# A Monte Carlo study of in-medium quark and gluon jet colour scaling

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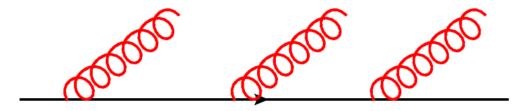
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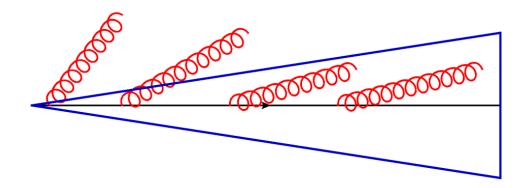
$$\left(\frac{E^{quark}}{E^{gluon}}\right)_{loss}^{AdS/CFT} = \left(\frac{C_F}{C_A}\right)^{\frac{1}{3}} \overset{QCD}{=} \left(\frac{4}{9}\right)^{\frac{1}{3}} = 0.7631...$$

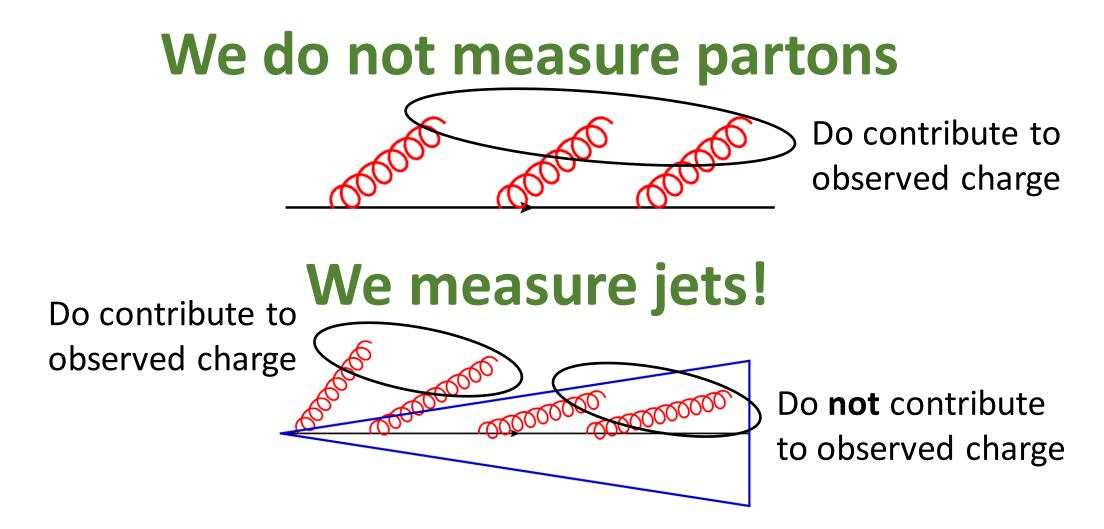
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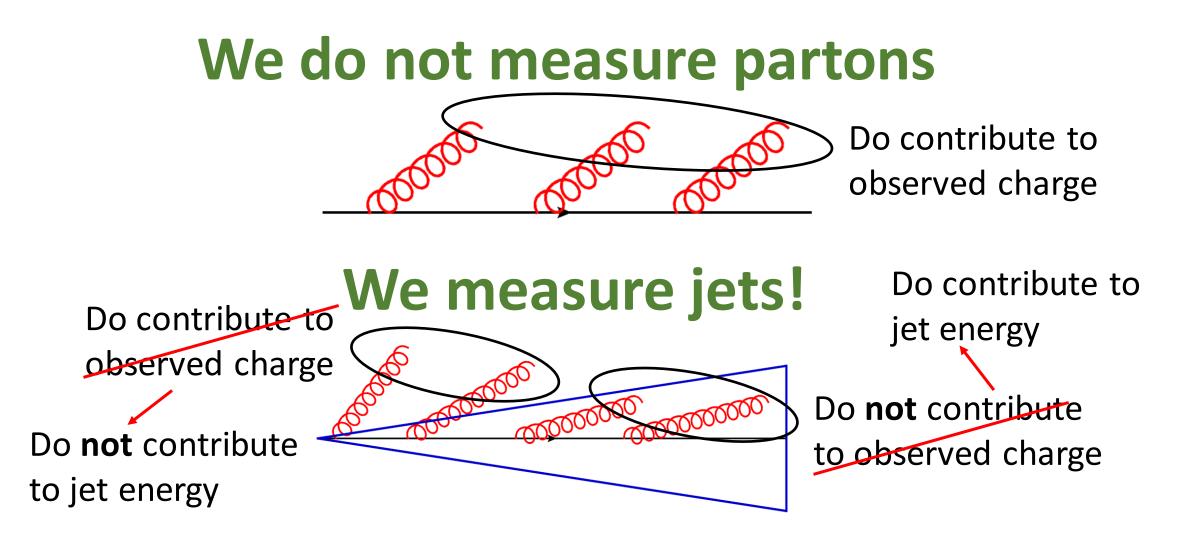
#### We do not measure partons



We measure jets!

