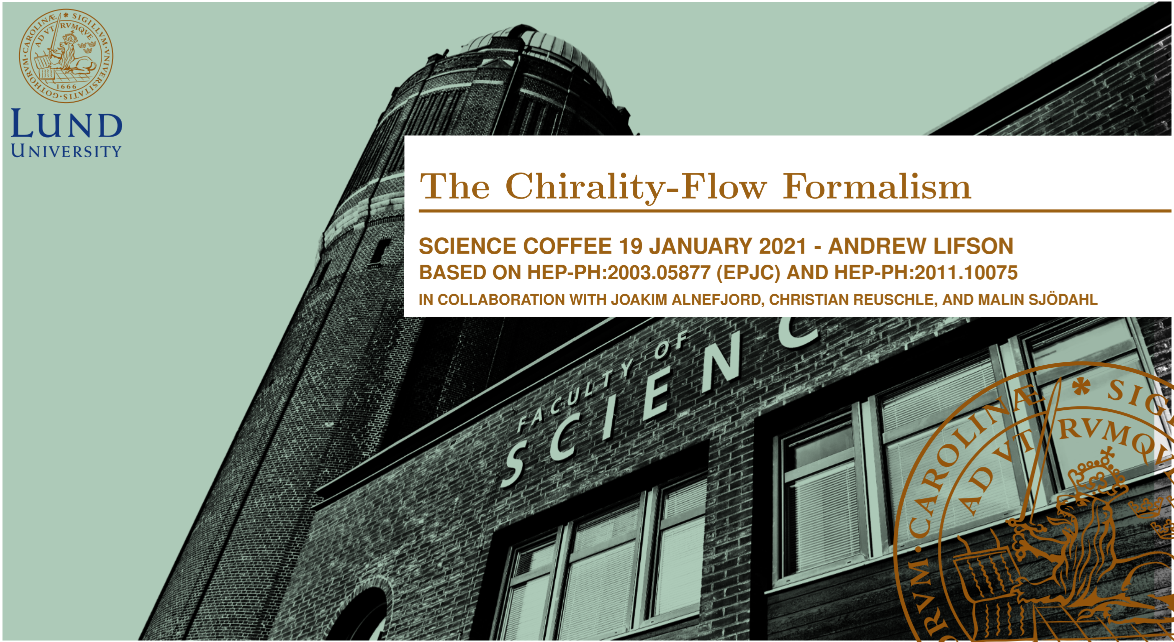
Lund University

The Chirality-Flow Formalism

SCIENCE COFFEE 19 JANUARY 2021 - ANDREW LIFSON

BASED ON HEP-PH:2003.05877 (EPJC) AND HEP-PH:2011.10075

IN COLLABORATION WITH JOAKIM ALNEFJORD, CHRISTIAN REUSCHLE, AND MALIN SJÖDAHL



Pre-Seminar

Lorentz Group Representations

Lorentz group elements: $e^{i(\theta_i J_i + \eta_i K_i)}$ $J_i \equiv$ rotations, $K_i \equiv$ boosts

- Lorentz group generators $\simeq$ 2 copies of su(2) generators
  - $so(3,1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

Group algebra defined by commutator relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} J_k$$

$$N_i^{\pm} = \frac{1}{2}(J_i \pm iK_i) , \quad [N_i^-, N_j^+] = 0 ,$$

$$[N_i^-, N_j^-] = i\epsilon_{ijk} N_k^- , \qquad [N_i^+, N_j^+] = i\epsilon_{ijk} N_k^+$$

- Representations (i.e. realisations of $N_i^{\pm}$)
  - $(0,0)$ scalar particles
  - $(\frac{1}{2}, 0)$ left-chiral and $(0, \frac{1}{2})$ right-chiral Weyl (2-component) spinors.
  - $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$, Dirac (4-component) spinors.
  - $(\frac{1}{2}, \frac{1}{2})$ vectors, e.g. gauge bosons

Recap of Little Group and Spin

Spin quantum numbers come from little group of a particle
Massive and massless particles have different little groups
$\Rightarrow$ they have fundamentally different properties

- Massless particles have two helicity states $\pm j$
  - j is the spin rep of Lorentz group, e.g. $j = 1$ for photons, $j = 1/2$ for electrons
- Massive particles have two types of spin quantum numbers:
  - total spin $j^2 = j(j+1)$ and
  - spin along an axis $j_s = j, j-1, \cdots, -j+1, -j$

These differences imply a fundamentally different treatment of massive and massless particles necessary

Massless and Massive Spinors

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Spinor $\psi(x) \sim \int d^4p(u(p)e^{-ip\cdot x} + v(p)e^{ip\cdot x})$

Massless

- Two helicity states $h = \pm\frac{1}{2}$
- Only one chirality per helicity state:
  - $u^+ = u_R$
  - $u^- = u_L$
- Two-component Weyl spinors
- Obey Weyl equations, e.g. $p^\mu \sigma_\mu u(p) = 0$
- At LHC used to describe leptons, light quarks

Massive

- Two fixed-spin states $j_s = \pm\frac{1}{2}$
- Both chiralities per spin-j_s state:
  - $u^+ \sim u_L + u_R$
  - $u^- \sim u_L + u_R$
- Four-component Dirac spinors
- Obey Dirac equations, e.g. $p^\mu \gamma_\mu u(p) = mu(p)$
- At LHC used to describe top and (sometimes) bottom quarks

LUND UNIVERSITY

The Helicity Basis: what and why?

Helicity basis means each particle has a specific helicity
Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles

- Ideal for (approximately) massless particles (e.g. most particles at LHC)
  - Helicity is the quantum number of the massless little group
  - For incoming (anti)spinors chirality $((\frac{1}{2},0)$ or $(0,\frac{1}{2})) \sim$ helicity $(-\frac{1}{2}$ or $+\frac{1}{2})$
  - For outgoing (anti)spinors chirality $((\frac{1}{2},0)$ or $(0,\frac{1}{2})) \sim$ helicity $(-\frac{1}{2}$ or $+\frac{1}{2})$
- Amplitude itself is a number rather than a matrix
  - Easy to square
- Different helicity amplitudes are orthogonal
  - Only sum over helicities after squaring

Spinor-Helicity: its Building Blocks

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

Spinors (use chiral basis):

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \qquad u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = \begin{pmatrix} [p| & 0 \end{pmatrix} \qquad \bar{u}^-(p) = \bar{v}^+(p) = \begin{pmatrix} 0 & \langle p| \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad P_{L/R} = \frac{1 \mp \gamma^5}{2}$$

- Amplitude written in terms of Lorentz-invariant spinor inner products

$$\langle ij\rangle = -\langle ji\rangle \equiv \langle i||j\rangle \text{ and } [ij] = -[ji] \equiv [i||j]$$

  - These are well known complex numbers, $\langle ij\rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

Spinor-Helicity: Vectors and Removing μ Indices

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

Dirac matrices in chiral basis

$$\gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\tau^\mu \\ \sqrt{2}\bar{\tau}^\mu & 0 \end{pmatrix} \qquad \sqrt{2}\tau^\mu = (1, \vec{\sigma}), \quad \sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma}),$$

Remove $\tau/\bar{\tau}$ matrices in amplitude with

$$\underbrace{\langle i|\bar{\tau}^\mu|j][k|\tau_\mu|l\rangle = \langle il\rangle[kj]}_{\text{Fierz identity}}, \qquad \underbrace{\langle i|\bar{\tau}^\mu|j] = [j|\tau^\mu|i\rangle}_{\text{Charge Conjugation}}$$

