Short-distance $(g\text{-}2)_\mu$ HLbL contributions in and beyond perturbation theory

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Science coffee
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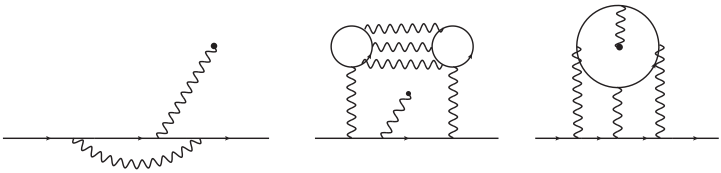
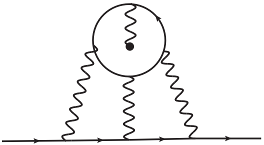
Muon g-2

- Muon response to weak magnetic field?
- Linear term in photon momentum of $\langle \mu^-_{p',\sigma'} | J^\mu_{\mathrm{EM}} e^{iS_{\mathrm{int}}} | \mu^-_{p,\sigma} \rangle$
- $S_{\mathrm{int}} \approx 0 \to g_\mu \approx 2$ $\qquad g_{\mu,\mathrm{exp}} \approx 2.002$
- $a_\mu^{\mathrm{exp}} \equiv (g_\mu^{\mathrm{exp}} - 2)/2 = 0.00116592089(63)$ PhysRevD.73.072003



HLbL: a multiscale problem

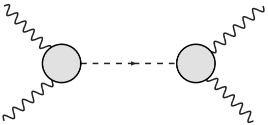


$$\Pi^q \sim \langle 0 | T(\prod_j^4 \int dx_j e^{-iq_j x_j} J^q(x_j)) e^{iS_{\rm int}} | 0 \rangle$$

$$J^\mu_q = Q_q \, \bar{q} \gamma^\mu q$$

$$a_\mu^{\rm HLbL} \sim \int_0^\infty dQ_{1,2} \int_{-1}^1 d\tau \sum_i T_i' \overline{\Pi}_i$$

- Nonperturbative problem: hadronic intermediate states
- Weights T_i enhance contributions near $Q_i \sim m_\mu$

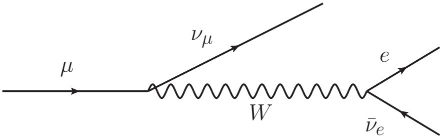


- From dispersion relations to resonance models, which need guidance $\rightarrow$ short-distance constraints?
- Basic question: asymptotic behaviour of (g-2) HLbL

The Operator Product Expansion

The basic idea Phys. Rev., 179:1499–1512, Annals Phys., 77:570–601

$$\mathcal{A}(x)\mathcal{B}(y) = \sum_i c_i'(x,y)\,\mathcal{O}'_i(x,y) \approx \sum_i c_i\,\mathcal{O}_i(y)(x-y)^{(d_{O_i}-d_A-d_B)}$$



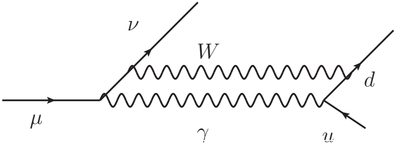
$$\langle e\bar{\nu}_e\nu_\mu|e^{iS}|\mu\rangle \sim \langle \int dx\, J(x)W(x) \int dy\, J(y)W(y)\rangle$$

$$\sim \langle \int dx \int dy\, J(x)D_W(x-y)J(y)\rangle \sim \langle \int dx \mathcal{L}_{eff}(x)\rangle$$

Basic EFT lagrangian (contains any possible mediator operator)

$$\mathcal{L}_{eff} \sim \sum_{i,D} \frac{c_{i,D}\,\mathcal{O}_{i,D}}{(\Lambda{=}M_W)^D}, \qquad D=6 \rightarrow \mathcal{L}_{G_F}^{\mathrm{Fermi}}$$

The Operator Product Expansion



$$\langle\rangle_{\text{fund}} \sim 1 + \frac{\alpha}{\pi}\left(a + b\ln\frac{M_W}{p}\right)$$

