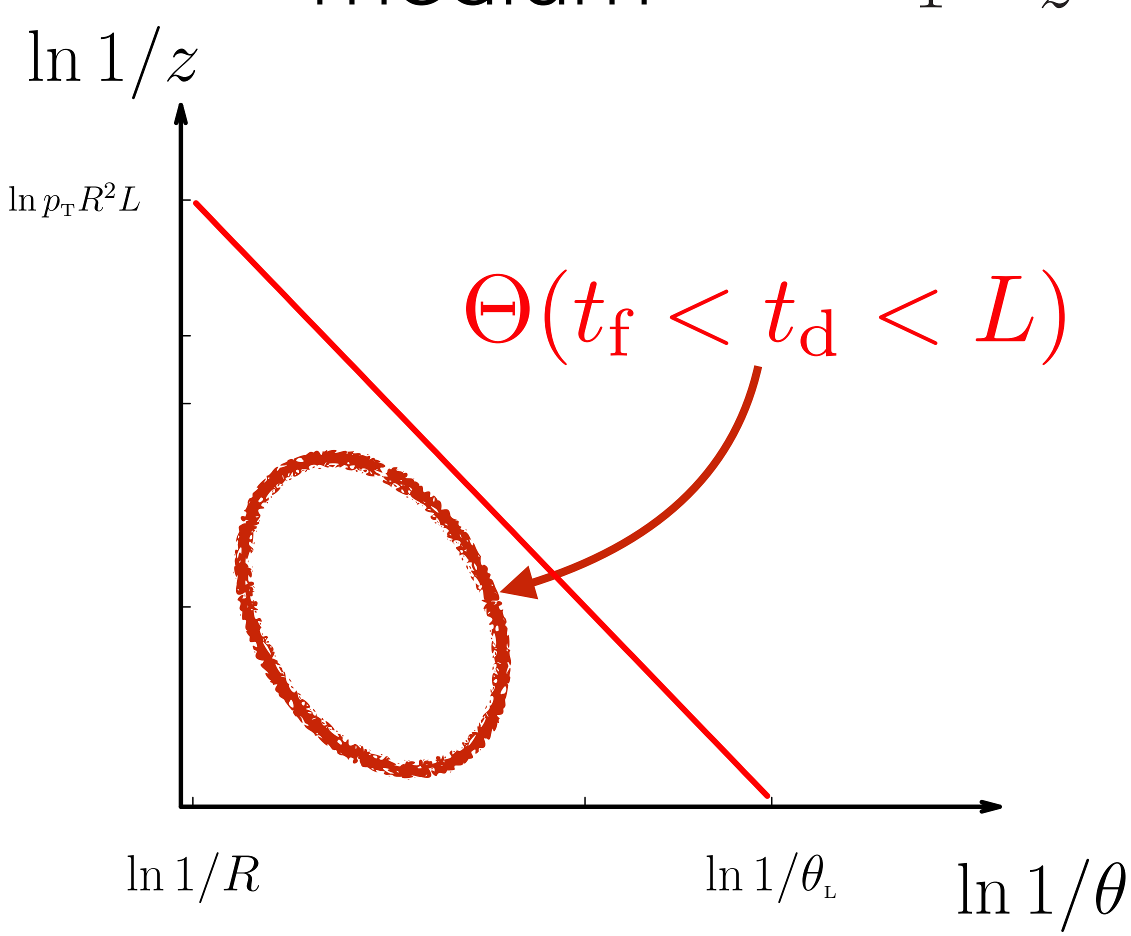
$$\Theta(t_{\rm d} < L < t_{\rm f}) + \Theta(\theta < \theta_c)$$

… is vacuum-like (AO).

GF CONTRIBUTIONS

Fragmentation **inside** the medium





… is quenched.

… is vacuum-like (AO).

