Non-abelian plasmas far-from-equilibrium - quasiparticle excitations and transport properties

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October 7th 2020

Hydrodynamization in the weak coupling framework

- Gluon occupation number $f_g \sim 1/g^2$, quarks $f_q \leq 1$. System dominated by effectively classical gluon fields.
- Interplay of expansion and interactions $\rightarrow f_g \sim 1. \rightarrow$ Kinetic theory becomes valid.

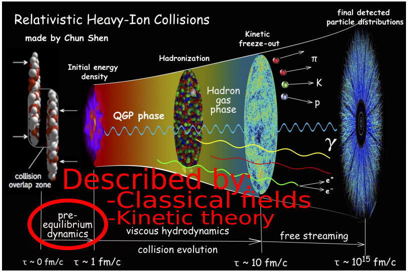


Figure: fig. Chun Shen

Sketch of the nonequilibrium evolution

- Classical Yang-Mills (→ real time lattice !) theory valid for initial overoccupied phase.
- Theories have a brief overlapping range of validity
- Final stages of out-of-equilibrium evolution described by kinetic theory (AMY).
- State of the art simulations: CYM+KT+Hydro (See e.g. Phys.Rev.Lett. 122 (2019) 12, 122302).

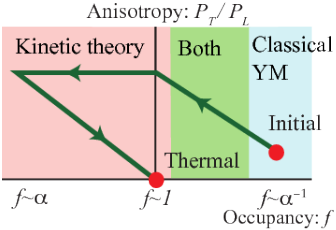


Figure: System's trajectory in the occupation number - anisotropy plane. Fig by. Aleksi Kurkela

Cherry picked topics in far-from equilibrium field theory

For the rest of the talk focus on classical dynamics and on the following phenomena:

- Plasmons, quasiparticles in the plasma. Properties such as mass and damping rate.
- Spectral functions in 3D plasma (Soon in 2D!). Also connected to quasiparticles.
- Heavy quark diffusion coefficient in far-from-equilibrium gluon plasma.
- Self-similarity in 2D.

- Initial stages, gluons $f \sim {}^{1}/_{g^2} \rightarrow$ classical fields $\rightarrow$ real time lattice simulations.
- Non-thermal fixed points and self-similar scaling. Occupation number scales as $f(t,p) = (Qt)^{\alpha} f_s((Qt)^{\beta} p)$.
- Dynamically generated hierarchy of scales reminiscent of PT:
  - Ultrasoft scale $g^2 T \iff$ magnetic scale $m_{\mathrm{mag}} \sim t^{-1/3}$
  - Soft scale $gT \iff$ mass scale $\omega_{\mathrm{pl}} \sim t^{-1/7}$
  - Temperature $T \iff$ hard scale $\Lambda \sim t^{1/7}$

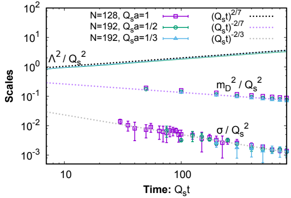


Figure: Phys.Rev.D 93 (2016) 7, 074036.

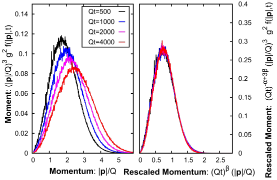


Figure: PRD 89 (2014) 11, 114007

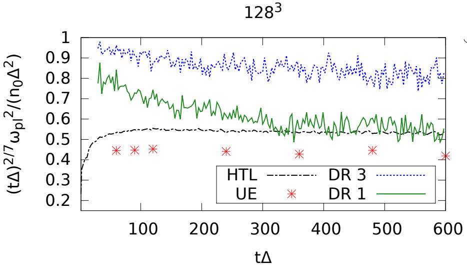
Plasmon mass scale, 3D

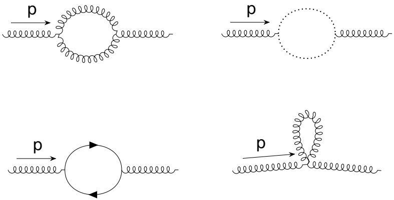
Perturbatively predict for self-similar regime:

$$\omega_{pl}^2 = \frac{4}{3} N_c \int \frac{d^3 p}{(2\pi)^3} \frac{g^2 f(t,p)}{\omega(p)} \sim t^{-2/7} \quad (1)$$