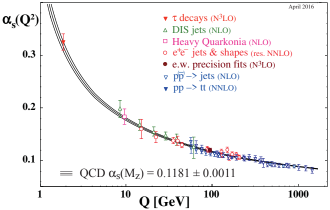
$$\langle\rangle_{\text{eff}} \sim \underbrace{c(\mu)}_{\ln\frac{M_W}{\mu}}\underbrace{\langle\mathcal{O}(\mu)\rangle}_{\ln\frac{\mu}{p}}$$

$$c(M_w) + \text{RGE} \to c(\mu = p)$$

1	$\alpha\ln$	$\alpha^2\ln^2$	$\alpha^3\ln^3$	...	$\alpha^n\ln^n$
	α	$\alpha^2\ln$	$\alpha^3\ln^2$	...	$\alpha^n\ln^{n-1}$
		α^2	$\alpha^3\ln$	...	$\alpha^n\ln^{n-2}$
			α^3	...	...
			...	...	α^n

Log resummation may not be enough

$$\langle\pi^-|J_P|0\rangle \sim f_\pi$$

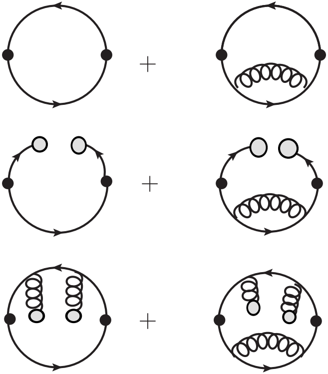


PDG '15

OPE in the vacuum

Asymptotic behaviour of two-point correlation functions

$$\Pi(q) = \int dx\, e^{-iqx} \langle 0 | T(J_1(x) J_2(0) | 0 \rangle; \;\; J_i \sim \bar{q} \Gamma_i q$$



$$+ \ldots = c(\text{pert})$$

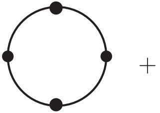
$$+ \ldots = c_{qq} \langle m_q \bar{q} q \rangle$$

$$+ \ldots = c_{GG} \langle GG \rangle$$

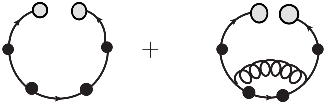
$$\Pi(Q) = \sum_{i,D} \frac{c_{i,D}(Q^2,\mu) \langle \mathcal{O}_{i,D}(\mu) \rangle}{Q^D}$$ Nucl.Phys.B 147 385-447

Back to the HLbL (Vacuum OPE?)

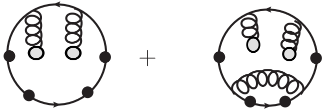
$$\Pi^{\mu_1\mu_2\mu_3\mu_4} = -i\int \frac{d^4q_3}{(2\pi)^4}\left(\prod_i^4 \int d^4x_i\, e^{-iq_ix_i}\right)\langle 0|\, T\left(\prod_j^4 J^{\mu_j}(x_j)\right)|0\rangle$$



$+ \ldots = c(\text{pert})$



$+ \ldots = c_{qq}\langle m_q\bar{q}q\rangle$



$+ \ldots = c_{GG}\langle GG\rangle$

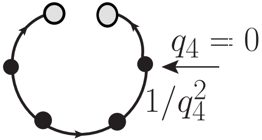
$$\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1}\ldots Q_4^{d_4}} \qquad \sum_i d_i = D$$

Back to the HLbL (Vacuum OPE?)

- $\Pi \sim \sum_{i,D} \frac{c_{i,D}(Q_i^2,\mu)\langle \mathcal{O}_{i,D}(\mu)\rangle}{Q_1^{d_1} \ldots Q_4^{d_4}}$ $\qquad \sum_i d_i = D$

- Static limit: $\lim_{q_4 \to 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\nu_4}}{\partial q_{4,\,\mu_4}} = -\lim_{q_4 \to 0} \frac{\partial \Pi^{\mu_1\mu_2\mu_3\mu_4}}{\partial q_{4,\,\nu_4}}$

- $\lim_{q_4 \to 0} \Pi \to \infty$



Phys.Lett.B 798 134994

- OPE in the vacuum only valid for large Euclidean Momenta!

