Interferometric Signatures of Hydrodynamics in Small Systems

Christopher J. Plumberg

Lund University

MPI @ LHC, Prague, Czech Republic

November 20, 2019

LUND UNIVERSITY

We’ve seen collectivity in small systems.

Where does it come from?

We've seen collectivity in small systems.

Where does it come from?

Possible explanations include:

- Initial-state correlations (e.g., CGC)
- String hadronization models (e.g., Lund string)
- Escape mechanisms
- *Hydrodynamics*

We've seen collectivity in small systems.

Where does it come from?

Possible explanations include:

- Initial-state correlations (e.g., CGC)
- String hadronization models (e.g., Lund string)
- Escape mechanisms
- *Hydrodynamics*

How can we discriminate between models?

Various observables:

- Anisotropic flow, strangeness enhancement, etc.
- **This talk**: Hanbury Brown–Twiss (HBT) interferometry
  - Collective signatures include K_T-scaling of radii, etc.
  - **Today**: *discuss scaling of radii with* $dN_{\mathrm{ch}}/d\eta$

HBT from hydrodynamics

$$C(\vec{p}_1, \vec{p}_2) \equiv E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \Big/ \left(E_{p_1} \frac{d^3 N}{d^3 p_1} E_{p_2} \frac{d^3 N}{d^3 p_2} \right)$$

HBT from hydrodynamics

$$
\begin{aligned}
C(\vec{p}_1, \vec{p}_2) &\equiv E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \Big/ \left(E_{p_1} \frac{d^3 N}{d^3 p_1} E_{p_2} \frac{d^3 N}{d^3 p_2} \right) \\
\to C_{\text{fit}}(\vec{q}, \vec{K}) &\equiv 1 + \lambda \exp \left(- \sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j \right) \\
\vec{q} &\equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2} \left(\vec{p}_1 + \vec{p}_2 \right)
\end{aligned}
$$

HBT from hydrodynamics

$$
\begin{aligned}
C(\vec{p}_1, \vec{p}_2) &\equiv E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \Big/ \left(E_{p_1} \frac{d^3 N}{d^3 p_1} E_{p_2} \frac{d^3 N}{d^3 p_2} \right) \\
\to C_{\mathrm{fit}}(\vec{q}, \vec{K}) &\equiv 1 + \lambda \exp\left(- \sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j \right) \\
\vec{q} &\equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2}\left(\vec{p}_1 + \vec{p}_2\right) \\
C_{\mathrm{th}}(\vec{q}, \vec{K}) &\approx 1 + \left| \frac{\int d^4x\, S(x,K) e^{iq\cdot x}}{\int d^4x\, S(x,K)} \right|^2
\end{aligned}
$$

HBT from hydrodynamics

$$
\begin{aligned}
C(\vec{p}_1, \vec{p}_2) &\equiv E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \Big/ \left(E_{p_1} \frac{d^3 N}{d^3 p_1} E_{p_2} \frac{d^3 N}{d^3 p_2} \right) \\
\to C_{\text{fit}}(\vec{q}, \vec{K}) &\equiv 1 + \lambda \exp\left(- \sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j \right) \\
\vec{q} &\equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2}\left(\vec{p}_1 + \vec{p}_2\right) \\
C_{\text{th}}(\vec{q}, \vec{K}) &\approx 1 + \left| \frac{\int d^4x\, S(x,K) e^{iq\cdot x}}{\int d^4x\, S(x,K)} \right|^2
\end{aligned}
$$

Hydrodynamic $S(x,p)$ defined by Cooper-Frye formula:

$$
S(x,p) = \frac{1}{(2\pi)^3} \int_{\Sigma(x_f)} \frac{p \cdot d^3\sigma(x_f) \delta^4(x - x_f)}{e^{(p\cdot u(x_f) - \mu)/T} \pm 1}
$$

HBT from hydrodynamics

$$
\begin{aligned}
C(\vec{p}_1, \vec{p}_2) &\equiv E_{p_1} E_{p_2} \frac{d^6 N}{d^3 p_1 d^3 p_2} \Big/ \left(E_{p_1} \frac{d^3 N}{d^3 p_1} E_{p_2} \frac{d^3 N}{d^3 p_2} \right) \\
\to C_{\mathrm{fit}}(\vec{q}, \vec{K}) &\equiv 1 + \lambda \exp\left(- \sum_{i,j=o,s,l} R^2_{ij}(\vec{K}) q_i q_j \right) \\
\vec{q} &\equiv \vec{p}_1 - \vec{p}_2,\ \vec{K} \equiv \frac{1}{2}\left(\vec{p}_1 + \vec{p}_2\right) \\
C_{\mathrm{th}}(\vec{q}, \vec{K}) &\approx 1 + \left| \frac{\int d^4x\, S(x,K) e^{iq\cdot x}}{\int d^4x\, S(x,K)} \right|^2
\end{aligned}
$$

For Gaussian sources:

$$
\begin{aligned}
\Longrightarrow R^2_{ij}(\vec{K}) &\equiv \left\langle (\tilde{x}_i - \beta_i \tilde{t})(\tilde{x}_j - \beta_j \tilde{t}) \right\rangle, \\
\langle f(x) \rangle &\equiv \frac{\int d^4x\, f(x) S(x,K)}{\int d^4x\, S(x,K)} \\
\tilde{x}_i &\equiv x_i - \langle x_i \rangle,\ \tilde{t} \equiv t - \langle t \rangle,\ \vec{\beta} \equiv \vec{K}/K^0
\end{aligned}
$$

In particular, we have these "pocket relations":

$$\begin{aligned} R_s^2 &= \left\langle \tilde{x}_s^2 \right\rangle \\ R_o^2 &= \left\langle \tilde{x}_o^2 \right\rangle - \beta_T \left\langle \tilde{x}_o \tilde{t} \right\rangle + \beta_T^2 \left\langle \tilde{t}^2 \right\rangle \\ R_l^2 &= \left\langle \tilde{x}_l^2 \right\rangle - \beta_L \left\langle \tilde{x}_l \tilde{t} \right\rangle + \beta_L^2 \left\langle \tilde{t}^2 \right\rangle \end{aligned}$$

- For $\beta_L = 0$, R_s^2 and R_l^2 dominated by *spatial* geometry
- For $\beta_T \neq 0$, R_o^2 contains both spatial *and* temporal information

