Initial Stages 2021 (highly biased and subjective) theory review

Jarkko Peuron(Lund University)

February 3rd 2021

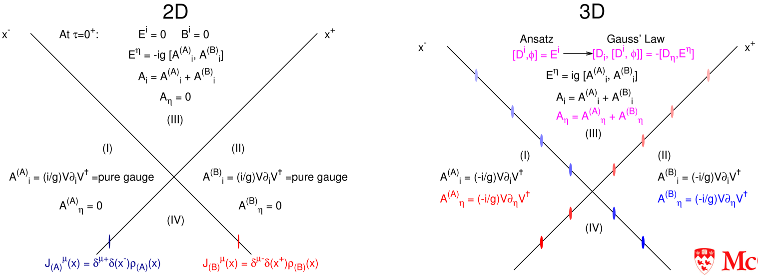
S. Jeon

- S. Jeon's plenary talk
- Try to generalize the boost invariant initial state to fully 3-dimensional IS.
- This **dramatically** changes certain features such as initial energy density and transverse/longitudinal pressures.

Finding the 3D Initial conditions

Goals

- Stay as close to the 2D initial conditions as possible
- Energy deposition only when there is overlap

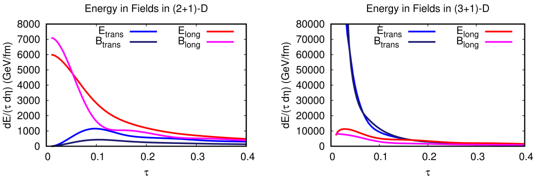


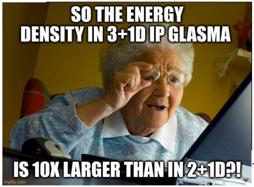
McGill

- Initial conditions 2D vs. 3D.

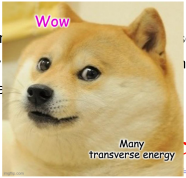
S. Jeon

2D vs 3D Evolutions





- Note the scale – 3D initial energy is *much* higher because of
  $$E = \int d\eta d^2x_\perp \tau \left(\frac{1}{2}\left((\mathcal{E}^\eta)^2 + (\mathcal{B}^\eta)^2\right) + \frac{1}{2\tau^2}\left(\mathbf{E}_\perp^2 + \mathbf{B}_\perp^2\right) \right)$$
- In 3D, one *cannot* set $\mathbf{E}_\perp = 0$ and $\mathbf{B}_\perp = 0$
- Large τ behaviours are similar

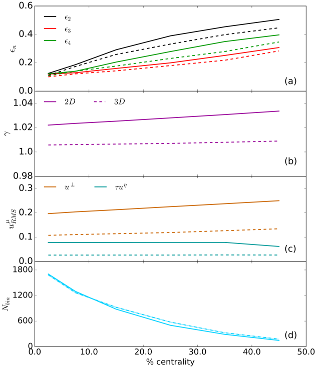


Jeon (McGill) | 3D IP-Glasma | 14/21

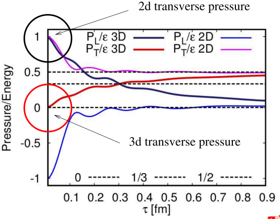
- Energy density increases by a factor of 10! → Increases discrepancy with Angantyr even further.

S. Jeon

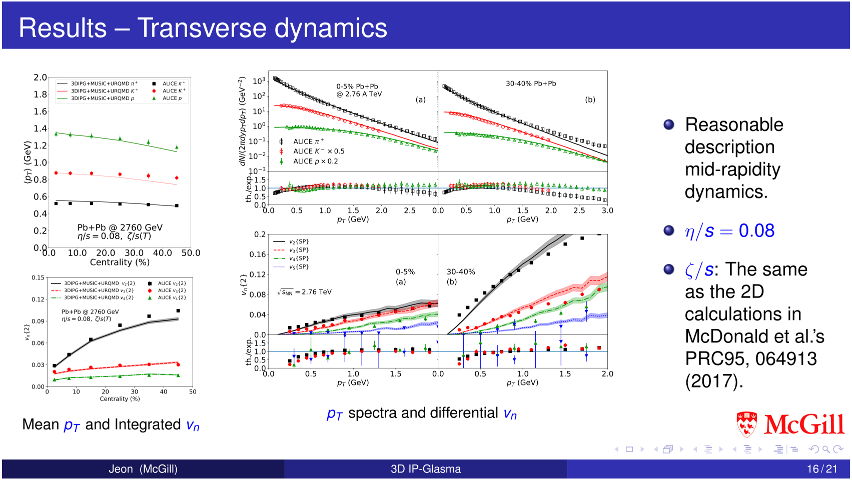
2D vs 3D Evolutions



- 3D evolution results in less developed initial state eccentricity and flows
- Why: Pressure