Rethinking the problem: soft static photon

$$\langle 0 | e^{iS} | \gamma_1 \gamma_2 \gamma_3 \gamma_4 \rangle \to \Pi^{\mu_1\mu_2\mu_3\mu_4}$$

One step backwards

$$\Pi^{\mu_1\mu_2\mu_3} = -\frac{1}{e} \int \frac{d^4 q_3}{(2\pi)^4} \left(\prod_i^3 \int d^4 x_i \, e^{-iq_i x_i} \right) \langle 0 | T \left(\prod_j^3 J^{\mu_j}(x_j) \right) e^{iS_{\text{int}}} | \gamma(q_4) \rangle$$

- $Q_{1,\cdots,3} \gg \Lambda_{\text{QCD}} \to$ OPE valid for the tensor
- We are looking for a static (soft photon contribution): $F^{\mu\nu}$
- From the OPE keep those operator contributions with the same quantum numbers as the static photon, $F^{\mu\nu}$

  Nucl.Phys.B 232 109-142, Phys.Lett.B 129 328-334, Phys.Rev.D 67 073006

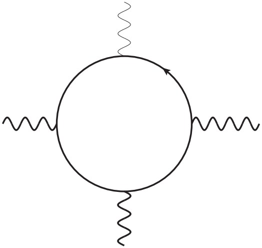
Operators

$$
\begin{aligned}
S_{1,\,\mu\nu} &\equiv e\, e_q F_{\mu\nu} \\
S_{2,\,\mu\nu} &\equiv \bar{q}\sigma_{\mu\nu} q \\
S_{3,\,\mu\nu} &\equiv i\;\bar{q} G_{\mu\nu} q \\
S_{4,\,\mu\nu} &\equiv i\,\bar{q}\bar{G}_{\mu\nu}\gamma_5 q \\
S_{5,\,\mu\nu} &\equiv \bar{q}q\;e\,e_q F_{\mu\nu} \\
S_{6,\,\mu\nu} &\equiv \frac{\alpha_s}{\pi}\,G_a^{\alpha\beta} G^a_{\alpha\beta}\;e\,e_q F_{\mu\nu} \\
S_{7,\,\mu\nu} &\equiv \bar{q}(G_{\mu\lambda}D_\nu + D_\nu G_{\mu\lambda})\gamma^\lambda q - (\mu \leftrightarrow \nu) \\
S_{\{8\},\,\mu\nu} &\equiv \alpha_s\,(\bar{q}\,\Gamma\,q\;\bar{q}\Gamma q)_{\mu\nu}
\end{aligned}
$$

$$
\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \tfrac{1}{e}\vec{C}^{\,T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\langle\vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = ee_q X_S^i\langle F_{\mu\nu}\rangle
$$

Completing the long-short distance separation

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \frac{1}{e}\vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\langle\vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = e e_q X_S^i \langle F_{\mu\nu}\rangle$$



$$c_{m_q^2} \sim \alpha_s^n(Q^2)\log^n\left(\frac{Q^2}{m_q^2}\right)$$

- Separation of long and short distance not complete
- Tree-level operators should be dressed and renormalized

Rev.Mod.Phys. 68 1125-1144, Z.Phys.C 60 569-578

Mixing

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \frac{1}{e}\vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\langle\vec{S}_{\mu_4\nu_4}\rangle; \quad \langle S_{i,\mu\nu}\rangle = e e_q X_S^i \langle F_{\mu\nu}\rangle$$

Dressing...

$$\vec{Q}_0^{\mu\nu} = \hat{M}(\epsilon)\vec{S}^{\mu\nu}; \qquad \hat{M}(\epsilon) = \mathbb{I} + \frac{m_q^{-2a\hat{\epsilon}}}{\hat{\epsilon}}\hat{M}_{\hat{\epsilon}} + \hat{M}_{\text{rem}}$$

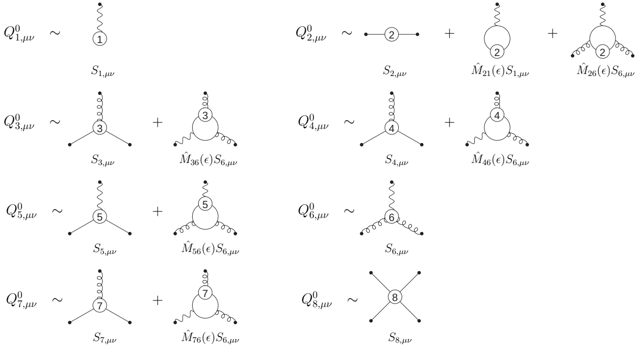
...and renormalizing

$$\vec{Q}_0^{\mu\nu} = \hat{Z}_{\overline{MS}}(\mu,\epsilon)\,\vec{Q}_{\overline{MS}}^{\mu\nu}(\mu)\,, \qquad \hat{Z}_{\overline{MS}}(\mu,\epsilon) = \mathbb{I} + \frac{\hat{M}_{\hat{\epsilon}}\,\mu^{2a\epsilon}}{\hat{\epsilon}}$$