In particular, we have these "pocket relations":

$$\begin{aligned} R_s^2 &= \left\langle \tilde{x}_s^2 \right\rangle \\ R_o^2 &= \left\langle \tilde{x}_o^2 \right\rangle - \beta_T \left\langle \tilde{x}_o \tilde{t} \right\rangle + \beta_T^2 \left\langle \tilde{t}^2 \right\rangle \\ R_l^2 &= \left\langle \tilde{x}_l^2 \right\rangle - \beta_L \left\langle \tilde{x}_l \tilde{t} \right\rangle + \beta_L^2 \left\langle \tilde{t}^2 \right\rangle \end{aligned}$$

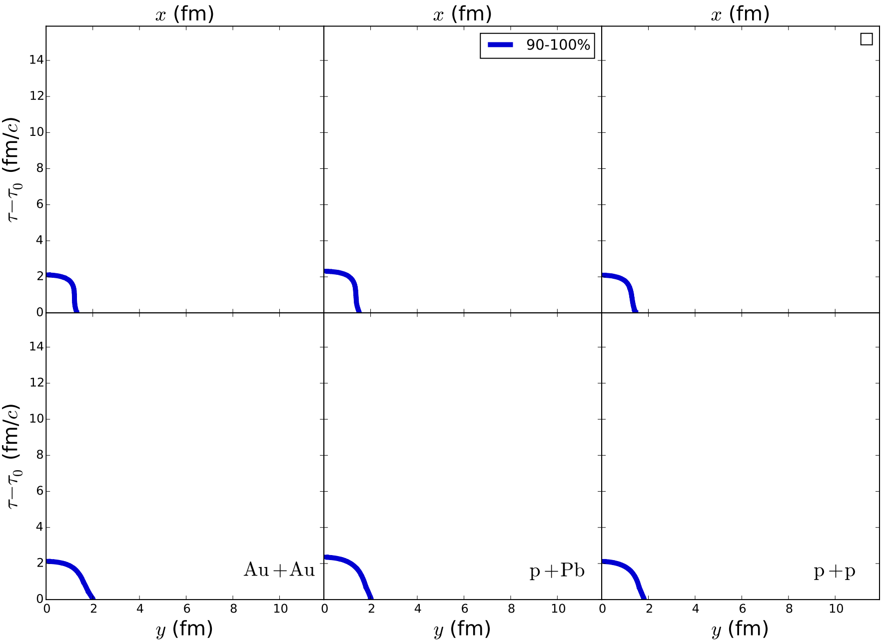
- For $\beta_L = 0$, R_s^2 and R_l^2 dominated by *spatial* geometry
- For $\beta_T \neq 0$, R_o^2 contains both spatial *and* temporal information

---

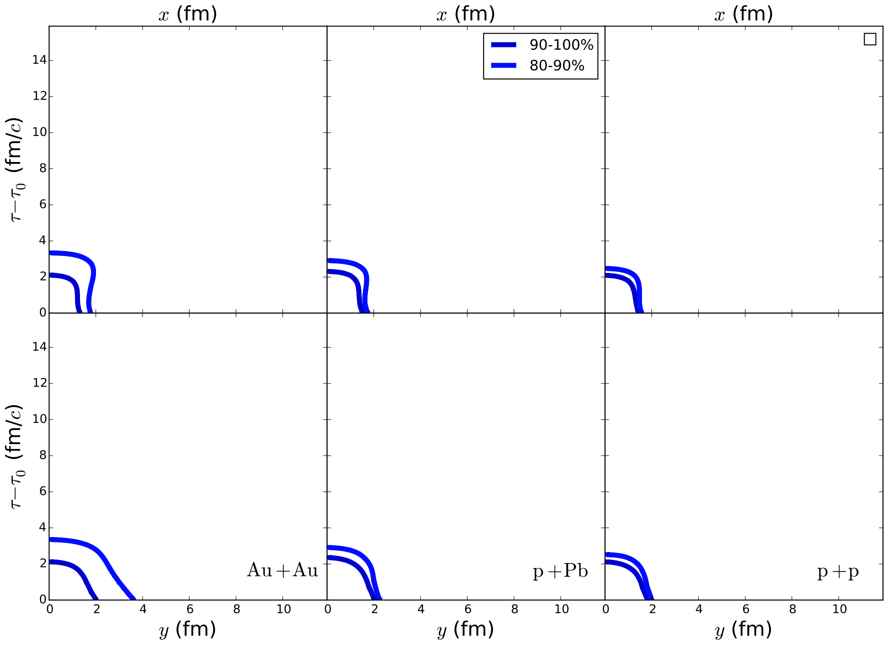
In this talk:

- **Run** hydrodynamics for p+p (7 TeV), p+Pb (5.02 TeV), and Au+Au (200 GeV)
- **Compute** the HBT radii from the correlation function $C(\vec{q}, \vec{K})$
- **Compare** with data
- **Interpret** the results using pocket relations

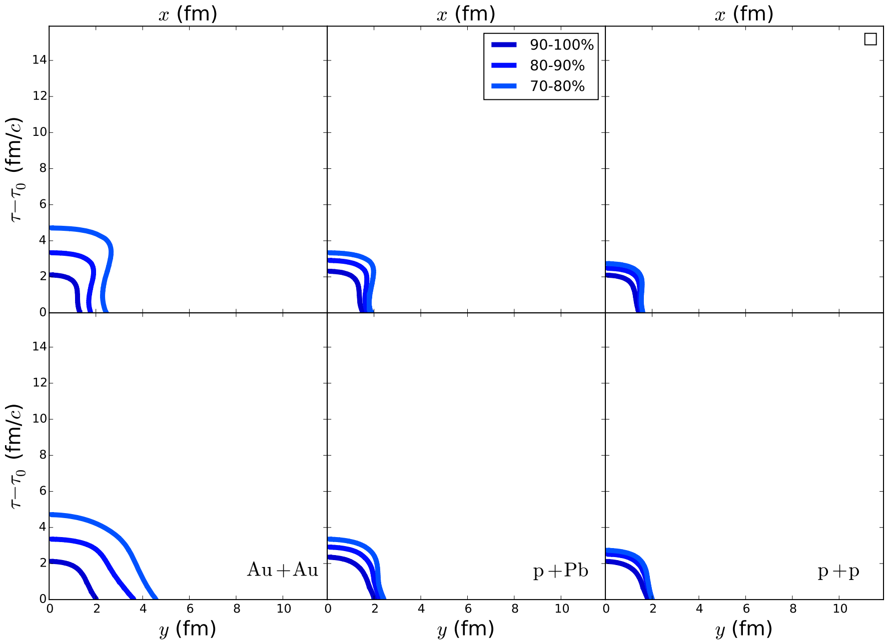
First: what kinds of geometries does hydrodynamics predict?