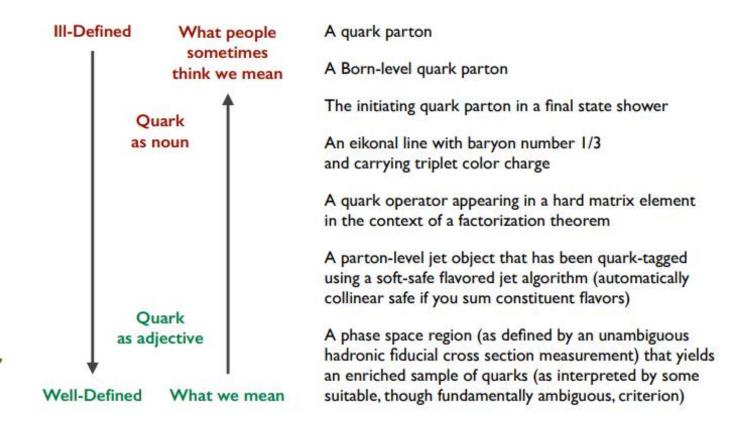






#### What is a Quark Jet?

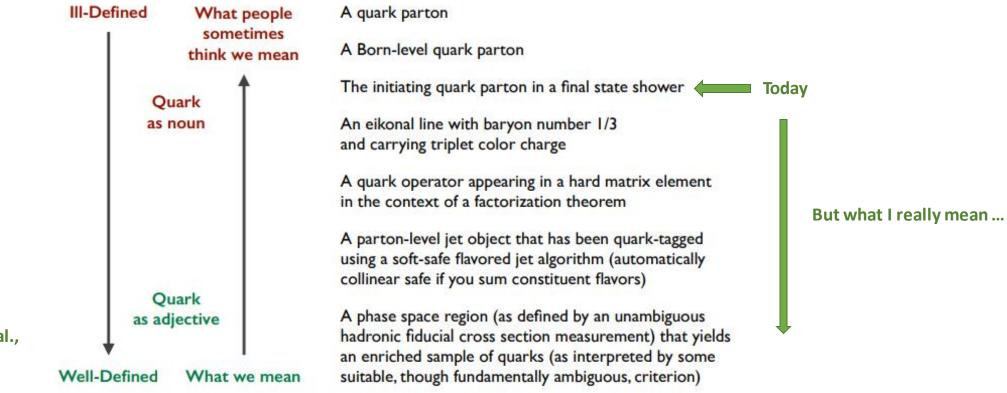
From lunch/dinner discussions



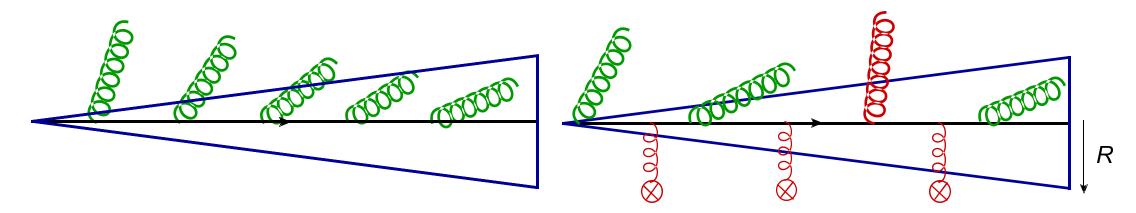
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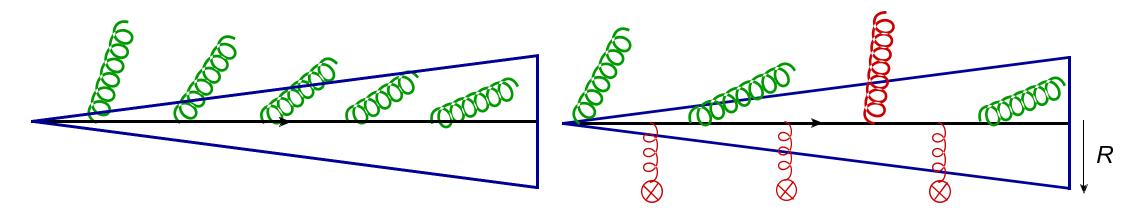


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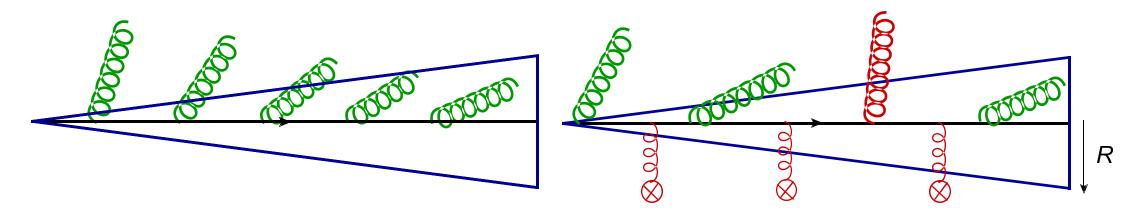
Vacuum:

Even in vacuum the transition from partons to jets is not trivial



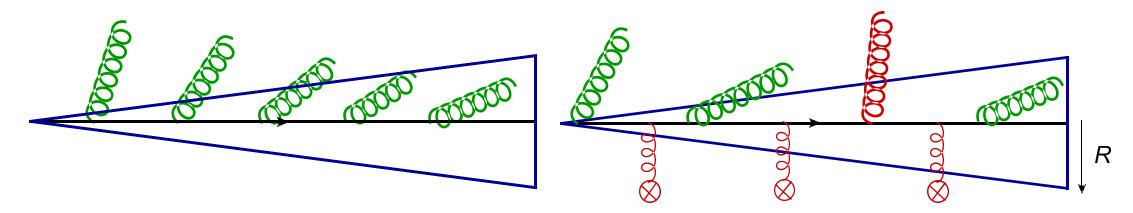
Vacuum:

Even in vacuum the transition from partons to jets is not trivial However, nowadays, many jet observables can be computed  $dProbability\left(\frac{1}{i}\right) = \frac{2\alpha_s C_i}{\pi} \frac{d\theta}{\theta} \frac{d\omega}{\omega}$ 



#### Medium:

Assuming decoupling between vacuum and medium emissions

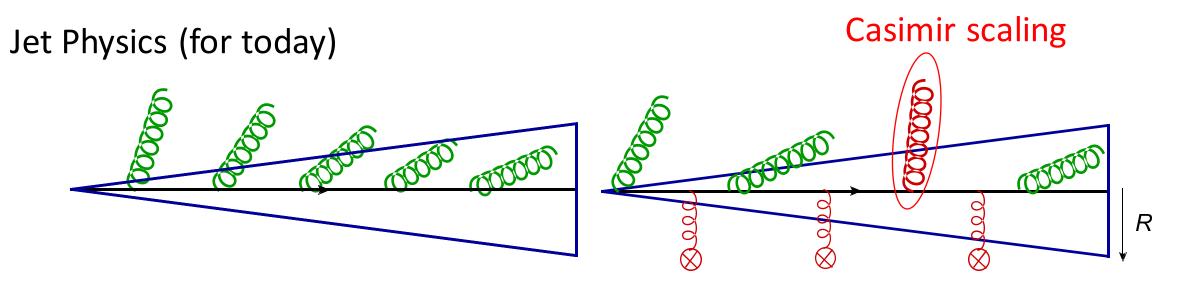


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Naively we expect the breaking of Casimir scaling for medium energy loss mechanisms



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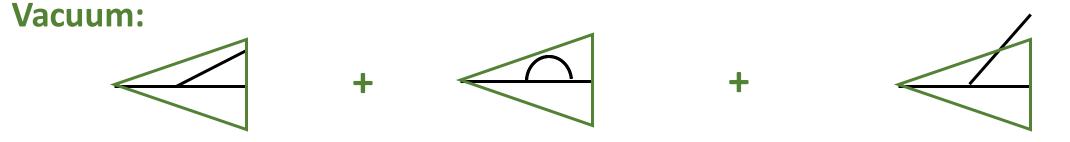
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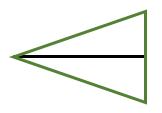
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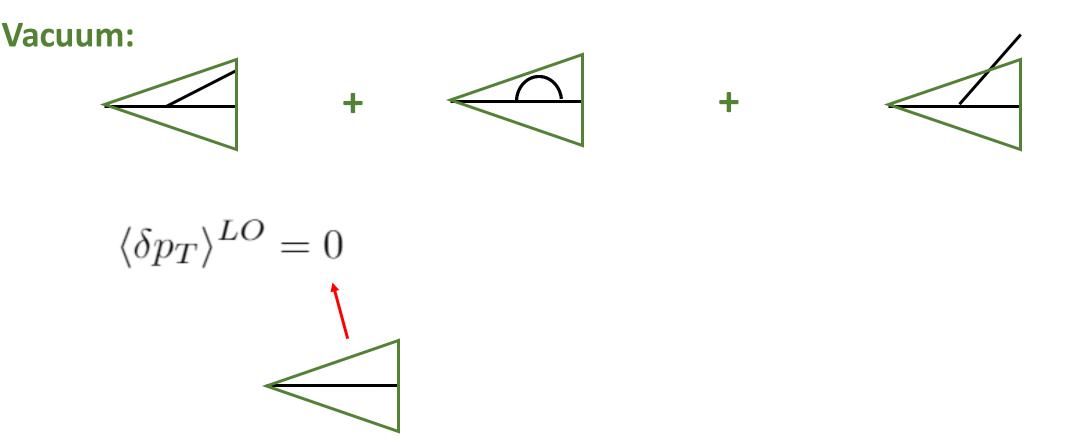
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A simple qualitative calculation to estimate Casimir breaking effects