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]
Caucal, Iancu, Mueller, Sozey PRL (2018)

GF NORMALIZATION

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]

Probability is no longer conserved!

$$Z(p, R; u = 1) = \mathcal{C}(p, R)$$

Non-trivial normalization: collimator function!

$$C(p, R) = 1 + \bar{\alpha} \int_0^R \frac{\mathrm{d}\theta}{\theta} \int_0^1 \mathrm{d}z \, P(z) \; \underline{\Theta(t_\mathrm{f} < t_\mathrm{d} < L)} \times \left[C(zp, \theta) \, C((1-z)p, \theta) \, \mathcal{Q}^2(p_T) - C(p, \theta) \right]$$

Strong quenching limit $Q \ll 1$ (Sudakov factor):

$$\mathcal{C}(p_\mathrm{T}, R) \simeq \exp\left[-2\bar{\alpha} \ln \frac{R}{\theta_c} \left(\ln \frac{p_\mathrm{T}}{\omega_c} + \frac{2}{3} \ln \frac{R}{\theta_c} \right) \right]$$

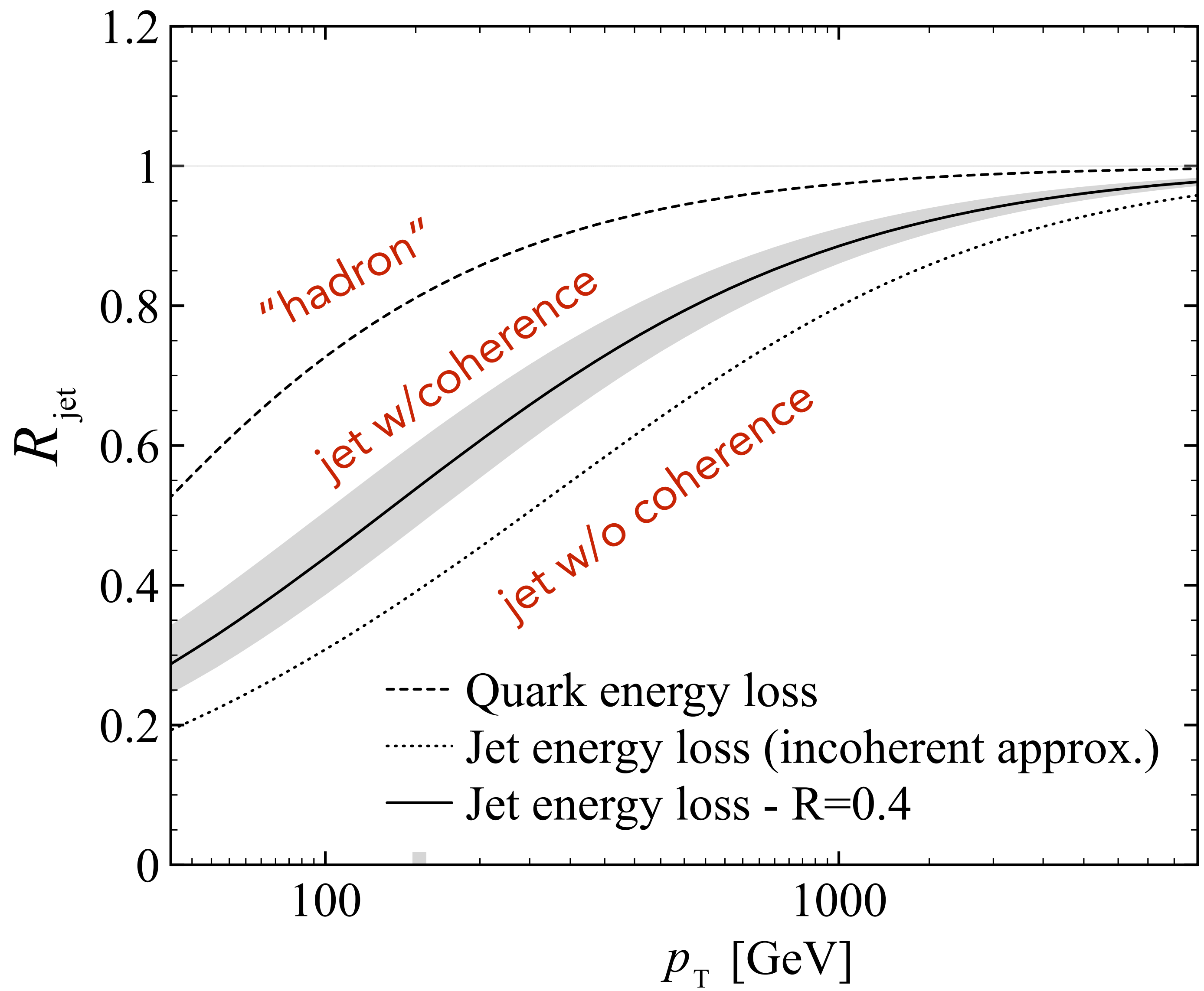
-for "in/out" model, the logs are different → sensitivity to coherence