- $\vec{Q}_{\overline{MS}}^{\mu\nu}(\mu) = \hat{U}_{\overline{MS}}(\mu)\,\vec{S}^{\mu\nu}; \quad \vec{C}_{\overline{MS}}(q_1,q_2,\mu) = \hat{U}_{\overline{MS}}^{-T}(\mu)\vec{C}(q_1,q_2)$
- $\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \frac{1}{e}\vec{C}_{\overline{MS}}^{T}(q_1,q_2,\mu)\langle\vec{Q}_{\overline{MS}}^{\mu\nu}(\mu)\rangle$

Mixing

- Need $\vec{Q}_0^{\mu\nu} = \hat{M}(\epsilon) S^{\mu\nu}$
- $\hat{M}(\epsilon)$ perturbation in m_q, e, g_s



Matrix element estimates

$$\langle 0|Q_i^{\mu\nu}|\gamma(q_4)\rangle = X_i \langle 0|e\, e_q F^{\mu\nu}|\gamma(q_4)\rangle \text{ e.g. focus on } Q_i^{\mu\nu} = \bar{q}\sigma^{\mu\nu} q$$

First try: χpT with tensor sources JHEP 09 (2007) 078

$$\mathcal{L} = \mathcal{L}_0 + t^j_{i,\mu\nu}(\mu)\bar{q}^i\sigma^{\mu\nu}q_j(\mu) \qquad \mathcal{L}^{\chi pT} = \Lambda_1(\mu)\,\langle\, t_+^{\mu\nu}(\mu)\, f_{+\mu\nu}\rangle + \cdots$$

$$e^{iZ} = \int \mathcal{D}[q,\bar{q},G]e^{iS} \; = \; \int \mathcal{D}U\, e^{iS_{eff}} \to X(\mu) \sim \Lambda_1(\mu) + \mathcal{O}\left(\frac{M_K^2}{\Lambda_\chi^2}\right) \ldots \Lambda_1 ?$$

Second try: expand EM vertex

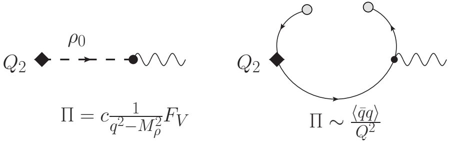
$$\langle 0|Q_{i,0}^{\mu\nu}|\gamma(q_4)\rangle \sim \langle 0|Q^{\mu\nu}_{i,0(\mathrm{QCD})}\int d^4x\, \mathcal{L}^{\mathrm{int}}_{\mathrm{EM}}(x)|\gamma(q_4)\rangle = -\langle 0|ee_qF^{\mu\nu}|\gamma(q_4)\rangle \Pi^{\mathrm{QCD}}_{JQ_i}(0)$$

So $X_i = -\Pi(0)$, where $\Pi^{\mathrm{QCD}}_{JQ_i}(q) \sim \int d^4x\, e^{-iqx}\, T(J(x)Q_{i,0(\mathrm{QCD})}(0))$

Matrix element estimates

$X_i = -\Pi(0)$, where $\Pi_{JQ_i}^{\mathrm{QCD}}(q) \sim \int d^4x\, e^{-iqx}\, T(J(x)Q_{i,0(\mathrm{QCD})}(0))$

- χpT? Again, $\Pi(0) \sim \Lambda_1 + \cdots$, but Λ_1?
- Large-N_c limit saturated by the exchange of (vector) resonances
  Nucl.Phys.B 72 461, Nucl.Phys.B 160 57-115
- Low-energy spectrum$\sim N_c$. High-energy spectrum flat



Eur.Phys.J.C 52 325-338

$$X_2 = \frac{2}{M_\rho^2}\langle\bar{q}q\rangle, X_3 = -\frac{m_0^2}{6M_\rho^2}\langle\bar{q}q\rangle\,, X_4 = -\frac{m_0^2}{6M_\rho^2}\langle\bar{q}q\rangle$$

