Composite Goldstone Dark Matter (and the Higgs)

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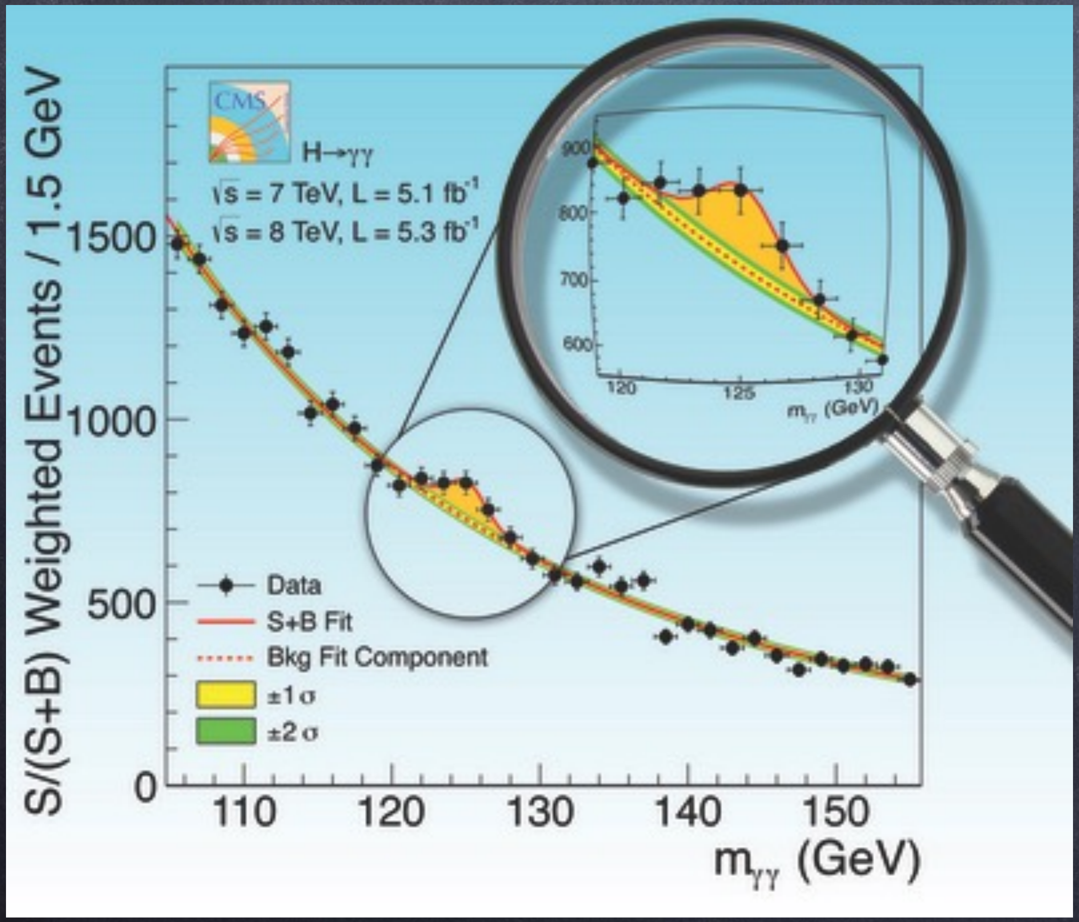
Monsieur le Higgs: enfin!

