FROM MICRO TO MACRO QCD PHENOMENA:

Origins of Collectivity in Nuclear Collisions

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COST Workshop Mini-School

February 26, 2019

Setting the stage

Goal: understand high-energy nuclear collisions

- "Small" systems (e.g., p+p and p+Pb collisions)
- "Intermediate" systems (e.g., O+O collisions)
- "Large" systems (e.g., Pb+Pb collisions)

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[1] Cf. Volodymyr Vovchenko's talk (Tuesday)
[2] Cf. Ilkka Helenius's talk (Tuesday)
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- *Light*: what can we learn from photons?[4]

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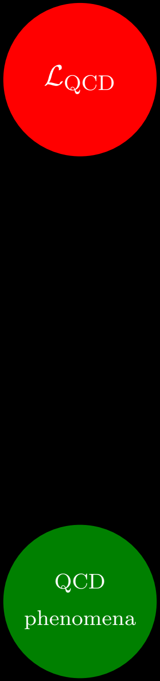
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QCD
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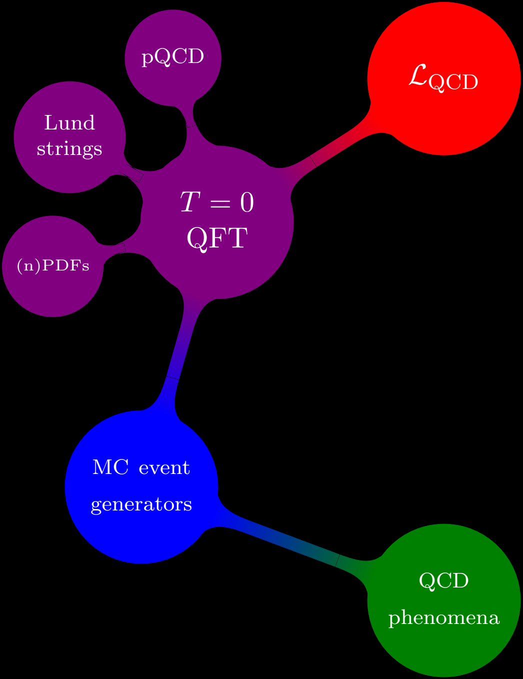
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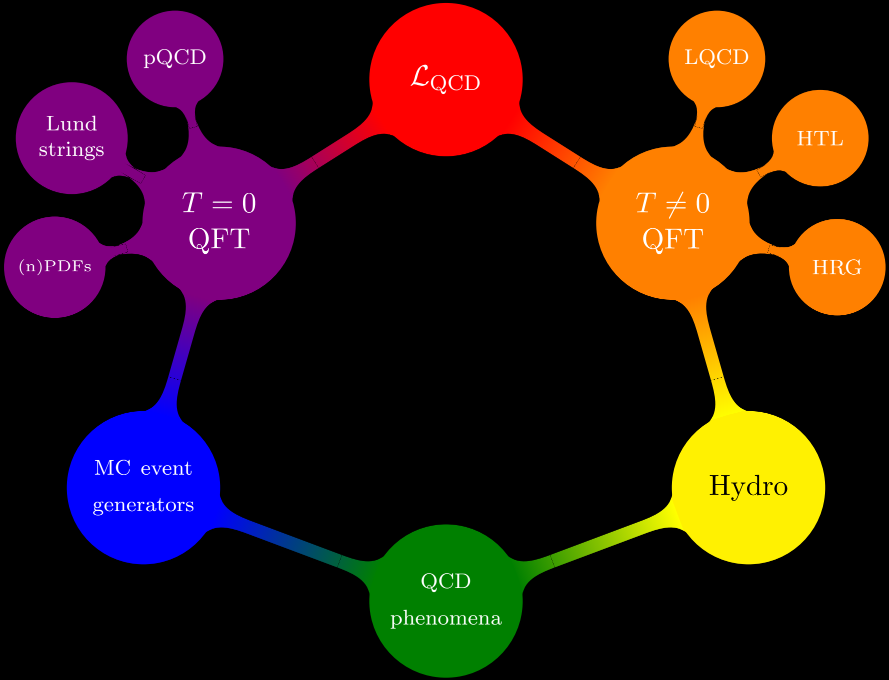
pQCD
Lund strings
$T = 0$ QFT
(n)PDFs
MC event generators
QCD phenomena



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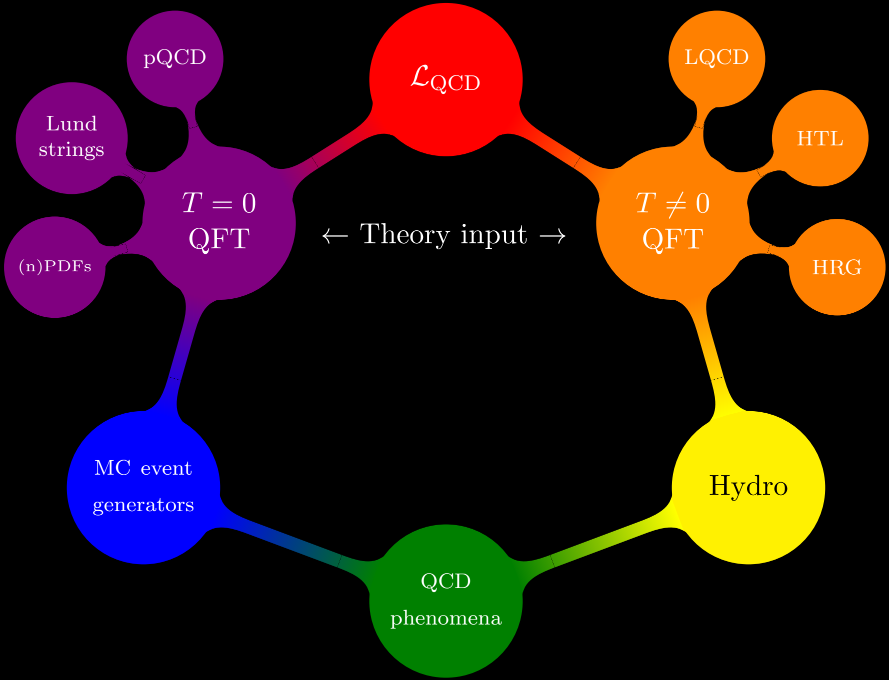

