Local Unitarity

A representation of differential cross-sections that is locally free of IR singularities at any order

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Automation

For what concerns perturbative computation of physical cross-sections

Reliable evaluation of finite integrands reproducing the x-section ⟶ **Automation**

IR singularities

Amplitudes have pinched **IR singularities**, so they cannot be Monte Carlo integrated.

However, **we know that physical cross-sections are finite (Bloch-Nordsieck, KLN)**

So there must exist a local representation of the cross-section that is finite, i.e.

$$\sigma = \sum_{L=0}^{\infty} \int d\Pi_L f_L$$

f_L is **Lebesgue integrable**

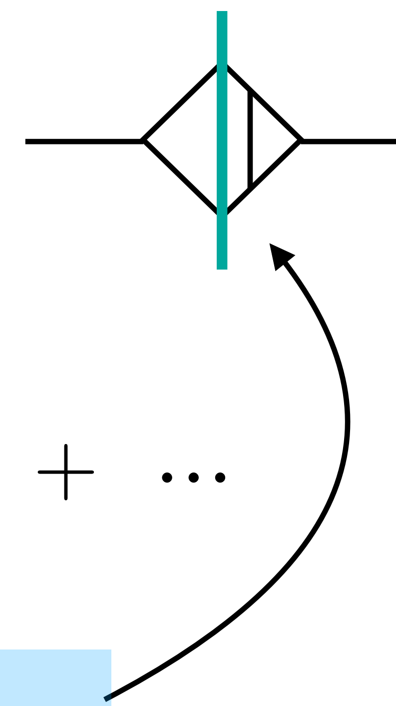
Actually, there are infinite local representations…

In relativistic quantum mechanics we measure cross-sections. Cross-sections can be expressed as **power-series in the coupling of the theory**

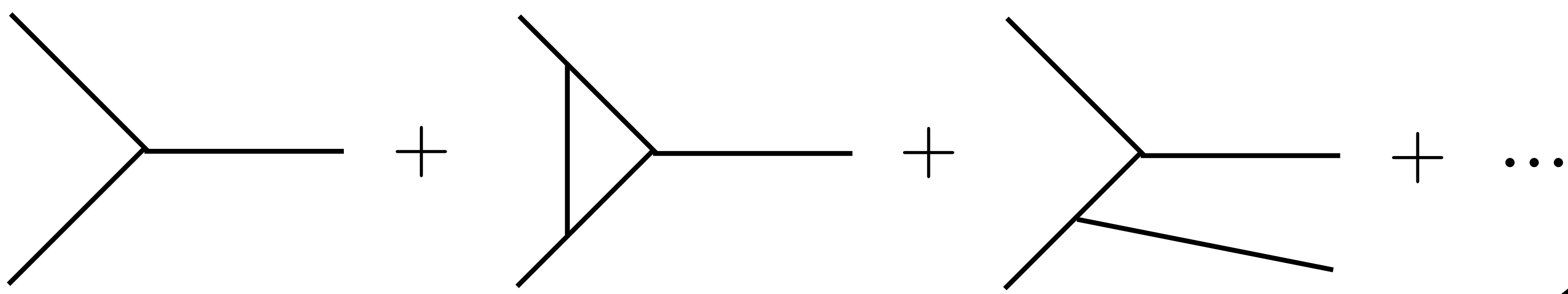
$$\sigma = \sum_{i=1}^{L} g^L \sigma^{(L)}$$

The correction can itself be expressed as the integral over the phase space of external particles of two **amplitudes**, which themselves are **sums of Feynman diagrams**