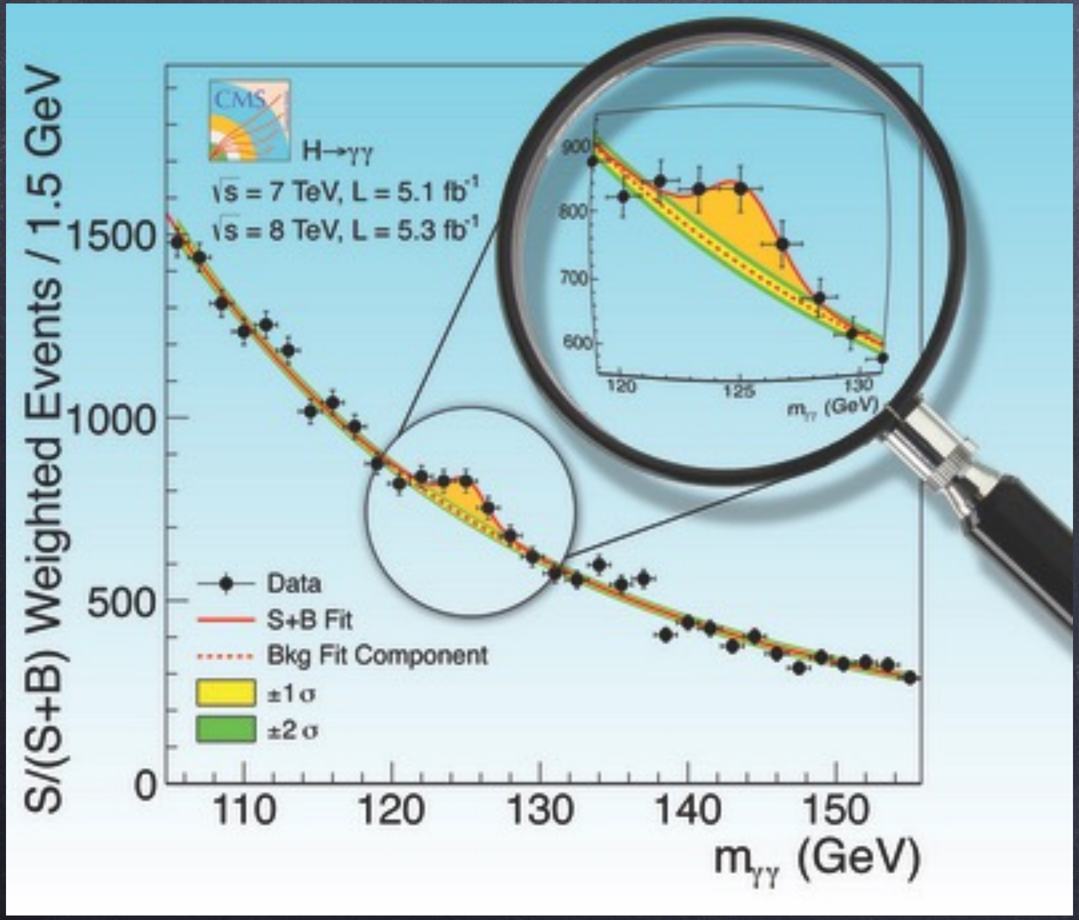
...and the LHC crew is happy!



Monsieur le Higgs: enfin!

For a theorist, this is the beginning of a new era!

We have a new toy,



a new probe in the EW sector!

What do we know about it?

- Theoretical Modelling, i.e. the Standard Model Higgs

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda\, (\phi^\dagger \phi)^2$$

"wrong sign"

It well <u>describes</u> the symmetry breaking, but no dynamical insight!

$$\tau^i = \frac{\sigma^i}{2}$$

Pauli matrices

$$\phi = e^{i\pi^i \tau^i} \cdot \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix}$$

$$v = \frac{\mu}{\sqrt{2\lambda}} \sim 246\ \text{GeV}$$

What do we know about it?

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2\ \phi^\dagger \phi - \lambda\ (\phi^\dagger \phi)^2$$

- Custodial symmetry as a lucky accident:

$$\phi = \begin{pmatrix} \varphi_u \\ \varphi_d \end{pmatrix} \qquad \tilde{\phi} = (i\sigma^2) \cdot \phi^* = \begin{pmatrix} \varphi_d^* \\ -\varphi_u^* \end{pmatrix}$$

Both transform as doublets of SU(2) [pseudo-real irrep]

- We can rewrite the Lagrangian as:

$$\Phi = \left(\tilde{\phi}\ \phi\right) = \begin{pmatrix} \varphi_d^* & \varphi_u \\ -\varphi_u^* & \varphi_d \end{pmatrix} \qquad \mathcal{L}_{\text{Higgs}} = \frac{1}{2}\text{Tr}\left[(D_\mu \Phi)^\dagger (D^\mu \Phi)\right] + \frac{\mu^2}{2}\text{Tr}\left[\Phi^\dagger \Phi\right] + \ldots$$

$$\Phi \to U_L \cdot \Phi \cdot U_R^\dagger$$

uncovers a "hidden" invariance under a global SU(2)L x SU(2)R

What do we know about it?