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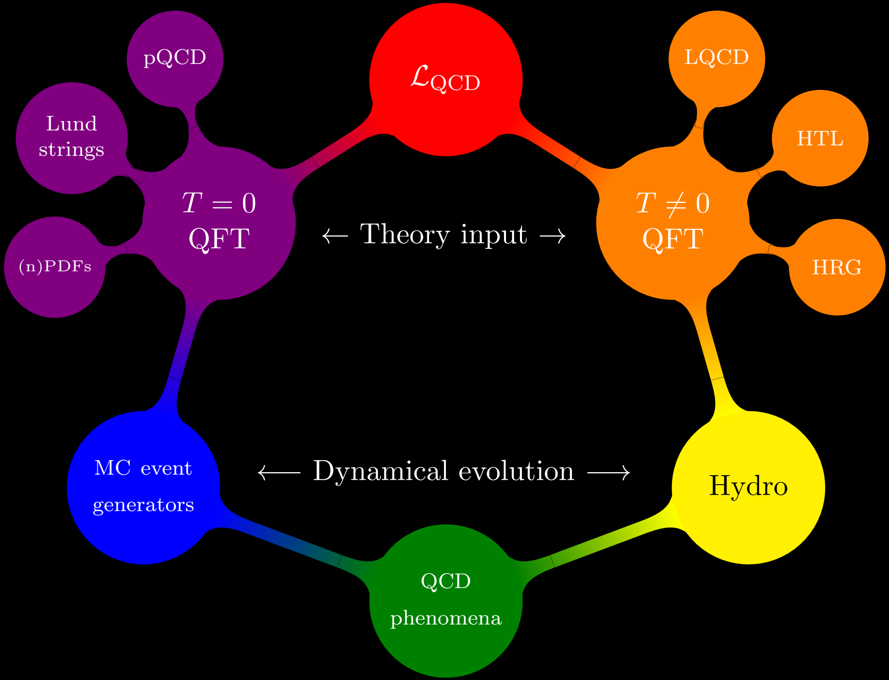

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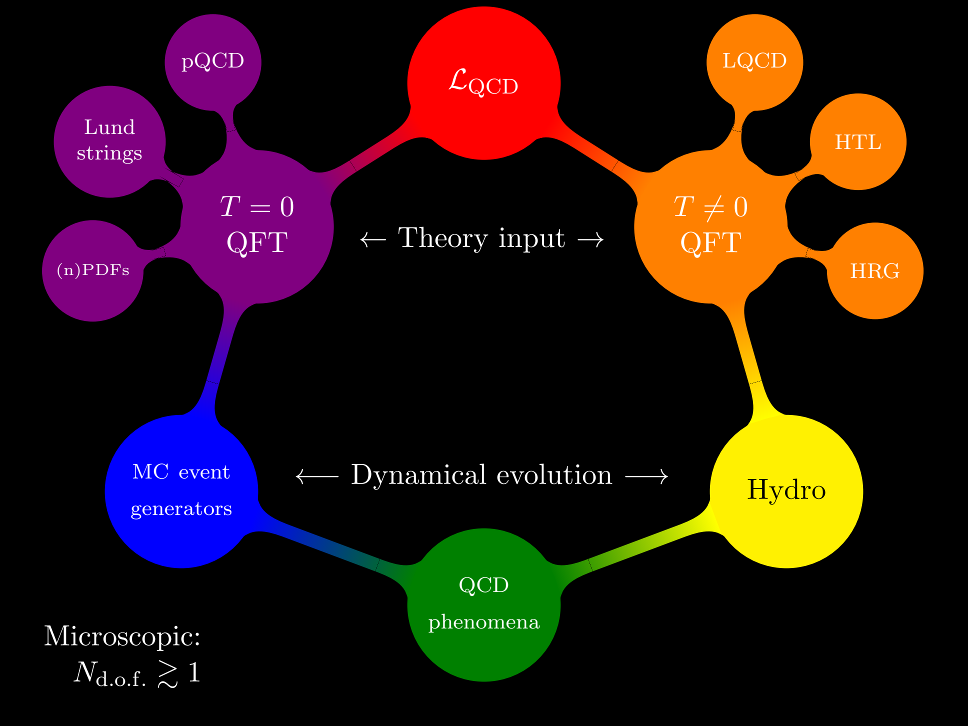
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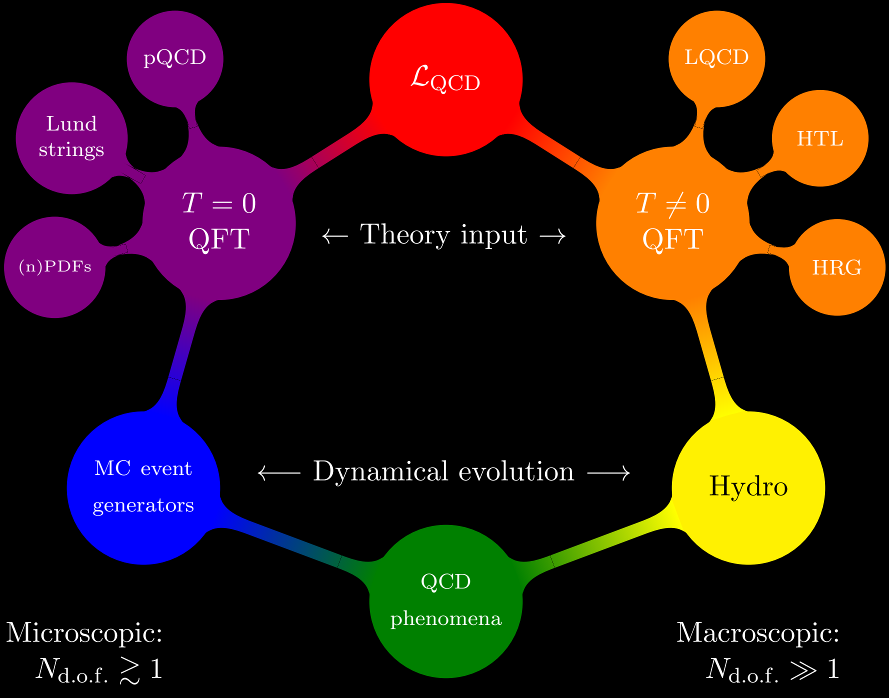