SINGLE-INCLUSIVE SPECTRUM

Y. Mehtar-Tani, KT arXiv:1707.07361 [hep-ph]

$$R_{\rm jet} = \mathcal{Q}_q(p_{\rm T}) \times \mathcal{C}(p_{\rm T}, R)$$

jet loses energy via total charge & resolved substructure fluctuations



Outlook

- probabilistic setup combining jet & medium scales
  - generating functional
  - systematically improvable
- test bed to compare against models & MCs
- collimator function
  - non-linear evolution of quenching

JET TOOLS 2019

2nd Heavy-Ion Jet Substructure Workshop

13 - 17 May
University of Bergen

- new tools for jet physics at the frontiers
- jet substructure and heavy flavor
- splitting maps of the shower
- jet modifications in small systems
- interplay between jets and underlying event
- statistical & machine-learning techniques

https://jettols.w.uib.no

Back-up

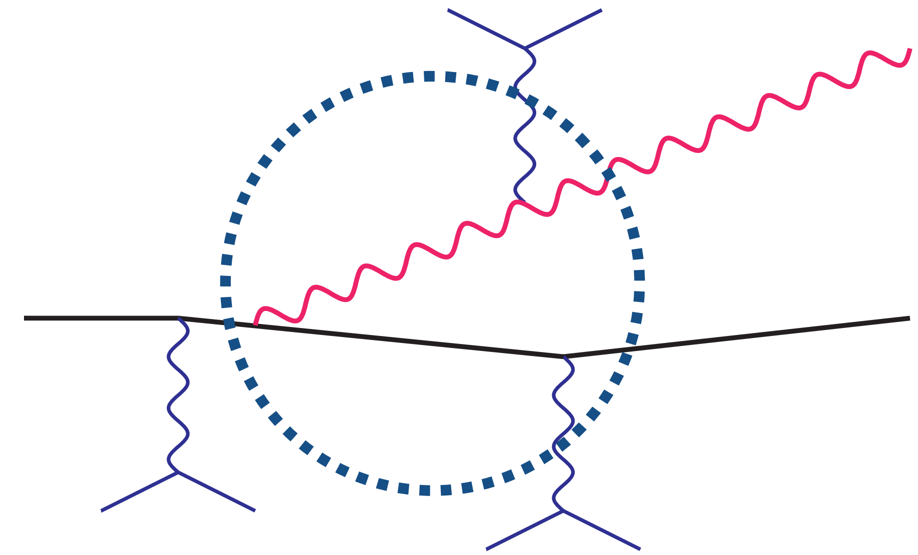
Medium-induced gluons

Baier, Dokshitzer, Mueller, Peigné, Schiff (1997-2000); Zakharov (1996);…

momentum broadening $\langle k_\perp^2 \rangle \sim \hat{q} t$

modified splitting kinematics
lack of collinear singularity!

$$t_{\mathrm{f}} = \frac{\omega}{k_\perp^2} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{\alpha_s C_R}{2\pi} \sqrt{\frac{\hat{q}L^2}{\omega}}$$

rare, small-angle emission

$$\omega_c = \hat{q}L^2$$

$$\theta_{\mathrm{br}}(\omega_c) \sim \sqrt{\frac{1}{\hat{q}L^3}} \equiv \theta_c$$