Vacuum: ╉ **KLN**  $-\frac{\alpha_s}{\pi} \int_{\mathcal{B}}^{\mathcal{O}(1)} \frac{d\theta}{\theta} \int_{0}^{1} dz P(z)(-1+f(z))$  $\langle \delta p_T \rangle^{LO} = 0$ **Recover: Dasgupta, Recover:** Dasgupta, Dreyer, Salam, Soyez Magnea, Salam, 0712.3014 1411.52182

A simple qualitative calculation to estimate Casimir breaking effects

Compute the transverse momentum imbalance at LO

Vacuum: **KLN**  $\langle \delta p_T \rangle^{LO} = 0 - \frac{\alpha_s}{\pi} \int_R^{\mathcal{O}(1)} \frac{d\theta}{\theta} \int_0^1 dz P(z)(-1+f(z))$  $P_q(z) = P_{gq}(z)$   $P_g(z) = \frac{1}{2}P_{gg}(z) + n_f P_{qg}(z)$ 

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Compute the transverse momentum imbalance at LO Vacuum:

To get Casimir scaling, one just computes quark to gluon ratio

Keeping track of logarithms of R (i.e. for small R and rigid cone jet algorithm)

$$\frac{\langle \delta p_T \rangle_{quark}^{LO}}{\langle \delta p_T \rangle_{gluon}^{LO}} = \frac{C_F \left( \log 4 - \frac{3}{8} \right)}{C_A \left( \log 4 - \frac{43}{96} \right) + n_f T_R \frac{7}{48}} \stackrel{QCD}{=} \frac{1.01129 C_F}{0.938378 C_A + 0.21875} = \frac{1.34839}{3.03388} \approx \frac{C_F}{C_A}$$

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We just aim for a very rough qualitative numerical estimate. We consider

$$\langle \delta p_T \rangle^{LO} = \int_0^1 dz P(z) f(z) \int \frac{d\theta}{\theta} \int_{\mathbf{q}} \frac{q^2}{q^2 + \tilde{\mu}^2 \tilde{k}^2 + 2q \tilde{\mu} \tilde{k} \cos \alpha}$$

up to overall factors

A simple qualitative calculation to estimate Casimir breaking effects

→ Compute the transverse momentum imbalance at LO Medium:

$$\langle \delta p_T \rangle = \int_0^1 dz P(z) f(z) \left( \int_q dq \log \frac{1}{R} + \int_q \int_R^{\mathcal{O}(1)} \frac{dq}{q} \tilde{\mu} \tilde{k}^2 \right) \text{ up to overall factors}$$
$$k_T = \tilde{k} \tilde{\mu} = (z(1-z)\theta)(E_{Jet})$$

A simple qualitative calculation to estimate Casimir breaking effects

# $\longrightarrow \text{ Compute the transverse momentum imbalance at LO}$ Medium: $\langle \delta p_T \rangle = \int_0^1 dz P(z) f(z) \left( \int_q dqq \log \frac{1}{R} + \int_q \int_R^{\mathcal{O}(1)} \frac{dq}{q} \tilde{\mu} \tilde{k}^2 \right) \text{ up to overall factors}$

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The first term is logarithmic in R, so it is the leading contribution

A simple qualitative calculation to estimate Casimir breaking effects

 $\rightarrow$  Compute the transverse momentum imbalance at LO Medium:

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Ignoring subleading terms

$$\langle \delta p_T \rangle = \int_0^1 dz P(z) f(z) \int_q dq \log \frac{1}{R}$$

A simple qualitative calculation to estimate Casimir breaking effects

Compute the transverse momentum imbalance at LO
Medium:

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$$\langle \delta p_T \rangle = \int_0^1 dz P(z) f(z) \int_q dq \log \frac{1}{R}$$

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$$\frac{\langle \delta p_T \rangle_{quark}}{\langle \delta p_T \rangle_{gluon}} = \frac{\int_0^1 dz P_q(z) f(z) \quad \alpha(z(1-z))^2}{\int_0^1 dz P_g(z) f(z) \quad \alpha(z(1-z))^2}$$

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- The jet result should differ from parton case jets have a finite extension
- An analytical treatment can be complicated due to medium screening (and technical complications not present in the vacuum)
- However, a Monte Carlo (MC) study is valuable to understand Casimir scaling
   This is the focus of today

Zapp, Krauss, Wiedemann, <u>1212.1599</u>

### **Analysis details**

- JEWEL to produce Z+qjets or Z+gjets with default parameters (1 million events for each)
- Vacuum and in-medium events (0-10% centrality), no recoil, no hadronization, no ISR, E<sub>CMS</sub>=5.02 TeV
- Z is used as a proxy for the initial parton