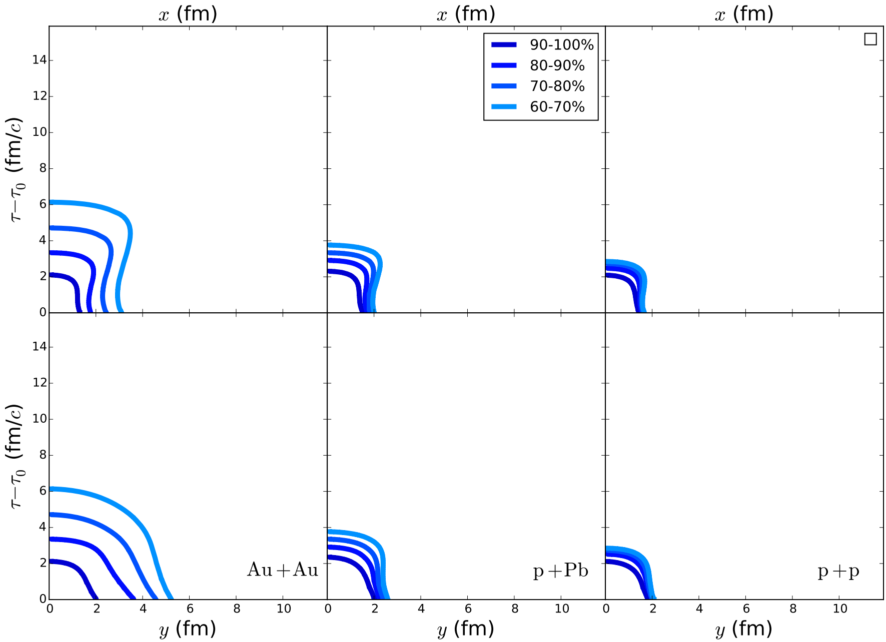
x (fm)
x (fm)
x (fm)
90-100%
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
0
2
4
6
8
10
12
14
Au+Au
p+Pb
p+p
0
2
4
6
8
10
y (fm)
y (fm)
y (fm)



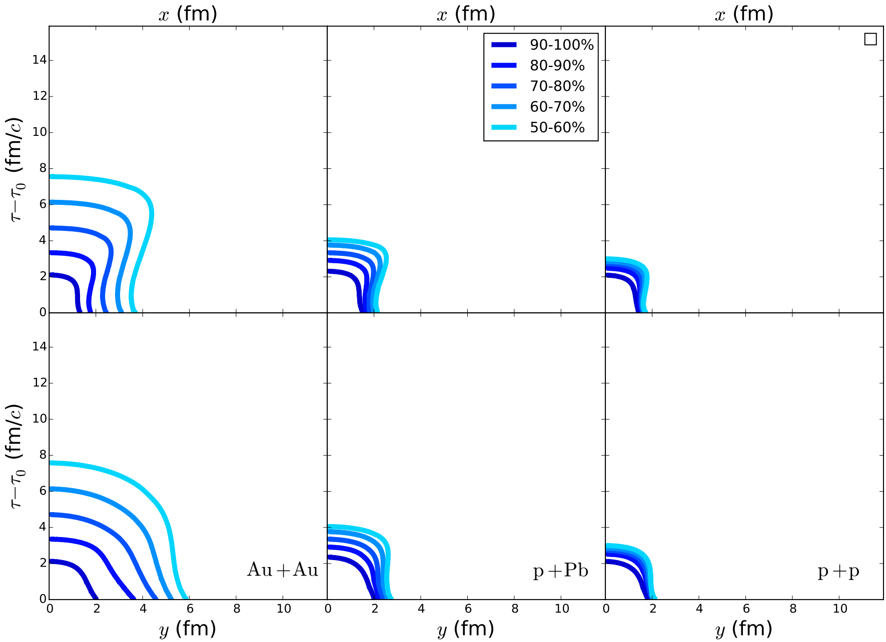
x (fm)
x (fm)
x (fm)
90-100%
80-90%
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
0
2
4
6
8
10
12
14
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)



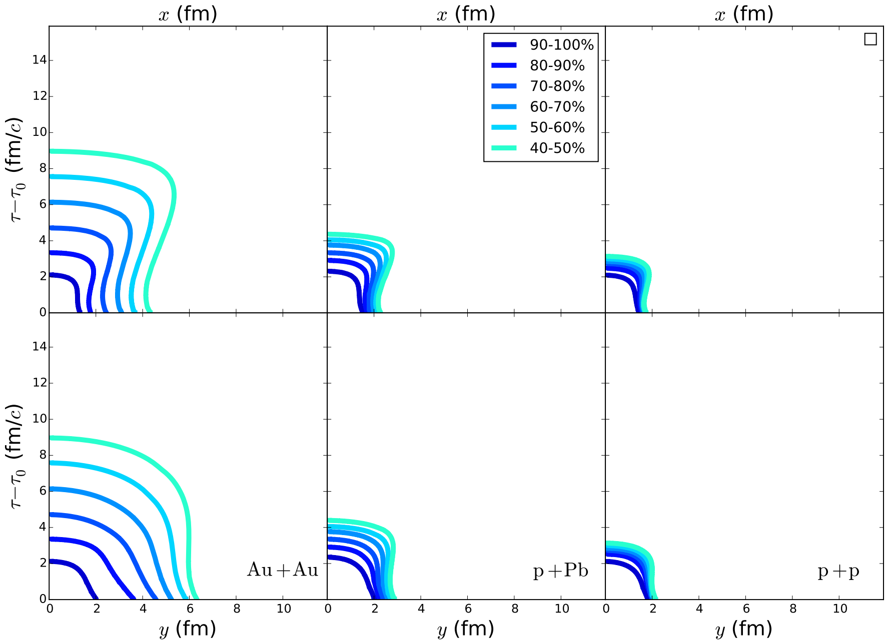
x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)