- Non-linear description:

$$\Sigma = e^{i\pi^i \tau^i} \cdot \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \mathcal{L}_{NL} = f(h)\,(D_\mu \Sigma)^\dagger (D^\mu \Sigma) - V(h)$$

- It correctly describes the symmetry breaking.
- The coupling of h to gauge bosons ARE proportional to the mass (but not determined).
- However: trilinear h coupling is not determined!

Do we still need BSM?

Compositeness is a way to dynamically protect
the Higgs mechanism!

E

No scalars

=

No hierarchy problem!

3 TeV $\Lambda_C \sim 4\pi v_{\rm SM}$

Composite scalars

$v_{\rm SM} \sim 246\ \mathrm{GeV}$

$$\delta m_h^2 \sim \frac{g^2}{16\pi^2} M_{\rm NPh}^2$$

Compositeness scale

The QCD template

Symmetry breaking by compositeness
is an experimentally tested mechanism!

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\langle \bar{q} q \rangle = \langle \bar{q}_R q_L \rangle = (2,2)_{\mathrm{SU(2)_L \times SU(2)_R}}$$

The quark condensate in QCD
breaks the EW symmetry!

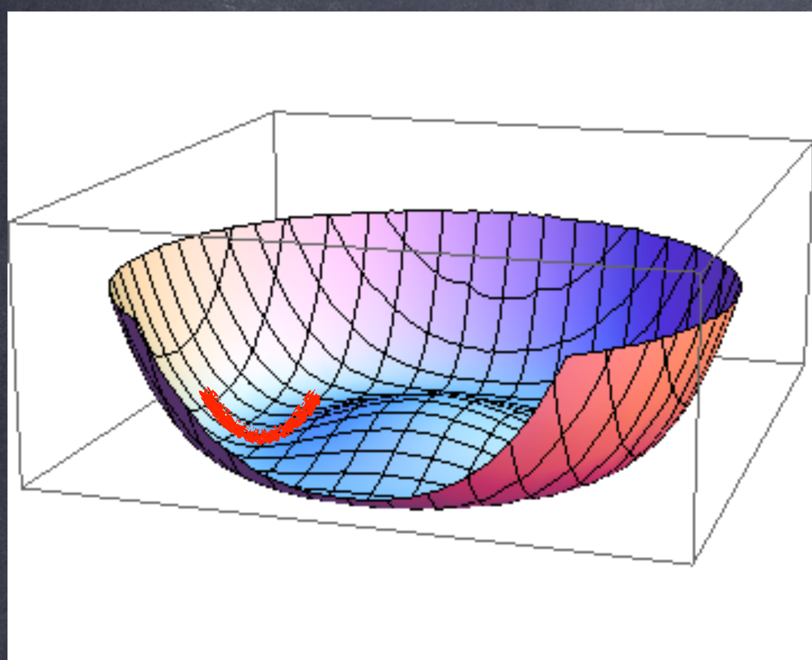
$$m_W = \frac{g f_\pi}{2} \sim 40 \text{ MeV}$$