Our MC study is based on JEWEL

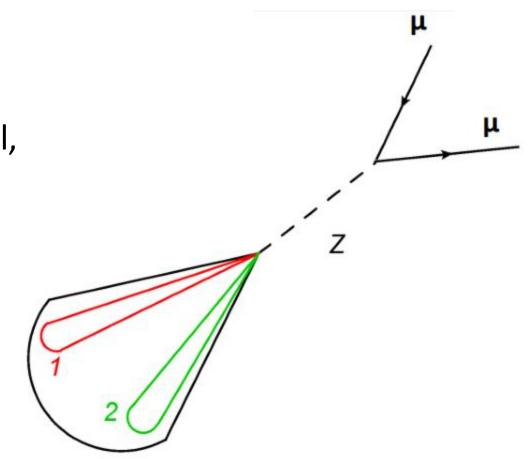
**Analysis details** 

**Z cuts:**  $p_T$ >50 GeV ,  $|\eta|$ <3 , di-muon channel, 70 GeV<Z-mass< 100 GeV

**Jet cuts:**  $p_T$ >20 GeV ,  $|\eta|$ <3 , anti-kt R=0.2-0.5

Order Jets by  $p_T$  and require b.t.b. with Z

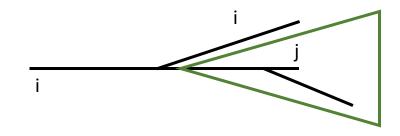
Require Jet and Z  $|\eta| < 3-R_{max}$ 

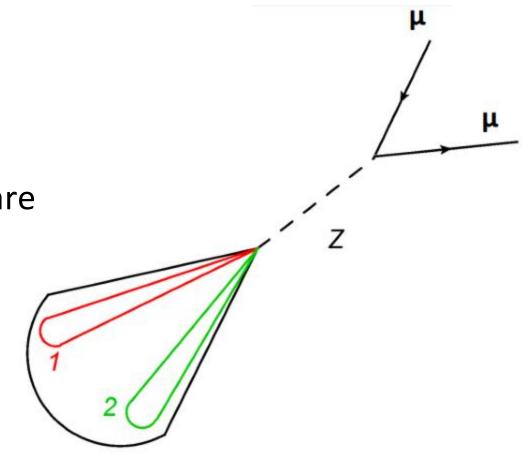


**Analysis details** 

### A possible issue:

Having a Z+i jet does not mean that we are picking the right parton



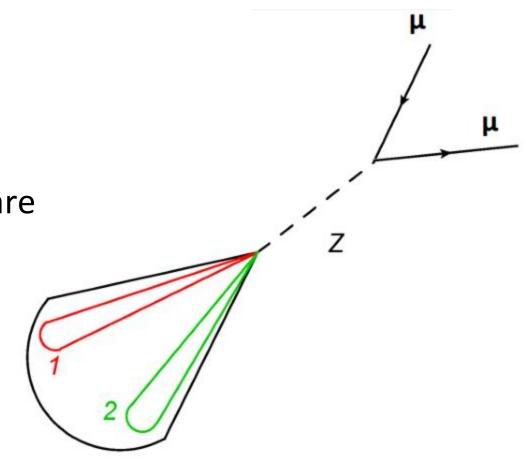


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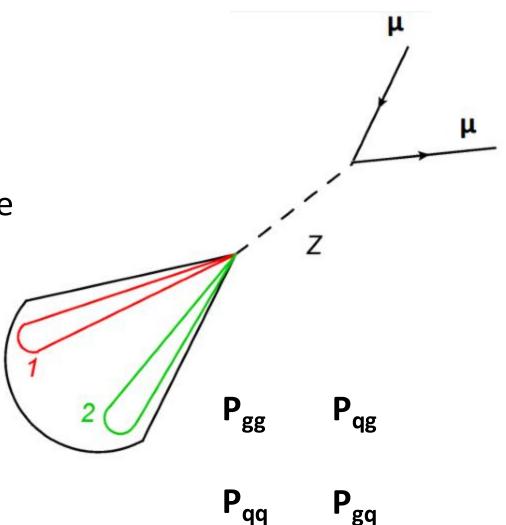


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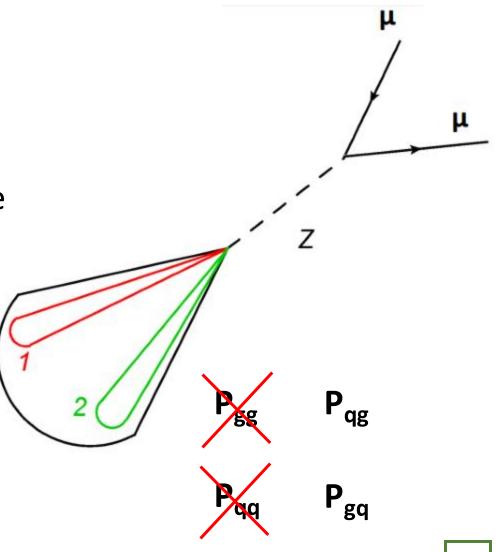


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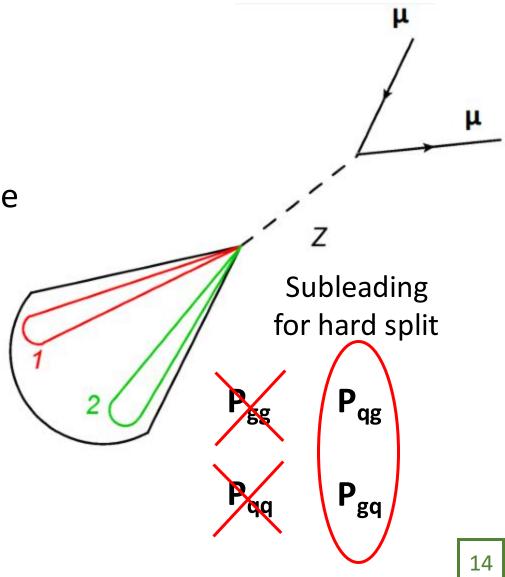


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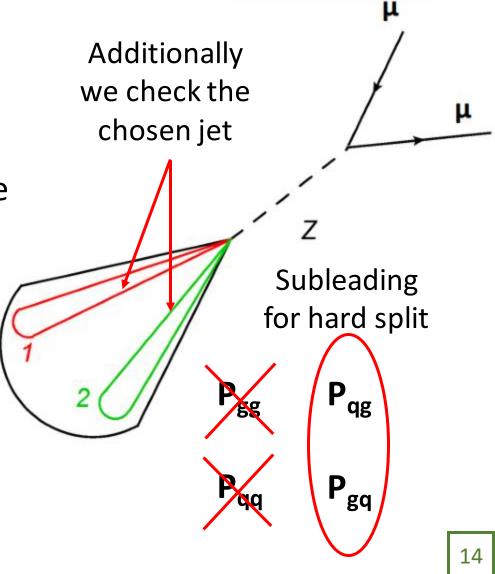


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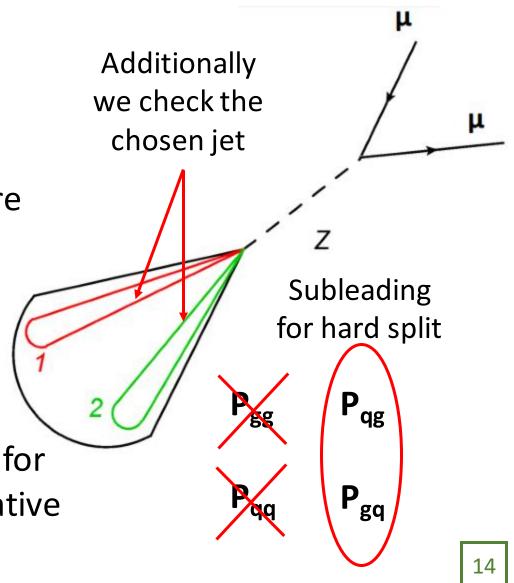


**Analysis details** 

# A possible issue:

Having a Z+i jet does not mean that we are picking the right parton

- JEWEL uses LO matrix elements
- This issue should be more relevant for small R, but we do not see a qualitative difference in the results



We make use of the following variable  $\Delta p_T = p_T^Z - p_T^{leading jet}$ 

$$\frac{\Delta p_T^{medium} - \Delta p_T^{vacuum}}{p_T^Z} = \frac{(p_T^{vacuum} - p_T^{medium})^{jet}}{p_T^Z} = \frac{medium \ imbalance}{initial \ p_T}$$

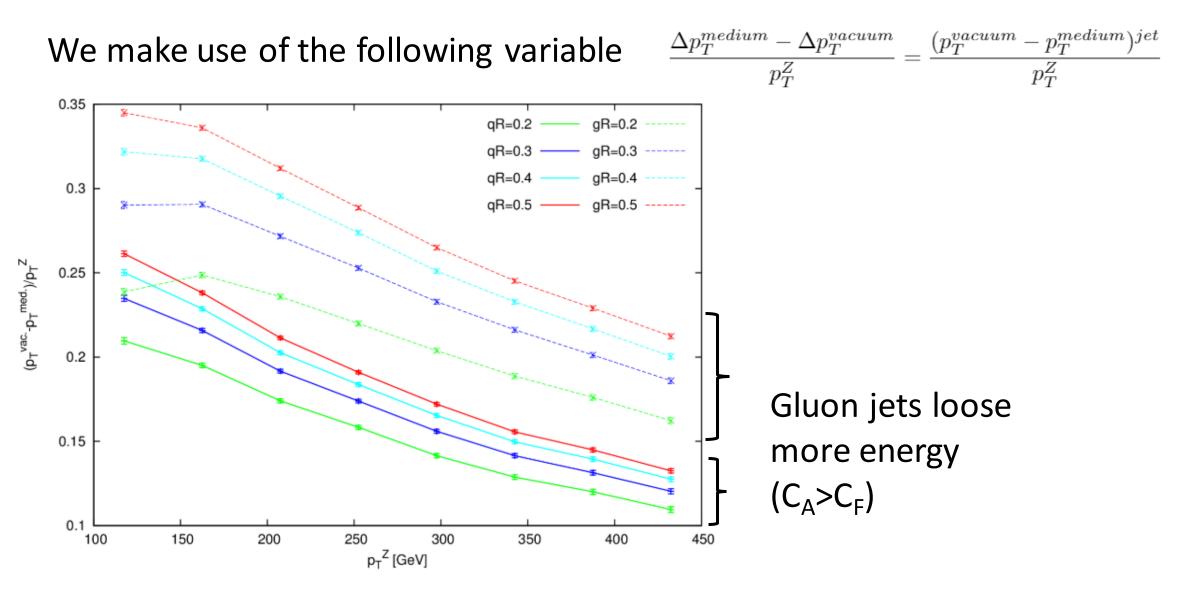
At the parton level this should scale with the Casimir associated to the parton