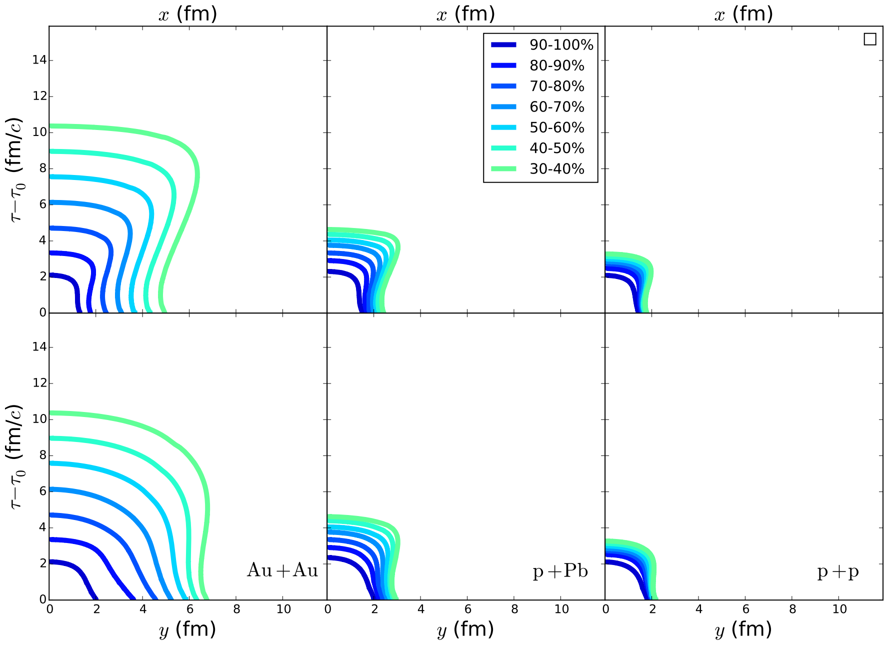
x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
τ−τ0 (fm/c)
τ−τ0 (fm/c)
y (fm)
y (fm)
y (fm)
Au+Au
p+Pb
p+p
0
2
4
6
8
10
12
14



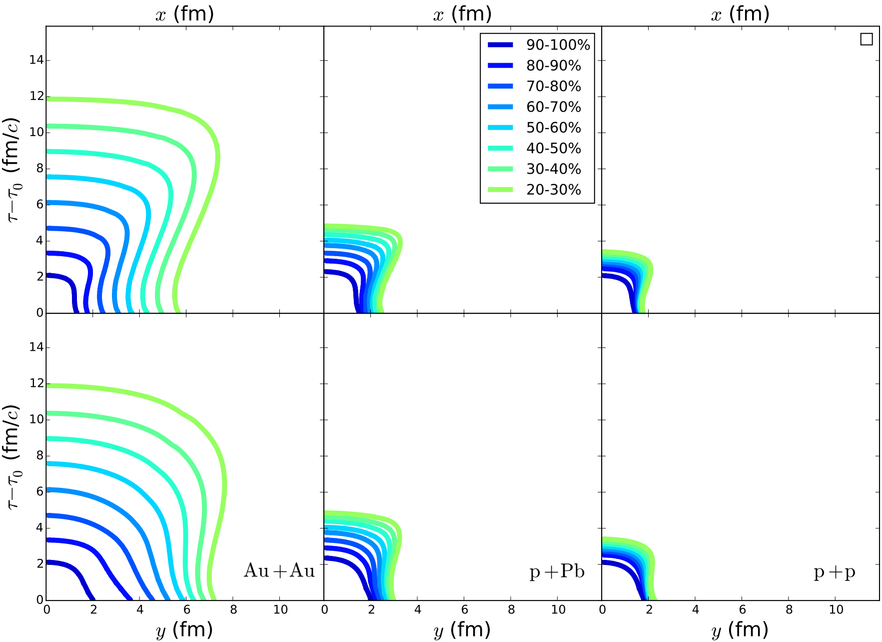
x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
τ−τ0 (fm/c)
τ−τ0 (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)

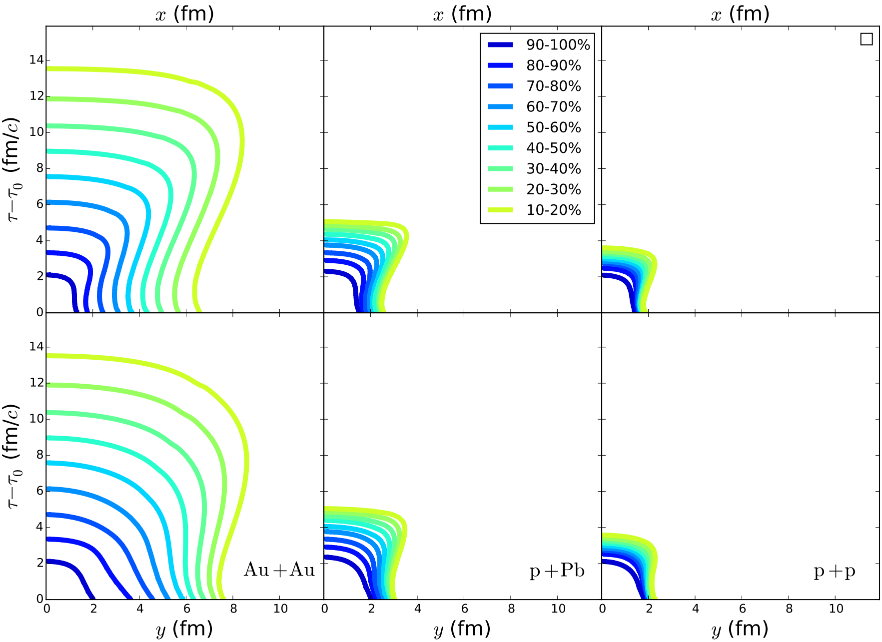


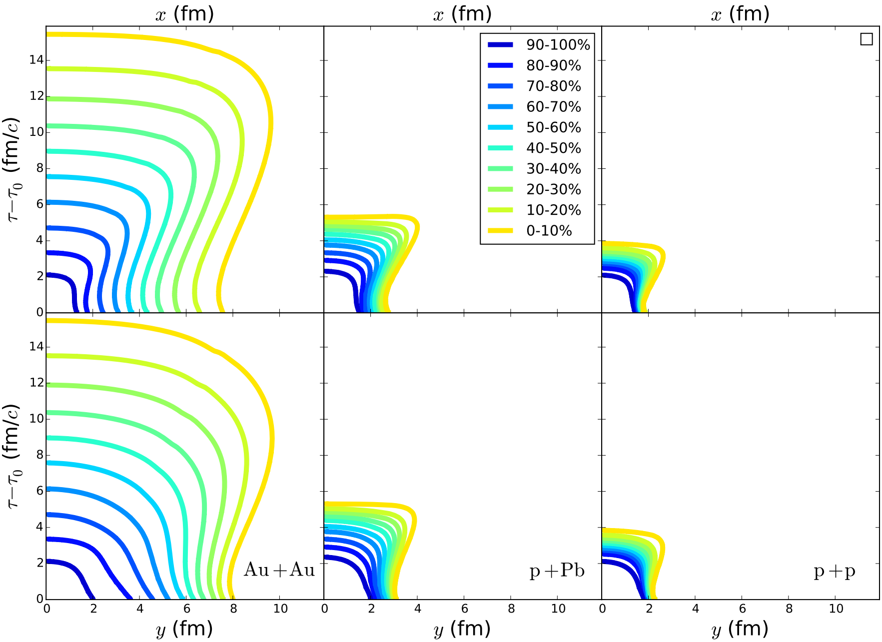
x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)

