Scale and Scheme Variations in NLO Merging

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Proton-Proton Collisions: Overview

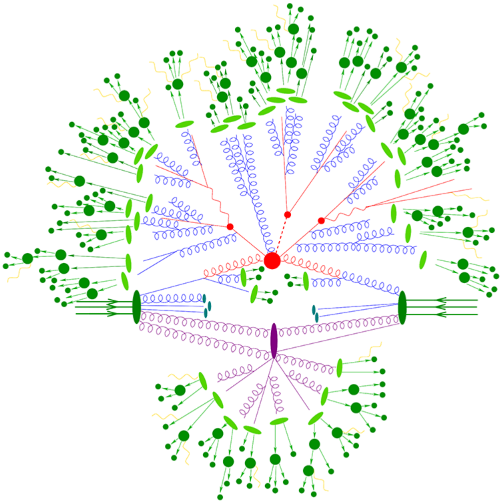


Figure from Stefan Höche

- Hard Process, resonant decays
- Parton Shower
- PDFs: Pick a parton from a hadron
- Hadronisation
- Hadron Decays
- Hadronic rescattering
- MPIs
- Beam Remnants/UE

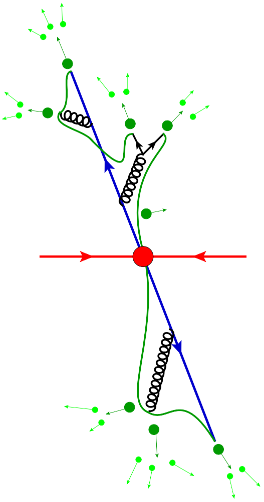
Fixed Order vs. All Order

Hard interaction: Matrix Elements (LO/NLO)

- Fixed order expansion in strong coupling α_s
- Fails for soft/collinear emissions, terms $\propto \alpha_s \log^2(\rho_0/\rho_{\mathrm{cut}}) > 1$
- $\Rightarrow$ Suitable for few well separated partons

Parton Shower: Radiative corrections

- Based on soft/collinear approximation
- Iterated, ordered in "hardness" ρ
- All order (in α_s) expression
- Only leading logarithmic terms $\propto \alpha_s^n \log^{2n}(\rho_0/\rho_{\mathrm{cut}})$ correct, but for any n
- $\Rightarrow$ Suitable for multiple soft/collinear emissions



Multi-jet Merging

Combine strengths of Matrix Elements and Parton Showers

Experiments measure exclusive event: need to describe all emissions

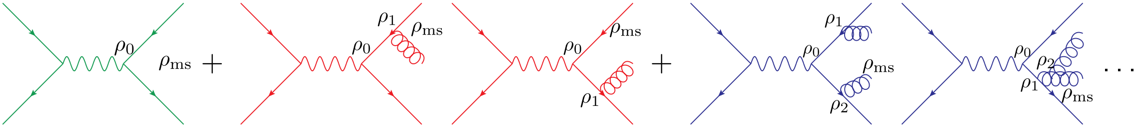
- Describe hard emissions by fixed order predictions (including interference effects)
- Add further emissions from parton shower

Want to improve PS emissions for more than hardest emission. Naive approach:

- Generate $[X]_{\mathrm{ME}}$ + parton shower
- Generate $[X + 1\,\text{jet}]_{\mathrm{ME}}$ + parton shower
- Generate $[X + 2\,\text{jets}]_{\mathrm{ME}}$ + parton shower
- ...

And combine everything into one sample. Does not work, double counting!
$\Rightarrow$ Forbid hard PS emissions and take no-emission probabilities Π into account

Multi-jet Merging: Illustration of CKKWL [Lönnblad (2001)] [Catani, Krauss, Kuhn, Webber (2001)]



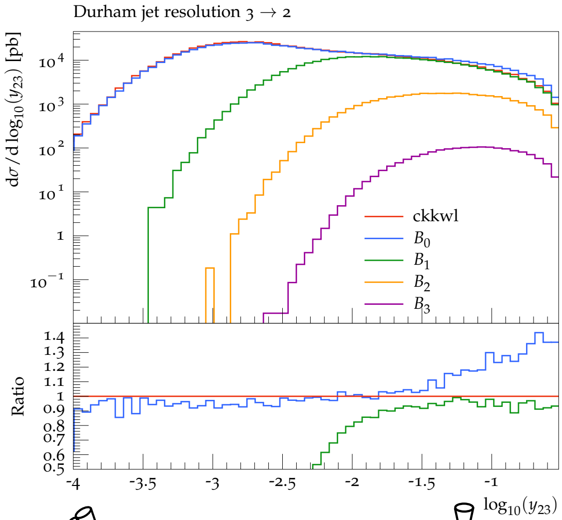
Combine MEs with different multiplicities, avoid overlap by reweighting