The occupation number is extracted as

$$f(t,p) = \frac{1}{N_c^2 - 1} \frac{\langle EE \rangle_T (t,t,p)}{\sqrt{p^2 + m^2}} . \quad (2)$$



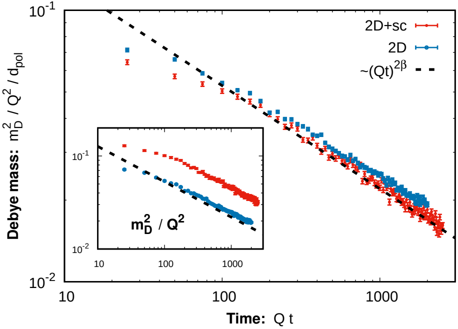


Plasmon mass scale, 2D

Perturbation theory + kinetic theory arguments:

$$\omega_{pl}^2 = \frac{4}{3} N_c \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{g^2 f(t,p)}{\omega(p)} \sim t^{-2/5} \tag{3}$$

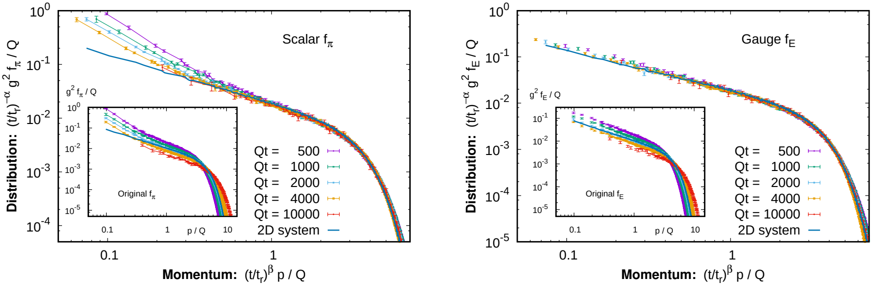
$\beta = -1/5$



Self-similarity in 2D

Two theories:

- Pure 2D
- 3+1D independent of z-direction, ∼ boost-invariant system. A_z becomes effectively a scalar field.



- Self-similar scaling also in 2D. However for scalar excitations at low momenta see some violations.

Spectral functions

Divide gauge field into background and a fluctuation
$A \to A + a, E \to E + e$.

$$\langle a(t,\boldsymbol{p})\rangle = \int \mathrm{d}t\, G_R(t,t',\boldsymbol{p}) j(t',\boldsymbol{p}) \tag{4}$$