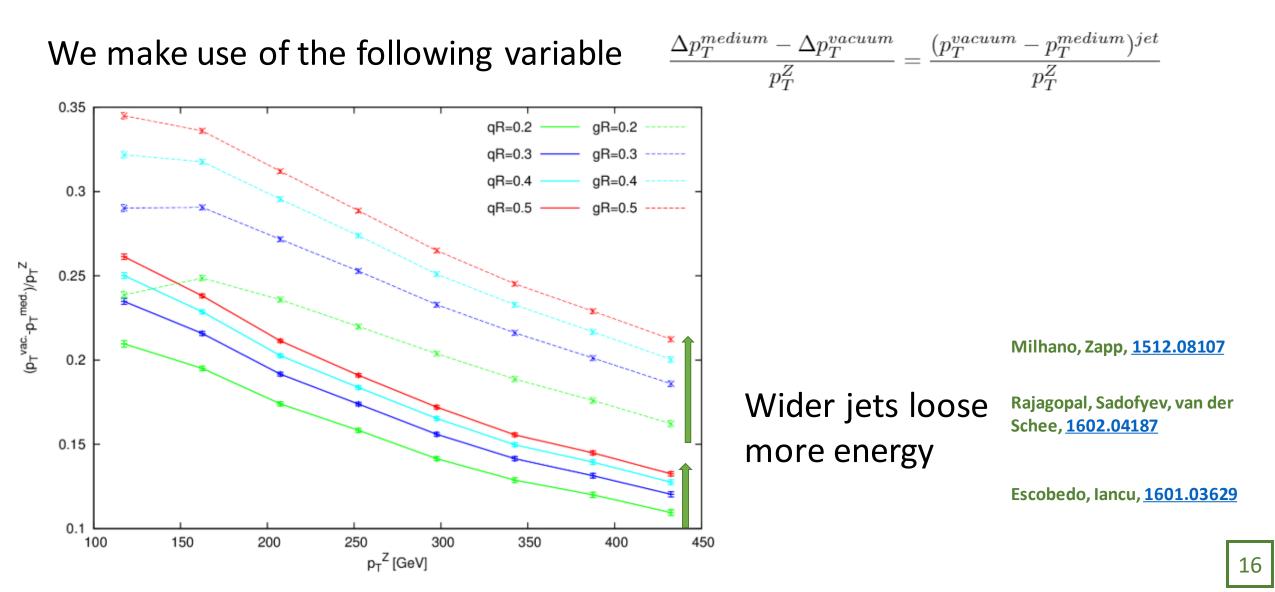
 $\Delta p_T^{medium} - \Delta p_T^{vacuum}$  $(p_T^{vacuum})$ We make use of the following variable  $p_T^Z$ 0.35 gR=0.2 gR=0.2 aR=0.3 aR=0.3 aR=0.4 aR=0.4 0.3 aR=0.5 aR=0.5  $(p_T^{vac.}-p_T^{med.})/p_T^Z$ 0.25 0.2 0.15 0.1 300 150 200 250 100 350 400 450 p<sub>T</sub><sup>Z</sup> [GeV]



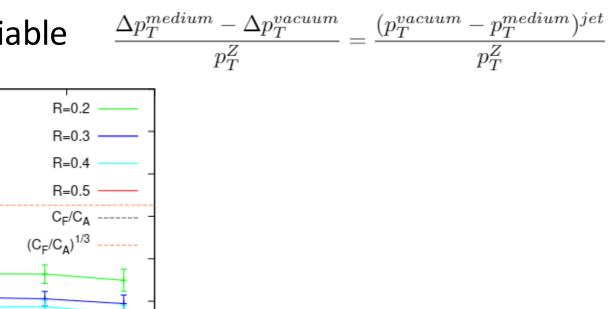
 $-p_T^{medium})^{jet}$ 

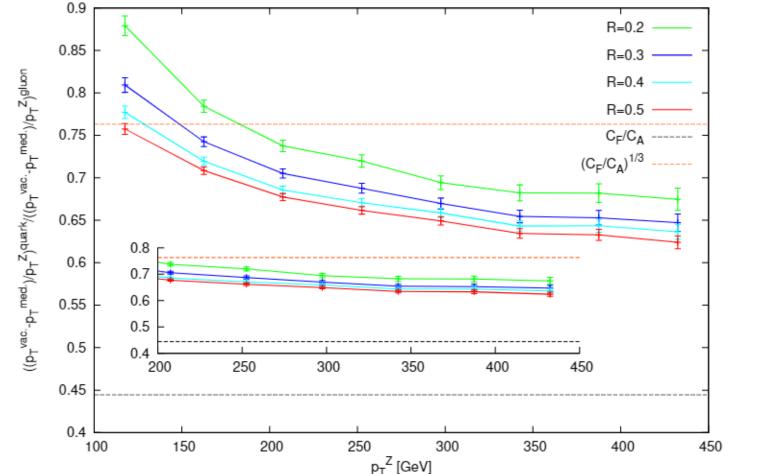
 $p_T^Z$ 





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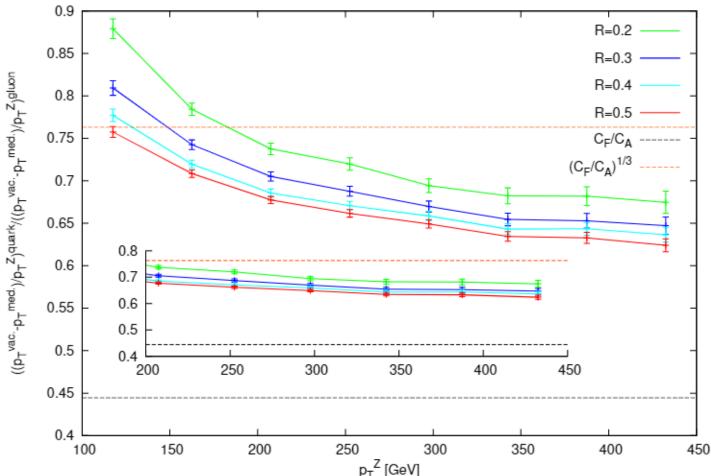