Express (massless) p^μ in terms of spinors

$$p^\mu = \frac{[p|\tau^\mu|p\rangle}{\sqrt{2}} = \frac{\langle p|\bar{\tau}^\mu|p]}{\sqrt{2}}, \quad \sqrt{2}p^\mu\tau_\mu \equiv \not{p} = |p]\langle p|, \quad \sqrt{2}p^\mu\bar{\tau}_\mu \equiv \bar{\not{p}} = |p\rangle[p|$$

LUND UNIVERSITY

Spinor-Helicity: Gauge Bosons in Terms of Spinors

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

Outgoing polarisation vectors:

$$\epsilon_+^\mu(p,r) = \frac{\langle r|\bar{\tau}^\mu|p]}{\langle rp\rangle}, \qquad \epsilon_-^\mu(p,r) = \frac{[r|\tau^\mu|p\rangle}{[pr]}$$

- r is a (massless) arbitrary reference momentum ($p \cdot r \neq 0$)
- Different r choices correspond to different gauges

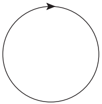
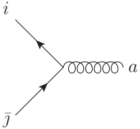
$$\epsilon_+^\mu(p,r') - \epsilon_+^\mu(p,r) = -p^\mu \frac{\langle r'r\rangle}{\langle r'p\rangle\langle rp\rangle}$$

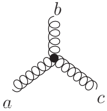
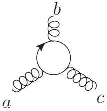
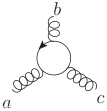
- Gauge invariant quantities must be r-invariant
  - Choose r as conveniently as possible (remember $\langle ij\rangle = -\langle ji\rangle$ s.t. $\langle ii\rangle = 0$)
  - Variance under $r \to r'$ good check of gauge invariance of (partial) amplitude

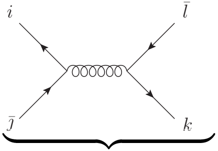
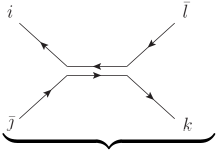
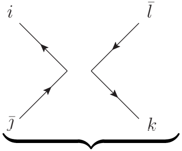
Colour Flow: a Quick Introduction

Standard method in $SU(N)$-colour calculations:

Write all objects in terms of $\delta_{i\bar{\jmath}} \equiv$ flows of colour

$\delta_{i\bar{\jmath}} =$  , $\mathrm{T_R} = 1$, $\sum_i \delta_{ii} = N =$  , $t^a_{i\bar{\jmath}} =$ 

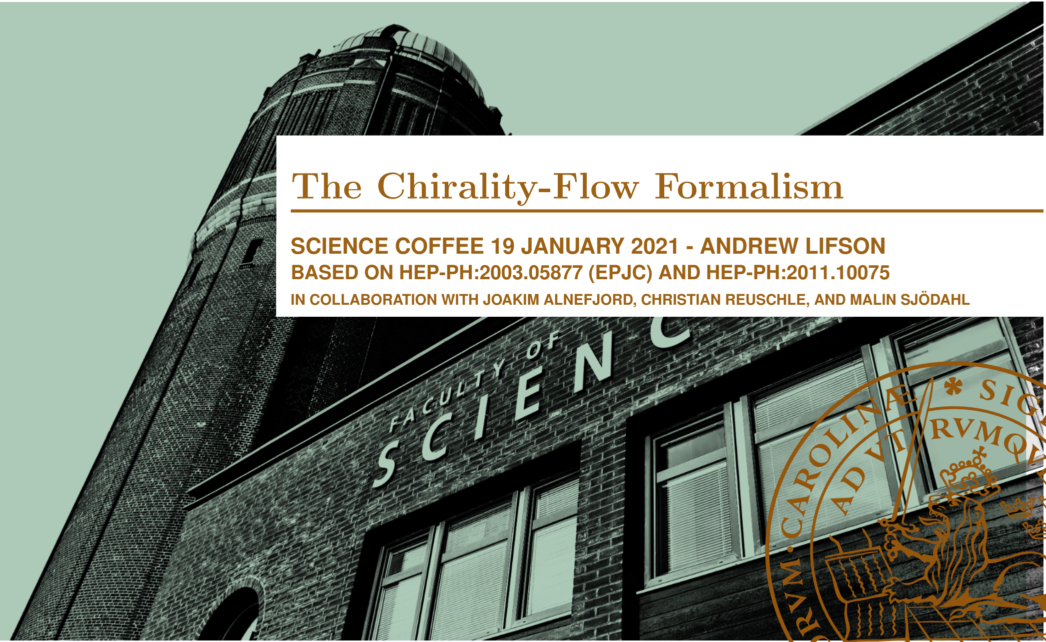
$if^{abc} =$  $=$  $-$  $= \mathrm{Tr}(t^a[t^b, t^c])$

 $=$  $- \frac{1}{N}$ 

$t^a_{i\bar{\jmath}} t^a_{k\bar{l}}$, $\delta_{i\bar{l}}\delta_{k\bar{\jmath}}$, $\delta_{i\bar{\jmath}}\delta_{k\bar{l}}$

Questions?

LUND UNIVERSITY



The Chirality-Flow Formalism

SCIENCE COFFEE 19 JANUARY 2021 - ANDREW LIFSON

BASED ON HEP-PH:2003.05877 (EPJC) AND HEP-PH:2011.10075

IN COLLABORATION WITH JOAKIM ALNEFJORD, CHRISTIAN REUSCHLE, AND MALIN SJÖDAHL

Introduction

LUND UNIVERSITY

Outline of the Seminar

- Last week: Seminar on little group and spinor helicity
  - Spinor helicity forms basis of this week's knowledge
  - Summarised in pre-seminar

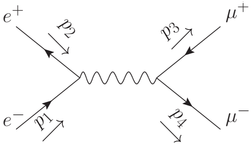
- Will first remind how to calculate Feynman diagrams with standard methods

- Then show our new chirality-flow method
  - Will first show why valid in massless QED and examples of how to implement it
  - Then massless QCD and example
  - Then massive particles and examples

- More information available at hep-ph:2003.05877 (published in EPJC) and hep-ph:2011.10075 (recently submitted to EPJC)

Traditional QFT: a Simple Example

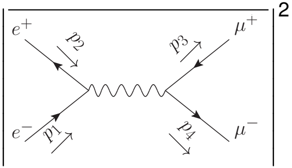
- Keep all particles unpolarised
- Obtain amplitude as matrix



$$\sim [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)]$$

Traditional QFT: a Simple Example

- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal

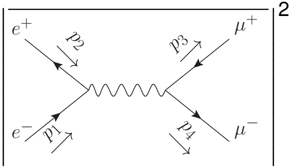


$$\sim \sum_{r,s,t,w} [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)]$$

$$\times [\bar{u}_s(p_1)\gamma^\nu v_r(p_2)][\bar{v}_w(p_3)\gamma_\nu u_t(p_4)]$$

Traditional QFT: a Simple Example

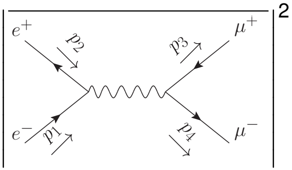
- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal
- Move components around



$$\sim \sum_{r,s,t,w} [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)]$$
$$\times [\bar{u}_s(p_1)\gamma^\nu v_r(p_2)][\bar{v}_w(p_3)\gamma_\nu u_t(p_4)]$$
$$\sim \sum_{r,s,t,w} [\gamma^\nu v_r(p_2)\bar{v}_r(p_2)\gamma^\mu u_s(p_1)\bar{u}_s(p_1)]$$
$$\times [\gamma_\nu u_t(p_4)\bar{u}_t(p_4)\gamma_\mu v_w(p_3)\bar{v}_w(p_3)]$$