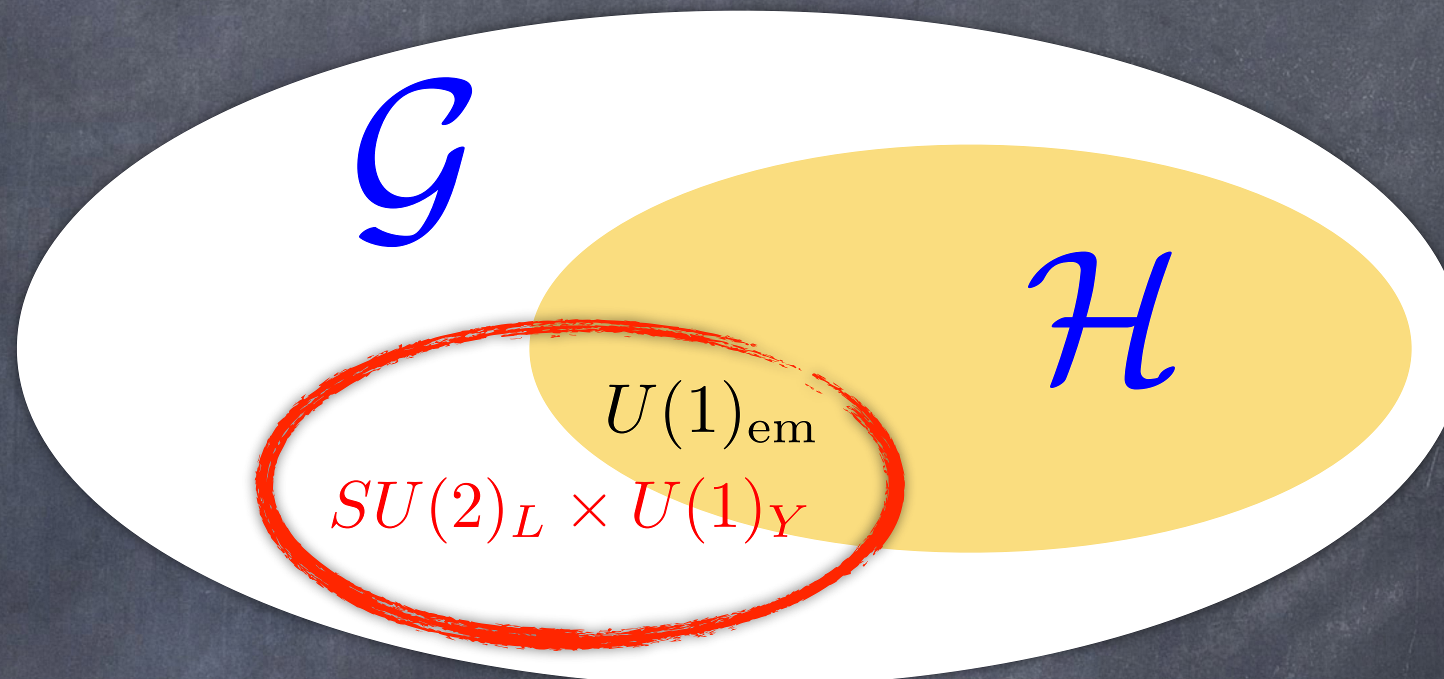
This observation led to the development of
Technicolor in 1979-80.

Note: this ideas is as old as the Standard Model itself!

- "Implication of dynamical symmetry breaking", S.Weinberg, Phys.Rev. D13 (1976) 974
- "Mass without scalars", S.Dimopoulos and L.Susskind, Nucl.Phys. B155 (1979) 237