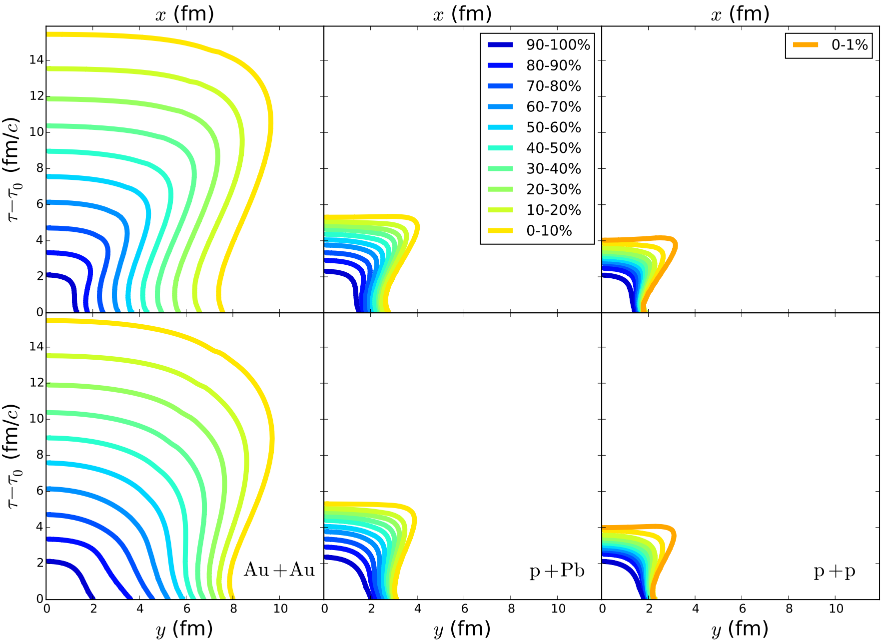


x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
τ−τ0 (fm/c)
τ−τ0 (fm/c)
y (fm)
y (fm)
y (fm)
Au+Au
p+Pb
p+p
0
2
4
6
8
10
12
14

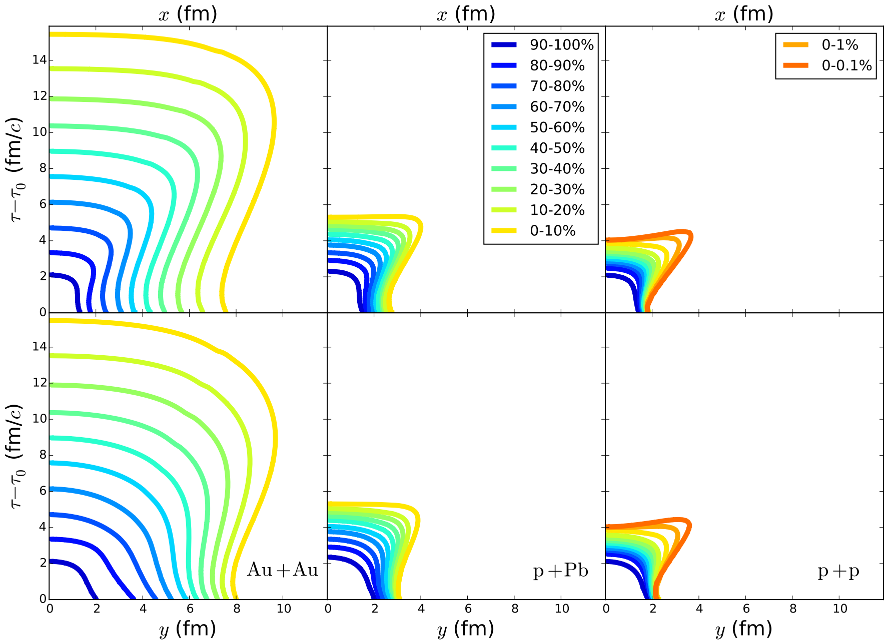

x (fm)
x (fm)
x (fm)
τ−τ0 (fm/c)
τ−τ0 (fm/c)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)

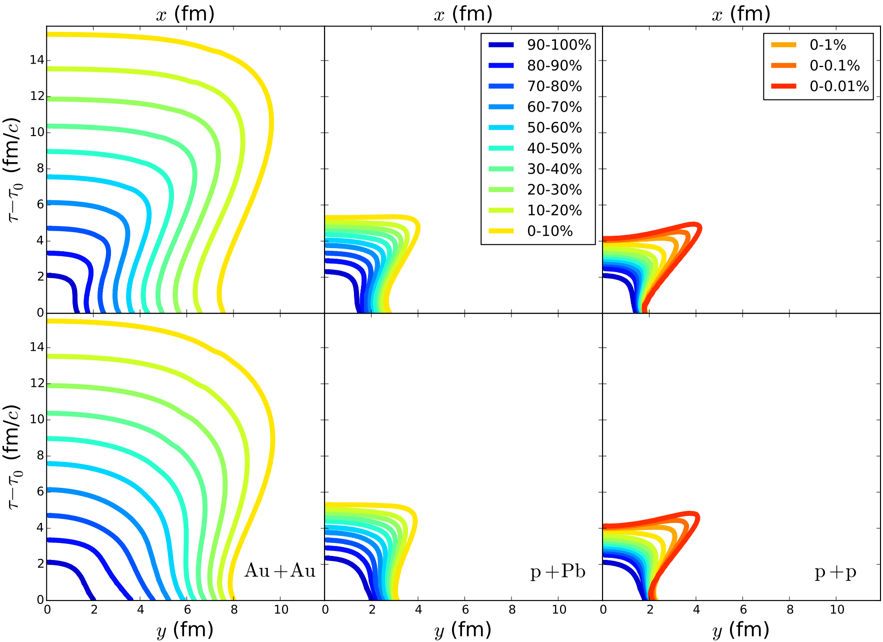

x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
τ−τ0 (fm/c)
τ−τ0 (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)