Use instantaneous perturbation ($j \sim \delta(t-t')$) that satisfies

$$\langle j j^* \rangle \sim \delta_{bb'}\delta_{pp'} \tag{5}$$

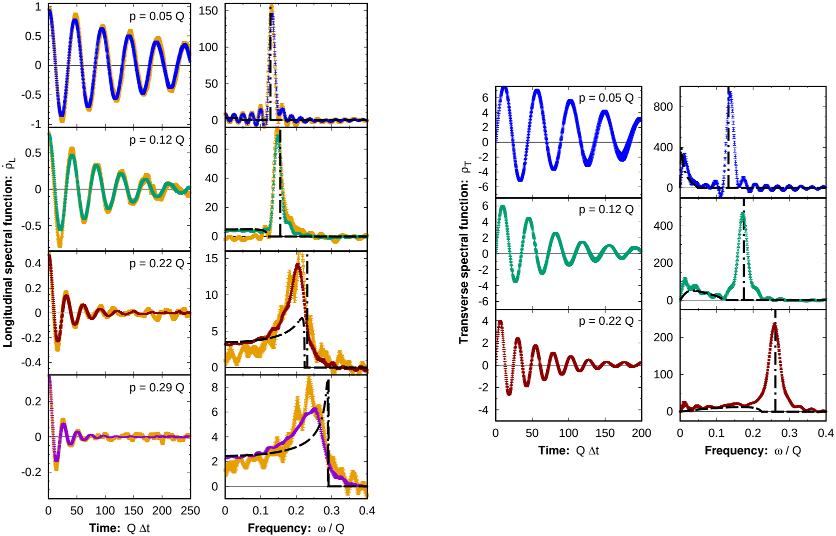
Thus we have

$$\langle a j^* \rangle = G_R \tag{6}$$

The spectral function is given by

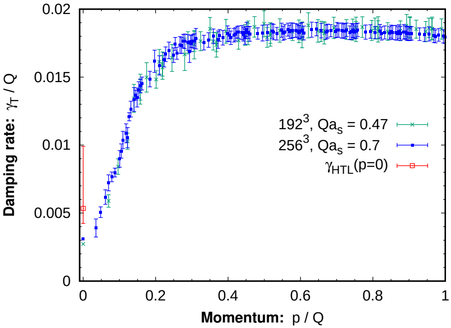
$$\rho = 2\mathfrak{Im} G_R \tag{7}$$

Spetral functions: numerical results



- See quasiparticle peaks with finite width + Landau cuts!

Quasiparticle damping rate → width of the QP peak



- First nonperturbative estimate of QP damping!
- Rough agreement with the perturbative (NLO) estimate (red point).

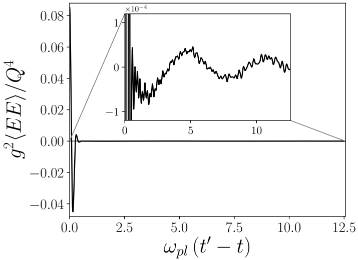
Heavy quark in a color field

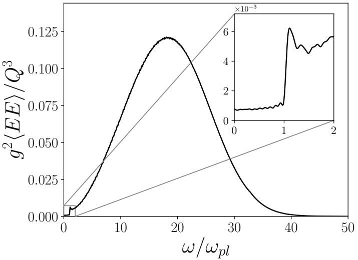
Classical equation of motion

$$\dot{p}_i(t) = \mathscr{F}_i(t)\,. \tag{8}$$

The force exerted on the heavy quark by chromoelectric field with $\langle \dot{p} \rangle = 0$ but with nonzero variance

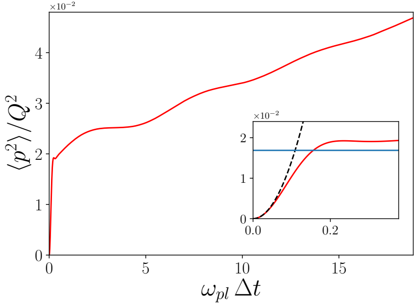
$$\langle \dot{p}_i(t)\dot{p}_i(t')\rangle = \frac{g^2}{N_c}\mathrm{Tr}\langle E_i(t)U_0(t,t')E_i(t')U_0(t',t)\rangle = \frac{g^2}{2N_c}\langle EE\rangle(t,t') \tag{9}$$





Momentum broadening

$$\langle p^2(t,\Delta t)\rangle = \frac{g^2}{2N_c}\int_t^{t+\Delta t} \mathrm{d}t' \int_t^{t+\Delta t} \mathrm{d}t''\, \langle EE\rangle(t',t''). \qquad (10)$$



3 distinct features

- Rapid growth $\Delta t \approx 2\pi/\Lambda$, broad peak dominates the integration
- Damped oscillations $\Delta t \approx 2\pi/\omega_{pl}$, new feature!
- Linear growth in Δt, $1/\Lambda \ll \Delta t \ll t$, consistent with Langevin description.