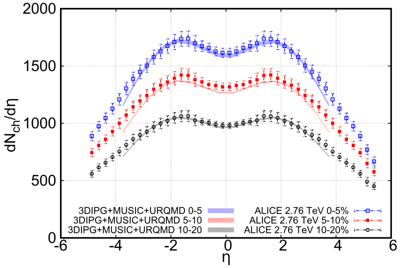


S. Jeon



- Agrees well with the data.

Results – Longitudinal dynamics



- Can capture global longitudinal dynamics
- Not yet applicable to $|\eta| \gtrsim 4$

- Agrees well with the data.

Strickland & Almaalol

- Almaalol plenary talk
- Almaalol poster
- Strickland invited parallel talk

Strickland

Enter the attractors

Work over the course of the last decade has shown that there exist three dynamical attractors for the non-equilibrium dynamics of the QGP:

1. An **early-time non-thermal attractor** associated with **Classical Yang-Mills (CYM) evolution**; never thermalizes and system generates ever growing momentum anisotropy; however, can be used as IC for the next stage o evolution

   Berges, Boguslavski, Schlichting, Venugopalan, …

2. A **QCD effective kinetic theory (QCD EKT) attractor** that can be matched onto both the early-time CYM non-thermal and late-time hydrodynamical attractors; numerical realization of bottom-up thermalization

   Kurkela, Zhu, Keegan, Romatschke, van der Schee, Mazeliauskas, Almaalol, MS, Schlicting, Du, Arnold, Moore, Yaffe, Baier, Mueller, Son, Schiff, …

3. A "late time" **universal dissipative hydrodynamical attractor**

   Heller, Spalinski, Romatschke, Kurkela, Svensson, Denicol, Noronha, MS, Almaalol, Martinez, Brewer, Blaizot, Yan, …

M. Strickland 5

- Recap: attractors

Almaalol

General moments of the distribution function

M. Strickland, JHEP2018, 128; 1809.01200.

Solving higher moments of the Boltzmann equation

$\mathrm{N_C} = 3$ in $0+1\mathrm{d}$ Bjorken flow, 2D grid $\{x_i, p_j\}$ with 250×2000 grid points

(Kurkela and Zhu PRL 115, 182301 (2015))

- A general moment of the distribution function is defined by

$$\mathcal{M}^{nm}[f] \equiv \int dP\,(p.u)^n\,(p.z)^{2m}\,f(x,p)$$

- The corresponding equilibrium values using a Bose distribution,

$$\mathcal{M}^{nm}_{\mathrm{eq}} = \frac{T^{n+2m+2}\Gamma(n+2m+2)\zeta(n+2m+2)}{2\pi^2(2m+1)}$$

- the hydrodynamics degrees of freedom are
  $\mathcal{M}^{10} =$ number density
  $\mathcal{M}^{20} =$ energy density
  $\mathcal{M}^{01} =$ longitudinal pressure
- Study the deviations from equilibrium

.

$$\overline{\mathcal{M}}^{nm}(\tau) \equiv \frac{\mathcal{M}^{nm}(\tau)}{\mathcal{M}^{nm}_{\mathrm{eq}}(\tau)}$$

D. Almaalol - Kent State University - 14

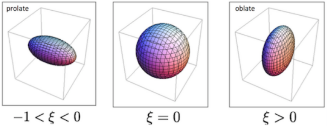
- Defining moments of the distribution function.
- Quantities with bar compare with equilibrium.

Almaalol

Initial distribution $-\frac{d\mathbf{f_p}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{2\leftrightarrow 2}[\mathbf{f_p}] + \mathcal{C}_{\exp}[\mathbf{f_p}]$.

