Properties of glasma color fields

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Outline

- Color Glass Condensate, saturation basics
- Glasma initial stage color field
- Glasma flux tubes and strings
- Non-flow particle correlations

Gluon saturation

A heavy ion event at the LHC



How does one understand what happened here?

Heavy ion collision in spacetime

The purpose in heavy ion collisions: to create QCD **matter**, i.e. system that is large and lives long compared to the microscopic scale



Heavy ion collision in spacetime

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Concentrate here on the earliest stage

Small x: the hadron/nucleus wavefunction is characterized by saturation scale $Q_s \gg \Lambda_{QCD}$.

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- ${f p}_{
 m T}\sim {m Q}_{
 m s}$: strong fields $A_{\mu}\sim 1/g$
 - occupation numbers $\sim 1/\alpha_s$
 - classical field approximation.
 - small α_s , but nonperturbative



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CGC: Effective theory for wavefunction of nucleus

- Large x = color charge ρ , **probability** distribution $W_{\gamma}[\rho]$
- Small x = classical gluon field A_{μ} + quantum flucts.

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Glasma: field configuration of two colliding sheets of CGC. (Here $y \sim \ln \sqrt{s}$)

Gluon saturation in IMF: nonlinear interactions

Saturation when phase space density of gluons large

- Number of gluons $xG(x, Q^2)$
- Size of one gluon $\sim 1/Q^2$
- Transverse space available πR_p^2
- Coupling α_s

Estimate in terms of gluon numbers

Nonlinearities important when

$$\pi R_p^2 \sim \alpha_s x G(x, Q^2) \frac{1}{Q^2}$$

Solve for $Q^2 = Q_s^2$; "saturation scale"



(LHC kinematics: $\textit{Q}_{s}\approx1$.. 2GeV)

Gluon saturation in dipole picture: unitarity



2-gluon exchange wrong when $Q^2 \sim \frac{1}{r^2} \lesssim Q_s^2 \sim \frac{\alpha_s x G(x, Q_s^2)}{\pi R_D^2}$

Gluon saturation in target rest frame

- Degree of freedom is scattering amplitude (not number of partons)
- Saturation appears as unitarity constraint

 \implies Built into formalism; does not look dynamical.

Wilson line

Classical color field described as Wilson line

$$V(\mathbf{x}_{T}) = P \exp \left\{ ig \int dx^{-} A^{+}_{cov}(\mathbf{x}_{T}, x^{-}) \right\} \in SU(3)$$

Relation to color charge $\rho : \nabla_{T}^{2} A^{+}_{cov}(\mathbf{x}_{T}, x^{-}) = -g\rho(\mathbf{x}_{T}, x^{-})$

Physical interpretation: Eikonal propagation of parton through target color field

Qs is characteristic momentum/distance scale

Precise definition used here:

$$C(\mathbf{x}_{T}) = \frac{1}{N_{c}} \left\langle \operatorname{Tr} V^{\dagger}(\mathbf{0}_{T}) V(\mathbf{x}_{T}) \right\rangle = \Theta^{-\frac{1}{2}} \iff \mathbf{x}_{T}^{2} = \frac{2}{Q_{c}^{2}}$$



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CGC color field is ISR

- In partonic picture: gluon field is cascade of ISR
- Gluons have k_T , but ordered in k^+ , not k_T
- EFT cutoff: color charge/field
- Many color charges: conservation only on average
- Quantitative calculations:
 - Do not count gluons
 - In stead: d.o.f is eikonal amplitude
 - ► Unitarity constraint, not Λ_{QCD} , cuts off $\int_0 \frac{dk_T}{k_T}$ \implies typical $k_T \sim Q_s$

Now: initial stage of heavy ion collision in this picture



Glasma fields

How to obtain intitial glasma fields

Now let two dense color field systems collide



Need LC gauge fields $A_{(1,2)}^i = \frac{i}{g} V_{(1,2)}(\mathbf{x}_T) \partial_i V_{(1,2)}^{\dagger}(\mathbf{x}_T)$ $V(\mathbf{x}_T) =$ Wilson line for nuclei (1) and (2) $\tau = 0$: match using $[D_{\mu}, F^{\mu\nu}] = J^{\nu}$:

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Need LC gauge fields
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 $V(\mathbf{x}_T) =$ Wilson line for nuclei (1) and (2)
 $r = 0$: match using $[D_{\mu}, F^{\mu\nu}] = J^{\nu}$:
 $A^i \Big|_{\tau=0} = A_{(1)}^i + A_{(2)}^i$
 $A^{\eta} \Big|_{\tau=0} = \frac{ig}{2} [A_{(1)}^i, A_{(2)}^i]$
 $A^{\tau} = 0$ gauge choice

 $au > \mathbf{0}$ Numerical **CYM** or approximations

This is the **glasma** field \implies Then average over initial Wilson lines.