Heavy quark diffusion

Diffusion coefficient connected to $\langle p^2 \rangle$

$$3\kappa(t,\Delta t) = \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle \tag{11}$$

As in thermal equilibrium, the diffusion coefficient $\kappa(t)$ defined at $\Delta t \to \infty$ limit

$$\frac{g^2}{2N_c}\langle EE\rangle(t,\omega=0) \ = \ 3\,\kappa_\infty(t)\,. \tag{12}$$

On the lattice $\kappa(t,\Delta t)$ given by

$$\kappa(t,\Delta t) \approx \frac{g^2}{3N_c}\int_t^{t+\Delta t}\mathrm{d}t'\int\frac{\mathrm{d}^3x}{V}\langle E_i^a(t,\mathbf{x})E_i^a(t',\mathbf{x})\rangle, \tag{13}$$

$\kappa(t)$ - time-evolution

- Expectation as (40) $\kappa(t) \approx At^{-5/7}(\log(t) + B)$, use as a fit.

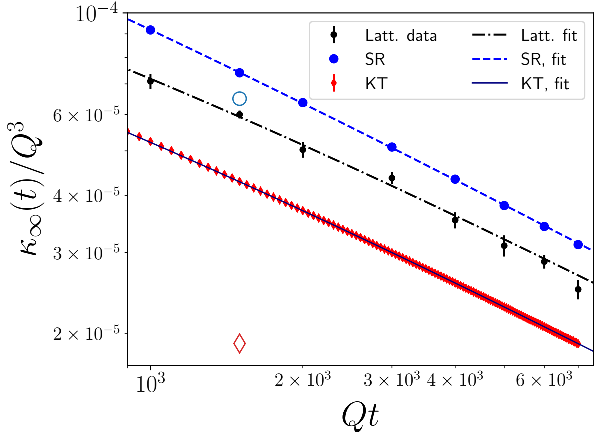


Figure: Extracted heavy quark diffusion coefficient vs. time

Conclusions

We have

- Measured quasiparticle properties (spectral function, mass, damping rate) in gluon plasma far-from-equilibrium
- Measured the heavy quark diffusion coefficient far-from-equilibrium.
- Demonstrated self-similarity also in 2+1D.

Future plans:

- Measure transport coefficients ($\kappa, \hat{q}$ etc.) out of equilibrium using KT.
- Spectral functions in 2D gluon plasma (extending PRD100 (2019), 094022 & PRD98 (2018), 014006).
- Quark production at the initial stages in heavy-ion collisions.
- ????

2. HTL method: main ingredients

$$\kappa = \frac{d_A}{6N_c}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{(2\pi)}\frac{2\sin(T\omega)}{\omega}\Bigg[2g^2 \frac{\ddot{F}_T(t,\Delta t=0,p)}{\dot{\rho}_T(t,\Delta t=0,p)}\,\dot{\rho}_T(t,\omega,p) + g^2 \frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)}\,\dot{\rho}_L(t,\omega,p)\Bigg]. \quad (14)$$

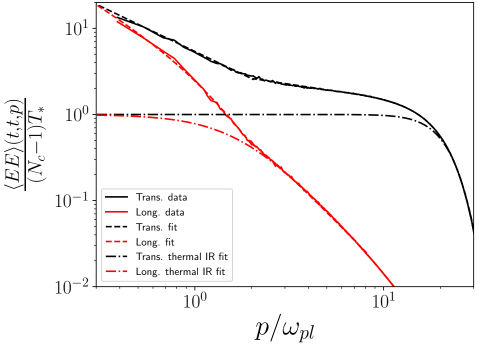
HTL expectation:

$$\ddot{F}_T^{HTL}(\bar{t},\Delta t=0,p) = T_*(\bar{t}) \quad (15)$$

$$\ddot{F}_L^{HTL}(\bar{t},\Delta t=0,p) = T_*(\bar{t})\,\frac{2m^2}{2m^2+p^2}. \quad (16)$$

2. HTL method: main ingredients

$$\kappa = \frac{d_A}{6N_c}\int \frac{d^3p}{(2\pi)^3}\int \frac{d\omega}{(2\pi)}\frac{2\sin(T\omega)}{\omega}\left[2g^2\frac{\ddot{F}_T(t,\Delta t=0,p)}{\dot{\rho}_T(t,\Delta t=0,p)}\dot{\rho}_T(t,\omega,p) + g^2\frac{\ddot{F}_L(t,\Delta t=0,p)}{\dot{\rho}_L(t,\Delta t=0,p)}\dot{\rho}_L(t,\omega,p)\right]. \quad (17)$$