X_2 in very good agreement with the lattice JHEP 07 (2020) 183

Matrix element estimates

$$\langle 0|Q_i^{\mu\nu}|\gamma(q_4)\rangle = X_i\langle 0|e\, e_q F^{\mu\nu}|\gamma(q_4)\rangle$$

$$\begin{aligned}
Q_{5,\,\mu\nu} &\equiv \bar{q}q\; e\, e_q F_{\mu\nu} \\
Q_{6,\,\mu\nu} &\equiv \frac{\alpha_s}{\pi}\, G_a^{\alpha\beta} G^a_{\alpha\beta}\; e\, e_q F_{\mu\nu} \\
Q_{7,\,\mu\nu} &\equiv \bar{q}(G_{\mu\lambda}D_\nu + D_\nu G_{\mu\lambda})\gamma^\lambda q - (\mu \leftrightarrow \nu) \\
Q_{\{8\},\,\mu\nu} &\equiv \alpha_s\,(\bar{q}\,\Gamma\, q\;\bar{q}\Gamma q)_{\mu\nu}
\end{aligned}$$

- X_5 and X_6 are the usual vacuum condensates.
- Q_8. $SU(3)_{\mathrm{V}}$ Nucl.Phys.B 234 (1984) 173-188, P, $C \rightarrow 12$
- Large-N_c $\rightarrow X_8 \sim X_2\langle\bar{q}q\rangle$
- Q_7? Dimensional guess

Start the game: Quark loop

$$\Pi^{\mu_1\mu_2\mu_3} = -\frac{1}{e}\int \frac{d^4 q_3}{(2\pi)^4} \left(\prod_{i=1}^{3} \int d^4 x_i \, e^{-iq_i x_i}\right) \langle 0| \, T\left(\prod_{j=1}^{3} J^{\mu_j}(x_j)\right) |\gamma(q_4)\rangle$$

- Direct $S_1^{\mu\nu} = ee_q F_{\mu\nu}$ contribution
- Take one extra $\mathcal{L}_{\mathrm{EM}}$ from $e^{iS_{int}}$
- $A^{\mu_4}(x_4) \sim -\frac{1}{2} x_{4\nu_4} F^{\mu_4\nu_4} \sim F^{\mu_4\nu_4} \lim_{q_4 \to 0} \partial^{q_4}_{\nu_4} e^{-iq_4 x_4}$
- The quark loop is actually the leading term of the correct OPE

Quark loop: some technicalities (sorry)

$$\lim_{q_4\to 0} \frac{\partial}{\partial q_4^{\nu_4}} \left[\sum_{\sigma(1,2,4)} \mathrm{Tr}\left(\gamma^{\mu_3} S(p+q_1+q_2+q_4)\gamma^{\mu_4} S(p+q_1+q_2)\gamma^{\mu_1} S(p+q_2)\gamma^{\mu_2} S(p)\right)\right]$$

$$\frac{\partial}{\partial q_4^{\nu_4}} S(p+q_4) = -S(p+q_4)\gamma^{\nu_4} S(p+q_4)$$

$$\Pi^{\mu_1\mu_2\mu_3}(q_1,q_2) = \vec{C}^{T,\mu_1\mu_2\mu_3\mu_4\nu_4}(q_1,q_2)\,\vec{X}\,\langle e_q F_{\mu_4\nu_4}\rangle$$

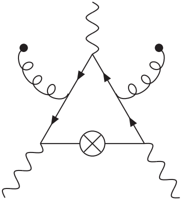
$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2} \int_{-1}^{1} d\tau \sum_i T_i' \overline{\Pi}_i$ JHEP 09 (2015) 074, JHEP 04 (2017) 161

1. Build general projectors P: $P_{\mu_1\mu_2\mu_3\mu_4\nu_4} C^{\mu_1\mu_2\mu_3\mu_4\nu_4} = \overline{\Pi}$
2. Reduce scalar integrals KIRA, REDUZE
3. Perform the g-2 integral

$$\hat{\Pi}^0_{m,S} \sim \sum_{i,j,k,n} \lambda^{-n} Q_1^{2i} Q_2^{2j} Q_3^{2k} \times \left[c_{i,j,k}^{(m,n)} + f_{i,j,k}^{(m,n)} F + g_{i,j,k}^{(m,n)} \log\left(\frac{Q_2^2}{Q_3^2}\right) + h_{i,j,k}^{(m,n)} \log\left(\frac{Q_1^2}{Q_2^2}\right)\right]$$

$$\hat{\Pi}^{m_q^2}_{m,S} \sim \sum_{i,j,k,n} \lambda^{-n} Q_1^{2i} Q_2^{2j} Q_3^{2k} \times \left[d_{i,j,k}^{(m,n)} + p_{i,j,k}^{(m,n)} F + q_{i,j,k}^{(m,n)} \log\left(\frac{Q_2^2}{Q_3^2}\right) + r_{i,j,k}^{(m,n)} \log\left(\frac{Q_1^2}{Q_2^2}\right) + s_{i,j,k}^{(m,n)} \log\left(\frac{Q_3^2}{m_q^2}\right)\right]$$

