

Color Field Fluctuations... or Not

An Open Discussion About Saturation Physics

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CLASH Seminar

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The Goal

- What this talk is **not**:
 - Comprehensive answers, finished stories, decisive conclusions
 - What this talk aims to be:
 - An ongoing **discussion** to better understand the nature of saturation physics and assess the evidence for it
- “Let us aim to be confused at a higher level”

Questions Offered for Discussion

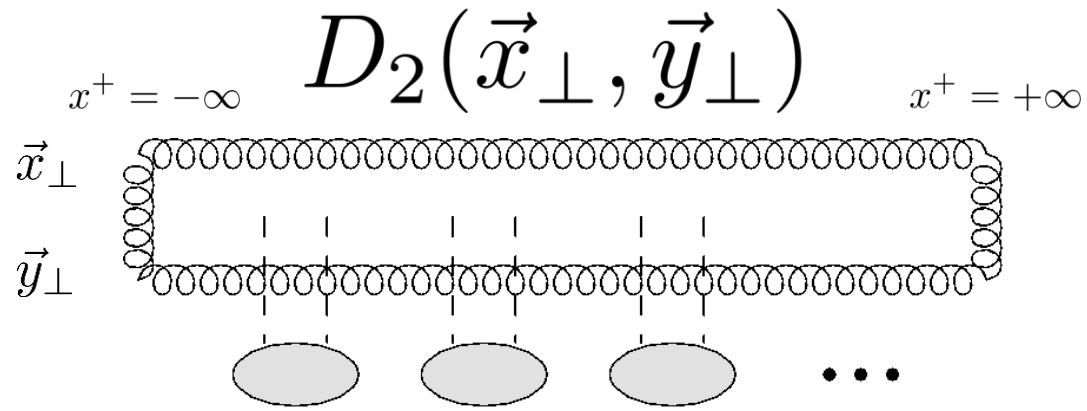
- This Talk:

How essential are the color field degrees of freedom associated with the Color Glass Condensate in heavy ion collisions?

- Other Potential Topics:

- ❖ How is saturation related with unitarity – in QCD and in other field theories?
- ❖ The CGC + Lund String model

Dipole Operators



$$D_2^{adj}(\vec{u}_\perp, \vec{b}_\perp) \equiv \frac{1}{N_c^2 - 1} \langle \text{tr}[U_{\vec{u}_\perp} U_{\vec{b}_\perp}^\dagger] \rangle$$

$$U_{\vec{x}_\perp} = \mathcal{P} \exp \left[ig \int dz_\mu A^\mu \right]$$

$$D_2(\vec{x}_\perp, \vec{y}_\perp) \approx D_2(|x - y|_T)$$

- Wilson lines: line integrals of the gluon fields
- After color averaging: exponentiate a 2-gluon block

$$D_2(\vec{x}_\perp, \vec{y}_\perp) \approx \exp \left[-\frac{1}{4} |x - y|_T^2 Q_s^2 \ln \frac{1}{|x - y|_T \Lambda} \right]$$

Application: The MSTV Calculation for Small Systems

Hierarchy of azimuthal anisotropy harmonics in collisions of small systems from the Color Glass Condensate

Mark Mace,^{1,2} Vladimir V. Skokov,³ Prithwish Tribedy,¹ and Raju Venugopalan¹

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³RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973

(Dated: August 8, 2018)

Systematics of azimuthal anisotropy harmonics in proton-nucleus collisions at the LHC from the Color Glass Condensate

Mark Mace,^{1,2} Vladimir V. Skokov,³ Prithwish Tribedy,¹ and Raju Venugopalan¹

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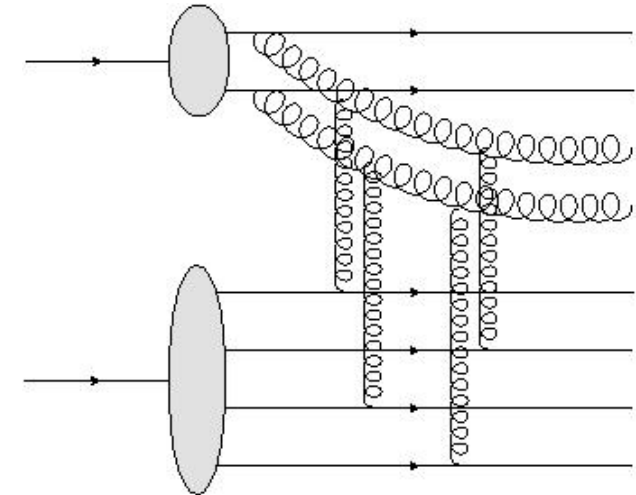
²Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA

³RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973-5000, USA

(Dated: July 4, 2018)

$$\frac{dN^{\text{even}}(\mathbf{k}_\perp)}{d^2k dy} [\rho_p, \rho_t] = \frac{2}{(2\pi)^3} \frac{\delta_{ij}\delta_{lm} + \epsilon_{ij}\epsilon_{lm}}{k^2} \Omega_{ij}^a(\mathbf{k}_\perp) [\Omega_{lm}^a(\mathbf{k}_\perp)]^*$$

$$\Omega_{ij}^a(\mathbf{k}_\perp) = g \int \frac{d^2p}{(2\pi)^2} \frac{p_i(k-p)_j}{p^2} \rho_p^b(\mathbf{p}_\perp) U_{ab}(\mathbf{k}_\perp - \mathbf{p}_\perp)$$



Gluon Correlations in MSTSV

Mace et al., Phys. Rev. Lett. **121** (2018), and arXiv:1901.10506

$$\frac{dN}{d^2k_{\perp}} [\rho_p, \rho_t] \stackrel{L.O.}{=} \frac{2}{(2\pi)^3} \frac{\delta^{ij} \delta^{\ell m} + \epsilon_T^{ij} \epsilon_T^{\ell m}}{k_T^2} \Omega_{(\vec{k}_{\perp})}^{a ij} [\rho_p, \rho_t] \left(\Omega_{(\vec{k}_{\perp})}^{a \ell m} [\rho_p, \rho_t] \right)^*$$

$$\frac{dN}{d^2k_{\perp 1} \cdots d^2k_{\perp n}} = \int [\mathcal{D}\rho_p] [\mathcal{D}\rho_t] W[\rho_p] W[\rho_t] \frac{dN}{d^2k_{\perp 1}} [\rho_p, \rho_t] \cdots \frac{dN}{d^2k_{\perp n}} [\rho_p, \rho_t]$$

- Event-by-event fluctuations:
 - MC Glauber **geometry**
 - Event-by-event color fields: **operators** in ρ_p, ρ_t
- Two-gluon correlations from **semi-dilute / dense** expressions
 - Inherently **factorized** at operator level (density-enhanced)

When is “Flow” not “Hydro”?

- Sampled **one color field at a time**:
 - The single-particle distribution is **anisotropic**
 - Many-body correlations arise only through **mutual correlation** to the direction of the color fields.

$$\frac{dN}{d^2k_{\perp 1} \cdots d^2k_{\perp n}} = \int [\mathcal{D}\rho_p] [\mathcal{D}\rho_t] W[\rho_p] W[\rho_t] \frac{dN}{d^2k_{\perp 1}} [\rho_p, \rho_t] \cdots \frac{dN}{d^2k_{\perp n}} [\rho_p, \rho_t]$$

$$v_n \equiv \frac{1}{N} \left| \int_p e^{in\phi} \frac{dN_1}{d^2p} \right|$$

$$\frac{dN_2}{d^2p_1 d^2p_2} \equiv \frac{dN_1}{d^2p_1} \frac{dN_1}{d^2p_2} + \delta_2(p_1, p_2)$$

“Flow” “Non-Flow”