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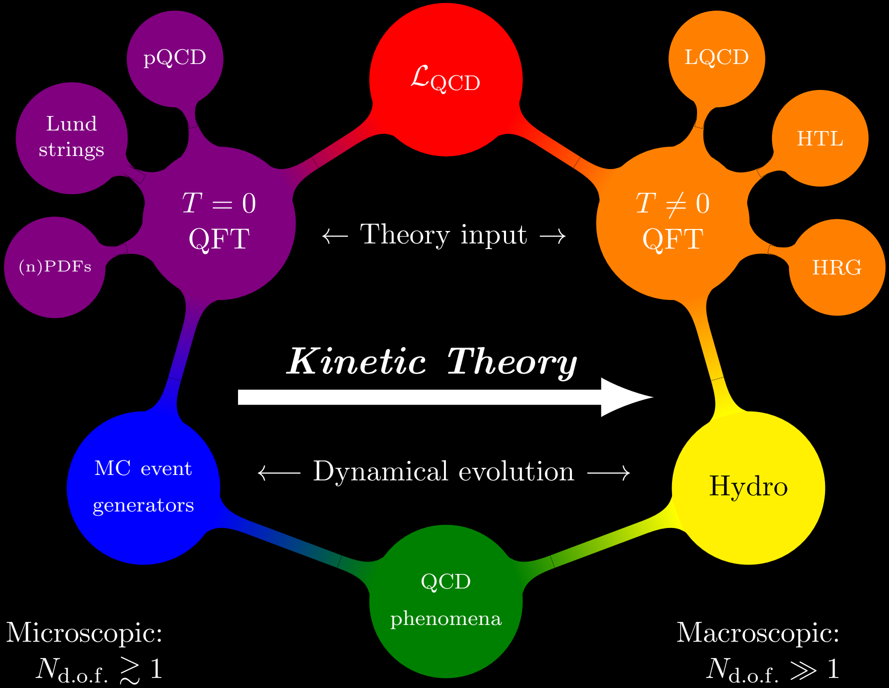
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This talk

Descriptions of nuclear collisions:

- *Microscopic approach*: kinetic theory
- *Macroscopic approach*: hydrodynamics and its relation to kinetic theory

Applications:

- When are kinetic theory and/or hydrodynamics valid?
- What is collectivity, and how is it related to hydrodynamics?

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Applications:

- When are kinetic theory and/or hydrodynamics valid?
- What is collectivity, and how is it related to hydrodynamics?

Conventions:

$$
\begin{aligned}
\hbar &= c = k_B = 1 \\
\sum_\mu x^\mu x_\mu &\equiv x^\mu x_\mu \\
g^{\mu\nu} &= \mathrm{diag}\{+1, -1, -1, -1\} \\
\text{“Local rest frame” (LRF)}: & \\
u^\mu &= u^\mu_{LRF} \equiv (1, \mathbf{0})
\end{aligned}
$$

Part I:

Introduction to kinetic theory and hydrodynamics

Introduction to kinetic theory and hydrodynamics

What is kinetic theory?

Kinetic theory:

- an approach to describing the evolution of systems
- composed of weakly coupled particles
- in terms of the *single-particle distribution* $f(\vec{r}, \vec{p}, t)$

Assumptions:

- System composed of weakly coupled particles
- Particle collisions are uncorrelated

$\Longrightarrow$ kinetic theory cannot describe strongly coupled systems!