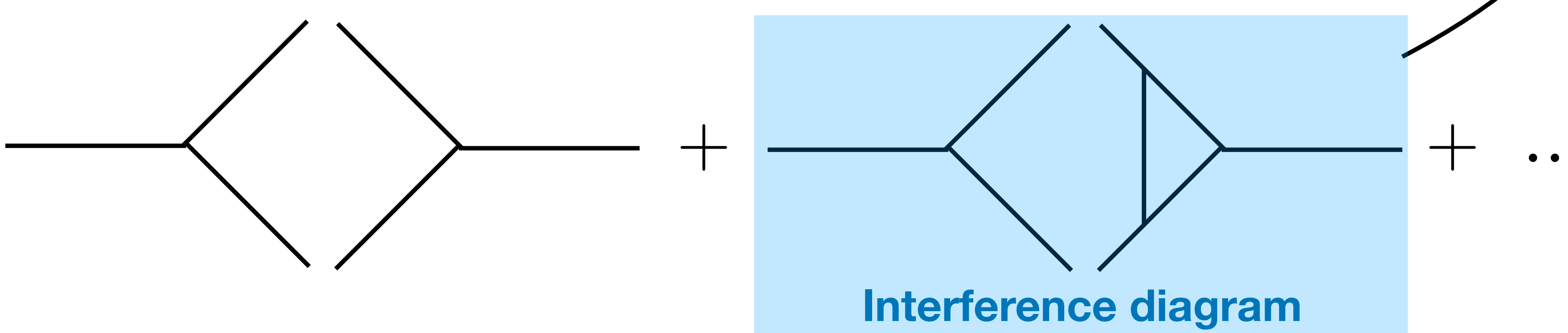
$$\sigma^{(L)} = \int d\Pi_{\text{ext}} A A^\star$$

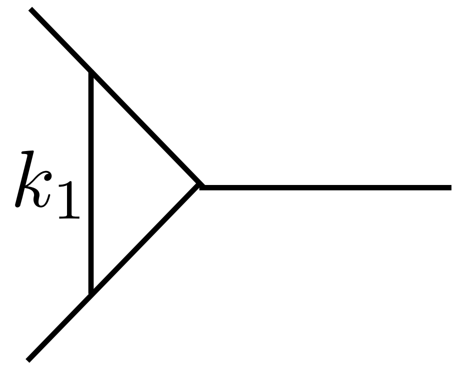


$$A =$$



$$AA^\star =$$

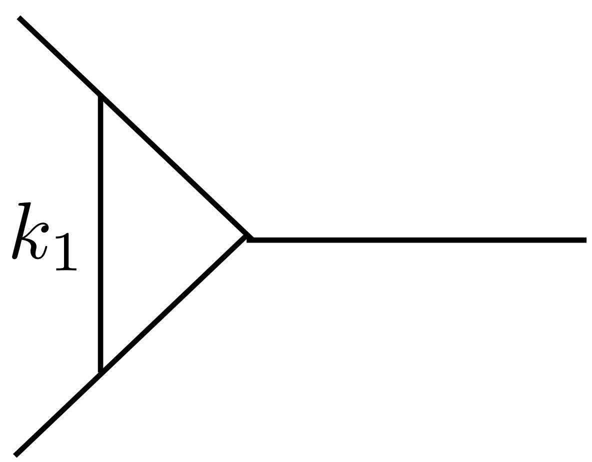




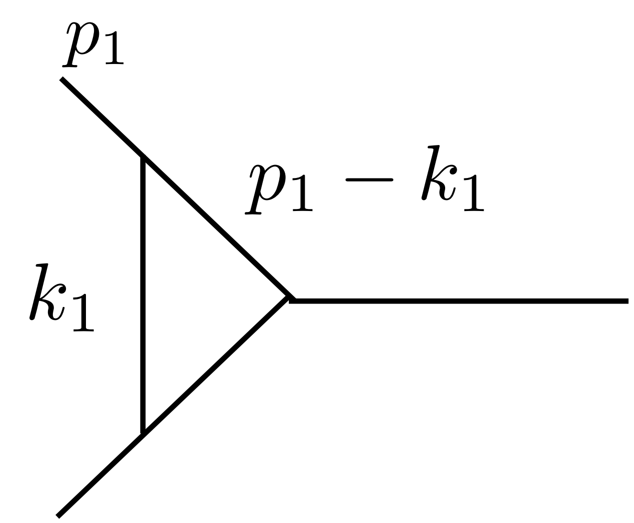
$$= \int d^4 k_1 g$$

This amplitude is plagued with infrared divergences, which correspond to particles going collinear or soft

e.g.