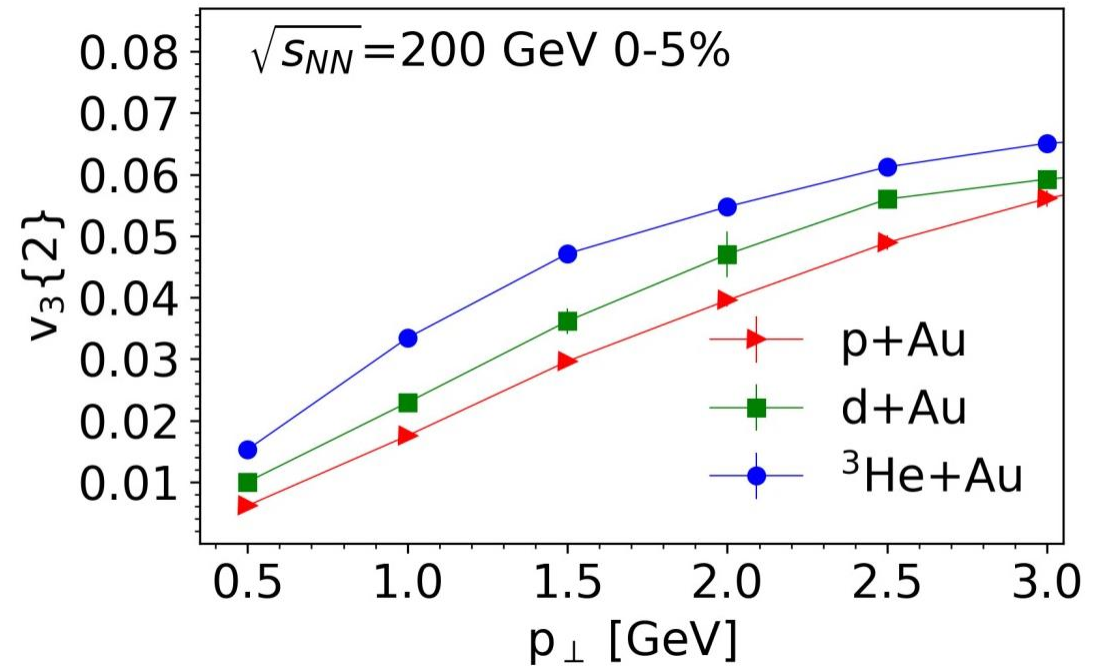
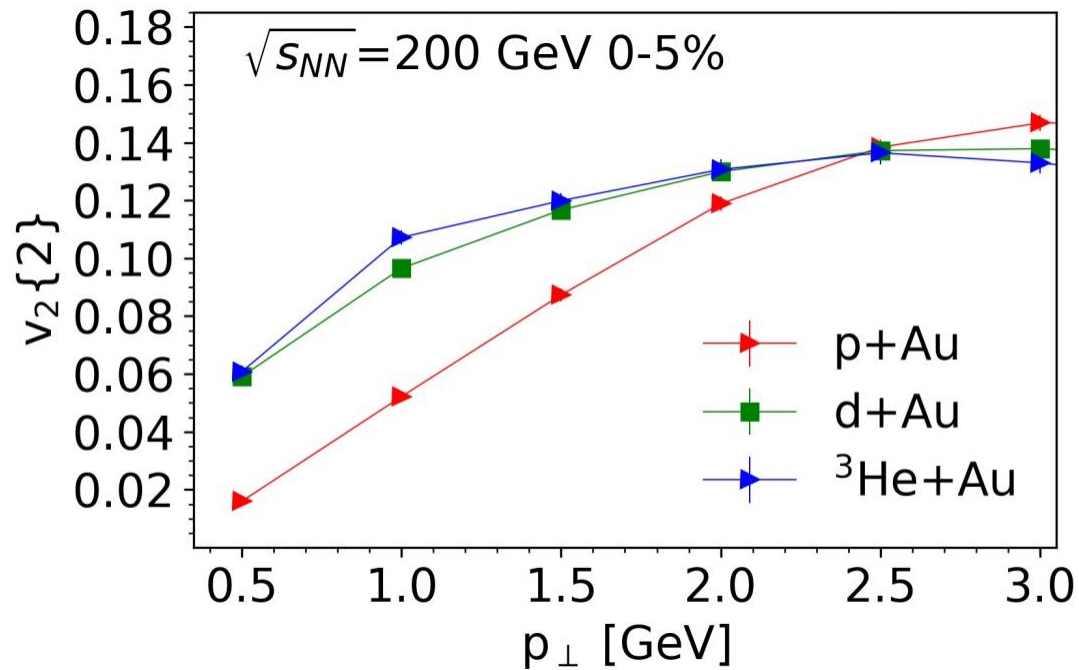
$$(v_n\{2\})^2 \stackrel{flow}{=} \langle v_n^2 \rangle \quad (v_n\{4\})^4 \stackrel{flow}{=} 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle$$

$$= (v_n\{2\})^4 - \text{Var}(v_n^2)$$

Similar Input, Similar Output

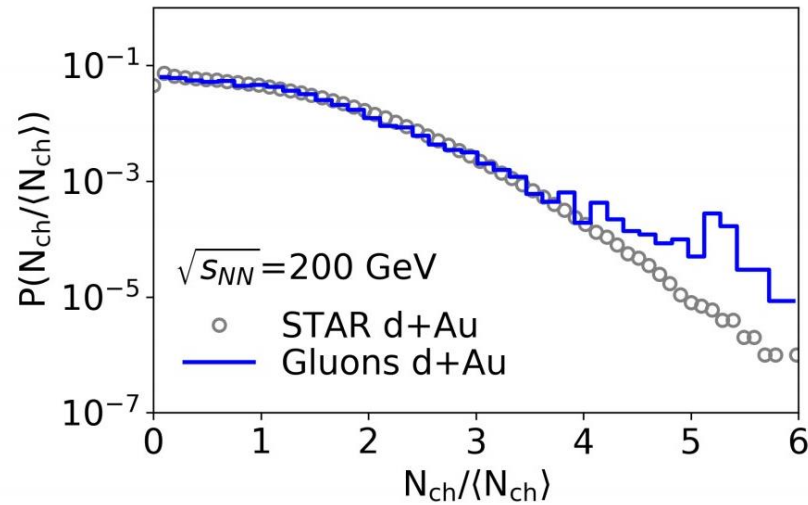
$$V_n(p_1, p_2) = \frac{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} e^{in\phi} \frac{dN(\mathbf{k}_\perp)}{d^2k dy} [\rho_p, \rho_t]}{\int_{p_1}^{p_2} k_\perp dk_\perp \frac{d\phi}{2\pi} \frac{dN(\mathbf{k}_\perp)}{d^2k dy} [\rho_p, \rho_t]}$$

$$v_n^2\{2\}(p_\perp) = \int \mathcal{D}\rho_p \mathcal{D}\rho_t W[\rho_p] W[\rho_t] \times V_n(p_\perp - \Delta/2, p_\perp + \Delta/2) V_n^*(0, \Lambda_{UV})$$



- The devil is certainly in the details, but: not surprising that MSTV gets systematics which resemble hydro (and hence the data)

Interesting Observations on Multiplicity Dependence



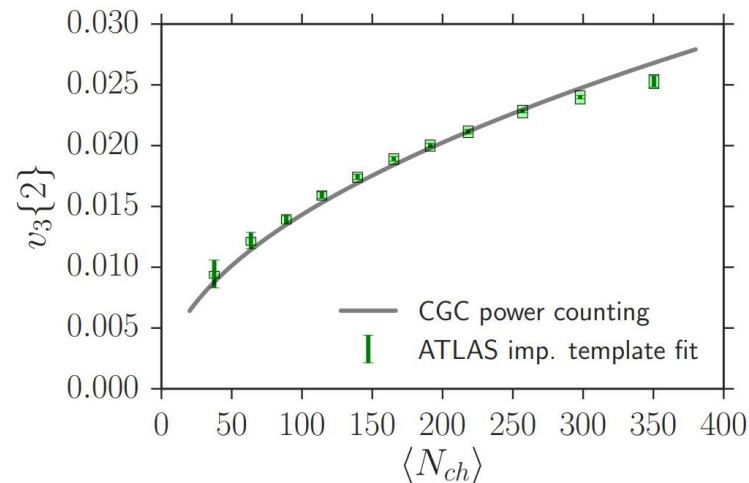
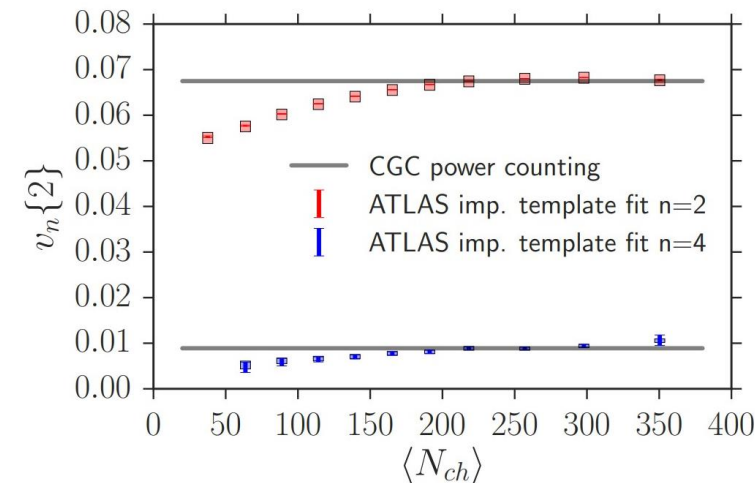
- Can fit gluon multiplicity distribution to data... with help