Traditional QFT: a Simple Example

- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal
- Move components around
- Use spin sums
- Take trace of fermionic structure
- Requires identities of γ^μ
- Simplify



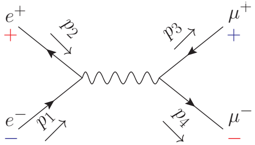
$$\sim \mathrm{Tr}\left[\gamma^\nu(\not{p}_2 - m_e)\gamma^\mu(\not{p}_1 + m_e)\right]$$
$$\times\, \mathrm{Tr}\left[\gamma_\nu(\not{p}_4 + m_\mu)\gamma_\mu(\not{p}_3 + m_\mu)\right]$$

$$\mathrm{Tr}\left[\gamma^{\mu_1}\gamma^{\mu_2}\right] = 4g^{\mu_1\mu_2}$$
$$\mathrm{Tr}\left[\gamma^{\mu_1}\ldots\gamma^{\mu_4}\right] = 4(g^{\mu_1\mu_2}g^{\mu_3\mu_4} - g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_3\mu_2})$$
$$\mathrm{Tr}\left[\gamma^{\mu_1}\ldots\gamma^{\mu_{2n+1}}\right] = 0$$

Same Simple Example using Spinor Helicity

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

- Explicit helicities for external particles
- $\gamma^\mu \to \tau^\mu$ or $\bar{\tau}^\mu \equiv$ Pauli matrices
- Now diagram is a complex number
  - $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$
  - Easy to square
- Square first, then sum over helicities
  - Some helicity configurations vanish
  - CP-invariance relates helicity configurations

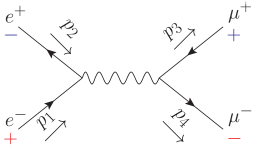


$$\sim \langle p_2 | \bar{\tau}^\mu | p_1] \langle p_4 | \bar{\tau}_\mu | p_3]$$
$$= [p_1 | \tau^\mu | p_2 \rangle \langle p_4 | \bar{\tau}_\mu | p_3]$$
$$= \langle p_4 p_2 \rangle [p_1 p_3]$$

Same Simple Example using Spinor Helicity

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

- Explicit helicities for external particles
- $\gamma^\mu \to \tau^\mu$ or $\bar{\tau}^\mu \equiv$ Pauli matrices
- Now diagram is a complex number
  - $\langle ij \rangle \sim [ij] \sim \sqrt{2 p_i \cdot p_j}$
  - Easy to square
- Square first, then sum over helicities
  - Some helicity configurations vanish
  - CP-invariance relates helicity configurations



$$\sim [p_2|\tau^\mu|p_1\rangle \langle p_4|\bar{\tau}_\mu|p_3]$$

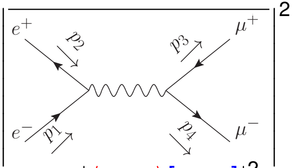
$$= \langle p_4 p_1 \rangle [p_2 p_3]$$

LUND UNIVERSITY

Same Simple Example using Spinor Helicity

Lorentz algebra $so(3,1) \cong su(2) \oplus su(2)$
Consider massless particles: chirality $\sim$ helicity

- Explicit helicities for external particles
- $\gamma^\mu \rightarrow \tau^\mu$ or $\bar{\tau}^\mu \equiv$ Pauli matrices
- Now diagram is a complex number
  - $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$
  - Easy to square
- Square first, then sum over helicities
  - Some helicity configurations vanish
  - CP-invariance relates helicity configurations



$$\sim 2|\langle p_4 p_2\rangle [p_1 p_3]|^2 + 2|\langle p_4 p_1\rangle [p_2 p_3]|^2$$

Define Problem

Kinematic part of amplitude slowed by spin and vector structures

- Can we still improve on this?
  - Deriving spinor inner products $\langle ij\rangle$, $[kl]$ requires at least 2 steps
    - Re-write every object as spinors
    - Use Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\alpha}\beta} = \delta_{\alpha}^{\ \beta}\delta^{\dot{\alpha}}_{\ \dot{\beta}}$
  - Not intuitive which inner products we obtain
- In SU(N) use graphical reps for calculations
  - E.g. using the colour-flow method (see pre-seminar)
  - (Also birdtracks etc.)
- Spinor-helicity $\equiv su(2) \oplus su(2)$
  - Can we do the same?

Massless Chirality Flow

Creating Chirality Flow: Building Blocks

A flow is a directed line from one object to another
$su(2)$ objects have dotted indices and $su(2)$ objects undotted indices

- Key difference:
  - Colour $\equiv$ single $SU(N)$: generators $t^a \to \delta$'s
  - Spinor-hel $\equiv su(2) \oplus su(2)$: $\tau^\mu, \bar{\tau}^\mu, |i\rangle_\alpha, |j]^{\dot{\alpha}}, \epsilon^\mu_\pm, \to \langle ij\rangle, [kl]$
- First step: Ansatz for spinor inner products (only possible Lorentz invariant)

$$\langle i|^\alpha |j\rangle_\alpha \equiv \langle ij\rangle = -\langle ji\rangle =$$ 

$$[i|_{\dot{\beta}} |j]^{\dot{\beta}} \equiv [ij] = -[ji] =$$ 

- Spinors and Kronecker deltas follow

$\langle i|^\alpha =$ 

, $|j\rangle_\alpha =$ 

$[i|_{\dot{\beta}} =$ 

, $|j]^{\dot{\beta}} =$ 

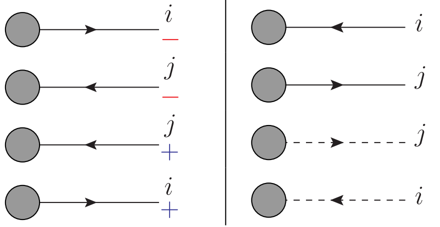
$\delta_\alpha^{\ \beta} \equiv \mathbb{1}_\alpha^{\ \beta} =$ 

, $\delta^{\dot{\beta}}_{\ \dot{\alpha}} \equiv \mathbb{1}^{\dot{\beta}}_{\ \dot{\alpha}} =$ 

Massless, Outgoing Spinors in Chirality Flow

Lorentz algebra $so(3,1) \cong \underbrace{su(2)}_{\text{dotted}} \oplus \underbrace{su(2)}_{\text{undotted}}$

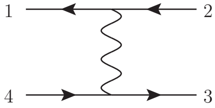
Spinor	Feynman	Flow
$\bar{u}^-(p_i) = \langle i\|$		
$v^-(p_j) = \|j\rangle$		
$v^+(p_j) = \|j]$		
$\bar{u}^+(p_i) = [i\|$		