Introduction to kinetic theory and hydrodynamics

Starting point: the *Boltzmann equation*:

$$\frac{\partial f}{\partial t} + \underbrace{\frac{\vec{p}}{m} \cdot \frac{\partial f}{\partial \vec{r}} - \frac{\partial V(\vec{r})}{\partial \vec{r}} \cdot \frac{\partial f}{\partial \vec{p}}}_{\text{“streaming terms”}} = \underbrace{C[f](\vec{r}, \vec{p}, t)}_{\text{“collision term”}}$$

Notation:

- $f(\vec{r}, \vec{p}, t)$: the single-particle distribution function
- $V(\vec{r})$: some external potential (e.g., gravity)

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Notation:

- $f(\vec{r}, \vec{p}, t)$: the single-particle distribution function
- $V(\vec{r})$: some external potential (e.g., gravity)
- Streaming terms describe evolution in absence of particle collisions
- Collision term describes internal interactions between particles in the system

Relativistic version:

$$\boxed{p^{\mu}\frac{\partial f}{\partial x^{\mu}} + mF^{\mu}\frac{\partial f}{\partial p^{\mu}} = C\left[f\right]}$$

From now on, I will assume that $F^{\mu} = 0$, for simplicity.

For $2 \to 2$ scattering, the collision term can be written

$$\begin{aligned} C\left[f\right] &= \frac{m}{2}\int d^3p_2\, d^3p_1'\, d^3p_2'\, w\left(12 \to 1'2'\right) \\ &\times \left[f\left(x, p_1'\right) f\left(x, p_2'\right) - f(x, p)\, f(x, p_2)\right] \end{aligned}$$

where $w\left(12 \to 1'2'\right)$ is the transition rate.

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where $w\left(12 \to 1'2'\right)$ is the transition rate.
In QCD, $C\left[f\right]$ also receives contributions from

- $1 \to 2$ scattering (e.g., gluon splitting)
- $2 \to 1$ scattering (e.g., gluon fusion)
- Etc.

What is hydrodynamics?

Introduction to kinetic theory and hydrodynamics

What is hydrodynamics?

Hydrodynamics:

- an approach to describing the evolution of systems
- based on collective flow of conserved quantities (e.g., $T^{\mu\nu}$)
- in terms of "course-grained," thermodynamic quantities like number density n and pressure P

Main assumption:

- System must be well-described by slowly-varying quantities in space and time

Next: let's illustrate some of these concepts with an example.

Example: gas of N non-interacting particles

[5]Recall that d^3p/p^0 is a Lorentz invariant.

[6]You can add more terms to this, but I will not worry about these today.

Example: gas of N non-interacting particles

Single-particle distribution:

$$f(x,p) \;=\; \sum_{i=1}^{N} \delta^3\left(\vec{x} - \vec{x}_i(t)\right) \delta^3\left(\vec{p} - \vec{p}_i\right)$$

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\text{where } \Delta^{\mu\nu}(x) &\equiv g^{\mu\nu} - u^\mu(x) u^\nu(x), \\
e(x) &= \left\langle \left(p^0\right)^2 \right\rangle, \text{ and } P(x) = \left\langle \frac{p^0}{3} \left(\vec{v} \cdot \vec{p}\right) \right\rangle .^7
\end{aligned}$$

[7] So far, $n(x)$, $e(x), \ldots$ still contain (microscopic) δ-functions and are not yet smooth, (macroscopic) thermodynamic variables. To convert them to genuinely smooth functions requires a procedure known as "*course graining*." From now on, I assume we have done this and will treat $n(x), \ldots$ as smoothly varying quantities in space-time.

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Kinetic theory and hydrodynamics

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Use the Boltzmann equation to find out!

$$\int d^3p \left(\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \frac{\partial f}{\partial \vec{r}} \right) = \int d^3p \, C\left[f\right] (\vec{r}, \vec{p}, t)$$

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$\rightarrow j^\mu$ is a conserved quantity! Similarly, $\partial_\mu T^{\mu\nu} = 0$.
We say that j^μ and $T^{\mu\nu}$ obey *conservation laws.*

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- **hydrodynamic lengthscale** $L_{\rm hydro}$:

$$L_{\rm hydro}^{-1} \sim \theta \sim \left|\partial_\mu \epsilon\right| / \epsilon \sim \cdots , \text{ where } \theta \equiv \partial_\mu u^\mu$$

  - the scale over which macroscopic quantities vary

2. When is kinetic theory or hydrodynamics valid?

A relativistic system can be characterized by **three lengthscales**:

- **mean free path** $\lambda_{\mathrm{mfp}} \sim 1/\left(\langle \sigma v \rangle n\right)$
  - the typical distance a particle travels before scattering
- **thermal de Broglie wavelength** $\lambda_{\mathrm{th}} \sim 1/T$
  - the typical size of a particle at temperature T
- **hydrodynamic lengthscale** L_{hydro}:

$$L_{\mathrm{hydro}}^{-1} \sim \theta \sim \left|\partial_\mu \epsilon\right| / \epsilon \sim \cdots \,, \text{ where } \theta \equiv \partial_\mu u^\mu$$

  - the scale over which macroscopic quantities vary

Ratios among these lengthscales *give us regimes of validity*:

- for kinetic theory: $\lambda_{\mathrm{mfp}}/\lambda_{\mathrm{th}}$
- for hydrodynamics: $\lambda_{\mathrm{mfp}}/L_{\mathrm{hydro}}$

Let's rewrite these ratios in a more illuminating form.

Kinetic theory and hydrodynamics

Kinetic theory estimates the *shear viscosity* η of a gas to be

$$\eta \sim \frac{1}{3} n \langle p \rangle \lambda_{\mathrm{mfp}} \sim s \langle p \rangle \lambda_{\mathrm{mfp}} \sim \lambda_{\mathrm{mfp}} T^4$$

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Micro-to-micro (kinetic theory):

$$\frac{\lambda_{\mathrm{mfp}}}{\lambda_{\mathrm{th}}} \sim \frac{\eta}{T^3} \sim \frac{\eta}{s} \quad \text{(the “specific shear viscosity”)}$$

$\rightarrow$ compares scale of inter-particle collisions with typical particle sizes

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$$\frac{\lambda_{\mathrm{mfp}}}{L_{\mathrm{hydro}}} \sim \frac{\eta\theta}{sT} \equiv \mathrm{Kn} \quad \text{(the “Knudsen number”)}$$

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What does this tell us about kinetic theory and hydrodynamics?