These include the mean of the ratio $\bar{Q}_S/g^2\mu$ taken to be 0.5, the variance of Gaussian fluctuations of $\ln(Q_S^2)$ [42] taken to be $\sigma = 0.5$, as well as an infrared cutoff scale for color fields taken to be $m = 0.3$ GeV. The effect of

- Interesting characteristic scaling with multiplicity

$$v_2\{2\} \stackrel{T_A T_B}{\sim} \sqrt{\frac{\int d^2x_\perp \left(\frac{dN}{d^2x}\right)^2}{\left[\int d^2x_\perp \frac{dN}{d^2x}\right]^2}} \sim \text{const}$$

$$v_3\{2\} \stackrel{T_A T_B}{\sim} \sqrt{\frac{\int d^2x_\perp \left(\frac{dN}{d^2x}\right)^3}{\left[\int d^2x_\perp \frac{dN}{d^2x}\right]^2}} \sim \sqrt{\frac{dN}{d^2x}}$$



...How Does this Single-Field Calculation Really Work?

$$\frac{dN}{d^2k_{\perp 1} \cdots d^2k_{\perp n}} = \int [\mathcal{D}\rho_p] [\mathcal{D}\rho_t] W[\rho_p] W[\rho_t] \frac{dN}{d^2k_{\perp 1}}[\rho_p, \rho_t] \cdots \frac{dN}{d^2k_{\perp n}}[\rho_p, \rho_t]$$

$$\left\langle \frac{d^m N}{d^2\mathbf{p}_1 \cdots d^2\mathbf{p}_m} \right\rangle \equiv \left\langle \frac{dN}{d^2\mathbf{p}_1} \cdots \frac{dN}{d^2\mathbf{p}_m} \right\rangle, \quad (7)$$

where the expectation value denotes an average over classical configurations of the target in a single event and over all events. Since each of the single-particle distributions inside the average here is a gauge-dependent functional of the classical field, we caution the reader that these distributions are qualitatively different from the gauge invariant single-particle distributions employed in hydrodynamic computations. No such simple product of gauge invariant distributions can be written in our case; indeed, as discussed at length in the Appendix, the Feynman diagrams corresponding to Eq. (7) are quantum interference diagrams.

*K. Dusling, M. Mace, R. Venugopalan
Phys.Rev.D 97 (2018)*

$$v_n \equiv \frac{1}{N} \left| \int_p e^{in\phi} \frac{dN_1}{d^2p} \right| \quad V_n(p_1, p_2) = \frac{\int_{p_1}^{p_2} k_{\perp} dk_{\perp} \frac{d\phi}{2\pi} e^{in\phi} \frac{dN(\mathbf{k}_{\perp})}{d^2k dy} [\rho_p, \rho_t]}{\int_{p_1}^{p_2} k_{\perp} dk_{\perp} \frac{d\phi}{2\pi} \frac{dN(\mathbf{k}_{\perp})}{d^2k dy} [\rho_p, \rho_t]}$$

$$\langle \rho^a \rho^b \rho^c \rho^d \rangle = \langle \rho^a \rho^b \rangle \langle \rho^c \rho^d \rangle + \langle \rho^a \rho^c \rangle \langle \rho^b \rho^d \rangle + \langle \rho^a \rho^d \rangle \langle \rho^b \rho^c \rangle$$

- Is this quantity well defined...?

The Alternative: Keeping vs. Smearing the Color Fields

- The operators in a calculation like MSTV are factorizing (flow-like) at the level of a single color field, but not after averaging the color fields out:

$$\left\langle \hat{D}_2(\vec{x}_\perp, \vec{y}_\perp) \hat{D}_2(\vec{z}_\perp, \vec{w}_\perp) \right\rangle_{\text{color}} \neq \left\langle \hat{D}_2(\vec{x}_\perp, \vec{y}_\perp) \right\rangle_{\text{color}} \left\langle \hat{D}_2(\vec{z}_\perp, \vec{w}_\perp) \right\rangle_{\text{color}}$$

- The picture before and after color averaging is very different:
 - Single-particle distribution $\frac{dN}{d^2k}$ directional vs isotropic
 - Factorizable vs irreducible multiparticle correlations

Systematics of “Flow” versus “Non-Flow”

- “**Flow**” refers to any source of anisotropy in the **single-particle** distribution **in an event**

$$v_n \equiv \frac{1}{N} \left| \int_p e^{in\phi} \frac{dN_1}{d^2p} \right|$$

- Multi-particle distributions arise both from **independent** production (**flow**) and dynamical **correlations** (**non-flow**)

$$\frac{dN_2}{d^2p_1 d^2p_2} \equiv \frac{\text{“Flow”}}{d^2p_1} \frac{dN_1}{d^2p_2} + \delta_2(p_1, p_2) \text{ “Non-Flow”}$$

- **Cumulants** are observables that are **differently sensitive** to **flow** and **non-flow**

$$(v_n\{2\})^2 \equiv \left\langle \frac{1}{N^2} \int_{p_1 p_2} e^{in(\phi_1 - \phi_2)} \frac{dN_2}{d^2p_1 d^2p_2} \right\rangle$$

M. Luzum, H. Petersen, J. Phys. G41 (2014)

$$(v_n\{4\})^4 \equiv 2 \left\langle \frac{1}{N^2} \int_{p_1 p_2} e^{in(\phi_1 - \phi_2)} \frac{dN_2}{d^2p_1 d^2p_2} \right\rangle^2 - \left\langle \frac{1}{N^4} \int_{p_1 p_2 p_3 p_4} e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \frac{dN_4}{d^2p_1 \cdots d^2p_4} \right\rangle$$

Multiparticle Cumulants: Flow Scenario

- Usually: a **flow-only** scenario

➤ **No dynamical correlations** ($\delta_2 = 0$)

“Flow” only

$$\frac{dN_2}{d^2p_1 d^2p_2} \equiv \frac{dN_1}{d^2p_1} \frac{dN_1}{d^2p_2}$$

➤ Multiparticle production **factorizes**

➤ All cumulants due to **single-particle anisotropy**

$$(v_n\{2\})^2 \stackrel{flow}{=} \langle v_n^2 \rangle$$

- All cumulants describe the **event-by-event distribution** of the **single-particle anisotropy** v_n

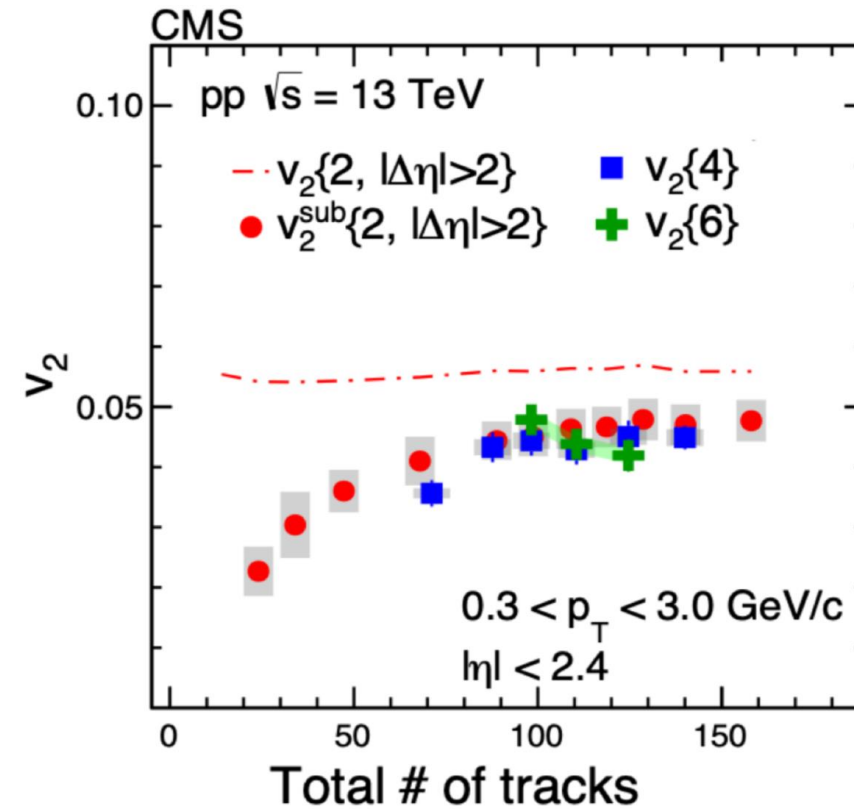
$$\begin{aligned} (v_n\{4\})^4 &\stackrel{flow}{=} 2\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle \\ &= (v_n\{2\})^4 - \text{Var}(v_n^2) \end{aligned}$$