 $-p_T^{medium_jet}$  $\Delta p_T^{medium}$  $(p_T^{vacuum})$  $\Delta p_T^{vacuum}$ We make use of the following variable  $p_T^Z$ 0.9 R=0.2 Casimir scaling is clearly broken 0.85 R=0.3 ((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)quark/((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)gluon R=0.4 0.8 R=0.5 0.75 C<sub>F</sub>/C<sub>A</sub> However the breaking is given by  $(C_{c}/C_{A})^{1/3}$ 0.7 a rescaling 0.65 0.8 0.6 0.7 0.6 0.55 0.5 0.5 0.4 200 250 300 350 400 450 0.45 0.4 200 300 350 400 100 150 250 450 17 p<sub>T</sub><sup>Z</sup> [GeV]

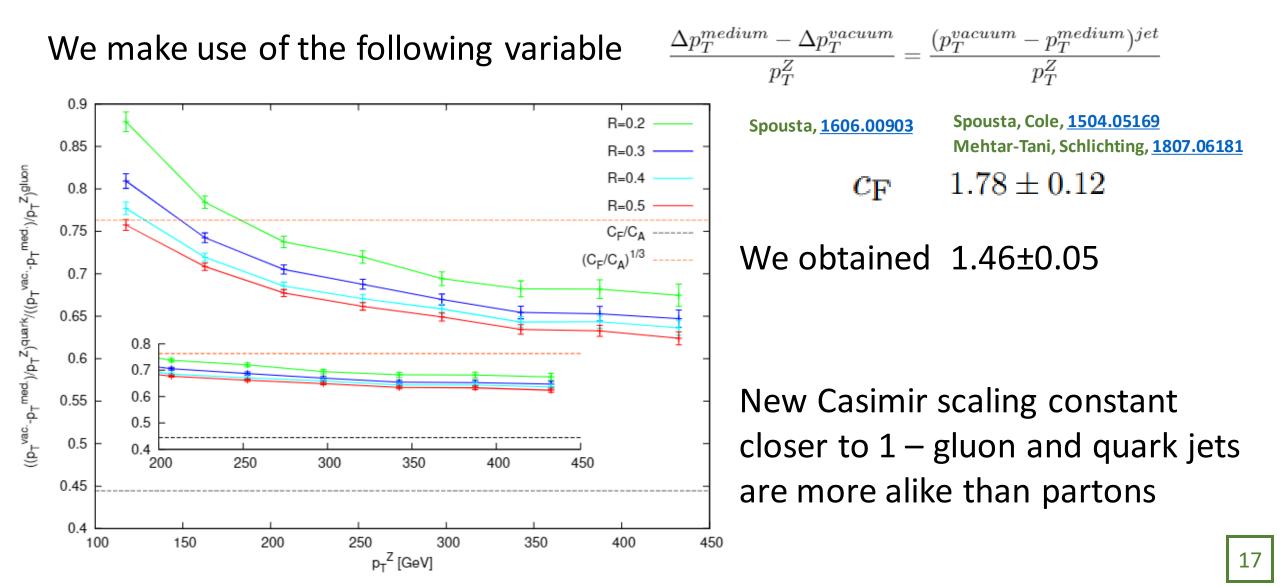
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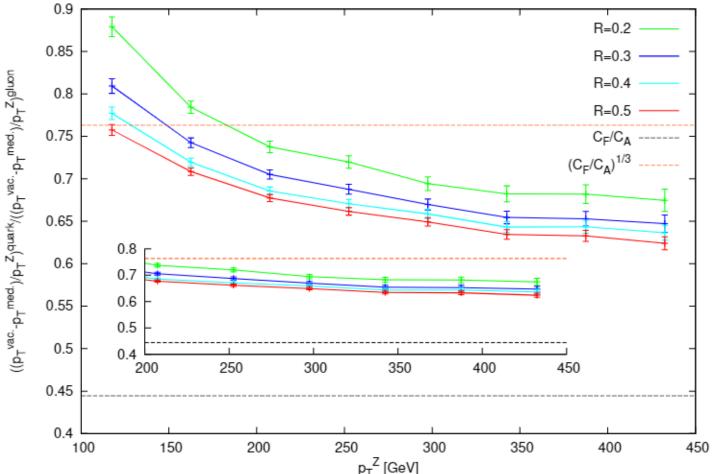


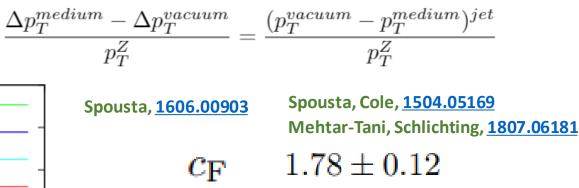
$\Delta p_T^n$	$\frac{p_T^{nedium} - \Delta p_T^{vacuum}}{p_T^Z} = \frac{q_T^{vacuum}}{p_T^Z}$	$\frac{p_T^{vacuum} - p_T^{medium})^{jet}}{p_T^Z}$
	Spousta, <u>1606.00903</u>	Spousta, Cole, <u>1504.05169</u>
	$\Delta p_T = c$	$_{\rm F} \cdot s \cdot \left(\frac{p_{T,\rm ini}}{p_{T,0}}\right)^{lpha}$
	$s = x \cdot N_{\text{part}} + y$	$x = (12.3 \pm 1.4) \cdot 10^{-3} \text{ GeV},$ $y = 1.5 \pm 0.2 \text{ GeV}$
	α	$0.52\pm0.02$
	$c_{ m F}$	$1.78\pm0.12$

New Casimir scaling constant closer to 1 – gluon and quark jets are more alike than partons



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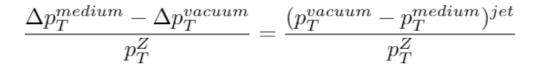


We obtained 1.46±0.05 However this value depends on the parametrization New Casimir scaling constant closer to 1 – gluon and quark jets are more alike than partons

17

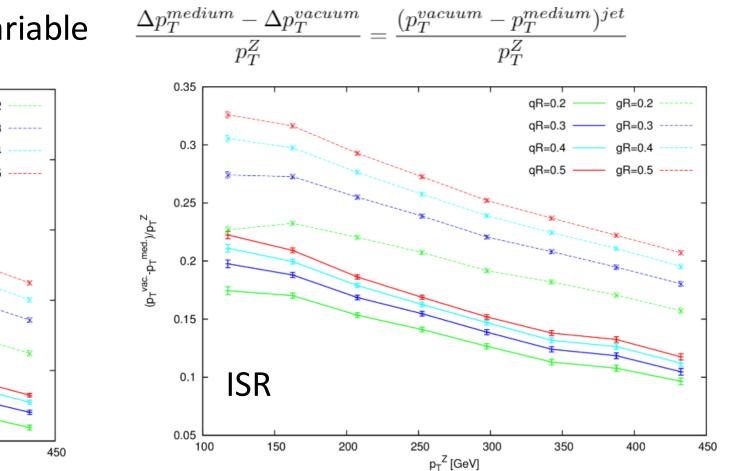
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 $\Delta p_T^{medium} -p_T^{medium_jet}$  $(p_T^{vacuum})$  $\Delta p_T^{vacuum}$ We make use of the following variable  $p_T^Z$  $p_T^Z$ 0.9 0.8 R=0.2 R=0.2 0.85 No ISR R=0.3 R=0.3 0.75  $((p_T^{vac}.-p_T^{med}.)/p_T^{-Z})^{quark}/((p_T^{vac}.-p_T^{med}.)/p_T^{-Z})^{gluon}$ ((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)quark/((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)gluon R=0.4 R=0.4 ISR 0.8 R=0.5 R=0.5 0.7 0.75 C<sub>F</sub>/C<sub>A</sub> C<sub>F</sub>/C<sub>A</sub> -----(C<sub>F</sub>/C<sub>A</sub>)<sup>1/3</sup>  $(C_{F}/C_{A})^{1/3}$ 0.65 0.7 0.65 0.6 0.8 0.8 0.6 0.7 0.55 0.7 0.6 0.55 0.6 0.5 0.5 0.5 0.5 0.4 0.4 250 200 300 350 400 450 200 250 300 350 400 450 0.45 0.45 0.4 0.4 150 200 350 400 100 250 300 450 150 300 350 100 200 250 400 450 p<sub>T</sub><sup>Z</sup> [GeV] p<sub>T</sub><sup>Z</sup> [GeV]