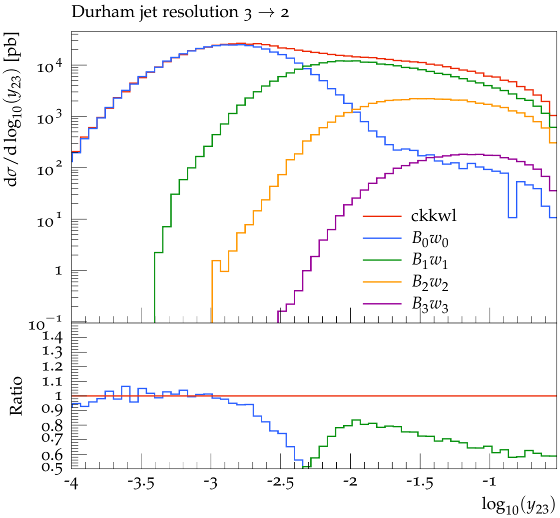
$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}_0 B_0 w_0 + \int d\phi_1 \mathcal{O}_1 B_1 w_1 + \int d\phi_1 \int d\phi_2 \mathcal{O}_2 B_2 w_2 \right\}$$

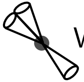
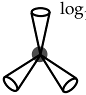
with the weights

$$w_0 = \Pi_0(\rho_0, \rho_{\rm ms}) \,, \;\; w_1 = \Pi_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Pi_1(\rho_1, \rho_{\rm ms}) \,,$$

$$w_2 = \Pi_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Pi_1(\rho_1, \rho_2) \frac{\alpha_s(\rho_2)}{\alpha_s(\mu_R)}$$





 Wrong without weights 

All plots generated with MG5_aMC@NLO + Pythia8

arXiv:1405.0301 arXiv:hep-ph/0603175

Unitarized Merging: UMEPS [Lönnblad, Prestel (2012)]

- Problem: CKKWL merging does not preserve inclusive cross section given by B_0 sample
- Fix by rewriting no-emission probability

$$B_0 w_0 = B_0 \Pi_0(\rho_0, \rho_1) = B_0 - \int_{\rho_1}^{\rho_0} \mathrm{d}\rho B_1(\rho) w_1$$

- Observables in unitarized multi-jet merging (UMEPS):

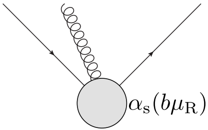
$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}_0 \left[B_0 - \int_S B_{1\to 0} w_1 \right] + \int d\phi_1 \mathcal{O}_1 B_1 w_1 \right\}$$

How Reliable are our Predictions?

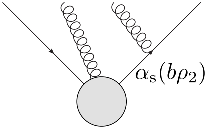
- Best answer: higher order calculations in α_s
- Strong coupling α_s depends on "hardness" scale ρ
- Choice of scale does not spoil fixed order accuracy, since $\alpha_s(\rho') = \alpha_s(\rho) + \mathcal{O}(\alpha_s^2)$
- Use ρ variations by factor $1/2$ and 2 to estimate higher order effects $\Rightarrow$ scale uncertainties

For consistency, do variation in three components of calculation simultaneously:

Hard process:
$\alpha_s(\mu_R)$ in matrix elements

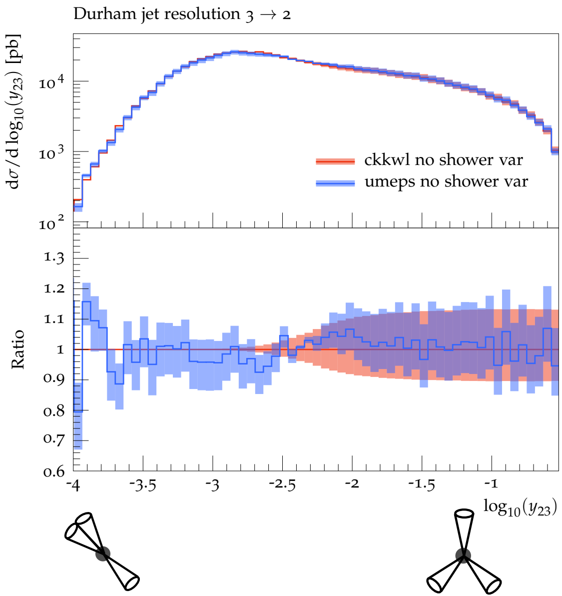


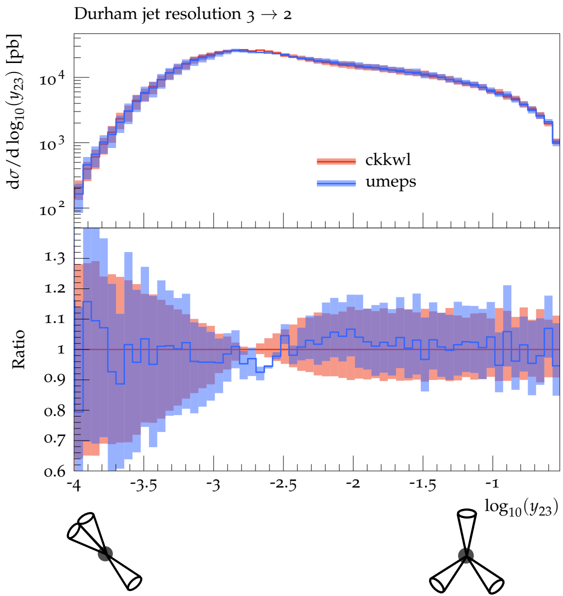
Parton shower:
$\alpha_s(\rho_i)$ in emissions



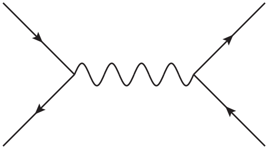
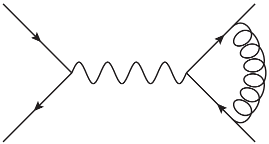
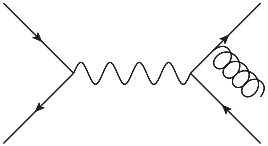
Merging weights:
No-emission probabilities and emissions

$$w_1 = \Pi_0(\rho_0, \rho_1; b)\frac{\alpha_s(b\rho_1)}{\alpha_s(b\mu_R)}$$


Durham jet resolution 3 → 2
dσ/d log10(y23) [pb]
ckkwl no shower var
umeps no shower var
Ratio
log10(y23)


Durham jet resolution 3 → 2
dσ/d log10(y23) [pb]
ckkwl
umeps
Ratio
log10(y23)

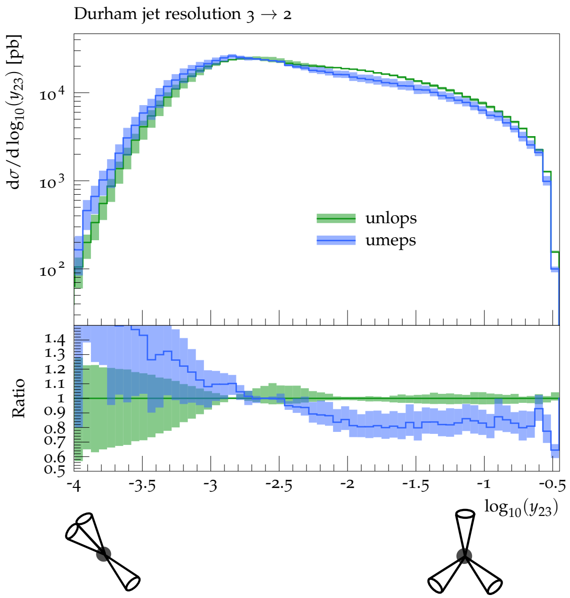
NLO: Improve Fixed Order Precision

Next-to-Leading Order in α_s: $\mathrm{d}\phi_n \bar{B}_n(\phi_n) = \mathrm{d}\phi_n[B_n(\phi_n) + \alpha_s V_n(\phi_n)] + \int_1 \mathrm{d}\phi_{n+1} \alpha_s R(\phi_{n+1})$

- UNLOPS [Lönnblad, Prestel (2013)]: Combine NLO matrix elements in unitary merging
- Subtract $\mathcal{O}(\alpha_s)$ from weights to preserve perturbative accuracy