M. Luzum, H. Petersen, J. Phys. G41 (2014)

Multiparticle Cumulants: Flow Scenario

➤ Natural **hierarchy**:

$$v_n\{2\} > v_n\{4\} \approx v_n\{6\} \approx \dots$$



Multiparticle Cumulants: Non-Flow Scenario

M. Luzum, H. Petersen, J. Phys. G41 (2014)

- Often in initial-state calculations:

- **Isotropic** single-particle distribution $(v_n = 0)$

$$\frac{dN_1}{d^2p} = \frac{1}{2\pi p_T} \frac{dN}{dp_T}$$

- Only **dynamical correlations**

$$\frac{dN_2}{d^2p_1 d^2p_2} \equiv \delta_2(p_1, p_2) \quad \text{“Non-Flow” only}$$

- Sequential **hierarchy** of correlations in N_c

- $\delta_2 \gg \delta_4 \gg \dots$

$$(v_n\{2\})^2 \stackrel{\text{nonflow}}{=} \langle \delta_{2,(n)} \rangle$$

- Usually **imaginary** $v_n\{4\}$

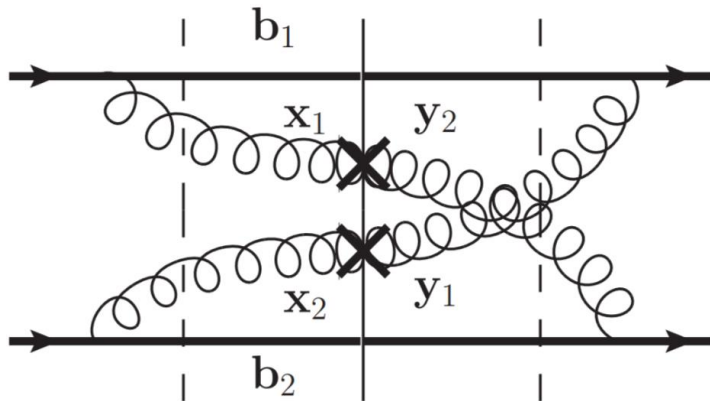
$$(v_n\{4\})^4 \stackrel{\text{nonflow}}{=} -2 \text{Var}(\delta_{2,(n)}) + \underbrace{\langle \delta_{4,(n)} \rangle}$$

Two-Gluon Correlations, After Color Averaging

Y. Kovchegov, D. Wertepny, Nucl. Phys. A906 (2013)

Full semi-dilute / dense

- E.g)



$$\begin{aligned} \frac{d\sigma_{crossed}}{d^2k_1 dy_1 d^2k_2 dy_2} &= \frac{1}{[2(2\pi)^3]^2} \int d^2B d^2b_1 d^2b_2 T_1(B - b_1) T_1(B - b_2) d^2x_1 d^2y_1 d^2x_2 d^2y_2 \\ &\times \left[e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{y}_2) - i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{y}_1)} + e^{-i\mathbf{k}_1 \cdot (\mathbf{x}_1 - \mathbf{y}_2) + i\mathbf{k}_2 \cdot (\mathbf{x}_2 - \mathbf{y}_1)} \right] \\ &\frac{16\alpha_s^2}{\pi^2} \frac{C_F}{2N_c} \frac{\mathbf{x}_1 - \mathbf{b}_1}{|\mathbf{x}_1 - \mathbf{b}_1|^2} \cdot \frac{\mathbf{y}_2 - \mathbf{b}_2}{|\mathbf{y}_2 - \mathbf{b}_2|^2} \frac{\mathbf{x}_2 - \mathbf{b}_2}{|\mathbf{x}_2 - \mathbf{b}_2|^2} \cdot \frac{\mathbf{y}_1 - \mathbf{b}_1}{|\mathbf{y}_1 - \mathbf{b}_1|^2} \\ &\times \left[Q(\mathbf{x}_1, \mathbf{y}_1, \mathbf{x}_2, \mathbf{y}_2) - Q(\mathbf{x}_1, \mathbf{y}_1, \mathbf{x}_2, \mathbf{b}_2) - Q(\mathbf{x}_1, \mathbf{y}_1, \mathbf{b}_2, \mathbf{y}_2) + S_G(\mathbf{x}_1, \mathbf{y}_1) \right. \\ &- Q(\mathbf{x}_1, \mathbf{b}_1, \mathbf{x}_2, \mathbf{y}_2) + Q(\mathbf{x}_1, \mathbf{b}_1, \mathbf{x}_2, \mathbf{b}_2) + Q(\mathbf{x}_1, \mathbf{b}_1, \mathbf{b}_2, \mathbf{y}_2) - S_G(\mathbf{x}_1, \mathbf{b}_1) \\ &- Q(\mathbf{b}_1, \mathbf{y}_1, \mathbf{x}_2, \mathbf{y}_2) + Q(\mathbf{b}_1, \mathbf{y}_1, \mathbf{x}_2, \mathbf{b}_2) + Q(\mathbf{b}_1, \mathbf{y}_1, \mathbf{b}_2, \mathbf{y}_2) - S_G(\mathbf{b}_1, \mathbf{y}_1) \\ &\left. + S_G(\mathbf{x}_2, \mathbf{y}_2) - S_G(\mathbf{x}_2, \mathbf{b}_2) - S_G(\mathbf{b}_2, \mathbf{y}_2) + 1 \right] \end{aligned}$$

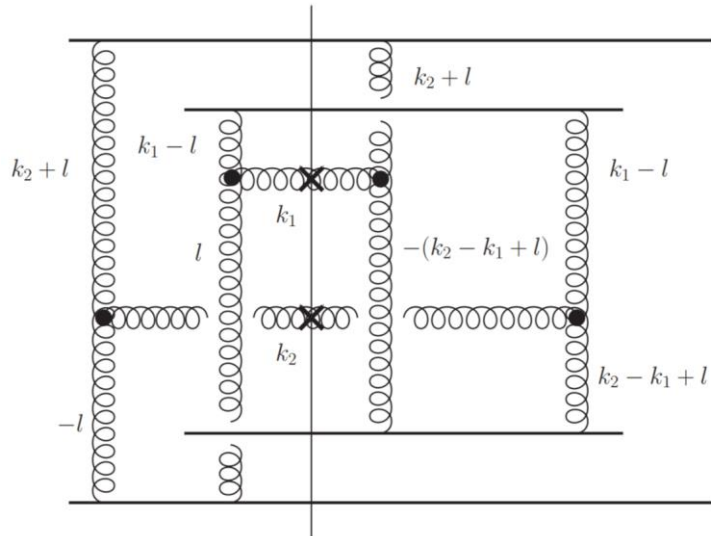
Two-Gluon Correlations, After Color Averaging

High pT:
semi-dilute / semi-dilute

Y. Kovchegov, D. Wertepny, Nucl. Phys. A906 (2013)

$$T_A^2(\vec{x}_\perp) \propto T_B^2(\vec{x}_\perp)$$

• E.g)