2. HTL method: main ingredients

$$\kappa = \frac{d_A}{6N_c} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}\omega}{(2\pi)} \frac{2\sin(T\omega)}{\omega} \left[2g^2 \frac{\ddot{F}_T(t, \Delta t = 0, p)}{\dot{\rho}_T(t, \Delta t = 0, p)} \dot{\rho}_T(t, \omega, p) + g^2 \frac{\ddot{F}_L(t, \Delta t = 0, p)}{\dot{\rho}_L(t, \Delta t = 0, p)} \dot{\rho}_L(t, \omega, p) \right]. \quad (18)$$

Equal time spectral functions:

$$\dot{\rho}_T^{HTL}(\Delta t = 0, p) = 2 \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \dot{\rho}_T^{HTL}(\omega, p) = 1 \quad (19)$$

$$\dot{\rho}_L^{HTL}(\Delta t = 0, p) = 2 \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \dot{\rho}_L^{HTL}(\omega, p) = \frac{2m^2}{2m^2 + p^2}. \quad (20)$$

2. HTL method: main ingredients

$$\rho_T(\omega,p) = (2\pi)\,\beta_T(\omega,p) + (2\pi)\,Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right] \tag{21}$$

$$\rho_L(\omega,p) = \frac{p^2}{\omega^2}(2\pi)\,\beta_L(\omega,p) + \frac{p^2}{\omega^2}(2\pi)\,Z_L(p)\left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p)\right]. \tag{22}$$

Quasiparticle peaks:

$$h_{T/L}\left(\omega \mp \omega_{T/L}(p),p\right) = \frac{1}{\pi}\frac{\gamma_{T/L}(p)}{\left(\omega \mp \omega_{T/L}(p)\right)^2 + \gamma^2_{T/L}(p)}\,. \tag{23}$$

$\omega_{T,L}$ Quasiparticle dispersion relation, $\gamma_{T,L}$ quasiparticle damping rate (inverse lifetime). Use our data, LO PT predicts $\gamma = 0$.

2. HTL method: main ingredients

$$\rho_T(\omega,p) = (2\pi)\,\beta_T(\omega,p) + (2\pi)\,Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right] \tag{24}$$

$$\rho_L(\omega,p) = \frac{p^2}{\omega^2}(2\pi)\,\beta_L(\omega,p) + \frac{p^2}{\omega^2}(2\pi)\,Z_L(p)\left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p)\right]. \tag{25}$$

Residues of the quasiparticle peaks:

$$Z_T(p) = \frac{\omega_T(p)\left(\omega_T^2(p)-p^2\right)}{3\omega_{\mathrm{pl}}^2\omega_T^2(p) - \left(\omega_T^2(p)-p^2\right)^2}\,, \tag{26}$$

$$Z_L(p) = \frac{\omega_L(p)\left(\omega_L^2(p)-p^2\right)}{p^2\left(p^2+2m^2-\omega_L^2(p)\right)}\,. \tag{27}$$

2. HTL method: main ingredients

$$\rho_T(\omega,p) = (2\pi)\,\beta_T(\omega,p) + (2\pi)\,Z_T(p)\left[h_T(\omega-\omega_T(p),p) - h_T(\omega+\omega_T(p),p)\right] \tag{28}$$

The Landau damping terms according to HTL at LO are given by:

$$\beta_T(\omega,p) = \left(\frac{m^2}{2}x(1-x^2)\,\theta\left(1-x^2\right)\right)\left(\left[[p^2\left(1-x^2\right) + m^2\left(x^2 + \frac{x\left(1-x^2\right)}{2}\ln\left|\frac{x+1}{x-1}\right|\right)\right]^2 + \frac{\pi^2}{4}m^4x^2\left(1-x^2\right)^2\right)^{-1} \tag{29}$$

where $x = \frac{\omega}{p}$

$$\rho_L(\omega,p) = \frac{p^2}{\omega^2}(2\pi)\,\beta_L(\omega,p) + \frac{p^2}{\omega^2}(2\pi)\,Z_L(p)\left[h_L(\omega-\omega_L,p(p)) - h_L(\omega+\omega_L(p),p)\right]. \tag{30}$$