Chirality-flow arrow opposite to fermion arrow

Chirality Flow for QED: Photon Exchange

- Second Step: Unpolarised photon exchange
- Split up standard term into helicity amplitudes



$$\mathcal{M}_i \sim \Big(\bar{u}(p_1)\gamma^\mu v(p_2)\Big) g_{\mu\nu} \Big(\bar{u}(p_3)\gamma^\nu v(p_4)\Big)$$

$$= \underbrace{\left([1|_{\dot\alpha}\tau^{\mu,\dot\alpha\beta}|2\rangle_\beta\right)\left([3|_{\dot\gamma}\tau_\mu^{\dot\gamma\eta}|4\rangle_\eta\right)}_{\mathcal{M}(1^+,2^-,3^+,4^-)} + \underbrace{\left([1|_{\dot\alpha}\tau^{\mu,\dot\alpha\beta}|2\rangle_\beta\right)\left(\langle 3|^\gamma\bar\tau_{\mu,\gamma\dot\eta}|4]^{\dot\eta}\right)}_{\mathcal{M}(1^+,2^-,3^-,4^+)}$$

$$+ \underbrace{\left(\langle 1|^\alpha\bar\tau^\mu_{\alpha\dot\beta}|2]^{\dot\beta}\right)\left([3|_{\dot\gamma}\tau_\mu^{\dot\gamma\eta}|4\rangle_\eta\right)}_{\mathcal{M}(1^-,2^+,3^+,4^-)} + \underbrace{\left(\langle 1|^\alpha\bar\tau^\mu_{\alpha\dot\beta}|2]^{\dot\beta}\right)\left(\langle 3|^\gamma\bar\tau_{\mu,\gamma\dot\eta}|4]^{\dot\eta}\right)}_{\mathcal{M}(1^-,2^+,3^-,4^+)}$$

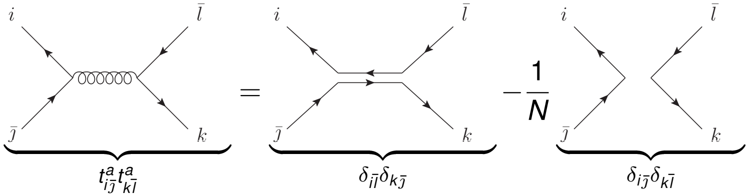
- 2 options: $\bar\tau^\mu\tau_\mu$ $(\tau^\mu\bar\tau_\mu)$, or $\bar\tau^\mu\bar\tau_\mu$ $(\tau^\mu\tau_\mu)$
  - $\bar\tau^\mu_{\alpha\dot\beta}\tau_\mu^{\dot\gamma\eta} = \delta_\alpha^{\ \eta}\delta^{\dot\gamma}_{\ \dot\beta}$ (Fierz)
  - $\bar\tau^\mu_{\alpha\dot\beta}\bar\tau_{\mu,\gamma\dot\eta} = \epsilon_{\dot\beta\dot\eta}\epsilon_{\alpha\gamma}$ ($\epsilon \equiv$ Levi Cevita)

Photon Exchange: What is a Flow?

- A flow:
  - Occurs when two indices are contracted via a Kronecker delta
  - Is represented by a line with an arrow to indicate flow direction
- Option 1: Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\gamma}\eta} = \delta_{\alpha}^{\ \eta}\delta^{\dot{\gamma}}_{\ \dot{\beta}}$
  - $\Rightarrow$ option 1 creates a flow
  - How to draw it?

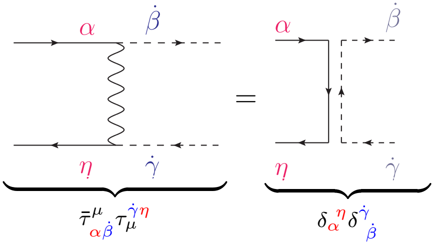
Photon Exchange: What is a Flow?

- A flow:
  - Occurs when two indices are contracted via a Kronecker delta
  - Is represented by a line with an arrow to indicate flow direction
- Option 1: Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\gamma}\eta} = \delta_{\alpha}^{\ \eta}\delta^{\dot{\gamma}}_{\ \dot{\beta}}$
  - $\Rightarrow$ option 1 creates a flow
  - How to draw it?
- Take inspiration from often-used SU(N) colour Fierz identity



Photon Exchange: What is a Flow?

- A flow occurs when two indices are contracted via a delta function
- A flow is represented by a line with an arrow to indicate flow direction
- Option 1: Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\gamma}\eta} = \delta_{\alpha}^{\ \eta}\delta^{\dot{\gamma}}_{\ \dot{\beta}}$
  - $\Rightarrow$ option 1 creates a flow
  - How to draw it?
- Take inspiration from often-used SU(N) colour Fierz identity
- Spinor Fierz in flow form is:



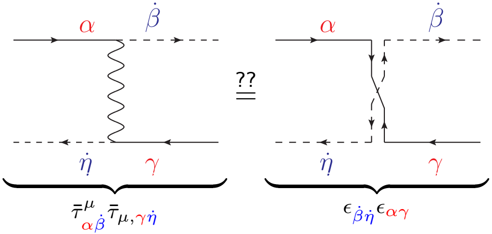
Photon Exchange: What is a Flow?

- A flow occurs when two indices are contracted via a delta function
- A flow is represented by a line with an arrow to indicate flow direction
- Option 2: $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\bar{\tau}_{\mu,\gamma\dot{\eta}} = \epsilon_{\dot{\beta}\dot{\eta}}\epsilon_{\alpha\gamma}$
  - $\Rightarrow$ option 2 does **not** create a flow!
  - How to draw it??

Photon Exchange: What is a Flow?

- A flow occurs when two indices are contracted via a delta function
- A flow is represented by a line with an arrow to indicate flow direction
- Option 2: $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\bar{\tau}_{\mu,\gamma\dot{\eta}} = \epsilon_{\dot{\beta}\dot{\eta}}\epsilon_{\alpha\gamma}$
  - $\Rightarrow$ option 2 does **not** create a flow!
  - How to draw it??
- Pictorially, problem seen by arrows pointing towards or away from each other

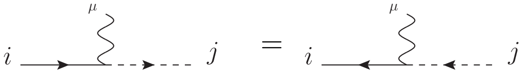
$$\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\bar{\tau}_{\mu,\gamma\dot{\eta}} =$$



LUND UNIVERSITY

Photon Exchange: The Arrow Flip

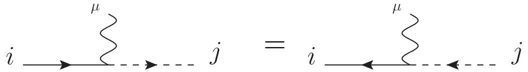
- Can fix with charge conjugation of a current
  - $\langle i|^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}|j]^{\dot{\beta}} = [j|_{\dot{\alpha}}\tau^{\mu,\dot{\alpha}\beta}|i\rangle_{\beta}$
- Or in pictures, charge conjugation = an arrow flip:



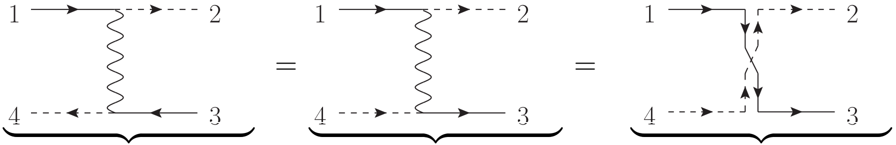
- Can replace $\tau \leftrightarrow \bar{\tau}$ if also replacing $|i\rangle_{\alpha} \leftrightarrow \langle i|^{\alpha}$, $[j|_{\dot{\alpha}} \leftrightarrow |j]^{\dot{\alpha}}$

Photon Exchange: The Arrow Flip

- Can fix with charge conjugation of a current
  - $\langle i|^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}|j]^{\dot{\beta}} = [j|_{\dot{\alpha}}\tau^{\mu,\dot{\alpha}\beta}|i\rangle_{\beta}$
- Or in pictures, charge conjugation $=$ an arrow flip:

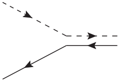
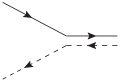