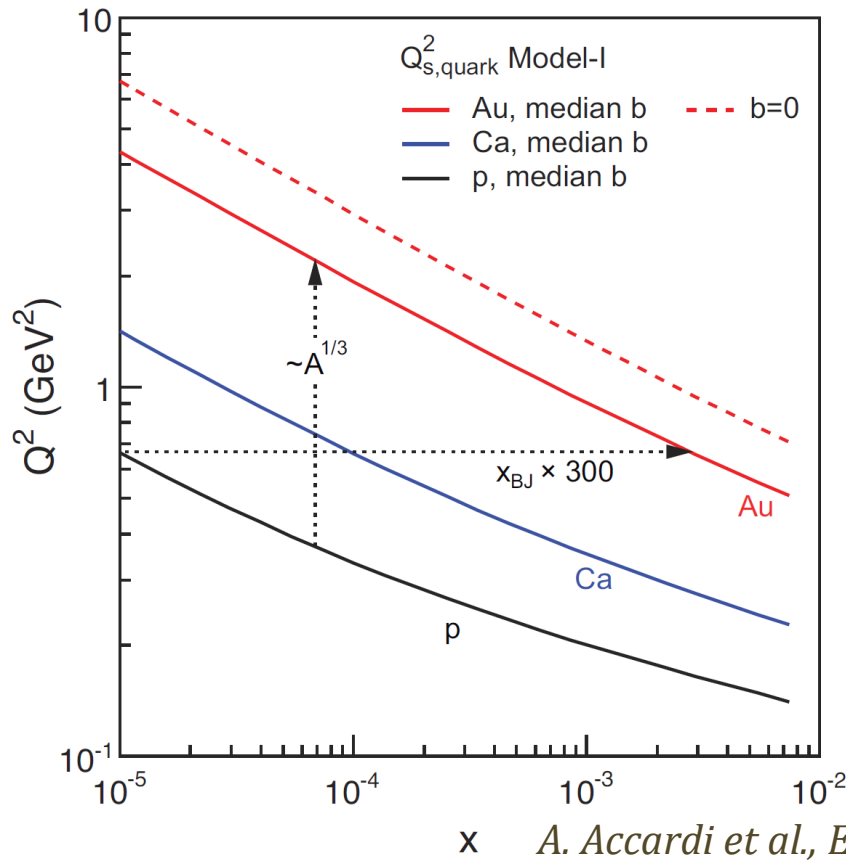
$$\left. \frac{d\sigma_{crossed}^{(corr)}}{d^2k_1 dy_1 d^2k_2 dy_2} \right|_{LO} = \frac{\alpha_s^2}{32\pi^4} \int d^2B d^2b [T_1(B-b)]^2 \frac{Q_{s0}^4(b)}{k_1^2 k_2^2}$$

$$\int \frac{d^2l}{(l^2)^2 ((l-k_1+k_2)^2)^2 ((k_1-l)^2)^2 ((k_2+l)^2)^2} \times \{ [l^2 (k_2+l)^2 + (k_1-l)^2 (l-k_1+k_2)^2 - k_1^2 (k_2-k_1+2l)^2] \times [l^2 (k_1-l)^2 + (k_2+l)^2 (l-k_1+k_2)^2 - k_2^2 (k_2-k_1+2l)^2] + 4l^2 (l-k_1+k_2)^2 [((k_1-l)^2)^2 + ((k_2+l)^2)^2] \} + (k_2 \rightarrow -k_2)$$

Two-Gluon Correlations, After Color Averaging

High pT:
semi-dilute / semi-dilute

M.D.S. et al., in preparation

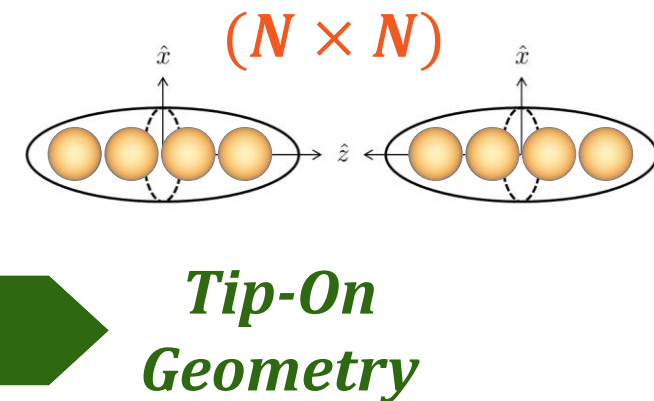
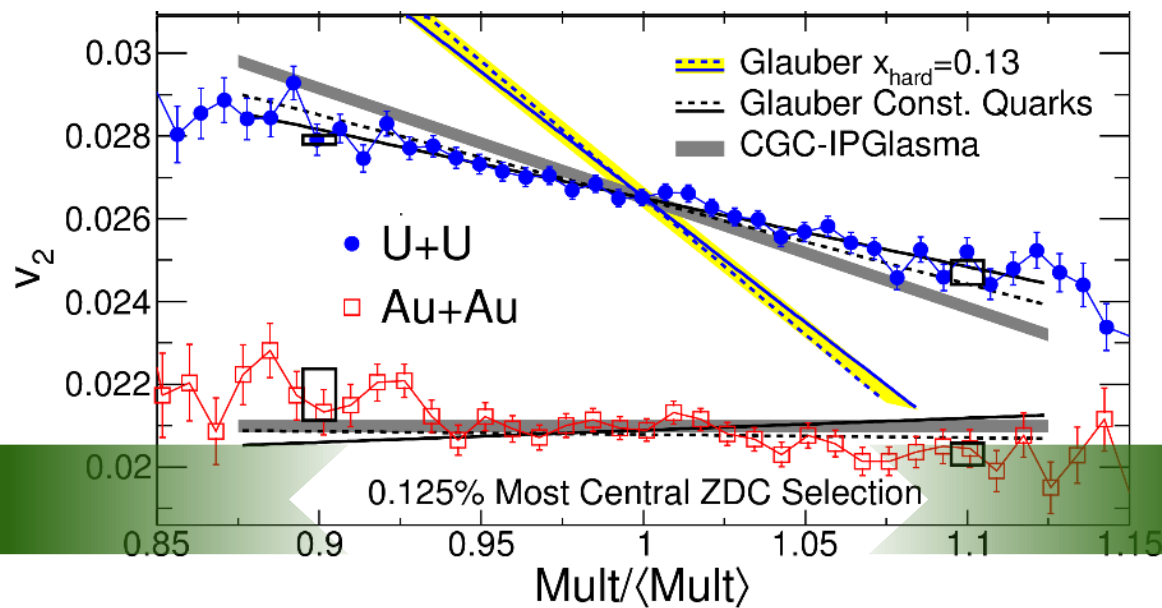
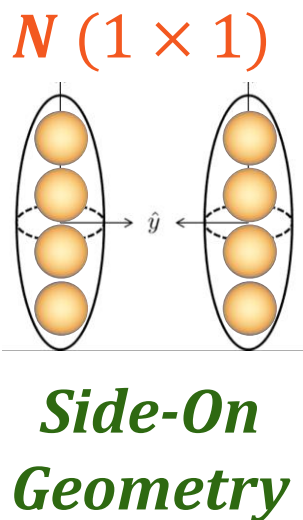


$$\delta_2(p_1, p_2) \stackrel{\text{L.O.}}{=} \left(\int d^2 x_{\perp} T_A^2(\vec{x}_{\perp}) T_B^2(\vec{x}_{\perp}) \right) f(p_1, p_2)$$

$$\delta_2(p_1, p_1) \stackrel{\text{N.L.O.}}{=} \left(\int d^2 x_{\perp} T_A^3(\vec{x}_{\perp}) T_B^3(\vec{x}_{\perp}) \right) g(p_1, p_2)$$

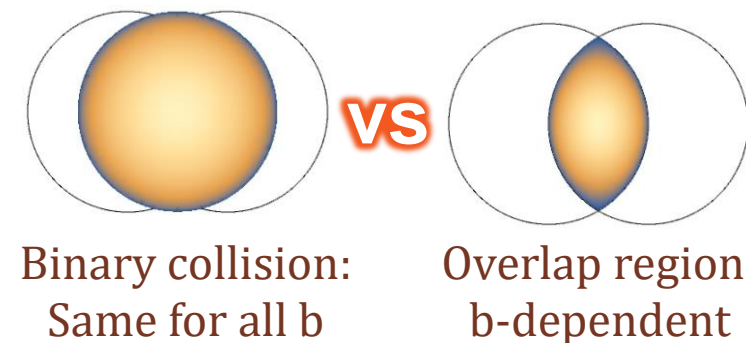
- Two-gluon correlations at high-pT
- Momentum dependent coefficients **cancel in ratios**
- **Can compare apples to apples between hydro and CGC**

Ultracentral Collisions of Deformed Ions

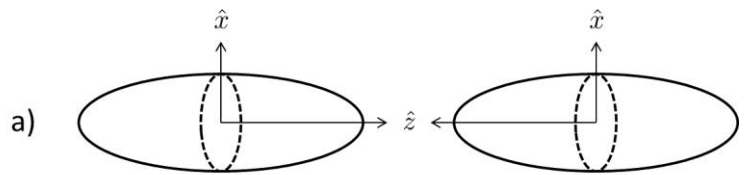


- Ultracentral ($b \approx 0$) collisions are sensitive to **nonspherical deformations** of the nuclear structure
- ❖ Sensitive to the microscopic **sub-nucleonic mechanisms of entropy deposition**