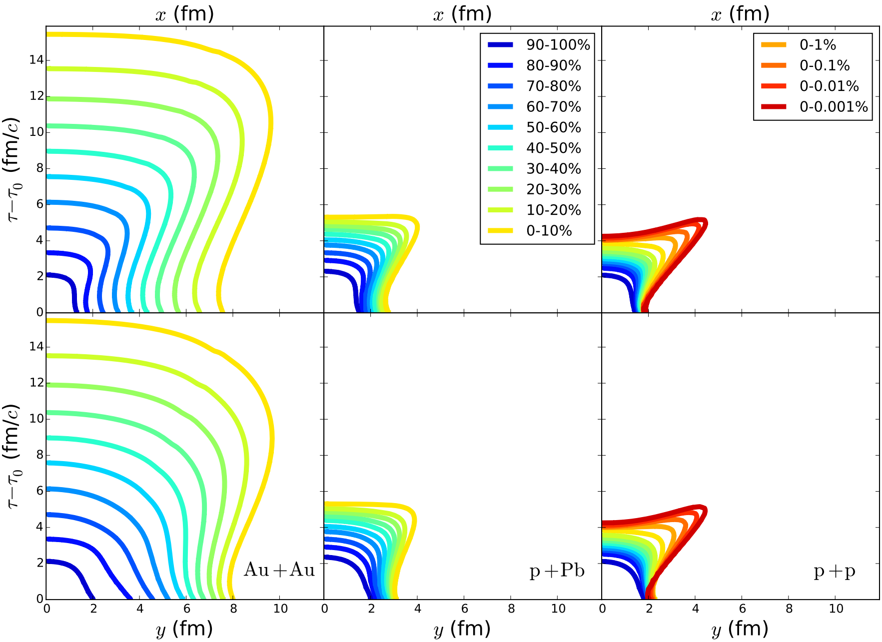

x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
0-10%
τ−τ0 (fm/c)
τ−τ0 (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)



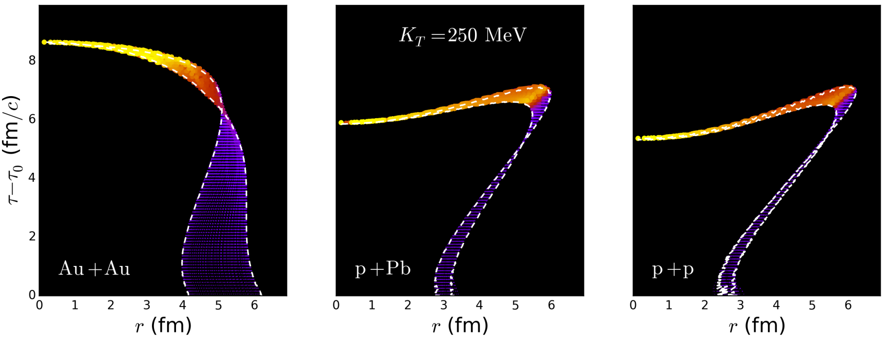
x (fm)
x (fm)
x (fm)
τ−τ0 (fm/c)
τ−τ0 (fm/c)
y (fm)
y (fm)
y (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
0-10%
0-1%
Au+Au
p+Pb
p+p


x (fm)
x (fm)
x (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
0-10%
0-1%
0-0.1%
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)


x (fm)
x (fm)
x (fm)
$\tau - \tau_0$ (fm/c)
$\tau - \tau_0$ (fm/c)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
0-10%
0-1%
0-0.1%
0-0.01%
Au+Au
p+Pb
p+p
y (fm)
y (fm)
y (fm)


x (fm)
x (fm)
x (fm)
τ−τ0 (fm/c)
τ−τ0 (fm/c)
y (fm)
y (fm)
y (fm)
90-100%
80-90%
70-80%
60-70%
50-60%
40-50%
30-40%
20-30%
10-20%
0-10%
0-1%
0-0.1%
0-0.01%
0-0.001%
Au+Au
p+Pb
p+p

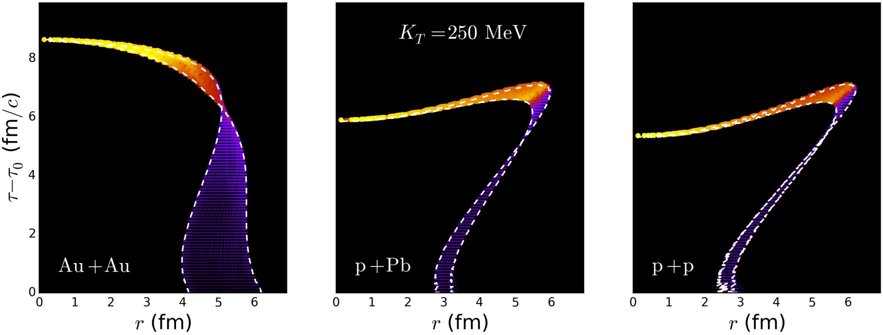
$dN_{\mathrm{ch}}/d\eta = 100$



Observations:

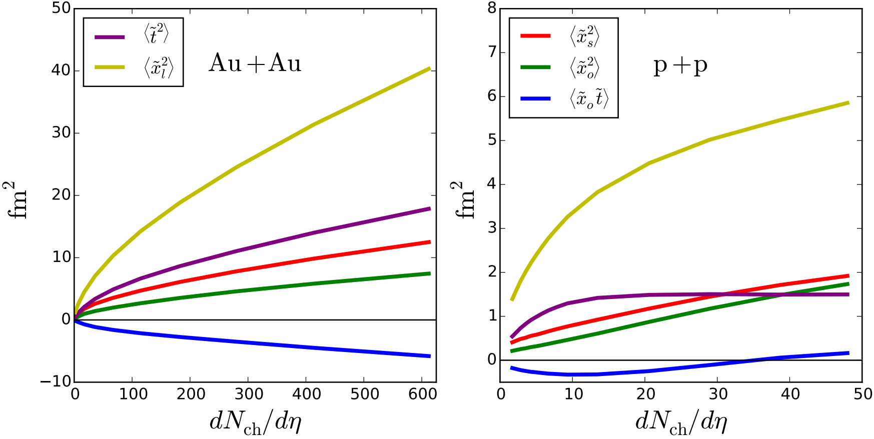
- Small systems highly elongated due to strong collective flow
- Geometric size of emission region $\langle \tilde{x}_i^2 \rangle$ similar across collision systems
- Emission duration $\langle \tilde{t}^2 \rangle$ decreases from Au+Au to p+Pb, p+p