When is kinetic theory valid?

1. *Dilute gas regime*:
$$\frac{\lambda_{\mathrm{mfp}}}{\lambda_{\mathrm{th}}} \sim \frac{\eta}{s} \gg 1$$
Particles travel a long time between collisions, meaning they are (mostly) on-shell and collisional broadening is negligible
$\rightarrow$ *kinetic theory works best here*
2. *Dense gas regime*:
$$\frac{\lambda_{\mathrm{mfp}}}{\lambda_{\mathrm{th}}} \sim \frac{\eta}{s} \sim 1$$
Particles collide frequently and are consistently off-shell
$\rightarrow$ *kinetic theory must be improved with quantum kinetic approach*
3. *Liquid regime*:
$$\frac{\lambda_{\mathrm{mfp}}}{\lambda_{\mathrm{th}}} \sim \frac{\eta}{s} \ll 1$$
No well-defined particle states
$\rightarrow$ *kinetic theory no longer applicable*

Kinetic theory and hydrodynamics

When is hydrodynamics valid?

Recall: $\mathrm{Kn} = \frac{\eta}{sT} \cdot \theta$

1. Ideal (perfect) hydrodynamics:

$$\mathrm{Kn} \approx 0 : \eta/s \approx 0 \text{ or } \theta \approx 0$$

   $\rightarrow$ *strong coupling and/or weak expansion ensures validity of hydrodynamics*

2. Viscous (non-ideal) hydrodynamics:

$$\mathrm{Kn} \lesssim 1 : \text{either } \eta/s \text{ or } \theta \text{ sufficiently small}$$

   $\rightarrow$ *moderate coupling and/or moderate expansion requires viscous hydrodynamics*

3. Hydrodynamics invalid:

$$\mathrm{Kn} \gg 1 : \eta/s \text{ and } \theta \text{ both large}$$

   $\rightarrow$ *coupling is too weak and/or expansion is too strong for hydrodynamics to work*

Hydrodynamics and kinetic theory: a short formulary

Kinetic theory:

$$\boxed{\text{Boltzmann equation: } p^\mu \frac{\partial f}{\partial x^\mu} = C\left[f\right]}$$

Hydrodynamics:

$$\boxed{\text{Conservation laws: } \partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j^\mu = 0}$$

Dictionary:

$$\begin{aligned}
\text{Particle current: } j^\mu(x) &= \int \frac{d^3p}{p^0} p^\mu f(x,p) \\
\text{Energy-momentum: } T^{\mu\nu}(x) &= \int \frac{d^3p}{p^0} p^\mu p^\nu f(x,p) \\
\text{Entropy flow: } s^\mu(x) &= \int \frac{d^3p}{p^0} p^\mu f(x,p)\left(1 - \ln f(x,p)\right)
\end{aligned}$$

Recap

- Kinetic theory describes systems composed of particles
  - Only describes systems composed of weakly coupled particles
  - The single-particle distribution f is described by the Boltzmann equation (BE)
  - Collision term implements relevant microscopic dynamics

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- Kinetic theory describes systems composed of particles
  - Only describes systems composed of weakly coupled particles
  - The single-particle distribution f is described by the Boltzmann equation (BE)
  - Collision term implements relevant microscopic dynamics
- Hydrodynamics replaces microscopic quantities (e.g., f) with macroscopic equivalents (n, $T^{\mu\nu}$, etc.)
  - Micro $\leftrightarrow$ macro transition effected by "course-graining"
  - Works best when $N_{\mathrm{d.o.f.}} \gg 1$
  - BE for $f \Longleftrightarrow$ conservations laws for j^{μ}, $T^{\mu\nu}$

Part II:

Hydrodynamics and collectivity

So what *is* collectivity?

Basically, it’s the difference between

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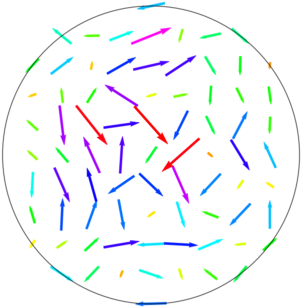
this



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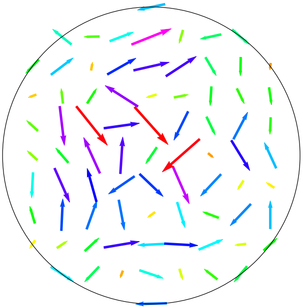

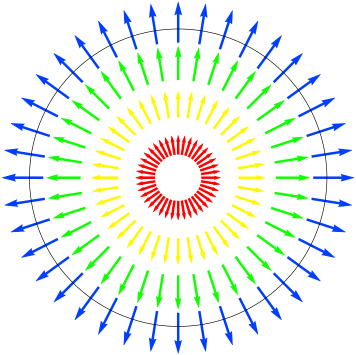

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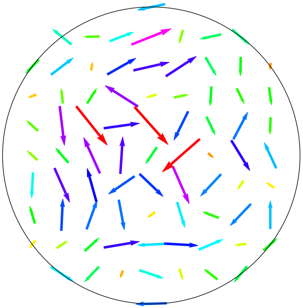