Compositeness, and the Higgs boson



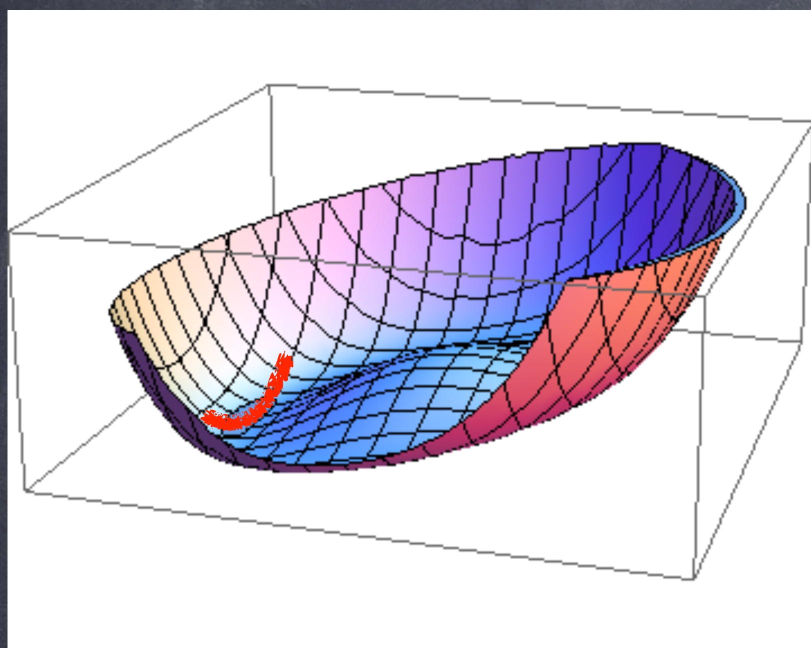


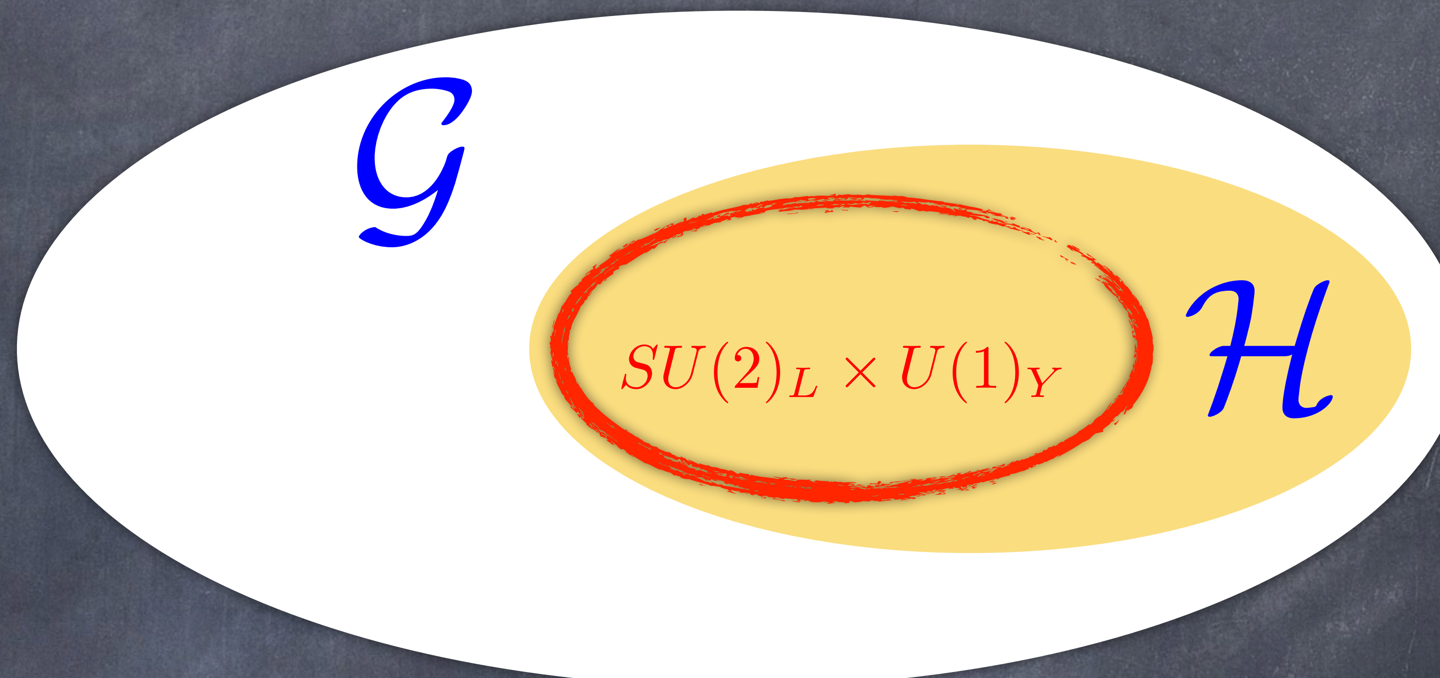
$\mathcal{G} \to \mathcal{H}$

QCD template:

- Goldstones include the longitudinal d.o.f. of W and Z ⟵ π pions
- the Higgs is a heavy bound state (singlet under H) ⟵ σ sigma