$dN_{\rm ch}/d\eta = 100$



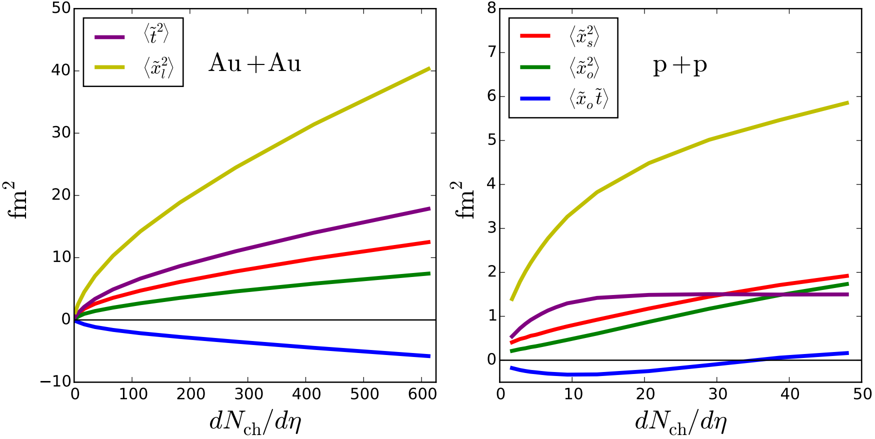
Expectations:

- Geometric contributions to R_i should grow with $dN_{\rm ch}/d\eta$
- Emission duration and spatial-temporal correlation behave differently in small vs. large systems



$$R_s^2 = \langle\tilde{x}_s^2\rangle \qquad R_o^2 = \langle\tilde{x}_o^2\rangle - \beta_T\langle\tilde{x}_o\tilde{t}\rangle + \beta_T^2\langle\tilde{t}^2\rangle \qquad R_l^2 = \langle\tilde{x}_l^2\rangle$$

- Geometry ($\langle\tilde{x}_s^2\rangle$, $\langle\tilde{x}_o^2\rangle$, $\langle\tilde{x}_l^2\rangle$) scales *monotonically* with $dN_{\rm ch}/d\eta$
- $\langle\tilde{t}^2\rangle$ monotonic in Au+Au, levels off in p+p
- $\langle\tilde{x}_o\tilde{t}\rangle < 0$ monotonic in Au+Au, changes sign in p+p

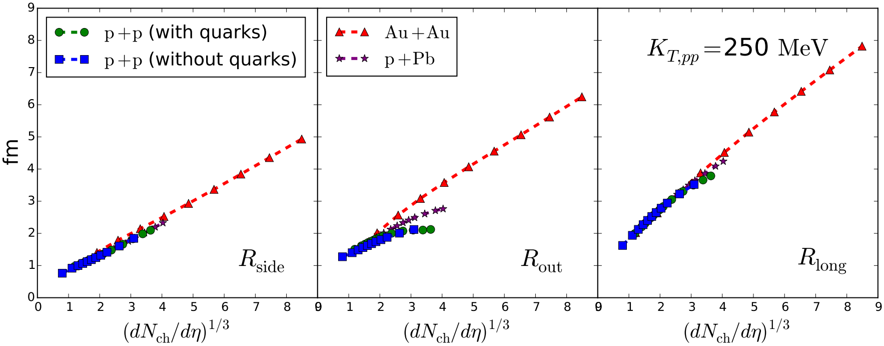


$$R_s^2 = \left\langle \tilde{x}_s^2 \right\rangle \qquad R_o^2 = \left\langle \tilde{x}_o^2 \right\rangle - \beta_T \left\langle \tilde{x}_o \tilde{t} \right\rangle + \beta_T^2 \left\langle \tilde{t}^2 \right\rangle \qquad R_l^2 = \left\langle \tilde{x}_l^2 \right\rangle$$

⇒ All R_i scale strongly in Au+Au

⇒ R_o scales more weakly than R_s, R_l in p+p

Full hydro results



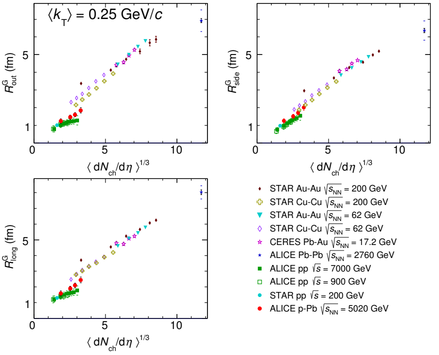
Hydrodynamics implies:

$$V \sim R_{\mathrm{out}} R_{\mathrm{side}} R_{\mathrm{long}} \propto dN_{\mathrm{ch}}/d\eta$$

Suggests: $$R_i \propto (dN_{\mathrm{ch}}/d\eta)^{1/3}$$

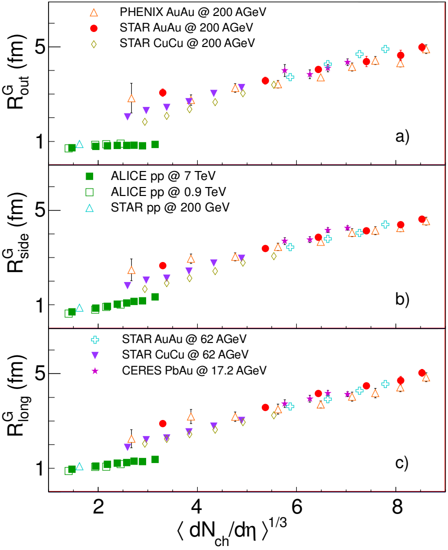
What does the data show?

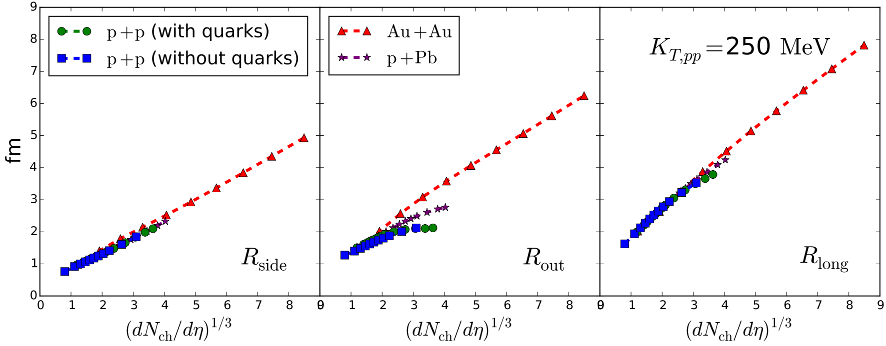
ALICE (2015)



- *Radii* different in pp vs. AA
- *Slopes* different in pp vs. AA
- Slope for pp R_{out} less than for R_{side}, R_{long}

ALICE (2011) - $K_T = 400\,\mathrm{MeV}$





Overall, hydrodynamics does fairly well:

	System radii	System slopes	Slope hierarchy
R_{out}	✓	✓	✓
R_{side}	✗(?)	✓	✓(?)
R_{long}	✗(?)	✗(?)	✓(?)