- Thermal Romatschke-Strickland

$$\mathrm{f}_{0,\mathrm{RS}}(\mathbf{p}) = f_{\mathrm{Bose}}\left(\sqrt{\mathbf{p}^2 + \xi_0 p_z^2}/\Lambda_0\right)$$



anisotropy parameter $(-1 < \xi_0 < \infty)$
Λ_0 is set by Landau matching
Romatschke,Strickland,PRD68, (2003)

- Non-thermal CGC

$$\mathrm{f}_{0,\mathrm{CGC}}(\mathbf{p}) = \frac{2A}{\lambda} \frac{\tilde{\Lambda}_0}{\sqrt{\mathbf{p^2} + \xi_0 p_z^2}} \exp^{-\frac{2}{3}(\mathbf{p}^2 + \xi_0 \hat{p}_z^2)/\tilde{\Lambda}_0^2}$$

The initial scale $\tilde{\Lambda}_0$ is related to the saturation scale $\tilde{\Lambda}_0 = \langle \mathrm{p_T} \rangle_0 \approx 1.8\,\mathrm{Q_s}$
A is set by fixing the initial energy density to match an expectation value estimated from a CYM simulation
A. Kurkela and Y. Zhu, Phys. Rev. Lett.115, 182301(2015)
T. Lappi, Phys. Lett.B703, 325-330 (2011)

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- Defining the ICs.

Almaalol

Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

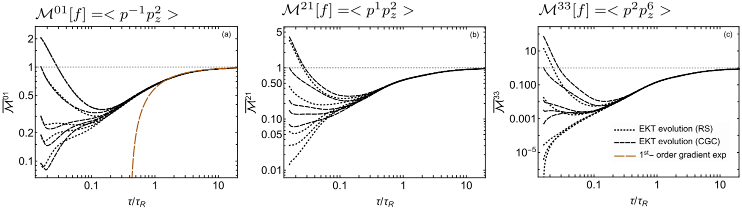
Non-equilibrium evolution becomes insensitive to initial conditions at very early times

$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

τ/τ_R	τ
0.2	0.32 fm/c
0.5	0.86 fm/c
1	1.88 fm/c
2	4.23 fm/c
5	14.1 fm/c
10	38.5 fm/c

- Forward attractor

Pressure anisotropy



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- Attractor 1.

Non-equilibrium QCD attractor at high temperature

DA, Kurkela,Strickland PRL. 125, (2020)

An attractor for the momentum phase space distribution function

$\tau_R(\tau) = 4\pi\bar{\eta}/T(\tau)$

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0.2	0.32 fm/c
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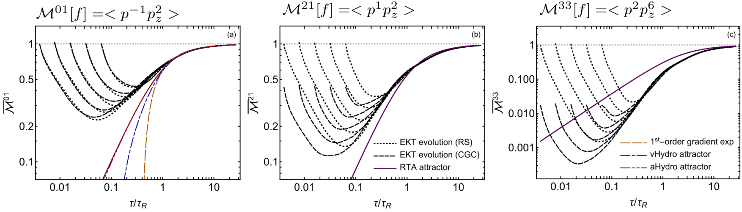
5	14.1 fm/c
10	38.5 fm/c

- Pullback attractor
- EKT extends beyond hydro degrees of freedom
- RTA fails to capture the dynamics at high moments

Pressure anisotropy



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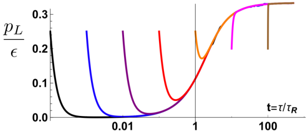
- Attractor 2.

Hint: different physical origin of early- and late-time attractors

relaxation time approximation



(qualitatively similar results for Israel-Stewart)

τ_R: timescale for collisions

Kurkela, van der Schee, Wiedemann, Wu [1907.08101]

Reduction in degrees of freedom driven by…

Rapid expansion without collisions

Collisions

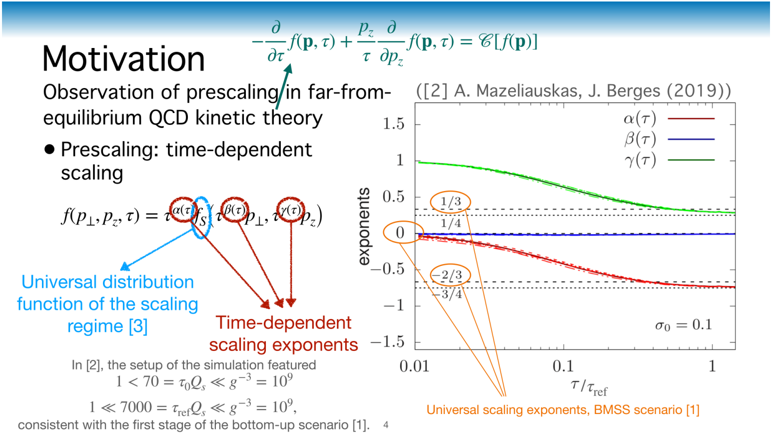
hydrodynamic modes

Suggests “slow mode” describing rapid expansion without collisions

Jasmine Brewer (CERN) 11

- Interpretation of the 2 different attractor phases: First stage is expansion dominated, where expansion is winning interactions (collisions). In the second phase interactions win, and the system approaches isotropy.