The Landau damping terms according to HTL at LO are given by:

$$\beta_L(\omega,p) = \frac{m^2 x\,\theta\left(1-x^2\right)}{\left[p^2+2m^2\left(1-\frac{x}{2}\ln\left|\frac{x+1}{x-1}\right|\right)\right]^2+\pi^2 m^4 x^2}\,, \tag{31}$$

where $x = \frac{\omega}{p}$

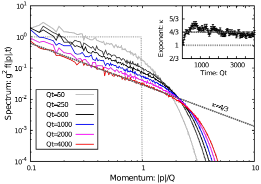


Figure: PRD 89 (2014) 114007

Comparison with thermal case

Compare far-from-equilibrium to equilibrium by defining an effective coupling via scale separation:

$$\epsilon \sim T^4, \quad m_D^2 \sim g^2 T^2 \quad \rightarrow \frac{m_D^2}{\sqrt{\epsilon}} \sim g_\epsilon^2 \tag{32}$$

Heavy quark diffusion - lattice extraction

$\kappa(t, \Delta t)$ oscillates in Δt with the frequency ω_{pl}

$$\omega_{pl}^2 = \frac{4}{3} N_c \int \frac{d^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{\omega(p)} \sim t^{-2/7} \tag{33}$$

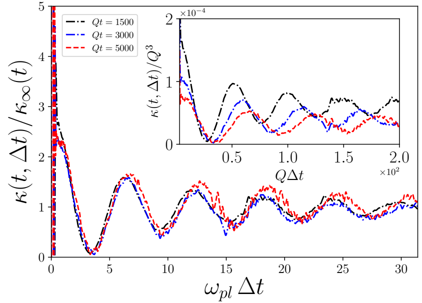


Figure: Lattice extraction of $\kappa(t, \Delta t)$

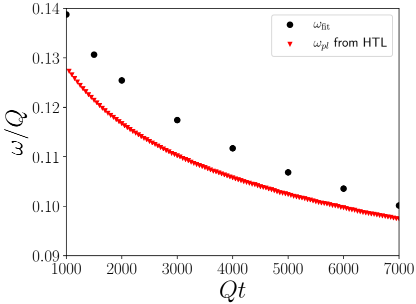


Figure: Frequency extracted from $\kappa(t, \Delta t)$ and using (33)

How κ depends on $\langle EE\rangle(t,\omega,p)$

$$3\kappa(t,\Delta t) \equiv \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle \tag{34}$$

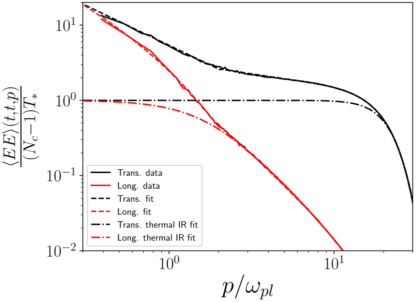
$$= \frac{g^2}{N_c}\int_t^{t+\Delta t} \mathrm{d}t' \langle EE\rangle(t+\Delta t, t')$$

$$\approx \frac{g^2}{N_c}\int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega}\,\langle EE\rangle(t,\omega,p), \tag{35}$$

Understanding the time dependence of $\langle EE \rangle$

Idea: start from $\langle EE \rangle(t,t,p)$ and construct $\kappa(t,\Delta t)$

- $\langle EE \rangle$ connected to particle distribution $f(t,p) \approx \frac{\langle EE \rangle_T(t,t,p)}{\sqrt{m^2+p^2}}$
- HTL: IR is thermal $\langle EE \rangle_T \approx T_* \to f(t,p) \approx \frac{T_*}{\sqrt{m^2+p^2}}$
- Far-from equilibrium lattice extraction: IR enhanced compared to thermal