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Initial glasma fields



- Initial condition is longitudinal *E* and *B* field, at $\tau \sim 1/Q_s$ evolves to $E_\tau^2 \sim B_\tau^2 \sim 2E_x^2 \sim 2B_y^2 \sim 2E_y^2 \sim 2B_y^2$
- Depend on transverse coordinate

with correlation length $1/Q_s \implies$ gluon correlations

- \blacktriangleright Fix gauge, Fourier-decompose: Gluons with $p_{T} \sim \textit{Q}_{s}$
- Configuration naturally has Chern-Simons charge

 $N_{\rm CS} \sim \varepsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} \sim E \cdot B$

Dilute limit, polarization, coherence

- $\blacktriangleright V(\mathbf{x}_T) \approx 1 i \frac{1}{\nabla_T^2} \rho(\mathbf{x}_T)$
- Gluon spectrum reduces to Bertsch-Gunion

How to understand longitudinal fields at $\tau = 0$?

- ▶ $\exists 2$ linear polarization states for gluon with \mathbf{k}_T :
 - $\blacktriangleright E_{\perp}(\perp \mathbf{k}_{T}) \& B_{z}$
 - $\blacktriangleright B_{\perp}(\perp \mathbf{k}_{T}) \& E_{z}$
- Both polarizations produced
- Boost invariant energy

$$\frac{\mathrm{d}E}{\mathrm{d}\eta} = \frac{1}{\tau} \operatorname{Tr} E^{i} E^{i} + \frac{\tau}{2} \operatorname{Tr} F_{ij} F_{ij} + \tau \operatorname{Tr} E^{\eta} E^{\eta} + \frac{1}{\tau} [D_{i}, A_{\eta}] [D_{i}, A_{\eta}]$$

- Only phase consistent with finite energy @ $\tau = 0$: 1 polarization in purely B_z and other in purely E_z
- Then decohere with au





(Epelbaum, Gelis, arXiv:1307.2214: quantum fluctuations change the picture)

Glasma flux tubes and strings

Color flux tubes and strings: where is the energy?



"2-component" picture partons + strings

- t = 0 Longitudinal (jet axis) momentum of receding partons
- Longitudinal motion: shift to potential energy of (homogenous?) color electric field
- String breaking: release to kinetic energy of newly created partons



Glasma fields: 1-scale problem

- Total $\sqrt{s} = \infty$, carried by beams
- Count energy /unit rapidity (spacetime rapidity η)
- ► Fields boost invariant ⇒ no longitudinal (beam direction) momentum
- Energy carried by k_T of gluons

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Color flux tubes and strings: transverse coordinate

For strings, I have only questions:

- What is "thickness" of string?
- Does a string have both momentum and position? Uncertainty relation? 2-scale argument?

Classical-Yang-Mills

▶ Fields in coordinate space:

 \implies k_T by Fourier-transform

- Heisenberg uncertainty relation satisfied
- For very small scale substructure (hot spots in proton), becomes limiting factor!





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Color flux tubes and strings: time and rapidity

- Volume of one unit $\Delta\eta$ grows as $\sim \tau$
- String with constant tension: $\frac{dE}{dn} \sim \tau \dots$ until string breaks
- ▶ Boost-invariant fields redshifted \implies after coherence lost $\frac{dE}{dn} \sim cst$



Azimuthal particle correlations

Azimuthal correlations, vn's

Experimental observation

Azimuthal anisotropy in particle production

▶ In AA, (high N_{ch}) pA & pp, LHC and RHIC



PHENIX, Nature Phys. 15 (2019) no.3, 214



Show as yield/trigger or as V_n: ATLAS, Phys. Rev. C **90** (2014) 4, 044906

What is the origin of the effect?

- Collective flow?
- Initial state gluon correlations?

Azimuthal correlations from flow

In large systems (nucleus-nucleus) this dominates



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Domains in the target color field

Physical origin for initial state particle correlations



How is color field seen by individual quark?