- Can replace $\tau \leftrightarrow \bar{\tau}$ if also replacing $|i\rangle_{\alpha} \leftrightarrow \langle i|^{\alpha}$, $[j|_{\dot{\alpha}} \leftrightarrow |j]^{\dot{\alpha}}$
- Considering the complete diagram we have:



$(\langle 1|^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}|2]^{\dot{\beta}})(\langle 3|^{\gamma}\bar{\tau}_{\mu,\gamma\dot{\eta}}|4]^{\dot{\eta}})$ $\quad$ $(\langle 1|^{\alpha}\bar{\tau}^{\mu}_{\alpha\dot{\beta}}|2]^{\dot{\beta}})(\langle 4|_{\dot{\eta}}\tau_{\mu}^{\dot{\eta}\gamma}|3]_{\gamma})$ $\quad$ $\langle 1|^{\alpha}\delta_{\alpha}^{\ \beta}|3\rangle_{\alpha}[4|_{\dot{\beta}}\delta^{\dot{\beta}}_{\ \dot{\alpha}}|2]^{\dot{\alpha}} \equiv \langle 13\rangle[42]$

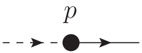
Quick Summary: Photon Exchange

- We are trying to build a spinor flow picture (contraction of spinors with delta functions)
- Have looked only at simple unpolarised photon exchange. Found:
  - Two terms have a ‘natural’ flow
  - Two terms can be changed into a flow
- We conclude for (at least) this process:
  - $\Rightarrow \tau^{\mu,\dot{\alpha}\beta} =$  , $\bar{\tau}^{\mu}_{\alpha\dot{\beta}} =$ 
  - $\Rightarrow g_{\mu\nu} =$  , or 
  - Fierz identity can be built into flow rule

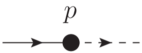
Creating Chirality Flow: Fermion Propagators

- Third step: Fermion propagators
- We split $\not{p}_{4d} \equiv p_\mu \gamma^\mu$ split into two terms

$$\not{p} \equiv \sqrt{2} p^\mu \tau_\mu^{\dot{\alpha}\beta} =$$

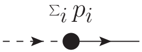


$$\bar{\not{p}} \equiv \sqrt{2} p_\mu \bar{\tau}^\mu_{\alpha\dot{\beta}} =$$



- Momentum dot defined to represent slashed momenta
- In a propagator, we have $p^\mu = \sum p_i^\mu \;,\; p_i^2 = 0$

$$\not{p} =$$



$$= \sum_i |i]^{\dot{\alpha}} \langle i|^{\beta} \quad \text{for} \quad p_i^2 = 0$$

$$\bar{\not{p}} =$$

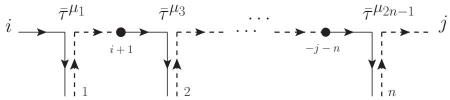
$$= \sum_i |i\rangle_{\alpha} |i]_{\dot{\beta}} \quad \text{for} \quad p_i^2 = 0$$

Fermion Lines with Multiple Photons

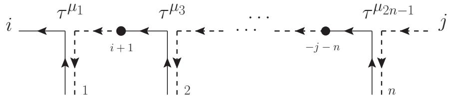
What if fermions emit more than one photon? Is flow picture valid?

- Yes (at least at tree level)
- Conjugation of a current holds for full fermion line

$$\langle i | \bar{\tau}^{\mu_1} \tau^{\mu_2} \dots \bar{\tau}^{\mu_{2n+1}} | j] =$$



$$=$$



$$= [j | \tau^{\mu_{2n+1}} \bar{\tau}^{\mu_{2n}} \dots \tau^{\mu_1} | i\rangle$$

i.e. arrow swap (and Fierz) works for any fermion line!

LUND UNIVERSITY

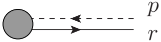
Creating Chirality Flow: External Gauge Bosons

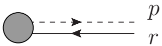
- Final step: External gauge bosons

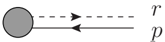
$$\epsilon_+^\mu(p,r) = \frac{[p|_{\dot\alpha}\tau^{\mu,\dot\alpha\beta}|r\rangle_\beta}{\langle rp\rangle}, \qquad \epsilon_-^\mu(p,r) = \frac{\langle p|^{\alpha}\bar\tau^{\mu}_{\alpha\dot\beta}|r]^{\dot\beta}}{[pr]}$$

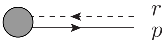
Pseudo vertex

External gauge bosons are just $f\bar f$ pairs with a denominator!
We can Fierz (with possible arrow swap) any external photon

$\epsilon_+^\mu(p,r) \to \frac{1}{\langle ri\rangle}$ 

, or $\epsilon_+^\mu(p,r) \to \frac{1}{\langle ri\rangle}$ 

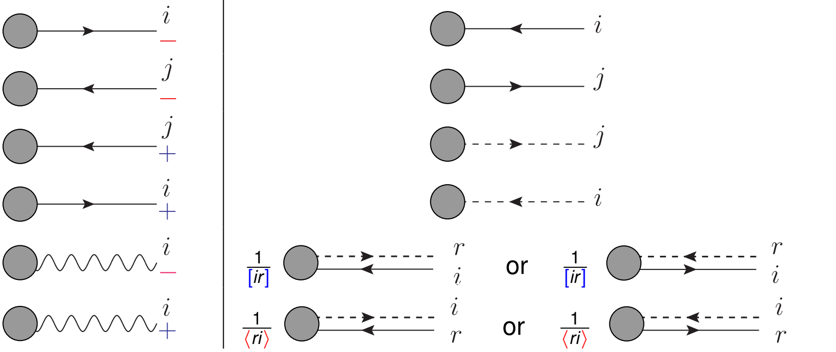
$\epsilon_-^\mu(p,r) \to \frac{1}{[ir]}$ 

, or $\epsilon_-^\mu(p,r) \to \frac{1}{[ir]}$ 

Note negative-hel particle solid, positive-hel dotted

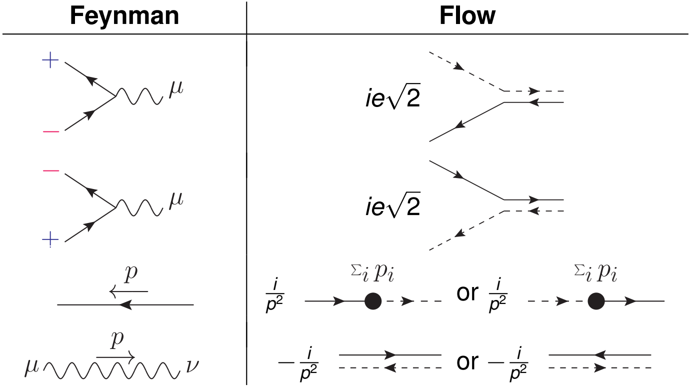
The QED Flow Rules: External Particles

Species	Feynman	Flow
$\bar{u}^{-}(p_i)$		
$v^{-}(p_j)$		
$v^{+}(p_j)$		
$\bar{u}^{+}(p_i)$		
$\epsilon^{\mu}_{-}(p_i, r)$		$\frac{1}{[ir]}$ or $\frac{1}{[ir]}$
$\epsilon^{\mu}_{+}(p_i, r)$		$\frac{1}{\langle ri \rangle}$ or $\frac{1}{\langle ri \rangle}$