Scheihing-Hitschfeld



- Self-similarity: system governed by 3 scaling exponents
- Pre-scaling: system governed by 3 time-dependent exponents, which converge to constants.

Brewer

Evolution of F can be described by effective "Hamiltonian"

$$\int_p p \left(\underbrace{\partial_\tau f}_{\partial_\tau F} - \underbrace{\frac{p_z}{\tau} \partial_{p_z} f}_{-\frac{1}{\tau}(\dots)F} = \underbrace{-C[f]}_{(\dots)F} \right)$$

(sometimes)

Evolution of F $\longleftrightarrow$ $\partial_y \psi = -\mathcal{H}(y)\psi$

$y = \log\left(\frac{\tau}{\tau_I}\right)$

Eigenstates give effective degrees of freedom

Slow modes: ground states

JB, Yan, Yin [1910.00021]

Non-linear collision kernel Scheihing-Hitschfeld Mon.

Jasmine Brewer (CERN) 14

- $F = \int_t pf(p)$
- Sometimes = with Relaxation Time Approximation.
- Interpret kinetic theory as Hamiltonian system.

Solution scheme

- At early times, we can use the small-angle scattering approximation, and that the typical gluon momenta satisfy $p_z \ll p_\perp \approx p$. It follows that

momentum diffusion constant [5]

$$\frac{\partial}{\partial \tau} f - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \approx \hat{q}(y) \frac{\partial^2 f}{\partial p_z^2} \approx \hat{q}(y) \nabla_{\mathbf{p}}^2 f,$$

where $\hat{q} = 4\pi N_c^2 \alpha_s^2 l_{Cb} \int_p (1+f)f \approx 4\pi N_c^2 \alpha_s^2 l_{Cb} \int_p f^2$.

over-occupied distribution $f \gg 1$

- To solve this, we will treat $\hat{q}$ as a time-dependent parameter and look for time-dependent scaling solutions $\langle p_\perp^m p_z^n \rangle = D_{n,m} A(\tau) B(\tau)^m C(\tau)^n$.

$$\implies \alpha = \alpha[\tau; \hat{q}, \partial_\tau \hat{q}; f_0], \quad \beta = \beta[\tau; \hat{q}, \partial_\tau \hat{q}; f_0], \quad \gamma = \gamma[\tau; \hat{q}, \partial_\tau \hat{q}; f_0].$$

- The only approximation we make is that $\partial \log \hat{q} / \partial \log \tau$ is varying slowly.

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- New idea: only one time-dependent parameter $\hat{q}$. Determines the exponents.

Results

Note that (pre)scaling implies that $\frac{\partial \log \hat{q}}{\partial \log \tau} = 2\alpha - 2\beta - \gamma$.

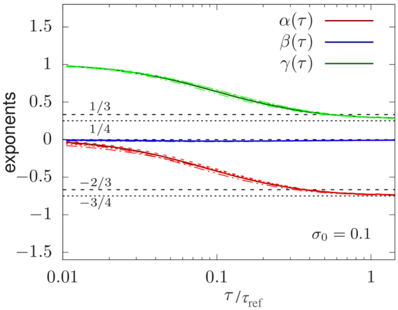
—> Then, replacing the expressions

$$\alpha = \alpha[\tau; \hat{q}, \partial_\tau \hat{q}]$$
$$\beta = \beta[\tau; \hat{q}, \partial_\tau \hat{q}]$$
$$\gamma = \gamma[\tau; \hat{q}, \partial_\tau \hat{q}]$$

one gets a 1st order ODE for $\hat{q}$.

Solving it, $\hat{q}$ fully determines α, β, γ.

Compare with [2]:



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- Turns out this model explains the pre-scaling phenomenon very well.

The adiabatic perspective: prescaling

Eigenvalues of $\mathscr{H}$: $E_n = 2n + 1 \implies$ Energy gap.