Spectral Reconstruction (SR) method

Generalized FDR: connecting spectrum and statistical properties (Boguslavski, Kurkela, Lappi, JP: Phys.Rev.D 98 (2018) 1, 014006):

$\langle EE\rangle_{T,L}(t,\omega,p) \approx \overbrace{\langle[\hat{E},\hat{A}]\rangle(t,\omega,p)}^{\dot{\rho}(t,\omega,p)}\langle EE\rangle(t,t,p)$. The spectral function $\dot{\rho}$ is defined as

$$\dot{\rho}(\omega,p) = 2\mathfrak{Im}G_R^{HTL}(\omega,p), \tag{36}$$

with $G_R^T = \dfrac{-1}{\omega^2 - p^2 - \Pi_T(\omega,p)}$ and $G_R^L = \dfrac{p^2}{\omega^2}\dfrac{-1}{p^2 - \Pi_L(\omega,p)}$

$$\Pi_T(x) = m^2 x\left(x + (1-x^2)Q_0(x)\right)$$
$$\Pi_L(x) = -2m^2\left(1 - x\,Q_0(x)\right), \tag{37}$$

and $(x = \omega/p)$

$$Q_0(x) = \frac{1}{2}\ln\frac{x+1}{x-1} = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| - \frac{i\pi}{2}\theta(1-x^2). \tag{38}$$

Spectral reconstruction method, quasiparticles and Landau damping

2 types of contributions:

- Particle-like excitations, at LO proportional to $\delta(\omega - \omega_{T,L})$.
- Landau damping contributions, nonzero only for $\theta(1 - \omega^2/p^2)$, damping of space charge waves in plasma.

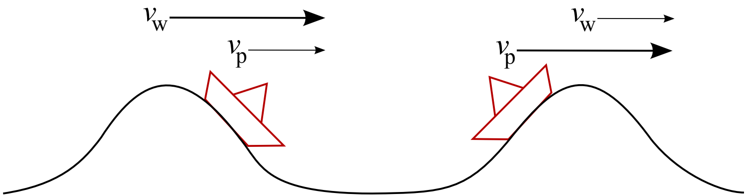


Figure: Stolen from Wikipedia

$\Delta t \to \infty$ limit in the SR framework

At the $\Delta t \to \infty$ limit only longitudinal Landau damping contributes. Using the thermal IR assumption

$$\langle EE \rangle_L^{LL}(t,t,p) = (N_c^2 - 1)\, T_* \frac{m_D^2}{p^2 + m_D^2}\, \theta(\Lambda - p)\,. \qquad (39)$$

We get

$$\kappa^{\mathrm{SR}}_{\infty,LL}(t) \approx \frac{N_c^2 - 1}{12\pi N_c} m_D^2(t) g^2 T_*(t) \log\left(\frac{\Lambda(t)}{m_D(t)}\right) \sim (Qt)^{-5/7} \log(Qt) \qquad (40)$$

This will be our expectation for the time dependence of $\kappa(t)$.

Breaking down the different HTL contributions

- Transverse QP contributions → oscillations at ω_{pl}.
- Longitudinal Landau cut → κ_∞

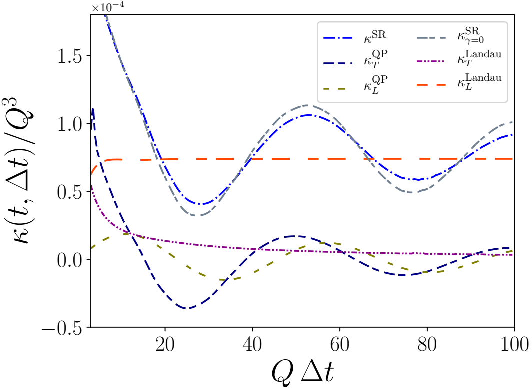


Figure: Different HTL contributions

3. Kinetic theory

In the kinetic theory framework κ is given by ($gq \rightarrow gq$, t-channel gluon exchange. Compton amplitude is suppressed)