$k_1 \to 0$



$p_1 - k_1 // k_1$

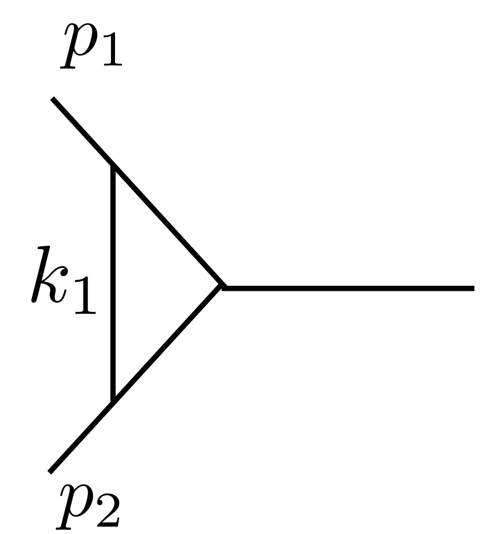
However, one discovers that $\sigma^{(L)}$ is **actually finite**

We want to construct an integral representation of $\sigma^{(L)}$ That is locally free of divergences, **so that we can feed it to a Monte Carlo integrator**

$$\sigma^{(L)} = \int d\Pi f$$

f is integrable

How to construct it? Let's start with LTD



We know that the external particle phase space measure looks like

$$d\Pi_{\text{ext}} = \delta\left(Q_0 - \sum_{i=1}^{m} p_i^0\right) \prod_{j=1}^{m} \frac{d^3\vec{p}_j}{2p_j^0(2\pi)^3}$$

Computed numerically

And the integration over virtual degrees of freedom of the left and right amplitude

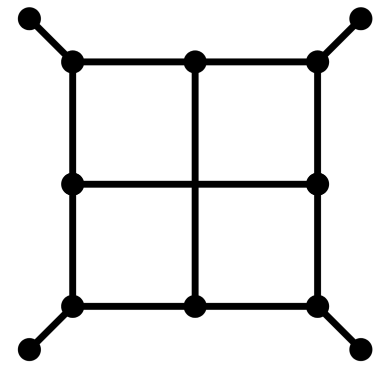
Computed analytically

$$\prod_{i=1}^{n_{\text{left}}} \frac{d^4 k_i}{(2\pi)^4} \qquad \prod_{i=1}^{n_{\text{right}}} \frac{d^4 k_i}{(2\pi)^4}$$

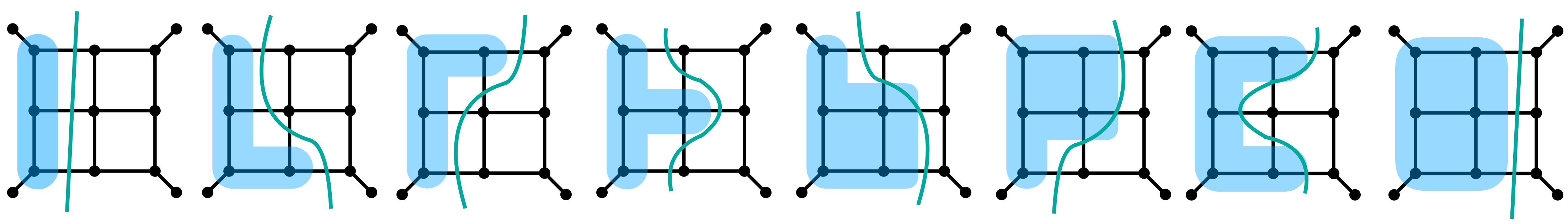
Then integrating the energies $d^4 k_i \to d^3 \vec{k}_i$ **(via LTD)** brings us closer to the objective, since **we have that integration on loops and external particles are on the same footing!**

The super-graph

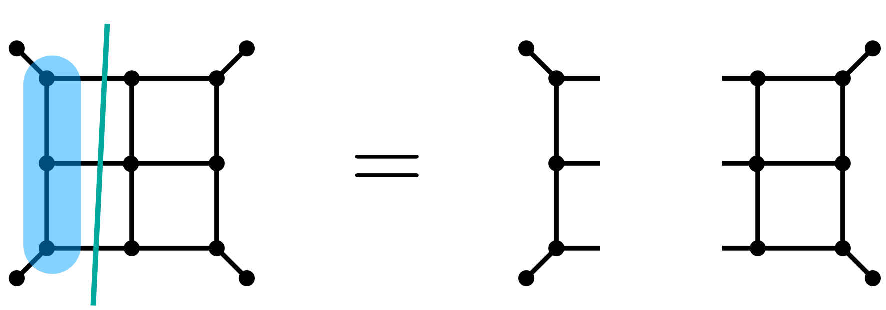
Given a graph with L loops



We can construct a class of interference diagrams



Instead of constructing interference diagrams from amplitudes, we do the opposite