—> After a sufficiently long time the state will be governed by the lowest modes.

$$\implies \text{initial condition}: |\psi\rangle = A_0|\psi_0\rangle + A_1|\psi_1\rangle.$$

—> Solving for the scaling exponents (perturbatively in A_1/A_0) gives

$$\gamma = -\frac{1}{2}\left(1 + \frac{\partial \log \hat{q}}{\partial \log \tau}\right) + \frac{A_1}{A_0}\frac{(\tau_I/\tau)^2}{4\tau\hat{q}}\left(3 + \frac{\partial \log \hat{q}}{\partial \log \tau}\right)^2, \quad \beta = 0, \quad \alpha = \gamma - 1.$$

"0th order" BMSS exponent

"Perturbative parameter"

"1st order" correction —> Prescaling

Now we solve the ODE $\frac{\partial \log \hat{q}}{\partial \log \tau} = 2\alpha - 2\beta - \gamma$

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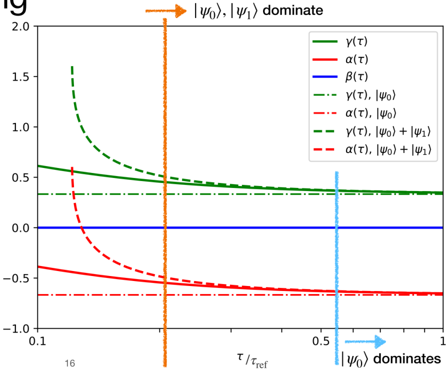
- Time dependence: perturbation theory.

Scheihing-Hitschfeld

Adiabatic prescaling

- Prescaling emerges as the lowest excited states decay.
- Appears before reaching the time-independent scaling regime, for any initial condition.
- For specific choices of initial conditions (which requires $f_0 \sim f_S$), prescaling can be extended to arbitrarily early times.



- Zeroth and first order terms give a good description of the data!

Müller

Momentum broadening

- Model early-time jet as single parton (quark or gluon)
- No backreaction, color field of glasma as background
- Very high initial momentum, no deflection
- McLerran-Venugopalan model

Wong equations

$$\frac{dp_\mu}{dt} = g\, Q^a(t) \frac{dx^\nu}{dt} F^a_{\mu\nu}(x(t))$$
$$\frac{dQ^a}{dt} = g \frac{dx^\mu}{dt} f^{abc} A^b_\mu(x(t)) Q^c(t)$$

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- Calculation with CYM, 2+1D boost invariant system.
- Q_a color charge of the quark.

Müller

Results

- Accumulated momenta at transition time $\tau_0 = 0.6\,\mathrm{fm}/c$

$$\langle p_\perp^2 \rangle \approx Q_s^2$$

- Infrared dependence m
- Gluonic jets

$$\frac{\langle p_\perp^2 \rangle_g}{\langle p_\perp^2 \rangle_q} = \frac{C_A}{C_F} = \frac{2N_c^2}{N_c^2 - 1}$$

- Broadening is anisotropic

$$\langle p_z^2 \rangle / \langle p_y^2 \rangle \approx 2$$

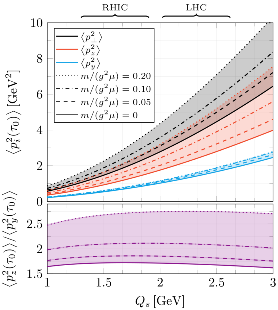

Fig. from A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [arXiv:2009.14206]

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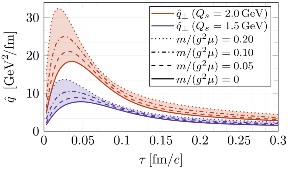
Müller

Jet broadening parameter $\hat{q}$

Jet broadening parameter $\hat{q}$: accumulated (squared) momentum per unit time

$$\hat{q}_\perp(\tau) = \frac{d\langle p_\perp^2 \rangle}{d\tau}$$



Most momentum is accumulated in the earliest stages of the glasma!

Fig. from A. Ipp, D. I. Müller, D. Schuh, PLB (2020) [arXiv:2009.14206]

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Czajka poster

Fokker-Planck equation

Evolution equation on the distribution function of heavy quarks: *Mrówczyński, Eur. Phys. J. A54, no 3, 43 (2018)*

$$\left(D - \nabla_p^i X^{ij}(\mathbf{v}) \nabla_p^j - \nabla_p^i Y^i(\mathbf{v})\right) n(t, \mathbf{x}, \mathbf{p}) = 0$$

$$\langle Q(t, \mathbf{r}, \mathbf{p}) \rangle = n(t, \mathbf{r}, \mathbf{p}) \qquad D = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

Collision terms:

$$X^{ij}(\mathbf{v}) \equiv \frac{1}{2N_c} \int_0^t dt' \left\langle F_a^i(t, \mathbf{x}) F_a^j\left(t', \mathbf{x} - \mathbf{v}(t - t')\right)\right\rangle$$

$Y^i(\mathbf{v}) = X^{ij}(\mathbf{v}) \frac{v^j}{T}$ - postulated so that the distribution function satisfies the Fokker-Planck equation in equilibrium

T - temperature of plasma that has the same energy density as in equilibrium

Collision terms determine energy loss and momentum broadening

Physical meaning: $\frac{\langle \Delta p^i \rangle}{\Delta t} = -Y^i(\mathbf{v}) \qquad \frac{\langle \Delta p^i \Delta p^j \rangle}{\Delta t} = X^{ij}(\mathbf{v}) + X^{ji}(\mathbf{v})$

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- Same initial condition Müller had, but now expand the IC in powers of τ and solve FP-equation for the heavy quark.