Conclusions

- Data show non-trivial hierarchy of $dN_{\mathrm{ch}}/d\eta$-scaling in p+p R_o^2 as compared with R_s^2, R_l^2
- Same feature is *not* present in Au+Au

Conclusions

- Data show non-trivial hierarchy of $dN_{\mathrm{ch}}/d\eta$-scaling in p+p R_o^2 as compared with R_s^2, R_l^2
- Same feature is *not* present in Au+Au
- Hydrodynamics generates this hierarchy automatically:
  - Small systems have much stronger collective flow than large systems
  - Flow generates *very* different geometries for different systems at same multiplicity

Conclusions

- Data show non-trivial hierarchy of $dN_{\mathrm{ch}}/d\eta$-scaling in p+p R_o^2 as compared with R_s^2, R_l^2
- Same feature is *not* present in Au+Au
- Hydrodynamics generates this hierarchy automatically:
  - Small systems have much stronger collective flow than large systems
  - Flow generates *very* different geometries for different systems at same multiplicity

Summary assessment: hydrodynamics reproduces qualitative features, but simple approach is quantitatively lacking

Conclusions

So what's missing?

- Initial-state fluctuations:
  - may shift multiplicity-dependence of average radii
  - enables new observables which probe fluctuations of shape and size
- Hadronic phase:
  - generates additional flow and smaller source sizes without (significantly) changing multiplicity
  - naturally relaxes of $\beta_L = 0$ assumption

Conclusions

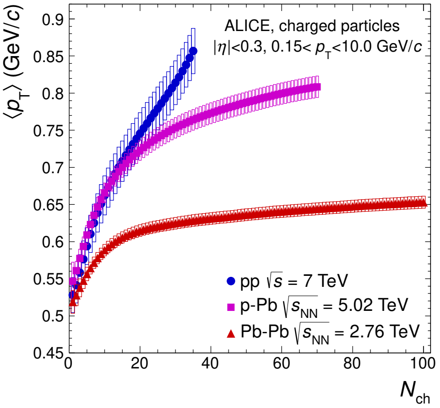
So what's missing?

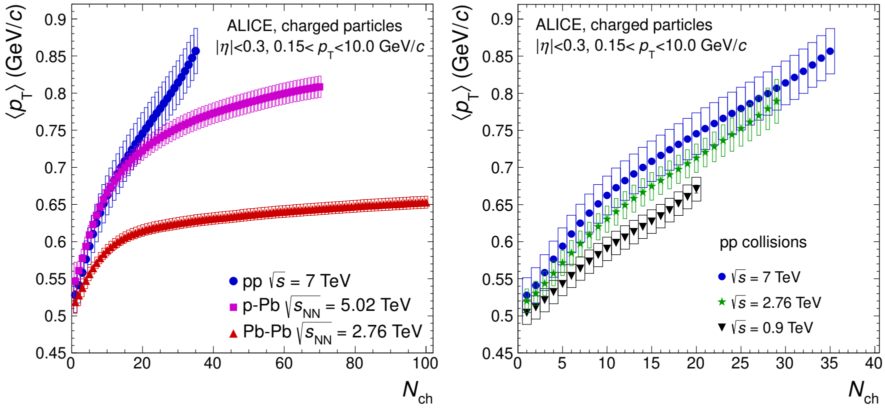
- Initial-state fluctuations:
  - may shift multiplicity-dependence of average radii
  - enables new observables which probe fluctuations of shape and size
- Hadronic phase:
  - generates additional flow and smaller source sizes without (significantly) changing multiplicity
  - naturally relaxes of $\beta_L = 0$ assumption

Děkuju!

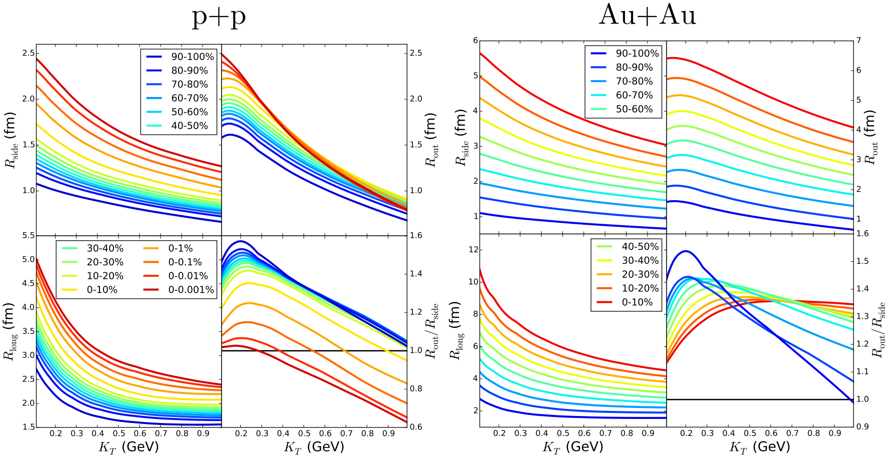
Knut and Alice Wallenberg Foundation

Backup slides


ALICE, charged particles
$|\eta|$<0.3, 0.15< p_T<10.0 GeV/c
$\langle p_T \rangle$ (GeV/c)
N_{ch}
pp $\sqrt{s}$ = 7 TeV
p-Pb $\sqrt{s_{NN}}$ = 5.02 TeV
Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV



ALICE, charged particles
|η|<0.3, 0.15< pT<10.0 GeV/c
⟨pT⟩ (GeV/c)
Nch
pp √s = 7 TeV
p-Pb √sNN = 5.02 TeV
Pb-Pb √sNN = 2.76 TeV
ALICE, charged particles
|η|<0.3, 0.15< pT<10.0 GeV/c
pp collisions
√s = 7 TeV
√s = 2.76 TeV
√s = 0.9 TeV



Generic consequences of strong collective flow:

- K_T-scaling
- Possibility of $R_o^2/R_s^2 < 1$

p + p (with quarks)