- Momentum transfer from target E-field
- "Domains" of size ~ 1/Q_s
 (Note actual calculation has Wilson lines, with correlation length, not explicit domains)
- Several probe particles inside domain: multiparticle azimuthal correlations.
- \blacktriangleright ~ $Q_{\rm S}^2 S_{\perp}$ domains (S_{\perp} = size of interaction area, πR_A^2 , πR_P^2)
- $\blacktriangleright \sim N_c^2$ colors
- ► Relative correlation $\sim \frac{1}{N_c^2 Q_s^2 S_{\perp}} \implies$ stronger in small systems
- Manifestation in momentum & coordinate space

 relevant for both flow & non-flow

Conclusions

- Gluon saturation & CGC: a weak-coupling & nonperturbative picture in the "BFKL" kinematic regime
- Classical color fields
 - Natural way to include gluon saturation
 - Measurable in dilute-dense processes like DIS
 - Can calculate dense-dense collision
- My personal opinion: glasma fields best thought of as perturbative gluons
 - Understand time dependence
 - High k_{T} -spectrum perturbative, k_{T} -factorization
 - But IR is regulated at $k_T \sim Q_{
 m s}$

 \implies requires nonperturbative calculation

- can be done with CYM



Glasma at $\tau = 0$

Energy density and Chern-Simons charge at $\tau = 0$

At exactly $\tau = 0$ can do a lot of things analytically (changing notation $\alpha^i \equiv A^i_{(1)}, \beta^i \equiv A^i_{(2)}$)

$$\langle \varepsilon(\mathbf{x}_{T}) \rangle = (-ig)^{2} \left(\delta^{ij} \delta^{kl} + \varepsilon^{ij} \varepsilon^{kl} \right) \left\langle \operatorname{Tr} \left([\alpha_{\mathbf{x}_{T}}^{i}, \beta_{\mathbf{x}_{T}}^{j}] [\alpha_{\mathbf{x}_{T}}^{k}, \beta_{\mathbf{x}_{T}}^{l}] \right) \right\rangle \quad (\sim \vec{E}^{2} + \vec{B}^{2} \underset{\tau=0}{=} E_{z}^{2} + B_{z}^{2})$$

$$\langle \dot{\nu}(\mathbf{x}_{T}) \rangle = (-ig)^{2} \quad \delta^{ij} \varepsilon^{kl} \qquad \left\langle \operatorname{Tr} \left([\alpha_{\mathbf{x}_{T}}^{i}, \beta_{\mathbf{x}_{T}}^{j}] [\alpha_{\mathbf{x}_{T}}^{k}, \beta_{\mathbf{x}_{T}}^{l}] \right) \right\rangle \quad (\sim \vec{E} \cdot \vec{B} = E^{z} \cdot B^{z})$$

Here $\dot{\nu}$ related to axial charge per unit area $\left. \frac{\mathrm{d}N_5}{\mathrm{d}^2 \mathbf{x}_T \,\mathrm{d}\eta} \right|_{\tau \lesssim 1/Q_s} \approx \frac{\tau^2}{2} \frac{g^2 N_f}{2\pi^2} \dot{\nu}(\mathbf{x}_T, \tau = 0^+)$

Nuclei independent \implies for $\langle \cdot \rangle$ factorize out Weizsäcker-Williams gluon distribution

$$\left\langle \operatorname{Tr}\left(\left[\alpha_{\mathbf{x}_{7}}^{i}, \beta_{\mathbf{x}_{7}}^{j} \right] \left[\alpha_{\mathbf{x}_{7}}^{k}, \beta_{\mathbf{x}_{7}}^{j} \right] \right) \right\rangle = \frac{1}{2} i f^{abc} i f^{a'b'c} \left\langle \alpha_{\mathbf{x}_{7}}^{i,a} \alpha_{\mathbf{x}_{7}}^{k,a'} \right\rangle \left\langle \beta_{\mathbf{x}_{7}}^{j,b} \beta_{\mathbf{x}_{7}}^{l,b'} \right\rangle ,$$

$$W^{ik}(\mathbf{x}_{T}, \mathbf{y}_{T}) \equiv \frac{\left\langle \alpha_{\mathbf{x}_{T}}^{i, a} \alpha_{\mathbf{x}_{T}}^{k, a} \right\rangle}{N_{c}^{2} - 1}$$

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Distributions in Gaussian approximation