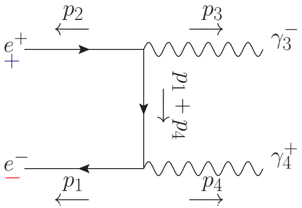
Lorentz algebra $so(3,1) \cong \underbrace{su(2)}_{\text{dotted}} \oplus \underbrace{su(2)}_{\text{undotted}}$

The QED Flow Rules: Vertices and Propagators



Lorentz algebra $so(3,1) \cong \underbrace{su(2)}_{\text{dotted}} \oplus \underbrace{su(2)}_{\text{undotted}}$

An Illuminating Example: $e^+e^- \to \gamma\gamma$



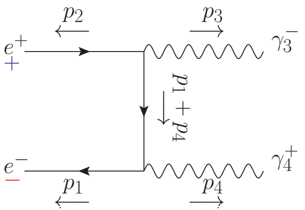
Spinor helicity:

$$\sim \langle p_1|\bar\tau^\mu \underbrace{(|p_1]\langle p_1| + |p_4]\langle p_4|)}_{\not{p}_1+\not{p}_4} \bar\tau^\nu|p_2] \underbrace{\frac{\langle r_3|\bar\tau_\nu|p_3]}{\langle r_3 3\rangle}}_{\epsilon_3^-} \underbrace{\frac{[r_4|\tau_\mu|p_4\rangle}{[4 r_4]}}_{\epsilon_4^+}$$

$$= \frac{(\langle p_1|\bar\tau^\mu|p_1] + \langle p_1|\bar\tau^\mu|p_4])[r_4|\tau_\mu|p_4\rangle \; (\langle p_1|\bar\tau^\nu|p_2] + \langle p_4|\bar\tau^\nu|p_2])[p_3|\tau_\nu|r_3\rangle}{\langle r_3 3\rangle[4 r_4]}$$

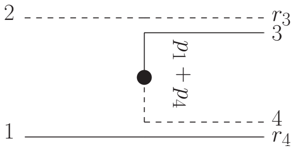
$$= \underbrace{\frac{\langle 1 r_4\rangle([41]\langle 13\rangle + [44]\langle 43\rangle)[r_3 2]}{\langle r_3 3\rangle[4 r_4]}}_{\text{Fierz identities like } \langle i|\bar\tau^\mu|j][k|\tau_\mu|l\rangle = \langle il\rangle[kj]} = \underbrace{\frac{\langle 1 r_4\rangle[41]\langle 13\rangle[r_3 2]}{\langle r_3 3\rangle[4 r_4]}}_{[ii]=0}$$

An Illuminating Example: $e^+e^- \to \gamma\gamma$

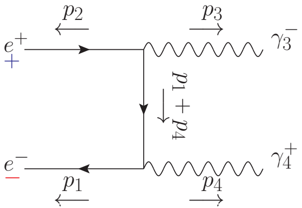


Chirality flow:

$$\sim \frac{1}{\langle r_3 3\rangle [4 r_4]}$$

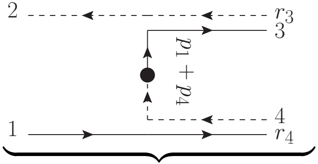


An Illuminating Example: $e^+e^- \to \gamma\gamma$



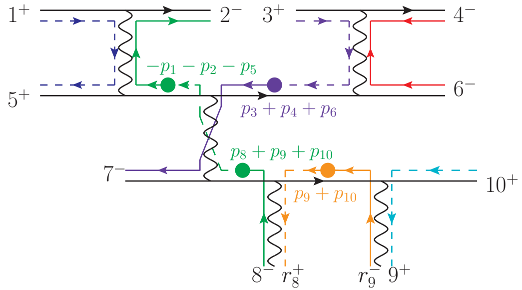
Chirality flow:

$$\sim \frac{1}{\langle r_3 3\rangle [4 r_4]}$$



$$\langle 1 r_4\rangle [41] \langle 13\rangle [r_3 2]$$

A complicated QED Example



Compare to:

- Standard QFT:
  $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_{10}})$,
  $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_4})$,
  $2 \times$ photon spin sum
- Standard spinor-helicity:
  5 charge conjugation/Fierz
  + rearranging

$$= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{s_{1\,2}\; s_{3\,4}\; s_{8\,9\,10}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{s_{1\,2\,5}\; s_{3\,4\,6}\; s_{8\,9\,10}\; s_{9\,10}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8r_8]\langle r_9 9\rangle}}_{\text{polarization vectors}} \; [15]\langle 64\rangle[10\;9]$$

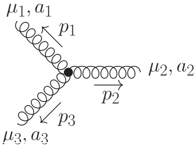
$$\times \Big(\langle r_9 9\rangle[9 r_8] + \langle r_9 10\rangle[10 r_8]\Big)\Big(\underbrace{[33]}_{0}\langle 37\rangle + [34]\langle 47\rangle + [36]\langle 67\rangle\Big)$$

$$\times \Big(-\langle 89\rangle[91]\langle 12\rangle - \langle 89\rangle[95]\langle 52\rangle - \langle 8\;10\rangle[10\;1]\langle 12\rangle - \langle 8\;10\rangle[10\;5]\langle 52\rangle\Big)$$

Extending to QCD

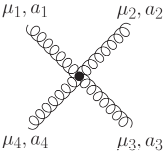
Non-abelian vertices only significant difference compared to QED

- 3-gluon:



$$= -\frac{g_s f^{abc}}{\sqrt{2}} g^{\mu_1\mu_2}(p_1 - p_2)^{\mu_3} + \circlearrowright$$

- 4-gluon:



$$= i g_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3} \left(g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_3} g^{\mu_2\mu_4}\right)$$

Need to understand p^μ in chirality flow

Momentum: The Last Piece of the Flow Puzzle

- Gordon identity: $p^\mu = \frac{1}{\sqrt{2}} \langle p|^{\alpha} \bar{\tau}^{\mu}_{\alpha\dot{\beta}} |p]^{\dot{\beta}} = \frac{1}{\sqrt{2}} [p|_{\dot{\alpha}} \tau^{\mu\dot{\alpha}\beta} |p\rangle_{\beta}$

p^μ is a pseudo vertex $\Rightarrow$ can be written as a flow

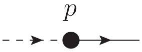
What does p^μ get contracted with?

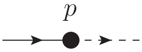
Momentum: The Last Piece of the Flow Puzzle

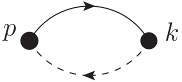
- Gordon identity: $p^\mu = \frac{1}{\sqrt{2}} \langle p|^{\alpha} \bar{\tau}^{\mu}_{\alpha\dot{\beta}} |p]^{\dot{\beta}} = \frac{1}{\sqrt{2}} [p|_{\dot{\alpha}} \tau^{\mu\dot{\alpha}\beta} |p\rangle_{\beta}$

p^μ is a pseudo vertex $\Rightarrow$ can be written as a flow

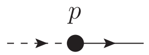
What does p^μ get contracted with?