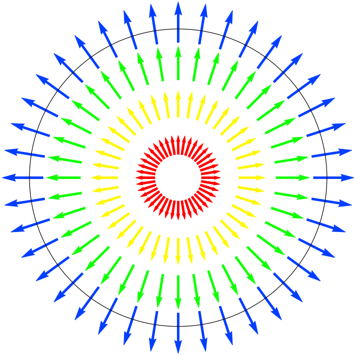

More precisely: "Collectivity" means

- fluid-like velocity profile and behavior
- strong position-momentum (x-p) correlations

Conservation laws

- *All* dynamics arises from requiring energy-momentum conservation and, if necessary, number (charge) conservation:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ \partial_\mu J_i^\mu &= 0, \end{aligned}$$

where i ranges over the conserved charges in the system

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\end{aligned}
$$

  where i ranges over the conserved charges in the system
- In general, must include all relevant J_i^μ, but ignore today for simplicity; focus on $T^{\mu\nu}$
- Two important questions about the conservation laws:
  - How do we solve them?
  - What do we learn from them?

Conservation laws: how to solve them

In general, 4 separate *equations of motion* (EoMs):

$$\begin{array}{rcl} \underline{\text{EoM}} & : & \underline{\#\text{ of constraints}} \\ \partial_\mu T^{\mu\nu} = 0 & : & 4 \quad (\nu = 0, 1, 2, 3) \end{array}$$

But 5 total unknowns:

$$\begin{array}{rcl} \underline{\text{Quantity}} & : & \underline{\#\text{ of unknowns}} \\ e(x) & : & 1 \\ P(x) & : & 1 \\ u^\mu(x) & : & 3 \quad (\text{since } u^\mu u_\mu = 1) \end{array}$$

Total unknowns > total constraints $\Longrightarrow$ *system is underdetermined!*

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Total unknowns > total constraints $\Longrightarrow$ *system is underdetermined!*
Need additional *equation of state* (EoS) to get unique solution:

$$P = P\left(e, \{n_i\}\right) \longleftrightarrow P\left(T, \{\mu_i\}\right)$$

EoS encodes the microscopic properties of the system.

Conservation laws: what we learn

What physics is implied by a hydrodynamic approach?

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Recall: $T^{\mu\nu}(x) = \left[e(x) + P(x)\right] u^{\mu}(x)u^{\nu}(x) - P(x)g^{\mu\nu}$

$$0 = \partial_{\mu}T^{\mu\nu}$$

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where $\theta \equiv \partial_\mu u^\mu$, $\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$, and

$$\dot{X} \equiv u^\mu \partial_\mu X, \quad \nabla^\mu X \equiv \Delta^{\mu\nu}\partial_\nu X$$

are covariant derivatives w.r.t. *time* and *space*, respectively;

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are covariant derivatives w.r.t. *time* and *space*, respectively; i.e.,

$$^{(*)}\text{in LRF } u^\mu = (1, \mathbf{0}): \ \dot{X} \to \partial_t X, \quad \nabla^\mu X \to \vec{\nabla} X$$

Conservation laws: what we learn

$$\partial_\mu T^{\mu\nu} = 0 \Longrightarrow u^\nu \dot{e} + (e + P)\left[u^\nu \theta + \dot{u}^\nu\right] - \nabla^\nu P = 0,$$

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Let's apply $u_\nu\left(\cdots\right)$ to this result.

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$$\begin{aligned}
u_\nu u^\nu &= 1 \\
u_\nu \dot{u}^\nu &= u_\nu u^\mu \partial_\mu u^\nu = \frac{1}{2} u^\mu \partial_\mu \left(u_\nu u^\nu\right) \\
&= \frac{1}{2} u^\mu \partial_\mu (1) = 0 \\
u_\nu \nabla^\nu X &= u_\nu \left(g^{\mu\nu} - u^\mu u^\nu\right) \partial_\mu X \\
&= \left(u^\mu - u^\mu\right) \partial_\mu X = 0
\end{aligned}$$

Conservation laws: what we learn

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$$\begin{aligned}
u_\nu u^\nu &= 1 \\
u_\nu \dot{u}^\nu &= u_\nu u^\mu \partial_\mu u^\nu = \frac{1}{2} u^\mu \partial_\mu (u_\nu u^\nu) \\
&= \frac{1}{2} u^\mu \partial_\mu (1) = 0 \\
u_\nu \nabla^\nu X &= u_\nu (g^{\mu\nu} - u^\mu u^\nu) \partial_\mu X \\
&= (u^\mu - u^\mu) \partial_\mu X = 0
\end{aligned}$$

Then you can show$^{(*)}$

$$\boxed{\dot{e} = -(e+P)\theta}$$

and

$$\boxed{\dot{u}^\nu = \frac{\nabla^\nu P}{e+P}}$$

Conservation laws: what we learn

- $\boxed{\dot{e} = -(e+P)\,\theta}$
  - $\dot{e}$: time-derivative of energy density e
  - θ: scalar expansion rate (i.e., four-divergence)
  - Since $e + P > 0$,

$$\theta > 0 \Longleftrightarrow \dot{e} < 0 \text{ and } \textit{vice versa},$$

$\rightarrow$ *expansion decreases the energy density and v.v.*

Conservation laws: what we learn

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- $\boxed{\dot{u}^\nu = \frac{\nabla^\nu P}{e + P}}$
  - $\dot{u}^\nu$: net acceleration of fluid element
  - $e + P$: relativistic "mass" of fluid element
  - $\nabla^\nu P$: net force of pressure gradient on fluid element