In Gaussian approximation WW-distribution from **dipole** $D_{\mathbf{x}_{T}\mathbf{y}_{T}} = \frac{1}{N_{c}} \left\langle \operatorname{Tr} \left(V_{\mathbf{x}_{T}} V_{\mathbf{y}_{T}}^{\dagger} \right) \right\rangle$

$$W^{ik}(\mathbf{x}_{T}, \mathbf{y}_{T}) = \frac{1}{2} \delta^{ij} G(r) + \frac{1}{2} \left(\delta^{ij} - 2 \frac{\mathbf{r}_{T}^{i} \mathbf{r}_{T}^{j}}{r^{2}} \right) h(r) = \frac{1}{g^{2} N_{c}} \left(\frac{\partial_{\mathbf{x}_{T}}^{j} \partial_{\mathbf{y}_{T}}^{k} \ln(D_{\mathbf{x}_{T} \mathbf{y}_{T}})}{\ln(D_{\mathbf{x}_{T} \mathbf{y}_{T}})} \right) \left(\left(D_{\mathbf{x}_{T} \mathbf{y}_{T}} \right)^{\frac{C_{A}}{C_{F}}} - 1 \right)$$
Unpolarized $G(r) = \frac{1}{g^{2} N_{c}} \frac{1 - \left(D(r) \right)^{C_{A}/C_{F}}}{\ln(D(r))} \left(\partial_{r}^{2} + \frac{1}{r} \partial_{r} \right) \ln(D(r))$
Linearly polarized $h(r) = \frac{1}{g^{2} N_{c}} \frac{1 - \left(D(r) \right)^{C_{A}/C_{F}}}{\ln(D(r))} \left(\partial_{r}^{2} - \frac{1}{r} \partial_{r} \right) \ln(D(r))$

 $\langle \varepsilon \rangle$ only depends on G(r = 0); correlations on both G(r) and h(r)

• GBW parametrization $G(r) = \frac{1 - e^{-(C_A/C_F)G_S^2 r^2/4}}{g^2 N_c r^2/4}$ & h(r) = 0

• MV model: In $D(r) \sim -r^2 \ln r \implies G(r=0)$ diverges, need to regulate

Glasma graph approximation

For correlations need 8-point functions

$$\langle \alpha^{i,a}_{\mathbf{x}_{T}} \alpha^{k,c}_{\mathbf{x}_{T}} \alpha^{i',a'}_{\mathbf{y}_{T}} \alpha^{k',c'}_{\mathbf{y}_{T}} \rangle, \qquad \alpha^{i}_{\mathbf{x}_{T}} = \frac{i}{g} V(\mathbf{x}_{T}) \partial_{i} V(\mathbf{x}_{T})$$

(This is why higher cumulants difficult)

T. L. and S. Schlichting, Phys. Rev. D 97 (2018) no.3, 034034 : used short distance "Glasma graph" approximation, factorize

$$\left\langle \alpha_{\mathbf{x}_{T}}^{i,a} \alpha_{\mathbf{x}_{T}}^{k,c} \alpha_{\mathbf{y}_{T}}^{j',a'} \alpha_{\mathbf{y}_{T}}^{k',c'} \right\rangle = \overbrace{\left\langle \alpha_{\mathbf{x}_{T}}^{i,a} \alpha_{\mathbf{x}_{T}}^{k,c} \right\rangle \left\langle \alpha_{\mathbf{y}_{T}}^{j',a'} \alpha_{\mathbf{y}_{T}}^{k',c'} \right\rangle}^{\text{disconnected}} \begin{bmatrix} \mathbf{I} = |\mathbf{x}_{T} - \mathbf{y}_{T}| \end{bmatrix} \\ + \underbrace{\left\langle \alpha_{\mathbf{x}_{T}}^{i,a} \alpha_{\mathbf{y}_{T}}^{j',a'} \right\rangle \left\langle \alpha_{\mathbf{x}_{T}}^{k,c} \alpha_{\mathbf{y}_{T}}^{k',c'} \right\rangle}_{\text{connected}} \left[\left\langle \alpha_{\mathbf{x}_{T}}^{i,a} \alpha_{\mathbf{y}_{T}}^{k',c'} \right\rangle \right\rangle \right\rangle$$

Full more complicated result obtained by J. L. Albacete, P. Guerrero-Rodríguez and C. Marquet, JHEP 1901 (2019) 073, P. Guerrero-Rodríguez, arXiv:1903.11602 (hep-ph).

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Correlation functions

- Glasma graph correct for short distance
- Full result has larger longer-distance correlation (softer power law)



P. Guerrero-Rodríguez, arXiv:1903.11602 (hep-ph).

Time evolution

Time evolution

Analytical expression for Glasma fields at $\tau = 0^+$. How to go beyond?