Now we include Hadronization (but no ISR) in the samples to see the impact

 $\Delta p_T^{medium} (p_T^{vacuum} - p_T^{medium})^{jet}$  $\Delta p_T^{vacuum}$ We make use of the following variable  $p_T^Z$  $p_T^Z$ 0.9 0.9 R=0.2 R=0.2 Parton level 0.85 Hadron level 0.85 R=0.3 R=0.3 ((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)quark/((p<sub>T</sub><sup>vac.</sup>-p<sub>T</sub><sup>med.</sup>)/p<sub>T</sub><sup>Z</sup>)gluon  $((p_T^{vac},-p_T^{med})/p_T^Z)^{quark}/((p_T^{vac},-p_T^{med})/p_T^Z)^{gluon}$ R=0.4 R=0.4 0.8 0.8 R=0.5 R=0.5 0.75 0.75 CF/CV C<sub>F</sub>/C<sub>∆</sub>  $(C_{E}/C_{A})^{1/3}$ (C<sub>F</sub>/C<sub>A</sub>)<sup>1/3</sup> 0.7 0.7 0.65 0.65 0.8 0.8 0.6 0.6 0.7 0.7 0.6 0.55 0.6 0.55 0.5 0.5 0.5 0.4 0.5 0.4 200 250 300 350 400 450 200 250 300 350 400 450 0.45 0.45 0.4 0.4 150 200 250 300 350 400 100 450 150 100 200 250 300 350 400 450 p<sub>T</sub><sup>Z</sup> [GeV] p<sub>T</sub><sup>Z</sup> [GeV]

350

p<sub>T</sub><sup>Z</sup> [GeV]

300

400

350

Finally, we explore varying the centralities of the events

R=0.2

R=0.3

R=0.4

R=0.5

(C<sub>F</sub>/C<sub>A</sub>)<sup>1/3</sup>

450

400

450

We make use of the following variable

0-10%

300

250

0.9

0.85

0.8

0.75

0.7

0.65

0.6

0.55

0.5

0.45

0.4

100

0.8

0.7

0.6

0.5

0.4

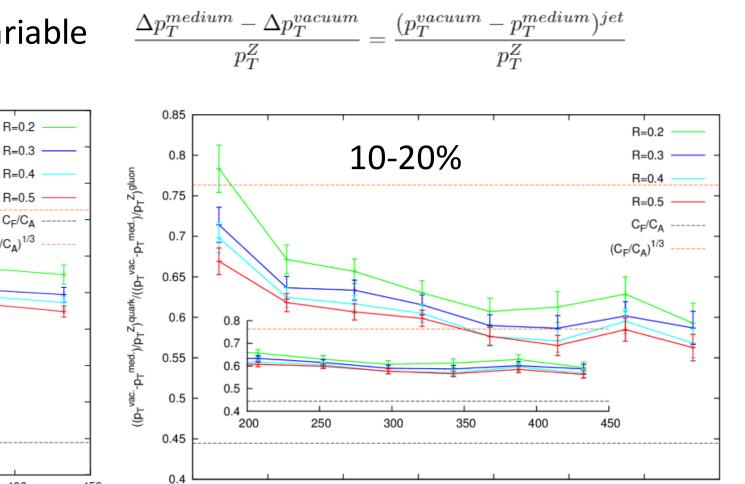
200

150

250

200

 $(p_{T}^{vac.}-p_{T}^{med.})/p_{T}^{Z})^{quark}/((p_{T}^{vac.}-p_{T}^{med.})/p_{T}^{Z})^{gluon}$ 



250

p<sub>T</sub><sup>Z</sup> [GeV]