$\rightarrow$ *relativistic hydrodynamics version of* $\vec{F} = m\vec{a}$

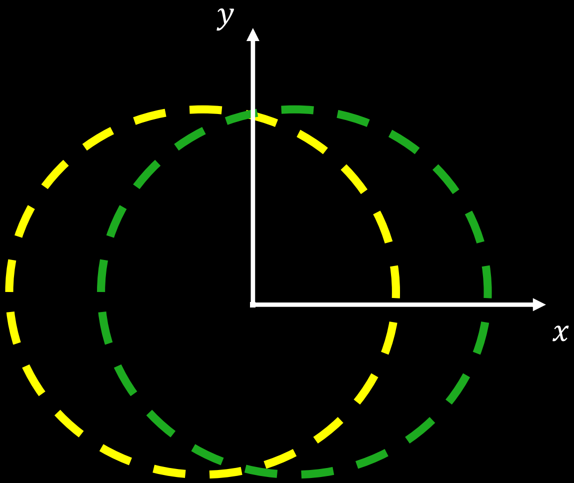
Conservation laws: what we learn

Relativistic Euler equation:

$$\boxed{\dot{u}^\nu = \frac{\nabla^\nu P}{e+P} = \frac{c_s^2}{1+c_s^2}\frac{\nabla^\nu e}{e}, \quad c_s^2 = \frac{\partial P}{\partial e}}$$

What does it mean?

- Hydrodynamics predicts a *collective*, momentum-space response to coordinate-space gradients of pressure or density
- Large speed-of-sound c_s^2 (“stiff” EoS) means a strong response; small c_s^2 (“soft” EoS) means a weak response



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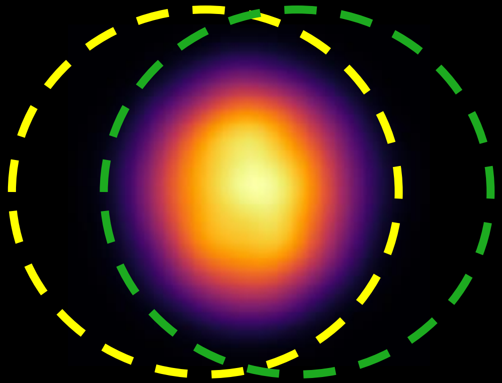
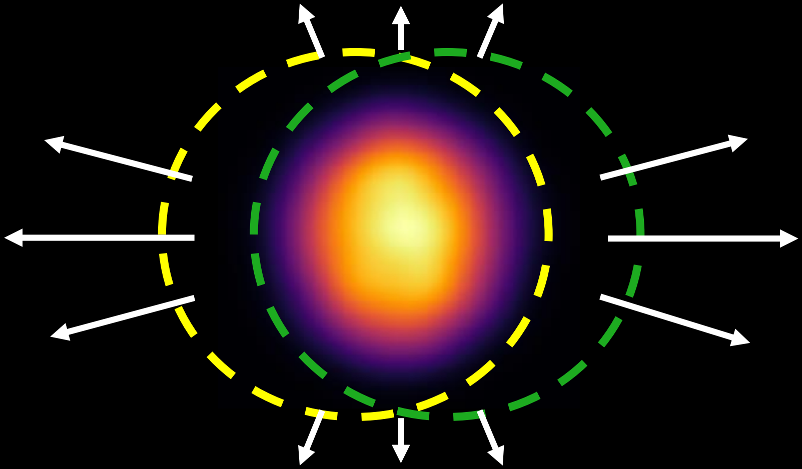
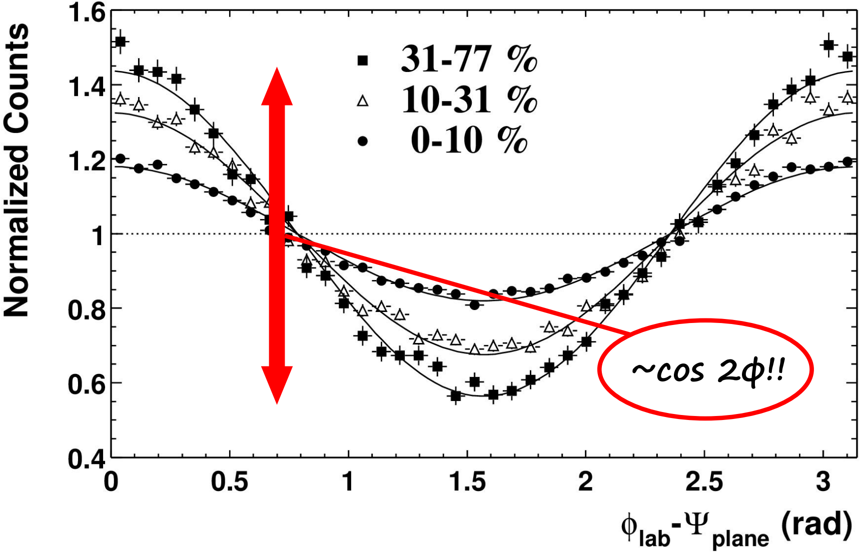
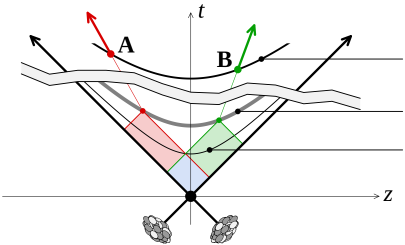


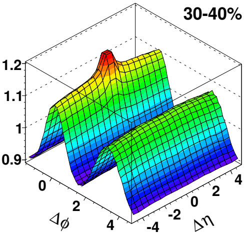




C. Adler *et al.* [STAR Collaboration], PRL **90**, 032301 (2003)

When is this collective response produced?