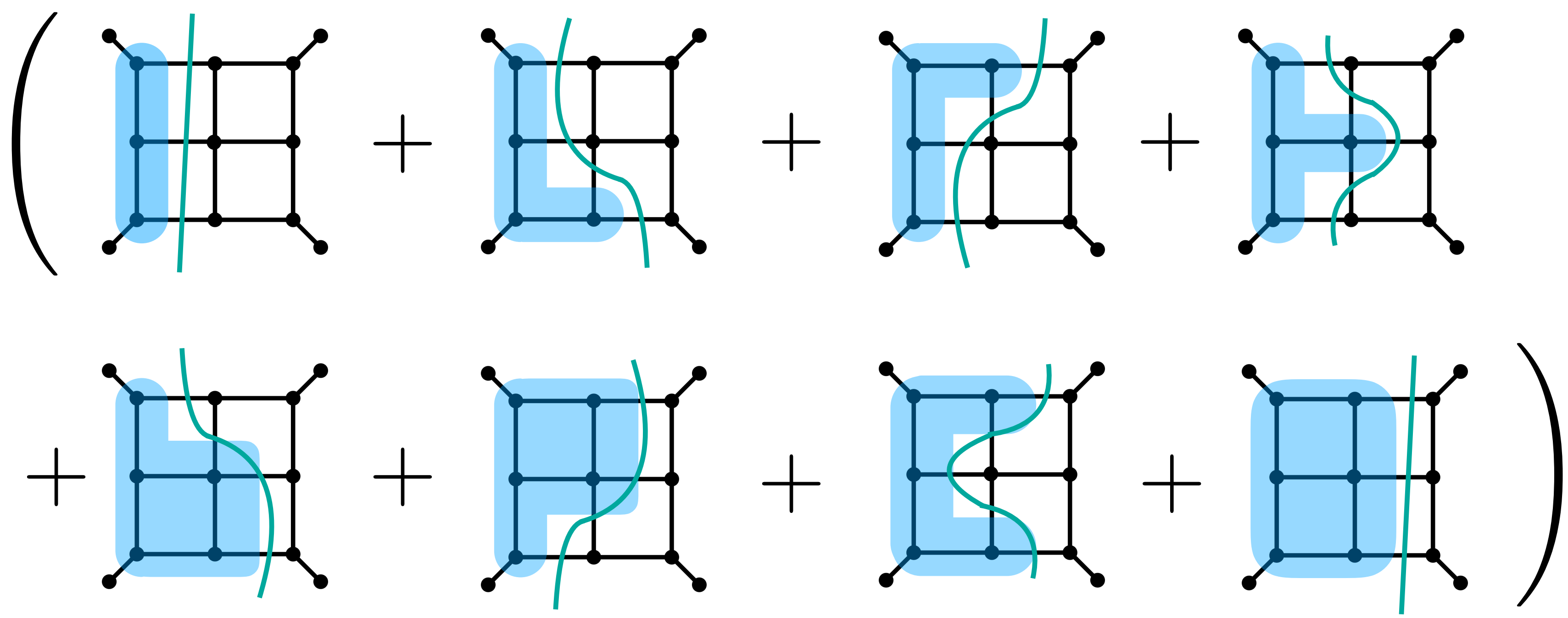


It turns out that the sum over all these interference diagrams is IR finite!

Each of the amplitudes can be expressed in the LTD representation.

We get something that looks like

$$\sigma \approx \int \left(\prod_{i=1}^{L} \frac{d^3\vec{k}_i}{(2\pi)^3}\right)$$



The integrand can be cast in a way that is **locally finite**, so it can be Monte Carlo integrated

This series of steps allows to identify the singularities of a cross-section and to construct an **integrand that is locally free of IR divergences**

- **No counter-terms**
- **No dimensional regularisation**
- **Not process specific**
- **Fully numerical and automatable**
- **Differential**

Also has a lot of interesting theoretical repercussions

- **Initial states with higher multiplicities**
- **Beyond LSZ**
- **Infrared scales from theory**
- **Classification of singularities and the systematics of their cancellations**