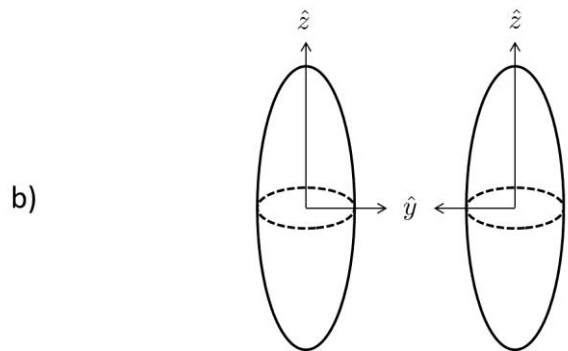
Model sensitivity



Deformed Ions as Model Discriminators



$$\rho(\vec{r}) = \rho_0 e^{-\frac{x^2}{R^2} - \frac{y^2}{R^2} - \frac{\lambda^2}{R^2} z^2}$$



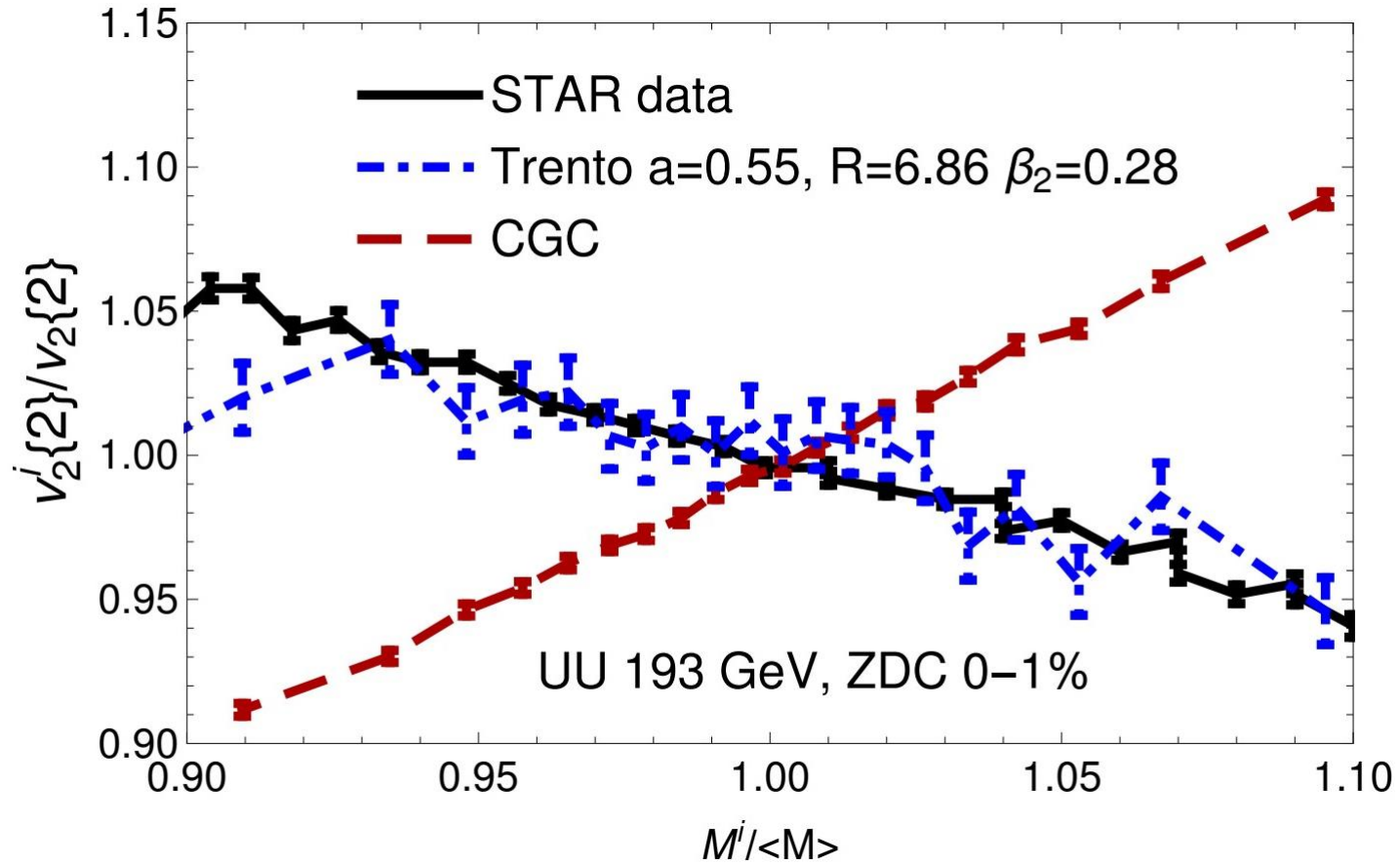
$$\frac{C_{tip-on-tip}(\mathbf{k}_1, y_1, \mathbf{k}_2, y_2)|_{LO}}{C_{side-on-side}(\mathbf{k}_1, y_1, \mathbf{k}_2, y_2)|_{LO}} = \frac{1}{\lambda} \approx 1.26 \quad (\text{for } U + U)$$

*Y. Kovchegov, D. Wertepny,
Nucl. Phys. A925 (2014)*

We conclude that, at least at the lowest order, the **two-gluon correlations behave in an exactly opposite way from hydrodynamics**: while hydrodynamic contribution to v_2 is ellipticity-driven, and is hence larger in the side-on-side

- For ultracentral collisions of ellipsoidal uranium, the multiplicity dependence is expected to be opposite from hydro
 - CGC doesn't care about the geometry itself; only the multiplicity

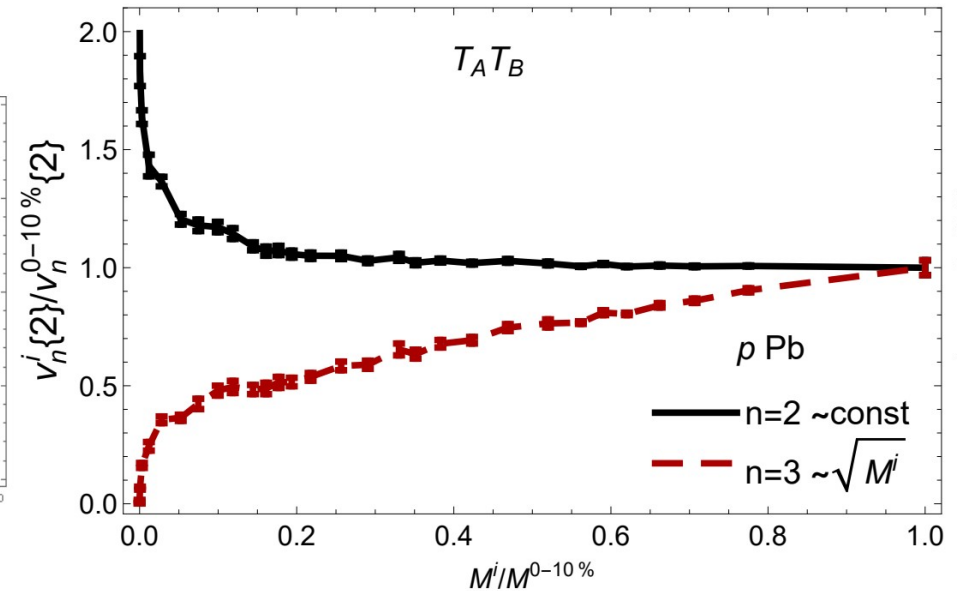
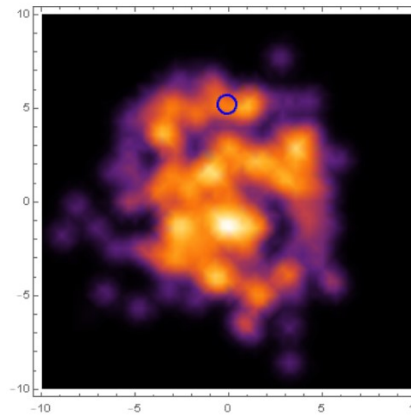
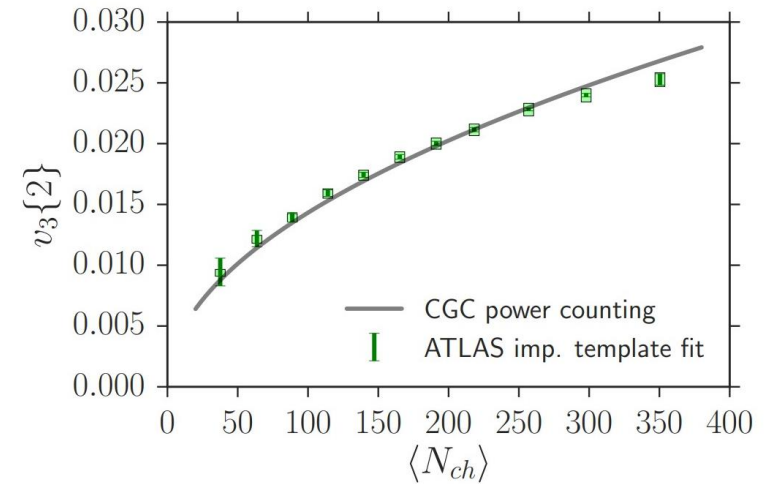
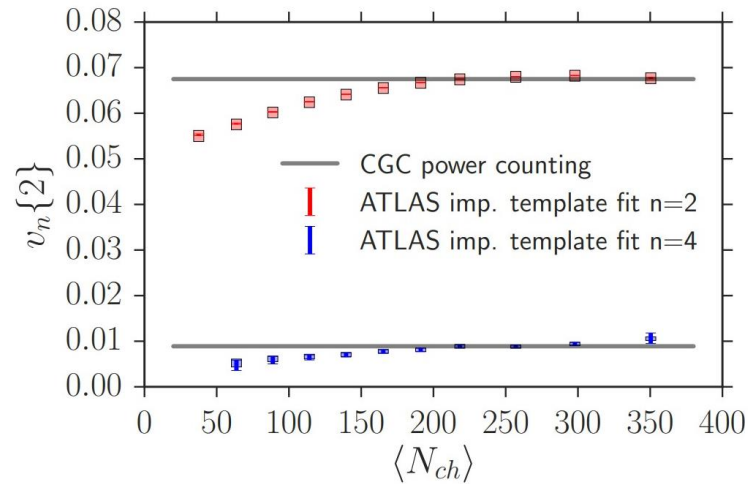
Qualitatively Different Multiplicity Dependence