- Classical Yang Mills on the lattice: Widely used (e.g. IPglasma), subject for another talk ...
- Series in τ ? G. Chen, R. J. Fries, J. I. Kapusta and Y. Li, Phys. Rev. C **92** (2015), 064912; M. E. Carrington, A. Czajka and S. Mrowczynski, (arXiv:2012.03042 (hep-ph)). Problem: $\varepsilon(\tau = 0)$ in the MV model is UV-divergent; although fine for $\tau > 0$ (On the lattice regulate by lattice spacing)

Boost invariant free field evolution:

$$E^{\eta}(\tau, k_{\perp}) = E_0^{\eta}(k_{\perp})J_0(k\tau) \qquad \qquad E^{i}(\tau, k_{\perp}) = -i\epsilon^{ij}\frac{k^{j}}{k}B_0^{\eta}(k_{\perp})J_1(k\tau)$$
$$B^{\eta}(\tau, k_{\perp}) = B_0^{\eta}(k_{\perp})J_0(k\tau) \qquad \qquad B^{i}(\tau, k_{\perp}) = -i\epsilon^{ij}\frac{k^{j}}{k}E_0^{\eta}(k_{\perp})J_1(k\tau).$$

P. Guerrero-Rodríguez and T.L., (arXiv:2102.09993 (hep-ph)). See also J. P. Blaizot, T.L. and Y. Mehtar-Tani, Nucl. Phys. A **846** (2010), 63-82; H. Fujii, K. Fukushima and Y. Hidaka, Phys. Rev. C **79** (2009), 024909

Free field time evolution

$$\langle \varepsilon(\tau, \mathbf{x}_{T}) \rangle = \frac{g^{2}}{2} (\delta^{ij} \delta^{kl} + \epsilon^{ij} \epsilon^{kl}) f^{abn} f^{cdn} \int \langle \alpha_{\mathbf{u}_{T}}^{i,a} \alpha_{\mathbf{v}_{T}}^{k,c} \rangle_{1} \langle \alpha_{\mathbf{u}_{T}}^{j,b} \alpha_{\mathbf{v}_{T}}^{l,d} \rangle_{2}$$

$$\times e^{i\mathbf{p}_{T} \cdot (\mathbf{x}_{T} - \mathbf{u}_{T})} e^{i\mathbf{k}_{T} \cdot (\mathbf{x}_{T} - \mathbf{v}_{T})} \left(J_{0}(\mathcal{P}\tau) J_{0}(k\tau) - \frac{\mathbf{p}_{T} \cdot \mathbf{k}_{T}}{\mathcal{P}_{T} k_{T}} J_{1}(\mathcal{P}\tau) J_{1}(k\tau) \right)$$

$$\langle \varepsilon(\tau, \mathbf{x}_{T}) \rangle = [\cdots] \int_{\mathbf{u}_{T}, \mathbf{v}_{T}} [\cdots] \frac{\delta(|\mathbf{x}_{T} - \mathbf{u}_{T}| - \tau)}{2\pi\tau} \frac{\delta(|\mathbf{x}_{T} - \mathbf{v}_{T}| - \tau)}{2\pi\tau} (1 + \cos(\theta_{\mathbf{x}_{T} - \mathbf{u}_{T}} - \theta_{\mathbf{x}_{T} - \mathbf{v}_{T}})).$$

Interpretation: interfering spherical expansing waves from $\mathbf{u}_{T}, \mathbf{v}_{T}$ to \mathbf{x}_{T}

Free field time evolution

For energy density correlator this structure generalizes: Correlators of charges at $\tau = 0 \implies$ correlators at $\tau > 0$





Explicit equation in paper.

(Tehcnical note: glasma graph approximation, WW distributions, work to do still ...)

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Subnucleon scales: IPglasma



Hydro description of pA data: subnucleon structure

- Color field fluctuations
- Hot spots inside nucleon

Bayesian fits of HI data: same conclusion



0.8

0.6

0.4

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Subnucleon scales: hotspot model

S. Demirci, T.L. and S. Schlichting, (arXiv:2101.03791 (hep-ph)) : Analytically tractable dilute-dense model with proton hotspots

Parameters:

- Proton radius R
- Hot spot radius r
- Hot spot number N_q
- ▶ IR cutoff for color fields m
- \implies Energy density correlator

 $\langle \varepsilon(\mathbf{x}_{T}) \varepsilon(\mathbf{y}_{T}) \rangle - \langle \varepsilon(\mathbf{x}_{T}) \rangle \langle \varepsilon(\mathbf{y}_{T}) \rangle$



Hotspot model eccentricities



For reasonable parameter values hot spots dominate eccentricity

Future: constrain from coherent and incoherent exclusive DIS J/Ψ data