$$\kappa = \frac{\left\langle \Delta k^2 \right\rangle}{\Delta t} = \frac{1}{6M} \int \frac{\mathrm{d}^3 \boldsymbol{k} \mathrm{d}^3 \boldsymbol{k'} \mathrm{d}^3 \boldsymbol{p'}}{(2\pi)^9 \, 8k^0 k'^0 M} (2\pi)^3 \, \delta^3 \left(\boldsymbol{p} + \boldsymbol{k'} - \boldsymbol{p'} - \boldsymbol{k} \right)$$
$$\times 2\pi\delta \left(k' - k \right) \boldsymbol{q}^2 \, |\mathscr{M}|^2_{\text{gluon}} f\left(k\right) f\left(k'\right). \quad (41)$$

k and k' gluon momenta, $q = k - k'$, p and p' incoming and outgoing heavy quark momenta.

$$|\mathscr{M}|^2_{\text{gluon}} = N_c C_H g^4 16 M^2 k_0^2 \left(1 + \cos^2 (\theta_{kk'}) \right) \frac{1}{\left(q^2 + m_D^2 \right)^2} \quad (42)$$

Generalize the previous formula, allow for a finite energy exchange

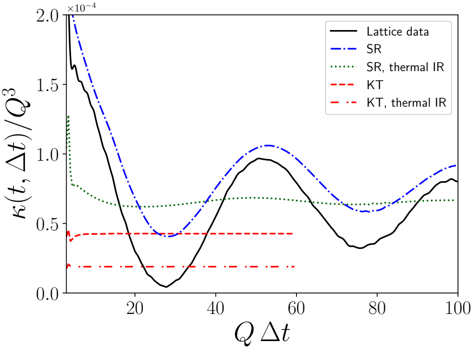
$$|\mathscr{M}|^2_{\text{gluon}}(\omega) = \left[N_c C_H g^4\right] \frac{4M^2(k_0 + k_0')^2\left(1 + \cos^2\theta_{\boldsymbol{k}\boldsymbol{k}'}\right)}{(q^2 - \omega^2 + m_D^2)^2}, \quad (43)$$

with $k_0' = k_0 + \omega$.

$$\kappa(\omega) = \frac{1}{6M}\int \frac{\mathrm{d}^3\boldsymbol{k}\,\mathrm{d}^3\boldsymbol{q}}{(2\pi)^6\, 8|\boldsymbol{k}||\boldsymbol{k}+\boldsymbol{q}|M} 2\pi\delta(|\boldsymbol{k}+\boldsymbol{q}| - |\boldsymbol{k}| - \omega)$$
$$\times\, \boldsymbol{q}^2\, |\mathscr{M}|^2_{\text{gluon}}(\omega) f(k) f(|\boldsymbol{k}+\boldsymbol{q}|) \quad (44)$$

Dependence Δt by Fourier transforming to time domain.

Dependence on Δt



- With IR enhancement see similar oscillations as in the data.
- KT: no oscillations. Trivialized frequency structure.
- Data vs. IR enhanced SR model → IR enhancement exists.

Comparison with thermal case

Expressing everything in tems of g:

$$\kappa^{\text{therm}} \approx 0.016\left(\ln\frac{1}{g} + 0.25\right) g^4 \varepsilon^{3/4} \tag{45}$$

$$\kappa_\infty(t) \approx 0.0050\left(\ln\frac{g}{\tilde{g}_\varepsilon^2} + 0.148\right) \tilde{g}_\varepsilon^5 g^{-1} \varepsilon^{3/4} \tag{46}$$

$$\kappa_\infty(t) \approx 0.0047\left(\ln\frac{1}{\tilde{g}_\Lambda} + 0.177\right) \tilde{g}_\Lambda^{5/2} g^{3/2} \varepsilon^{3/4}. \tag{47}$$

- $g_\epsilon, g_\Lambda \gg g$ for overoccupied system. Extrapolate to $f \sim 1$ to compare to thermal.
- Coefficient of κ^{therm} smaller than far-from equilibrium: IR enhancement increases m_D more than it increases κ.