M. D. S. et al, 1905.13323

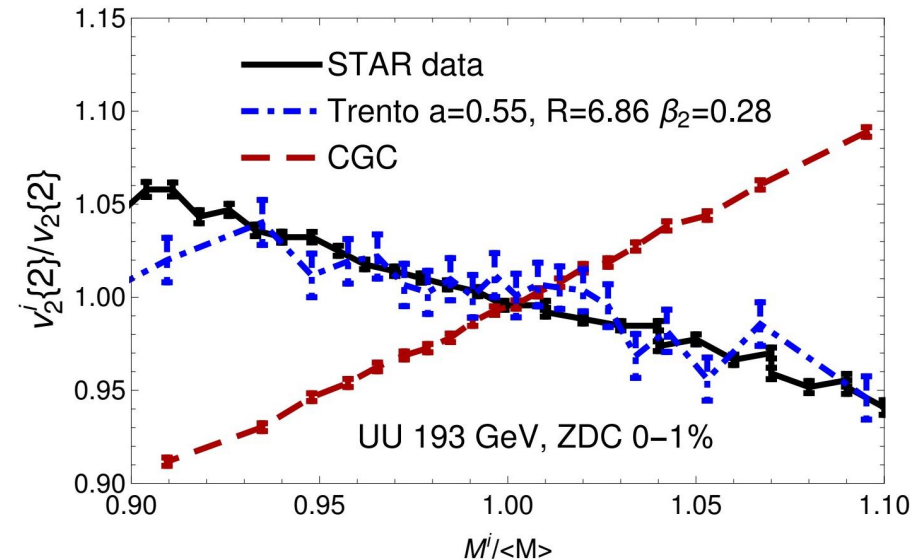
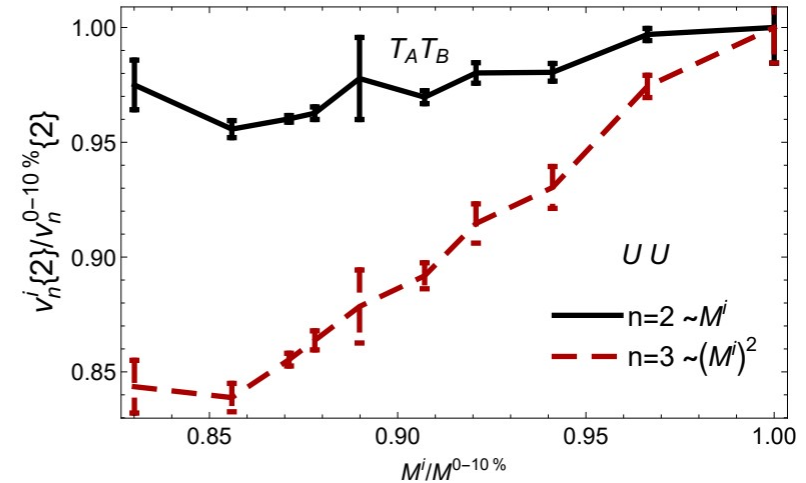
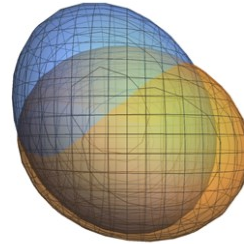
A Continuation of the MSTSV Multiplicity Story

- MSTV: Loose power counting based on gluon densities
- Even for a “lumpy” system, that scaling is seen in our simulations (here p Pb)