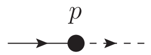
- $\tau_\mu \to \not{p}/\sqrt{2} = \frac{1}{\sqrt{2}}$ 

, $\bar{\tau}_\mu \to \bar{\not{p}}/\sqrt{2} = \frac{1}{\sqrt{2}}$ 

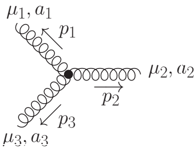
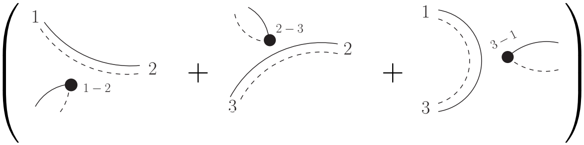
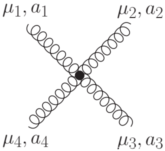
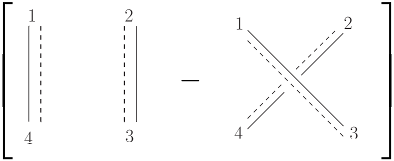
- $k_\mu \to p \cdot k = \underbrace{\frac{\mathrm{Tr}(\not{p}\bar{\not{k}})}{2}}_{\mathrm{Tr}(\tau^\mu \bar{\tau}^\nu) = g^{\mu\nu}} = \frac{1}{2}$ 

Chirality-flow rule for p^μ

$p^\mu \to \frac{1}{\sqrt{2}}$ 

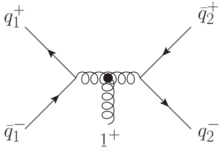
, or $p^\mu \to \frac{1}{\sqrt{2}}$ 

The Non-abelian Massless QCD Flow Vertices

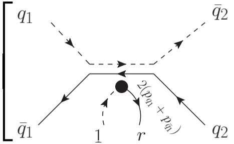
Feynman	Flow
	$-\frac{g_s f^{abc}}{2}$ 
	$ig_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3}$ 

Arrow directions only consistently set within full diagram
Double line $\equiv g_{\mu\nu}$, momentum dot $\equiv p_\mu$

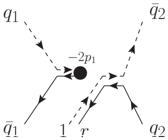
QCD Example: $q_1\bar{q}_1 \to q_2\bar{q}_2 g$



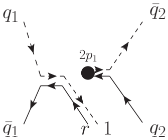
$$= \frac{ig_s^3}{2 s_{q_1\bar{q}_1} s_{q_2\bar{q}_2} \langle r1\rangle} \Bigg[$$



$+$



$+$



$\Bigg]$

$$\Big[\cdots\Big] \equiv \Big\{ 2[q_1\bar{q}_2]\langle q_2\bar{q}_1\rangle\big([1q_1]\langle q_1 r\rangle + [1\bar{q}_1]\langle 1r\rangle\big)$$

$$- 2[q_1 1]\langle 1\bar{q}_1\rangle\langle q_2 r\rangle[1\bar{q}_2] + 2[q_1 1]\langle r\bar{q}_1\rangle\langle q_2 1\rangle[1q_2]\Big\}$$

Massive Chirality Flow

LUND UNIVERSITY

Massive Spinor Helicity Basics

Decompose massive momentum p as sum of massless ones

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

Spin measured along $s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu) = \frac{1}{m}(p^\mu - 2\alpha q^\mu)$

- Massive spinors and polarisation vectors written in terms of massless Weyl spinors of momentum $p^\flat$, q
- We recycle results from massless chirality flow
  - E.g. slashed momentum written

$$\not{p} \equiv \sqrt{2} p^\mu \tau_\mu = |p^\flat]\langle p^\flat| + \alpha |q]\langle q|$$

$$\bar{\not{p}} \equiv \sqrt{2} p^\mu \bar{\tau}_\mu = |p^\flat\rangle [p^\flat| + \alpha |q\rangle [q|$$

- q is arbitrary but physical, as defines spin direction
  - Spin-summed amplitude independent of q choice

See e.g. hep-ph:0510157 for more details

LUND UNIVERSITY

The Helicity Basis in Massive Spinor Helicity

Decompose massive momentum p as sum of massless ones

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

Spin measured along $s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu) = \frac{1}{m}(p^\mu - 2\alpha q^\mu)$

- Consider eigenvectors/values of $\not{p}, \bar{\not{p}}$

$$\not{p}|p_{f/b}] = \lambda_{f/b}|p_{f/b}] \qquad \bar{\not{p}}|p_{f/b}\rangle = \lambda_{f/b}|p_{f/b}\rangle$$

$$\lambda_{f/b} = p^0 \pm |\vec{p}| \qquad p^\mu_{f/b} = \frac{\lambda_{f/b}}{2}(1, \pm\hat{p})$$

See e.g. hep-ph:9805445, hep-ph:2011.10075 for more details

LUND UNIVERSITY

The Helicity Basis in Massive Spinor Helicity

Decompose massive momentum p as sum of massless ones

$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$

Spin measured along $s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu) = \frac{1}{m}(p^\mu - 2\alpha q^\mu)$

- Consider eigenvectors/values of $\not{p}, \bar{\not{p}}$

$$\not{p}|p_{f/b}] = \lambda_{f/b}|p_{f/b}] \qquad \bar{\not{p}}|p_{f/b}\rangle = \lambda_{f/b}|p_{f/b}\rangle$$

$$\lambda_{f/b} = p^0 \pm |\vec{p}| \qquad p^\mu_{f/b} = \frac{\lambda_{f/b}}{2}(1, \pm\hat{p})$$

Conclusion: in helicity basis!

$p^\mu = p^\mu_f + p^\mu_b \,, \quad p_f^2 = p_b^2 = 0 \,, \qquad p^\flat \to p_f, \quad \alpha \to 1, \quad q \to p_b$

Spin measured along $s^\mu = \frac{1}{m}(p^\mu_f - p^\mu_b) = \frac{1}{m}(|\vec{p}|, p^0\hat{p}) \equiv$ direction of motion!

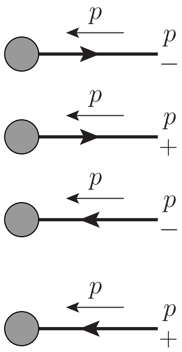
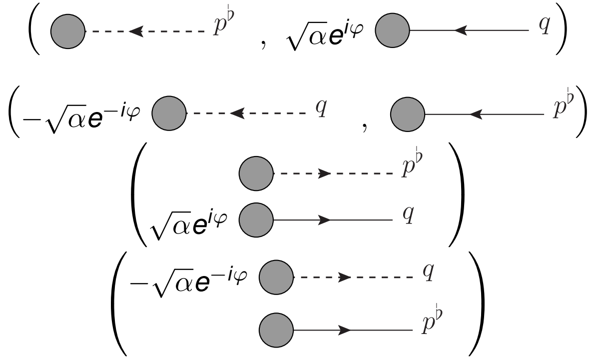
See e.g. hep-ph:9805445, hep-ph:2011.10075 for more details

LUND UNIVERSITY

Incoming Massive Spinors in Chirality Flow

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu\,, \quad (p^\flat)^2 = q^2 = 0\,, \quad e^{i\varphi}\sqrt{\alpha} = \frac{m}{\langle p^\flat q\rangle}\,, \qquad e^{-i\varphi}\sqrt{\alpha} = \frac{m}{[q p^\flat]}$$

$$\text{Spin operator } -\frac{\Sigma^\mu s_\mu}{2} = \frac{\gamma^5 s^\mu \gamma_\mu}{2}, \qquad s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu)$$

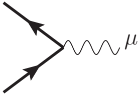
Spinor	Feynman	Flow
$\bar{v}^-(p)$		
$\bar{v}^+(p)$		
$u^-(p)$		
$u^+(p)$		

LUND UNIVERSITY

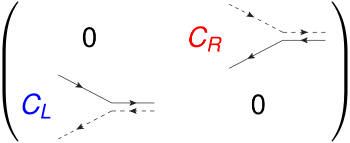
Fermion Vertices

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

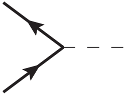
Fermion-vector vertex



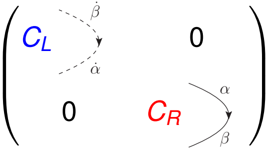
$$= ie(P_L C_L + P_R C_R)\gamma^\mu = ie\sqrt{2}$$