Gluon condensate



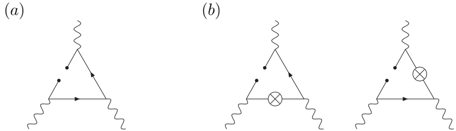
$$\Pi^{\mu_1\mu_2\mu_3}_{GG}(q_1,q_2) \sim e_q^4 F_{\nu_4\mu_4} \frac{4\pi^2\langle \alpha_s G_a^{\mu\nu} G^a_{\mu\nu}\rangle}{32 d(d-1)} \left(g_{\nu_5\nu_6} g_{\mu_5\mu_6} - g_{\mu_5\nu_6} g_{\nu_5\mu_6}\right) \left(\prod_{i=4}^{6} \lim_{q_i\to 0} \frac{\partial}{\partial q_i^{\nu_i}}\right)$$

$$\times \sum_{\sigma(1,2,4,5,6)} \mathrm{Tr}\Big(\gamma^{\mu_3} S(p+q_1+q_2+q_4+q_5+q_6)\gamma^{\mu_1} S(p+q_2+q_4+q_5+q_6)$$

$$\times \gamma^{\mu_2} S(p+q_4+q_5+q_6)\gamma^{\mu_4} S(p+q_5+q_6)\gamma^{\mu_5} S(p+q_6)\gamma^{\mu_6} S(p)\Big)$$

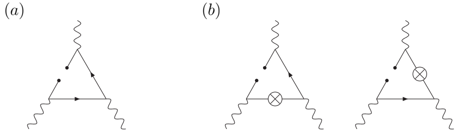
- Same procedure as in the quark loop Also for the two loops! (in progress)
- All infrared divergences exactly cancel after including mixing

One cut topologies



- More complicated than it looks.

One cut topologies



- More complicated than it looks. For example b) (sorry)

$$\bar{q}(x_1)\Big(B_\epsilon(u) + ie_q A_\epsilon(u)\Big)q(x_3) = \sum_{m,n}\frac{(-x_1)^{\nu_1}\cdots(-x_1)^{\nu_n}}{n!}\frac{x_3^{\nu'_1}\cdots x_3^{\nu'_m}}{m!} \times \sum_{p=1}\sum_{q=0}^{p}\frac{(-1)^{p-q+1}p\,u^{\omega_1}\cdots u^{\omega_p}}{(p+1)q!(p-q)!}$$
$$\times\bar{q}D^{\nu_1}\cdots D^{\nu_n}D^{\omega_1}\cdots D^{\omega_q}D^{\epsilon}D^{\omega_{q+1}}\cdots D^{\omega_p}D^{\nu'_1}\cdots D^{\nu'_m}q$$

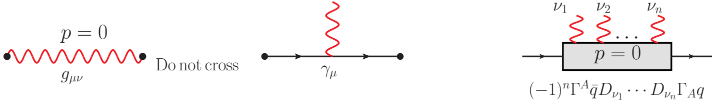
And after some algebra...