- Two space-time events A and B are correlated only if their past light-cones overlap
- For given separation in spatial rapidity $\eta_A - \eta_B$, the *latest* τ_0 when the correlations could have been produced is

$$\tau_0 = \tau_f \exp\left(-\frac{1}{2}\left|\eta_A - \eta_B\right|\right)$$

A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A **810**, 91 (2008)
G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. C **86**, 014907 (2012)

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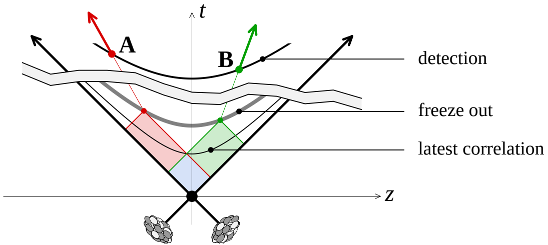

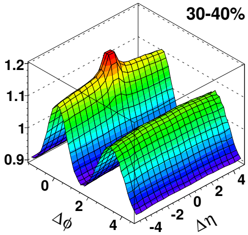

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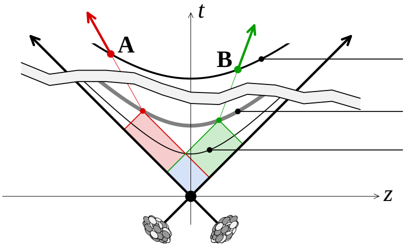

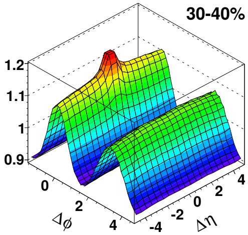

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  for $|y_A - y_B| = 5$ and $\tau_f = 10$ fm/c.
- Collective response is generated very early in collision evolution!

$\Longrightarrow$ Collectivity consistent with hydrodynamic response to initial geometry!

A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A **810**, 91 (2008)
G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. C **86**, 014907 (2012)

Recap

- Hydrodynamical conservation laws...
  - are coupled, non-linear differential equations for thermodynamic quantities (j^μ, $T^{\mu\nu}$, etc.)
  - require input from theoretical descriptions of QCD
  - predict tight correlations between geometry and final-state spectra which are established *early* in the collision

Conclusions

- Both kinetic theory and hydrodynamics are useful tools for describing the evolution of complex systems
  - Kinetic theory works best for weakly coupled, dilute systems
  - Hydrodynamics works best in systems with mean free path much smaller than the scale of variation in thermodynamic quantities
  - We can specify these regimes of validity quantitatively
  - The two regimes are not mutually exclusive!

[8] Cf. Leif Lönnblad's talk (Monday)

Conclusions

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  - Kinetic theory works best for weakly coupled, dilute systems
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  - We can specify these regimes of validity quantitatively
  - The two regimes are not mutually exclusive!
- Hydrodynamics *implies* collectivity, but not *vice versa*
  - There may be alternatives to hydrodynamics (e.g., initial-state correlations or string shoving) which generate collective behavior[8]
  - Is hydrodynamics responsible for collectivity in all collision systems? Or are there other mechanisms at play in smaller systems?

[8] Cf. Leif Lönnblad's talk (Monday)

Where to from here?

Outstanding questions:

- Do the conditions for hydrodynamics apply in high-energy nuclear collisions?
- What is the smallest possible system in which they can apply?
- Can a kinetic-theory approach explain all available data? Which data truly require a hydrodynamic approach?

Look forward to exciting progress this week!

Thanks for your attention!

Further reading

Introductions to Kinetic Theory and Hydrodynamics

- http://www.damtp.cam.ac.uk/user/tong/kintheory/kt.pdf
- https://courses.physics.ucsd.edu/2015/Fall/physics210b/LECTURES/CH05.pdf
- D. H. Rischke, Lect. Notes Phys. **516**, 21 (1999)
- https://www.phys.unideb.hu/mta-deparg/sites/default/files/seminar/etele_molnar_2012.pdf
- S. Jeon and U. Heinz, Int. J. Mod. Phys. E **24**, 1530010 (2015)

Collectivity and Nuclear Collisions

- U. W. Heinz, hep-ph/0407360.
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Backup slides

An aside on course-graining [Back]

To convert $n(x)$ from microscopics (δ-functions) to macroscopics (a smooth function), we *course-grain* in the following way:

$$n(t,\vec{x}) \to n_{\mathrm{CG}}(t,\vec{x}) \quad = \quad \lim_{\epsilon\to 0} \frac{3}{4\pi\epsilon^3} \int d^3x' \, \theta\left(\left|\vec{x}' - \vec{x}\right| - \epsilon\right) n(t,\vec{x}')$$

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In other words, "course-graining" means

1. "bin" all particles by their phase-space coordinates x and p
2. make the binwidth ϵ as small as possible
3. interpret the resulting histogram as a smooth-ish function in x and p

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Comments:

- Course-graining works best for $N \gg 1$, hence this condition for a macroscopic treatment to be valid.
- Course-graining generalizes to non-classical microscopics too
- Hereafter I will pretend that the course-graining has been carried out, and will treat $n(x)$ (etc.) as smooth functions.