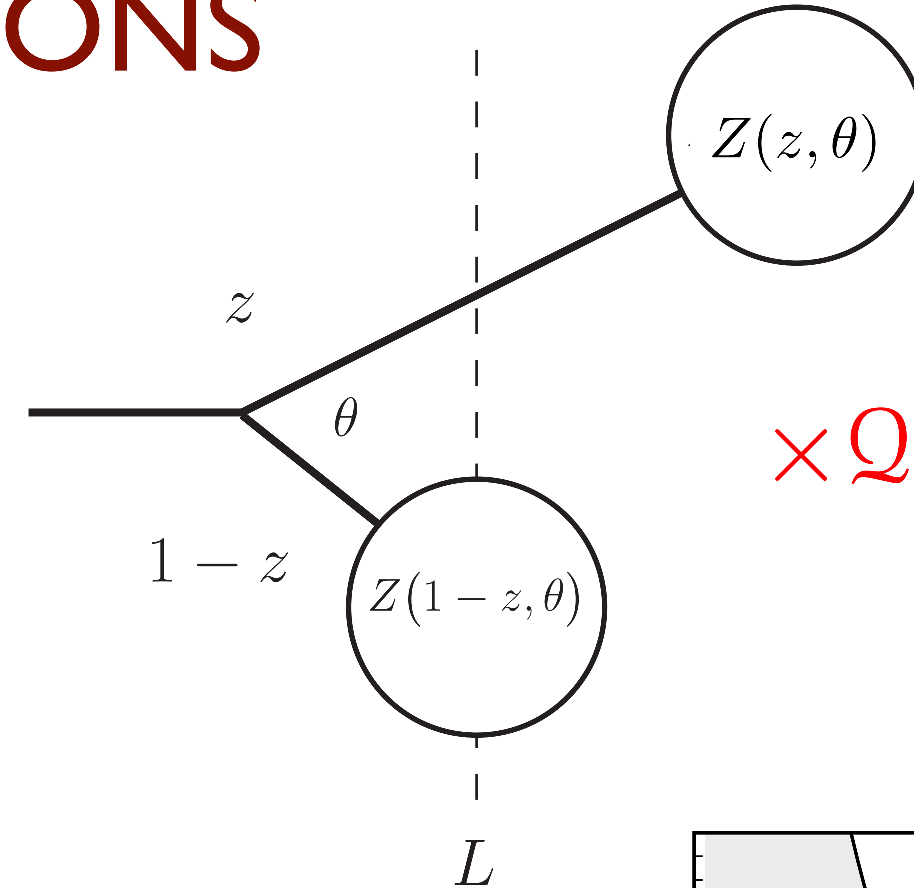
copious, large-angle emissions

$$\omega_s = \bar{\alpha}^2 \hat{q} L^2$$

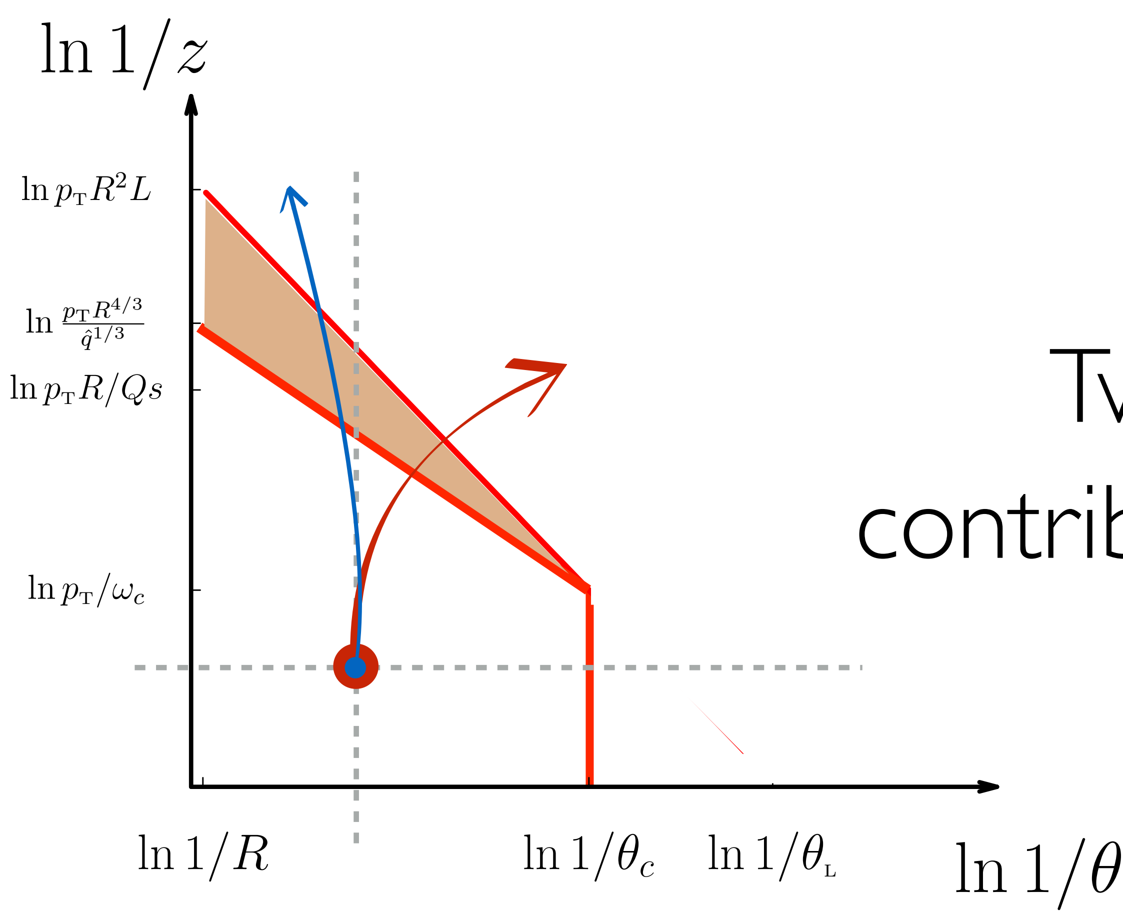