Czajka poster

CGC: before the collision

Classical Yang-Mills equations: $[D_\mu, F^{\mu\nu}] = J^\nu$ $\quad J^\mu_{1,2}(x^\mp, \vec{x}_\perp) = \delta^{\mu\pm}\rho_{1,2}(x^\mp, \vec{x}_\perp)$

Solutions of CYM equations:

$$A^\pm_{1,2}(x^\pm, \vec{x}_\perp) = 0$$

$$A^i_{1,2}(x^\pm, \vec{x}_\perp) = \theta(x^\mp)A^i_{1,2}(\vec{x}_\perp)$$

$$A^i_{1,2}(\vec{x}_\perp) = -\frac{1}{ig}U_{1,2}(\vec{x}_\perp)\partial^i U^\dagger_{1,2}(\vec{x}_\perp)$$

pure gauge transform of vacuum

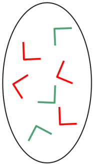
$U(\vec{x}_\perp) \equiv U[g, \rho, \vec{x}_\perp]$ - the unitary matrix

Chromoelectric and chromomagnetic fields are given by the respective components of the strength tensor. They are:

$$E^z_{1,2}(x^\pm, \vec{x}_\perp) = 0 \qquad E^i_{1,2}(x^\pm, \vec{x}_\perp) = -\frac{\delta(x^\mp)}{\sqrt{2}}A^i_{1,2}(\vec{x}_\perp)$$

$$B^z_{1,2}(x^\pm, \vec{x}_\perp) = 0 \qquad B^i_{1,2}(x^\pm, \vec{x}_\perp) = \mp\epsilon^{ij}E^j_{1,2}(x^\pm, \vec{x}_\perp)$$

Only transverse components are different from zero.



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- IC: standard 2+1D.

Czajka poster

After the collisions: glasma

Glasma fields develop in the forward light-cone region.

Analytical approach to solve CYN for the glasma fields proposed in:

Chen, Fries, Kapusta, Li, Phys. Rev. C 92, 064912 (2015)

CGC looses applicability soon after the collision ⟶ proper time of such an evolving system is small

τ -acts as an expansion parameter

$$A^i_\perp(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A^i_{\perp(n)}(\vec{x}_\perp)$$
$$A(\tau, \vec{x}_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(\vec{x}_\perp)$$

$$|A^i_{\perp(n)}| \sim Q_s^n |A|$$
$$|A_{(n)}| \sim Q_s^{n+1} |A|$$
$$|A| = \sqrt{A^i_1 A^i_1}$$

radius of convergence is set by the only time scale - saturation momentum scale as

$$1/Q_s$$

τQ_s -dimensionless

Boundary conditions connect different light-cone sectors:

$$A^i_\perp(\tau = 0, \vec{x}_\perp) = A^i_1(\vec{x}_\perp) + A^i_2(\vec{x}_\perp)$$
$$A(\tau = 0, \vec{x}_\perp) = -\frac{ig}{2}[A^i_1(\vec{x}_\perp), A^i_2(\vec{x}_\perp)]$$

Using the boundary conditions the system of coupled YM equations can be solved recursively.

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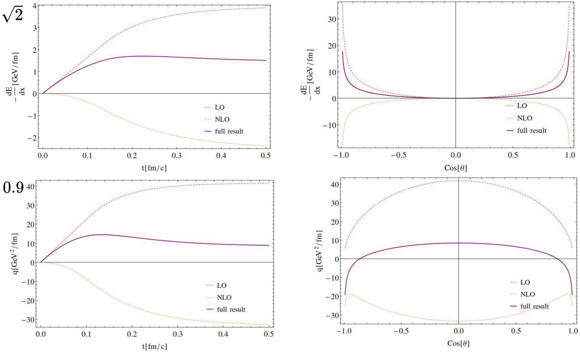
- Expansion in τ