300

150

100

200

20

450

400

350

Finally, we explore varying the centralities of the events

R=0.2

R=0.3

R=0.4

R=0.5

C<sub>F</sub>/C<sub>Δ</sub>

 $(C_{F}/C_{A})^{1/3}$ 

We make use of the following variable

0-10%

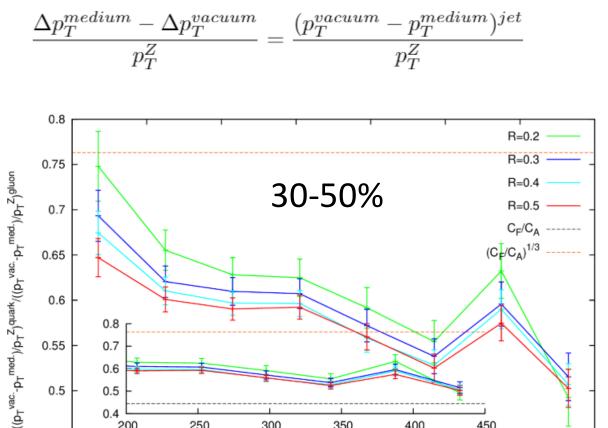
0.9

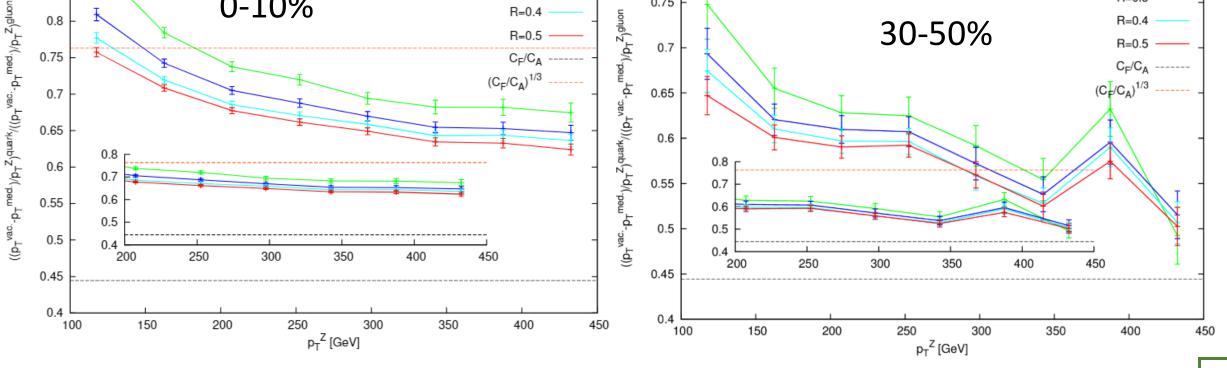
0.85

0.8

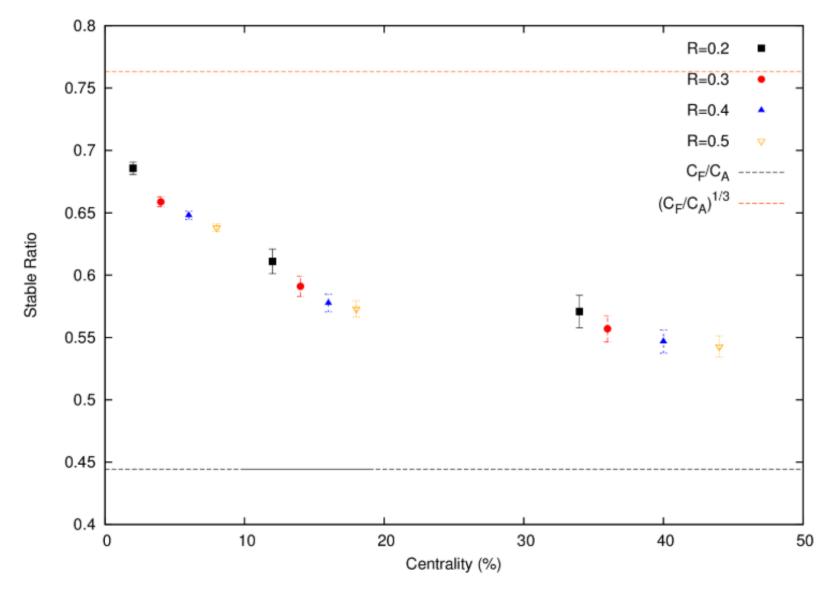
0.75

0.7



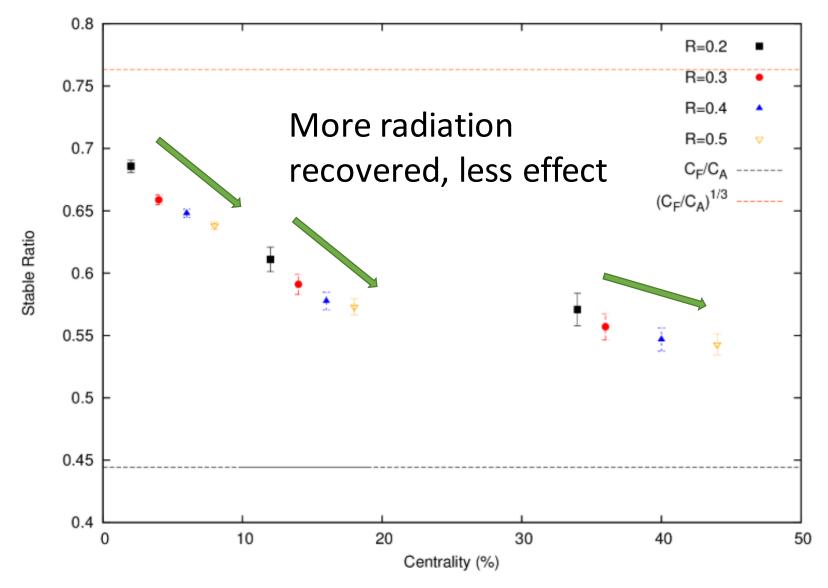


#### Finally, we explore varying the centralities of the events

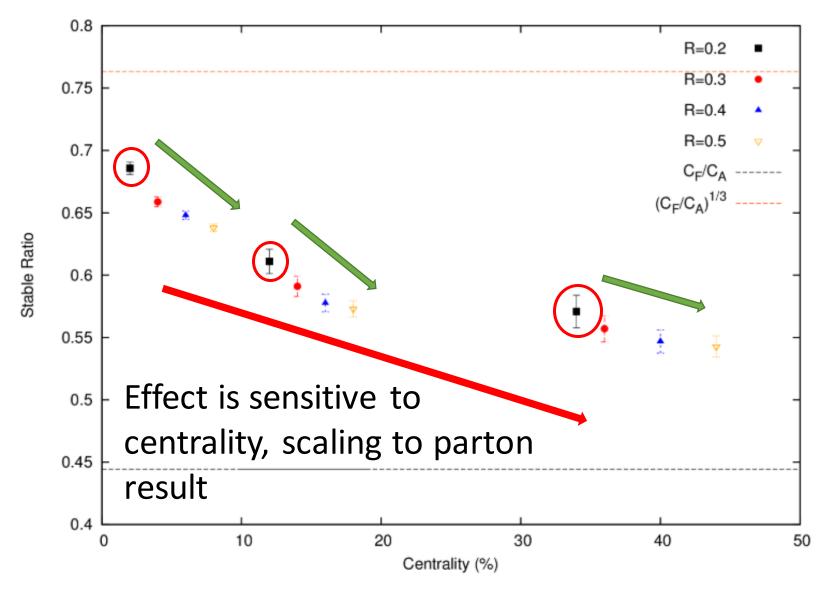


22





#### Finally, we explore varying the centralities of the events



## Conclusions

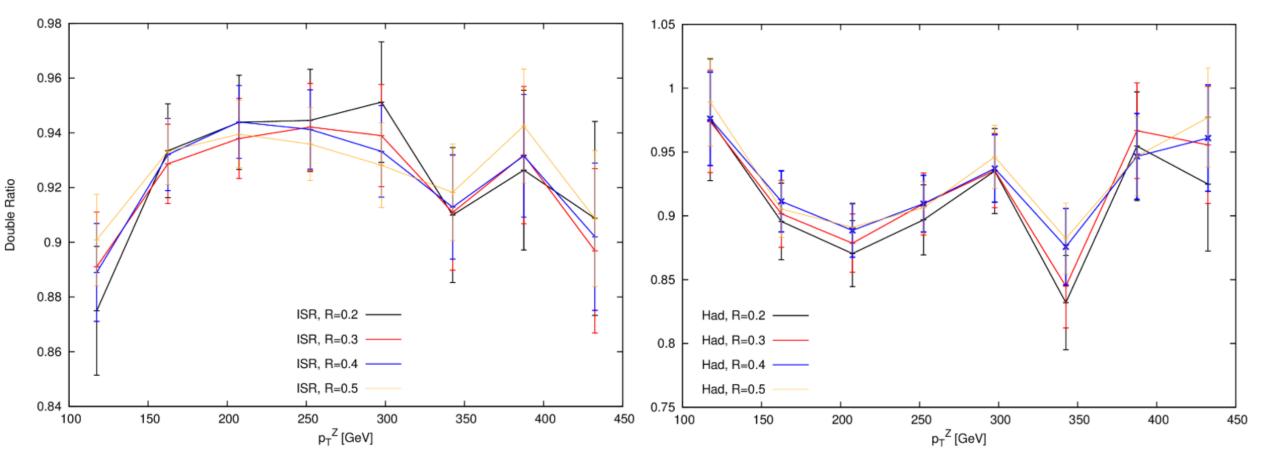
We recover a constant quark to gluon scaling but this is not given directly by the parton's Casimir

Initial flavor seems to be washed way by the medium possible physical picture: proliferation of gluons

Still to be done: include Q-Pythia for comparison

We make use of the following variable

$$\frac{\Delta p_T^{medium} - \Delta p_T^{vacuum}}{p_T^Z} = \frac{(p_T^{vacuum} - p_T^{medium})^{jet}}{p_T^Z}$$



We make use of the following variable