$$\theta_{\mathrm{br}}(\omega_s) \sim \frac{1}{\bar{\alpha}^{3/2}} \theta_c$$

GF CONTRIBUTIONS

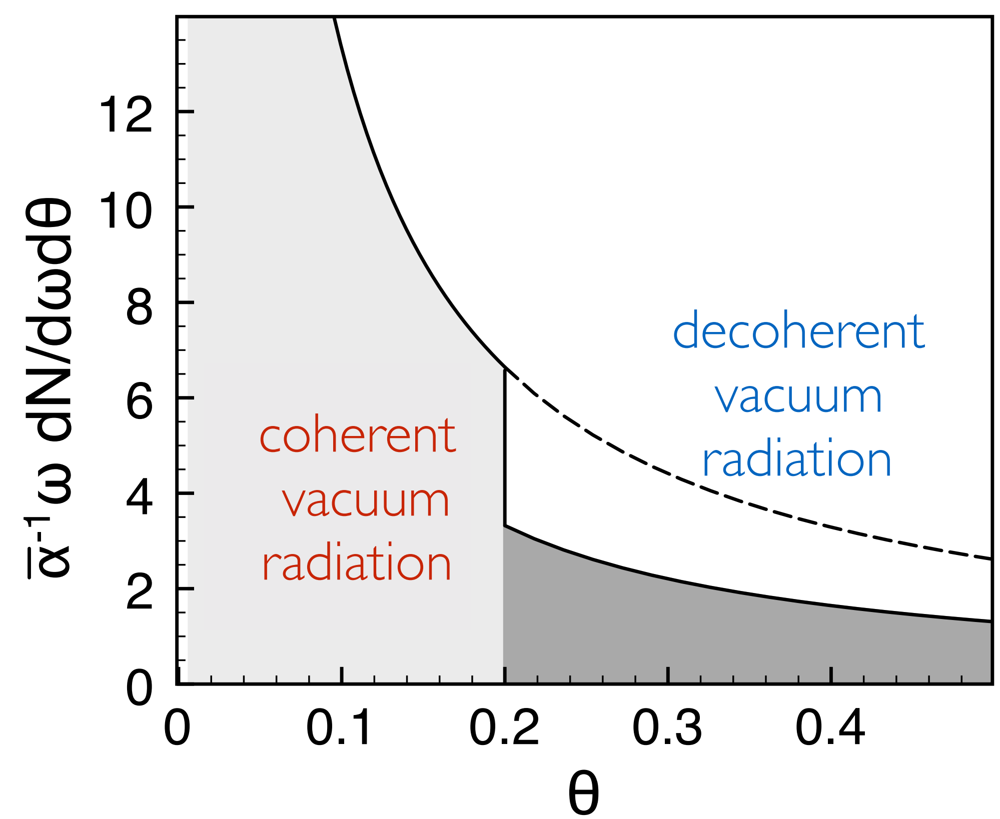
Splitting **inside** the medium, fragmentation **outside**