Energy loss and momentum broadening

$$m = 200\text{MeV},\ N_c = 3,\ g = 1,\ Q_s = 2\text{GeV}$$

$v_{||} = v_{\perp} = 1/\sqrt{2}$

$v_{||} = 0,\ v_{\perp} = 0.9$



energy loss is minimal when HQ moves perpendicularly to the axis

momentum broadening is maximal when HQ moves perpendicularly to the axis

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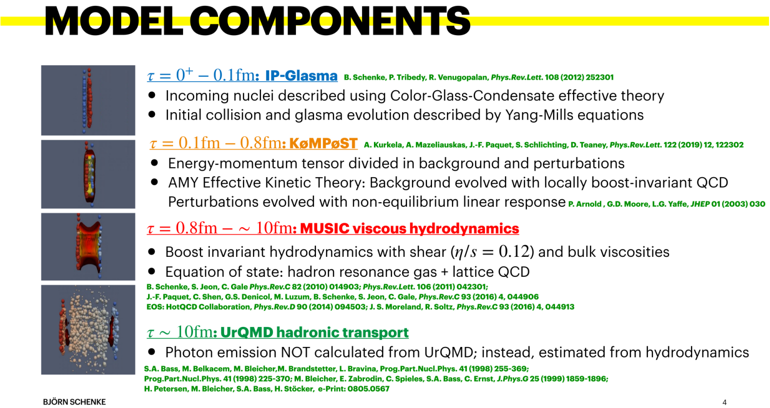
- Their conclusion $\hat{q}_{\mathrm{LO+NLO}} = 8.5\,\frac{\mathrm{GeV}^2}{\mathrm{fm}}$.

Schenke parallel talk



- Reminder: components of the model.

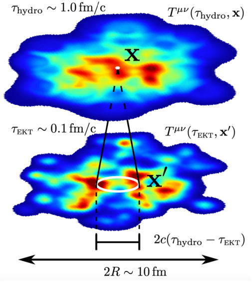
PRE-EQUILIBRIUM EVOLUTION

KøMPøST A. Kurkela, A. Mazeliauskas, J.-F. Paquet, S. Schlichting, D. Teaney, *Phys.Rev.Lett.* 122 (2019) 12, 122302

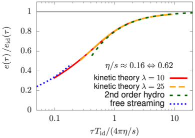
- Weak coupling limit (then extrapolated)
- Separate energy momentum tensor into average over causal circle and linear perturbations:

$$T^{\mu\nu}(\tau_{\rm EKT}, \vec{x}') = \bar{T}^{\mu\nu}_{\vec{x}}(\tau_{\rm EKT}) + \delta T^{\mu\nu}_{\vec{x}}(\tau_{\rm EKT}, \vec{x}')$$

- Nonequilibrium evolution of the background energy density exhibits a **universal scaling behavior**
- Time evolution of the perturbations is governed by **kinetic response functions**, that can be stored and used for different initial temperature or shear viscosity using scaling transformations



+ No need to solve BE every time
- Presently conformal
- Linear approximation breaks down for large fluctuations (small systems)



BJÖRN SCHENKE 5

- Reminder: kompost.

PHOTON PRODUCTION

Prompt photons — Compute perturbative QCD photons @ NLO INCNLO, CTEQ6.1m, BFG-2, Isospin, EPS09

NLO pQCD [INCNLO] [Could also use fit to p+p data]

$$\frac{dN_\gamma}{d^3p} = \frac{N_{binary}}{\sigma_{pp}^{inel}} \overbrace{f_{a/A} \otimes f_{b/B} \otimes d\hat{\sigma}_{ab\to\gamma/c+d}[\otimes D_{\gamma/c}]}$$

P. Aurenche, M. Fontannaz, J. Ph. Guillet, B. A. Kniehl, E. Pilon, *Eur.Phys.J.C* 9 (1999) 107-119

Pre-equilibrium photons

- KøMPøST provides energy momentum tensor
- Extract "effective temperature" assuming QCD equation of state
- Photon production estimated as if thermal photons (with viscous corrections)

Thermal photons — Integrate thermal photon rates (with viscous corrections where available) over spacetime volume

$$\frac{dN_\gamma}{d^3p} = \int d^4X \frac{d\Gamma_\gamma}{d^3p}(\mathrm{p}, T(X), u^\mu(X), \dots)$$

Late stage — While we switch to UrQMD at T~145 MeV for hadronic observables, we compute photon production using profiles from hydrodynamics down to T=105 MeV