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Big\{ \mathcal{O}_0 \Big[\bar{B}_0 - \int_S \bar{B}_{1\to 0} - \int_S B_{1\to 0}(w_1 - w_1|_{\mathcal{O}(\alpha_s)}) \Big] + \int d\phi_1 \mathcal{O}_1 \Big[\bar{B}_1 + B_1 (w_1 - w_1|_{\mathcal{O}(\alpha_s)}) \Big] \Big\}$$



- Central prediction changes
- Scale variation band reduces

Freedom in Choice of Merging Scheme

Merging scheme should

- preserve fixed order quantum interference model
- preserve parton shower state evolution model

Define three valid variants of UNLOPS, look at 1 jet contribution

UNLOPS-1

$$B_1 w_1 + \left[\bar{B}_1 - B_1 w_1|_{\mathcal{O}(\alpha_s)}\right]$$

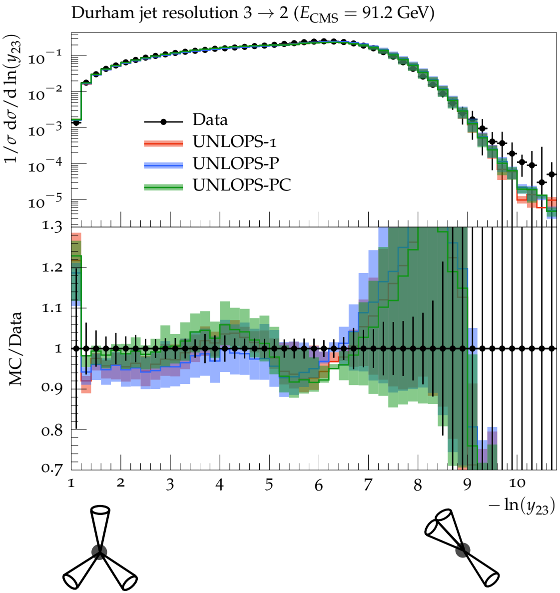
UNLOPS-P

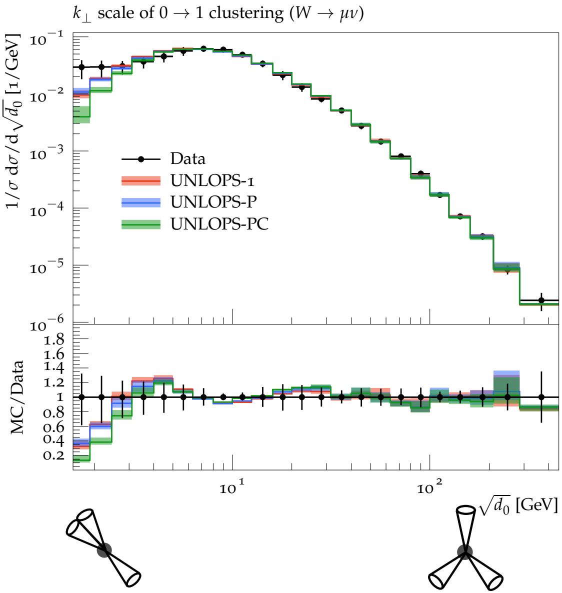
$$B_1 w_1 + \left[\bar{B}_1 - B_1 w_1|_{\mathcal{O}(\alpha_s)}\right] \Pi_0(\rho_0, \rho_1, b)$$

UNLOPS-PC

$$B_1 w_1 + \left[\bar{B}_1 - B_1 w_1|_{\mathcal{O}(\alpha_s)}\right] \Pi_0(\rho_0, \rho_1, b) \frac{\alpha_s(b\rho_1)}{\alpha_s(b\mu_R)}$$

[LG, Prestel (2020)]


Durham jet resolution 3 → 2 (E_{CMS} = 91.2 GeV)
$1/\sigma\, d\sigma / d\ln(y_{23})$
Data
UNLOPS-1
UNLOPS-P
UNLOPS-PC
MC/Data
$-\ln(y_{23})$


$k_\perp$ scale of 0 → 1 clustering ($W \to \mu\nu$)
$1/\sigma\, d\sigma / d\sqrt{d_0}$ [1/GeV]
Data
UNLOPS-1
UNLOPS-P
UNLOPS-PC
MC/Data
$\sqrt{d_0}$ [GeV]

Summary

- Precise predictions and realistic uncertainty estimations important for experiments
- Consistent renormalization scale variation good uncertainty estimate
- Freedom in choice of NLO merging scheme $\Rightarrow$ use as uncertainty on merging prescription
- Reliably estimate merging uncertainties by combining scale and scheme variations