$\times Q^2\,\Theta(t_{\rm f} < t_{\rm d} < L)$



Two contributions:

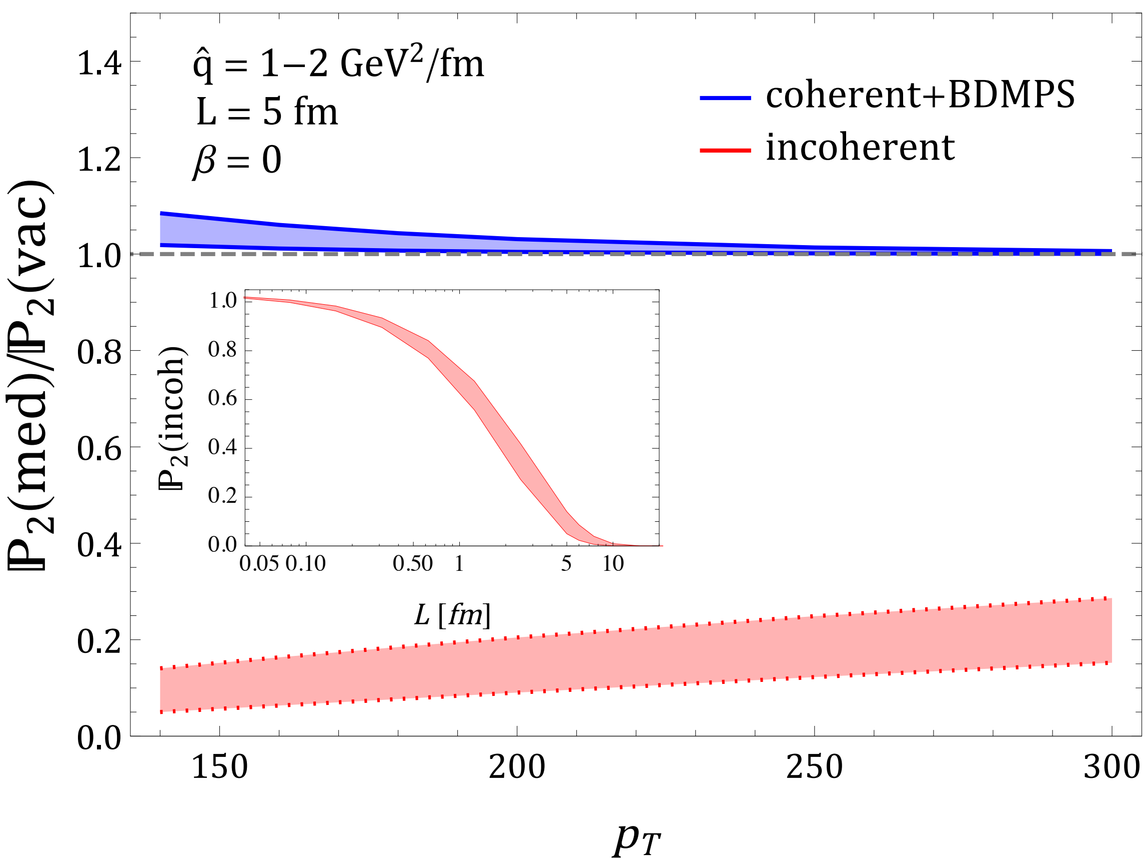


Mehtar-Tani, Salgado, KT PRL 106 (2011) 122002, PLB (2012)