More work on pre-equilibrium photons: O. Linnyk, V.P. Konchakovski, W. Cassing, E.L. Bratkovskaya, Phys.Rev.C 88 (2013) 034904; L. McLerran, B. Schenke, Nucl.Phys.A 929 (2014) 71-82; J. Berges, K. Reygers, N. Tanji, R. Venugopalan, Phys.Rev.C 95 (2017) 5, 054904; M. Greif, F. Senzel, H. Kremer, K. Zhou, C. Greiner, and Z. Xu, Phys. Rev. C95, 054903 (2017); A. Monnai, J.Phys.G 47 (2020) 7; O. Garcia-Montero, J.Phys.Conf.Ser. 1667 (2020) 1, 012008; J. Churchill, L. Yan, S. Jeon, C. Gale, arXiv:2008.02902 (PRC in press); ...

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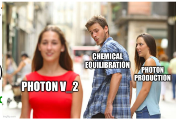
- How they compute photon production.

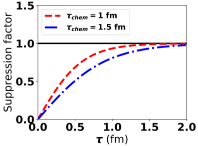
CHEMICAL EQUILIBRIUM?

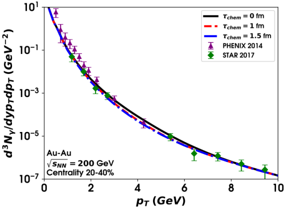
Account for chemical non-equilibration in KøMPøS
by suppressing the photon rate

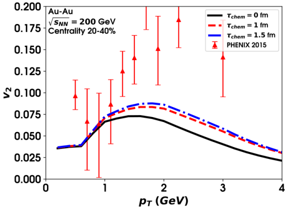
$$\frac{dN_\gamma}{d^3p} = Suppression(\tau)\int d^4X \frac{d\Gamma_\gamma}{d^3p}(p, T(X), u^\mu(X), \dots)$$

Estimated from Kurkela and Mazeliauskas, Phys.Rev.Lett. 122 (2









PHENIX Collaboration, Phys.Rev.C91 (2015) 6, 064904; Phys. Rev. C 94, 064901 (2016); STAR Collaboration, *Phys.Lett.B770* (2017) 451-458

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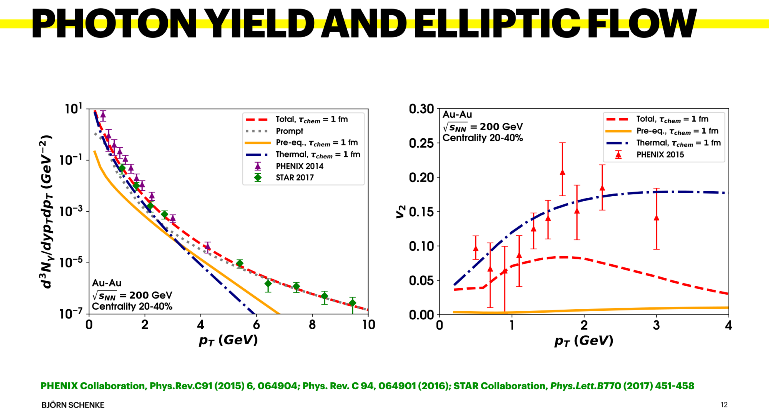
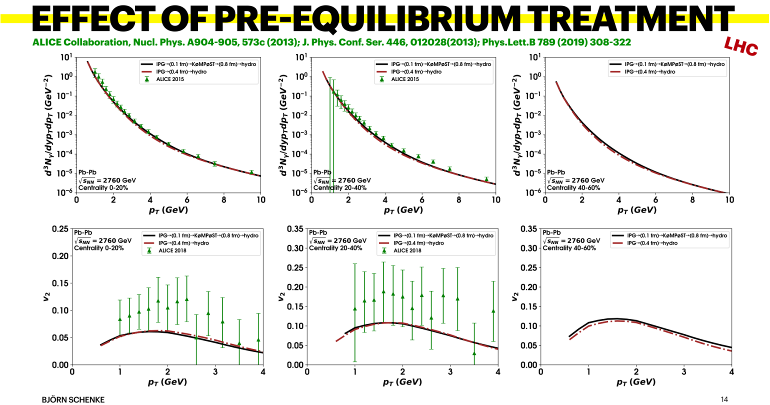
- Chemical equilibration seems to have negligible impact on photon production. However it's sensitive to photon v_2.

Schenke parallel talk



- Effect of the pre-equilibrium on photon production and v_2 small.

Schenke parallel talk



- Pre-equilibrium has negligible impact on photon production.