Fermion-scalar vertex



$$= ie(P_L C_L + P_R C_R) = ie$$



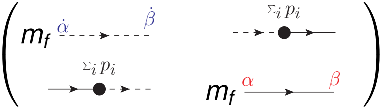
Left and right chiral couplings may differ

Fermion Lines with Multiple Emissions

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

Fermion propagator

$$\frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \delta^{\dot\alpha}{}_{\dot\beta} & \sqrt{2} p^{\dot\alpha\beta} \\ \sqrt{2}\bar{p}_{\alpha\dot\beta} & m_f \delta_\alpha{}^\beta \end{pmatrix} = \frac{i}{p^2 - m_f^2}$$



- Propagators and vertices don't always contribute factor $\tau/\bar\tau$
- Have to update arrow swap procedure to include even number of $\tau/\bar\tau$

$$\langle i|\bar\tau^{\mu_1}\tau^{\mu_2}\ldots\bar\tau^{\mu_{2n+1}}|j] = [j|\tau^{\mu_{2n+1}}\bar\tau^{\mu_{2n}}\ldots\tau^{\mu_1}|i\rangle$$
$$\langle i|\bar\tau^{\mu_1}\tau^{\mu_2}\ldots\tau^{\mu_{2n}}|j\rangle = -\langle j|\bar\tau^{\mu_{2n}}\bar\tau^{\mu_{2n-1}}\ldots\tau^{\mu_1}|i\rangle$$
$$[i|\tau^{\mu_1}\bar\tau^{\mu_2}\ldots\bar\tau^{\mu_{2n}}|j] = -[j|\tau^{\mu_{2n}}\bar\tau^{\mu_{2n-1}}\ldots\bar\tau^{\mu_1}|i]$$

Arrow flips may induce minus signs! Care must be taken

Massive Polarisation Vectors

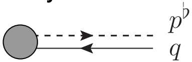
$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

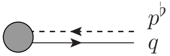
- External gauge bosons

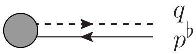
$$\epsilon_+^\mu(p) = \frac{[p^\flat|_{\dot\alpha}\tau^{\mu,\dot\alpha\beta}|q\rangle_\beta}{\langle qp^\flat\rangle} \,, \qquad \epsilon_-^\mu(p) = \frac{\langle p^\flat|^\alpha \bar\tau^\mu_{\alpha\dot\beta}|q]^{\dot\beta}}{[p^\flat q]}$$

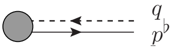
$$\epsilon_0^\mu(p) = s^\mu = \frac{1}{m}(p^{\flat,\mu} - \alpha q^\mu)$$

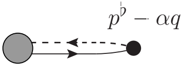
- Translate to chirality flow

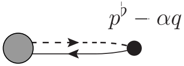
$\epsilon_+^\mu(p) \to \frac{1}{\langle ri\rangle}$ 

, or $\epsilon_+^\mu(p) \to \frac{1}{\langle ri\rangle}$ 

$\epsilon_-^\mu(p) \to \frac{1}{[p^\flat q]}$ 

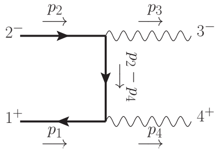
, or $\epsilon_-^\mu(p) \to \frac{1}{[p^\flat q]}$ 

$\epsilon_0^\mu(p) \to \frac{1}{m\sqrt{2}}$ 

, or $\epsilon_0^\mu(p) \to \frac{1}{m\sqrt{2}}$ 

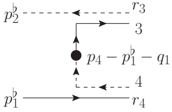
A Massive *Illuminating* Example

Consider the same diagram of $f_1^+ \bar{f}_2^- \to \gamma_3^+ \gamma_4^-$ as before but include mass m_f

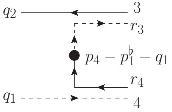


- Obtain 3 new terms
- Simplify with choices of q_1, q_2, r_3, r_4
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i \rangle}$, $e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$

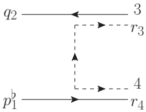
$$= \frac{-2ie^2}{(s_{23} - m_f^2)\langle r_3 3\rangle [4 r_4]} \Bigg\{$$



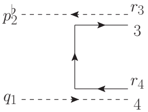
$$- \sqrt{\alpha_1 \alpha_2} e^{i(\varphi_2 - \varphi_1)}$$



$$+ m_f \Bigg(\sqrt{\alpha_2} e^{i\varphi_2}$$



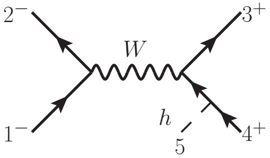
$$- \sqrt{\alpha_1} e^{-i\varphi_2}$$



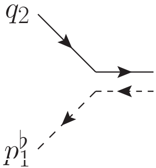
$$\Bigg) \Bigg\}$$

A Second Massive Example: $f_1 \bar{f}_2 \to W \to f_3 \bar{f}_4 h_5$

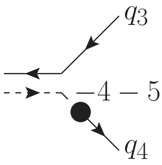
- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \cdots q_5$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i \rangle}$, $\quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$
- Scalar has no flow line



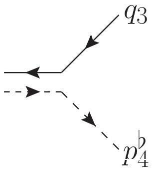
Step 1: Draw fermion lines: $\sim C_{L,12}\sqrt{\alpha_2}e^{i\varphi_2}$



$\times\, C_{L,34}\sqrt{\alpha_3}(-e^{i\varphi_3}) \Big[\sqrt{\alpha_4}(-e^{i\varphi_4})$



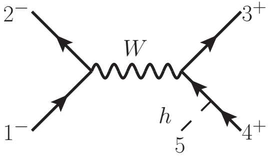
$+\, m_4$



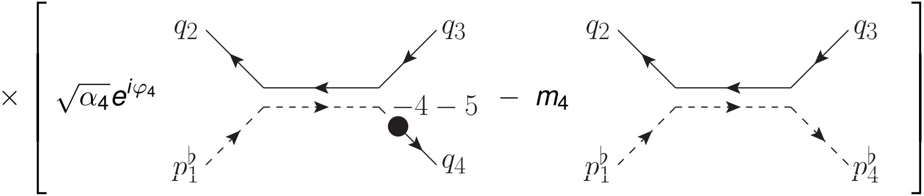
$\Big]$

A Second Massive Example: $f_1 \bar{f}_2 \to W \to f_3 \bar{f}_4 h_5$

- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \cdots q_5$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i \rangle}$, $e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$
- Scalar has no flow line



Step 2: Flip arrows and connect: $C_{L,12} C_{L,34} \sqrt{\alpha_2 \alpha_3} e^{i(\varphi_2+\varphi_3)}$



Conclusions

Conclusions and Outlook

- Chirality flow offers the shortest possible journey from Feynman diagram to complex number
  - Further simplifies the spinor helicity formalism
  - Calculations often performed in a single step, particularly for massless diagrams
- Useful at tree level for *any* model with only Dirac fermions and matries (Pauli matrices), Minkowski metric, momenta, spin 0 and 1 bosons in Feynman rules
  - E.g. full standard model at tree level understood
- Loops next on the agenda
- Useful for generators based on Feynman diagrams
- Useful for quick pen and paper calculations and checks

LUND UNIVERSITY