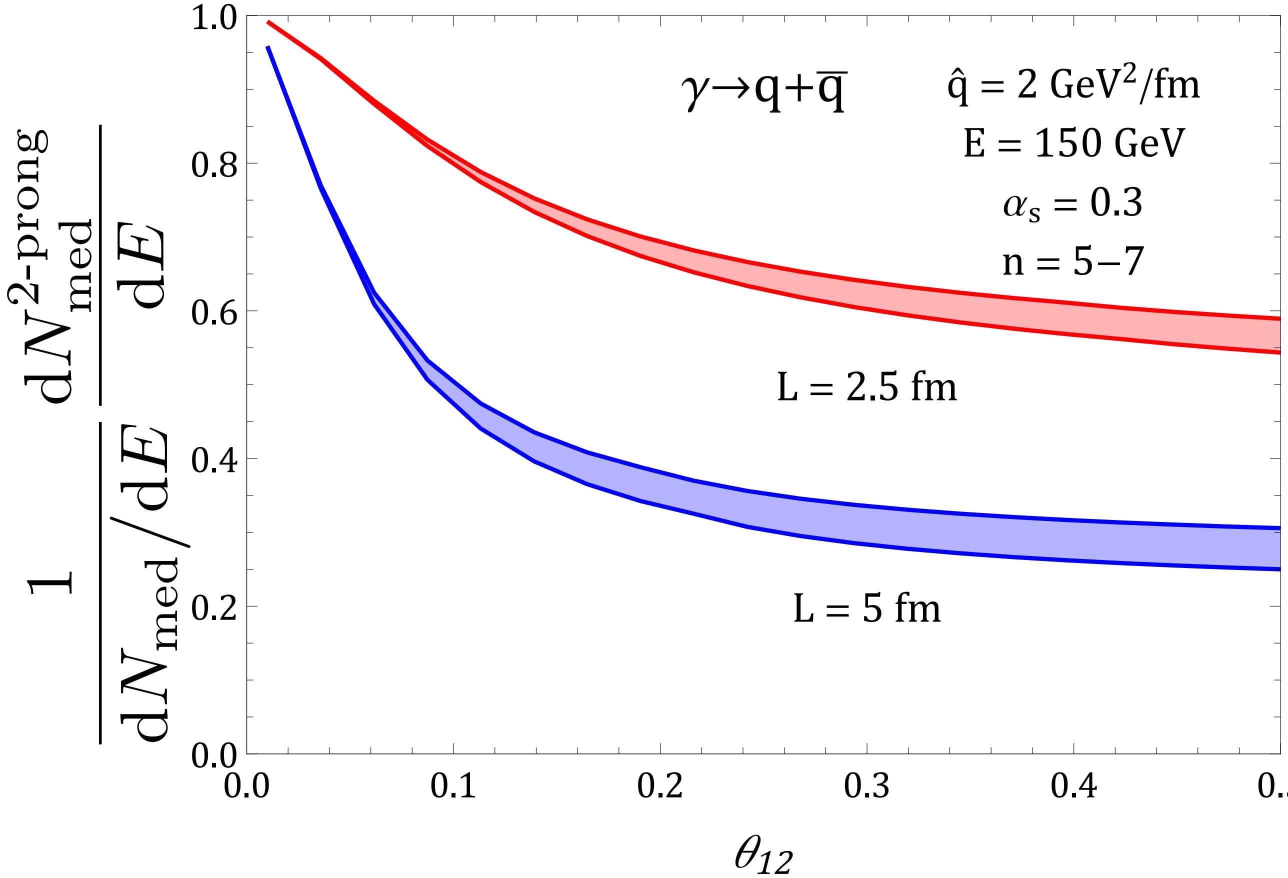
TWO-PRONG PROBABILITY

Choosing two-prong substructure using SoftDrop

Mehtar-Tani, KT arXiv:1610.08930



Casalderrey, Mehtar-Tani, Salgado KT QM2017 proceedings



Non-monotonic behavior indicative of coherence effects!

Suggestive: inclusive z_g distribution [Andrews et al. (ALICE) QM2018].