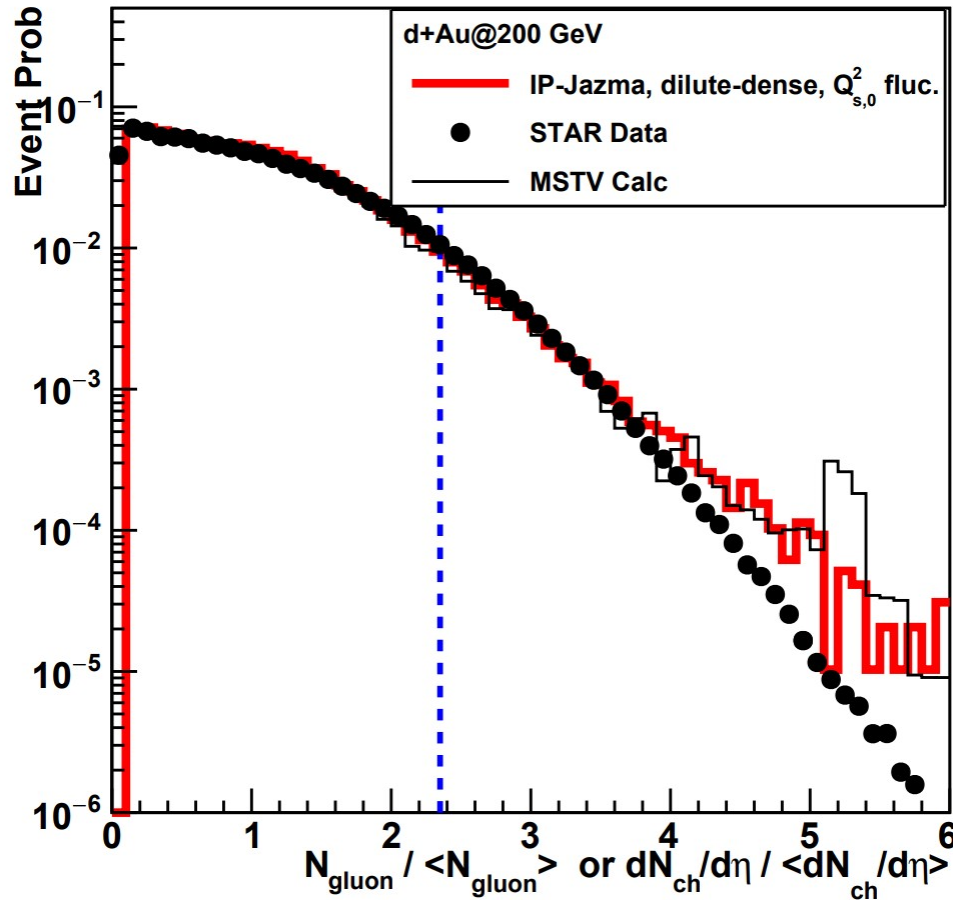


A Continuation of the MSTSV Multiplicity Story

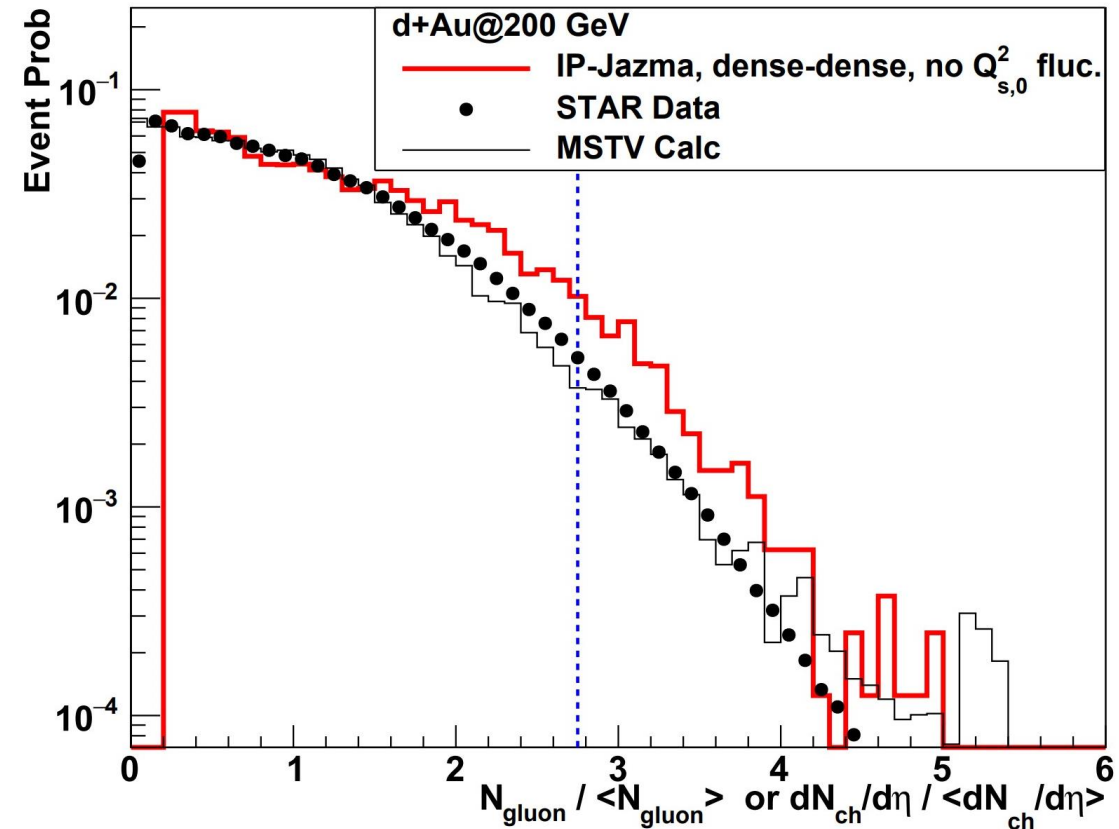
- MSTV: Loose power counting based on gluon densities
- Even for a “lumpy” system, that scaling is seen in our simulations (here p Pb)
- Non-spherical deformations qualitatively change the multiplicity dependence



IP-Jazma: The Null Hypothesis



J. Nagle and W. Zajc, Phys.Rev.C 99 (2019)



- Many crucial features of the CGC calculations can be reproduced **without** event by event color field fluctuations

Initial Eccentricities Without Color Fields

- Flow harmonics from IP-Glasma are well approximated by an initial-state model with no color field fluctuations

$$s_0 \sim \sqrt{T_A T_B}$$

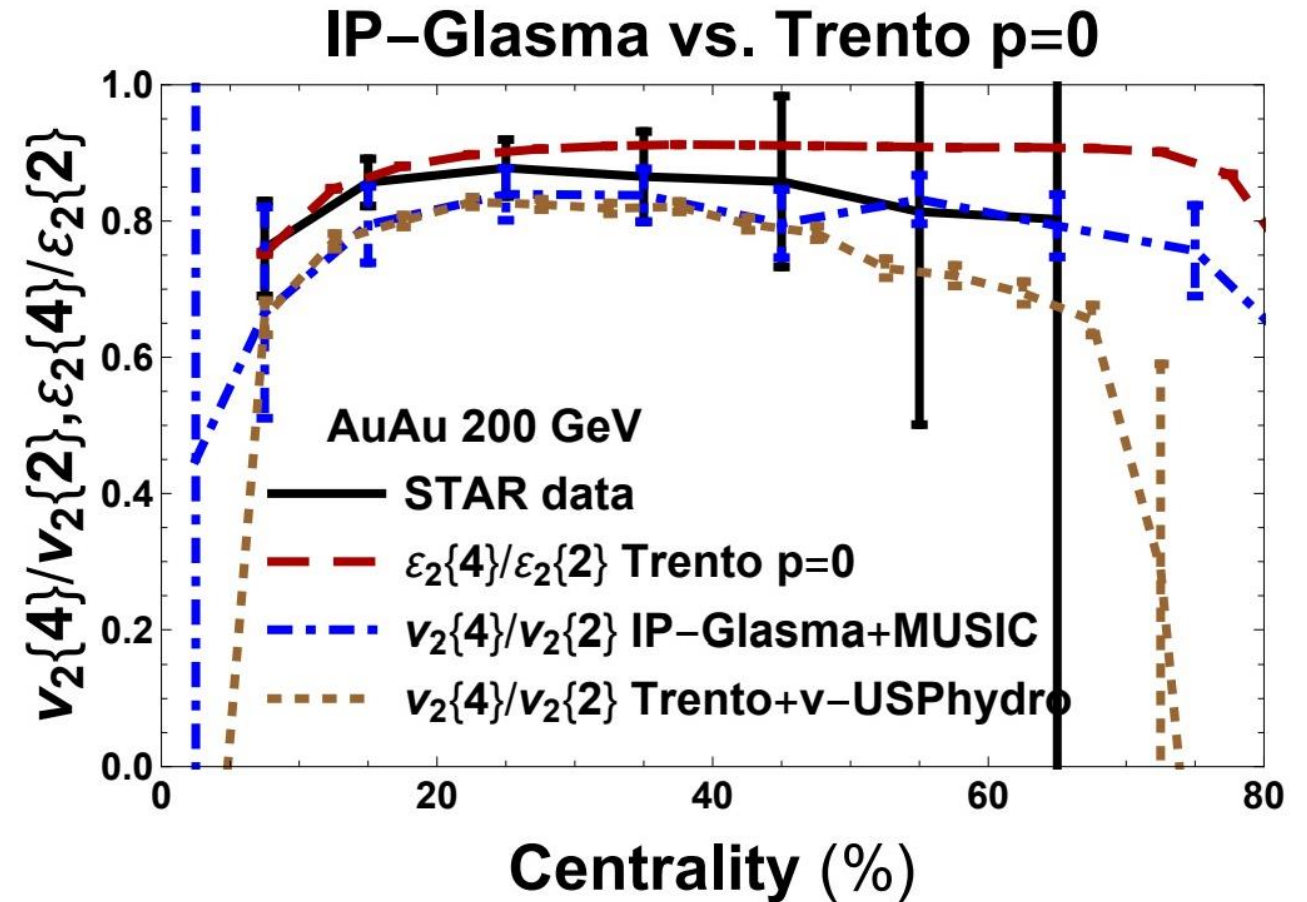
J. E. Bernhard et al., Phys.Rev.C 94 (2016)

Closest allowed functional form to the initial energy density?

$$\epsilon_0 \sim T_A T_B$$

G. Chen et al., Phys.Rev.C 92 (2015)

*M. D. S. et al,
arXiv:1910.03677*



Questions to Ponder...

- What are the implications for the many-body cumulants of keeping versus averaging over event-by event color fluctuations?
- Are individual color-field configurations well-defined “events”?
- Is the positive / negative slope of the multiplicity dependence $v_2(N_{ch})$ in ultracentral collisions of deformed ions a robust discriminator of hydro vs. non-hydro models?
- Are any color field fluctuations **required** to describe the gross features of the data?