$$\Pi_{\mathrm{B}}^{\mu_1\mu_2\mu_3} = -e_q^3\lim_{p_{1A,2A,3A}}\sum_A\sum_{m,n}\frac{(i\partial_{p_{1A}})^{\nu_1}\cdots(i\partial_{p_{1A}})^{\nu_n}}{n!}\frac{(i\partial_{p_{3A}})^{\nu'_1}\cdots(i\partial_{p_{3A}})^{\nu'_m}}{m!}\times$$
$$\sum_{p=1}\sum_{q=0}^{p}\frac{(i\partial_{p_{2A}})^{\omega_1}\cdots(i\partial_{p_{2A}})^{\omega_p}p(-1)^{p-q+1}}{(p+1)q!(p-q)!}\times\Big(\mathrm{Tr}[\gamma^{\mu_3}\Gamma^A\gamma^{\mu_1}iS(p_1^A-q_1)\gamma^\epsilon iS(p_1^A-p_2^A-$$
$$q_1)\gamma^{\mu_2}iS(q_3+p_3^A)] + \mathrm{Tr}[\gamma^{\mu_3}\Gamma^A\gamma^{\mu_1}iS(p_1^A-q_1)\gamma^{\mu_2}iS(p_2^A+p_3^A+q_3)\gamma^\epsilon iS(q_3+p_3^A)]\Big)\times$$
$$\langle 0|\bar{q}D^{\nu_1}\cdots D^{\nu_n}D^{\omega_1}\cdots D^{\omega_q}D^{\epsilon}D^{\omega_{q+1}}\cdots D^{\omega_p}D^{\nu'_1}\cdots D^{\nu'_m}c_A\Gamma^A q|\gamma(q_4)\rangle$$

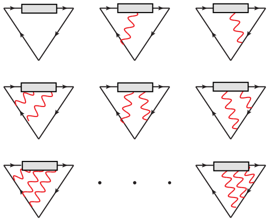
- Number of terms grows up very fast

One cut topologies: the emerging pattern

- Three simple rules:



- Our case:

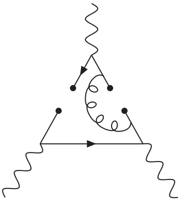


$$\Pi^{\mu_1\mu_2\mu_3} \sim \sum (-1)^n \langle 0|\bar{q} D_{\nu_1} \ldots D_{\nu_n} \Gamma^A q|\gamma(q_4)\rangle$$

$$\times \mathrm{Tr} \left\{ \gamma^{\mu_3} \Gamma^A \gamma^{\mu_1} iS(-q_1) \gamma^{\nu_1} iS(-q_1) \ldots \gamma^{\nu_p} iS(-q_1) \gamma^{\mu_2} iS(q_3) \gamma^{\nu_{p+1}} iS(q_3) \ldots \gamma^{\nu_n} iS(q_3) \right\}$$

- $\langle 0|\bar{q} D_{\nu_1} \ldots D_{\nu_n} \Gamma^A q|\gamma(q_4)\rangle$ can be decomposed into our basis

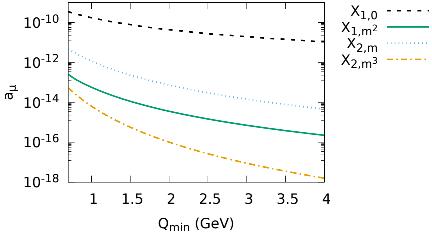
Four-quark operators

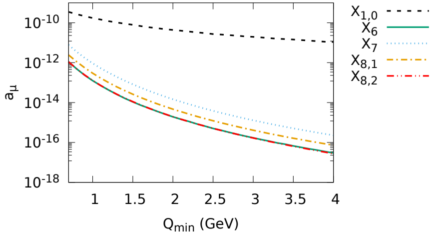


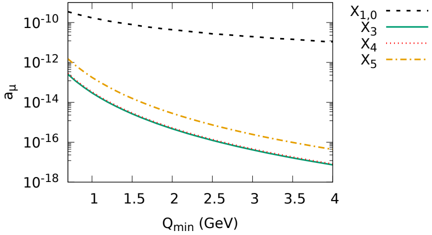
- Only contributions not associated to one flavor
- Simple to calculate
- Even simpler result

$$\hat{\Pi}_1 = \hat{\Pi}_4 = 8\overline{X}_{8,2}\frac{Q_1^2+Q_2^2}{Q_1^4 Q_2^4 Q_3^2}$$
$$\hat{\Pi}_{54} = 8\overline{X}_{8,2}\frac{Q_2^4-Q_1^4}{Q_1^6 Q_2^6 Q_3^2}$$
$$\hat{\Pi}_7 = \hat{\Pi}_{17} = \hat{\Pi}_{39} = 0$$

Numerical results







- Expected scaling
- Quark loop dominates
- Small power corrections, even at 1 GeV

Conclusions

- Static photon must be formulated as a soft degree of freedom
- When the other three momenta are large, an OPE can be built
- The quark loop is the leading term
- Power corrections have been computed and found to be small
- Perturbative corrections are underway

THANKS!