Compositeness, and the Higgs boson





$\mathcal{G} \to \mathcal{H}$

QCD template:

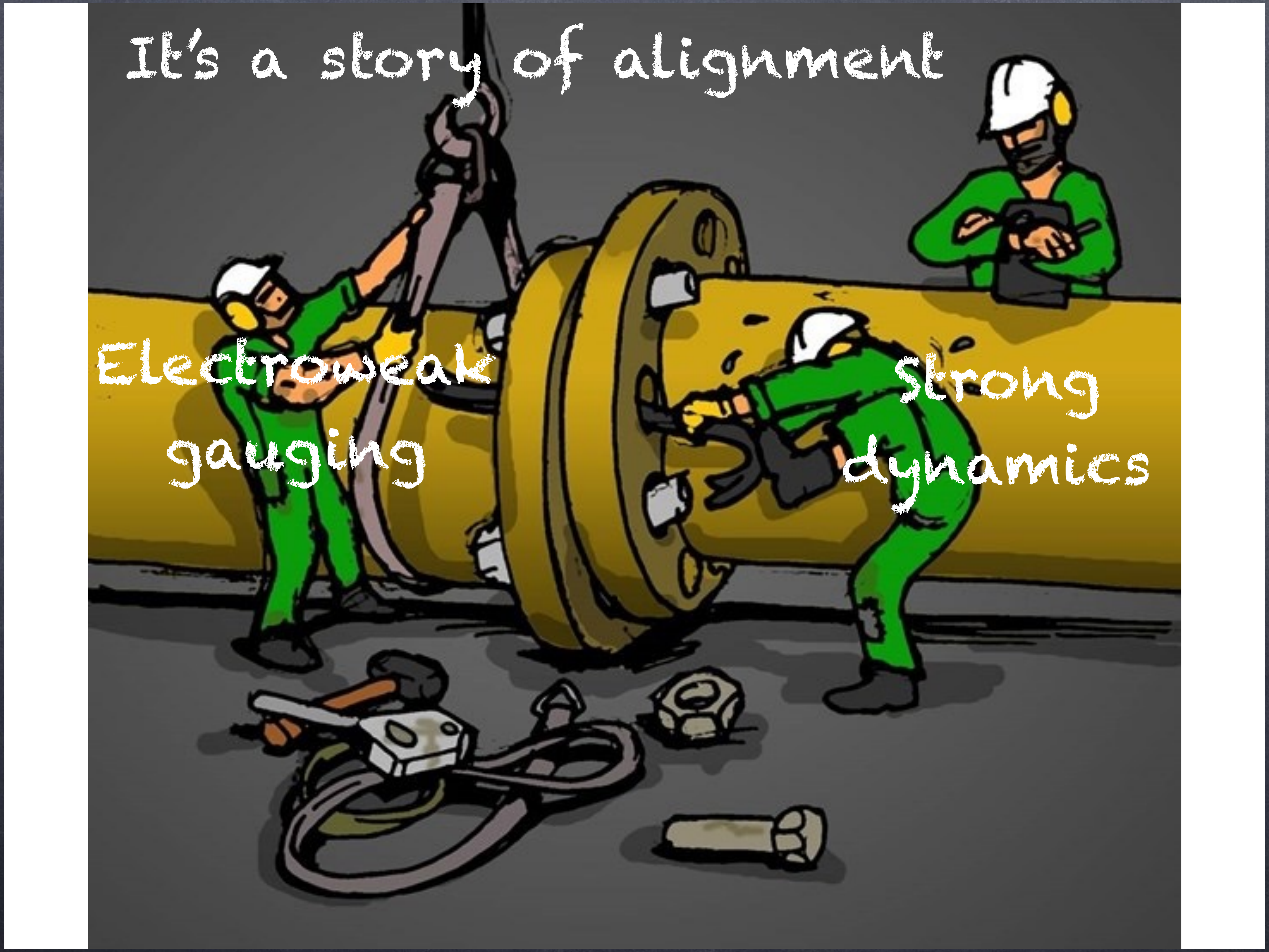
- Goldstones include the longitudinal d.o.f. of W and Z

π pions

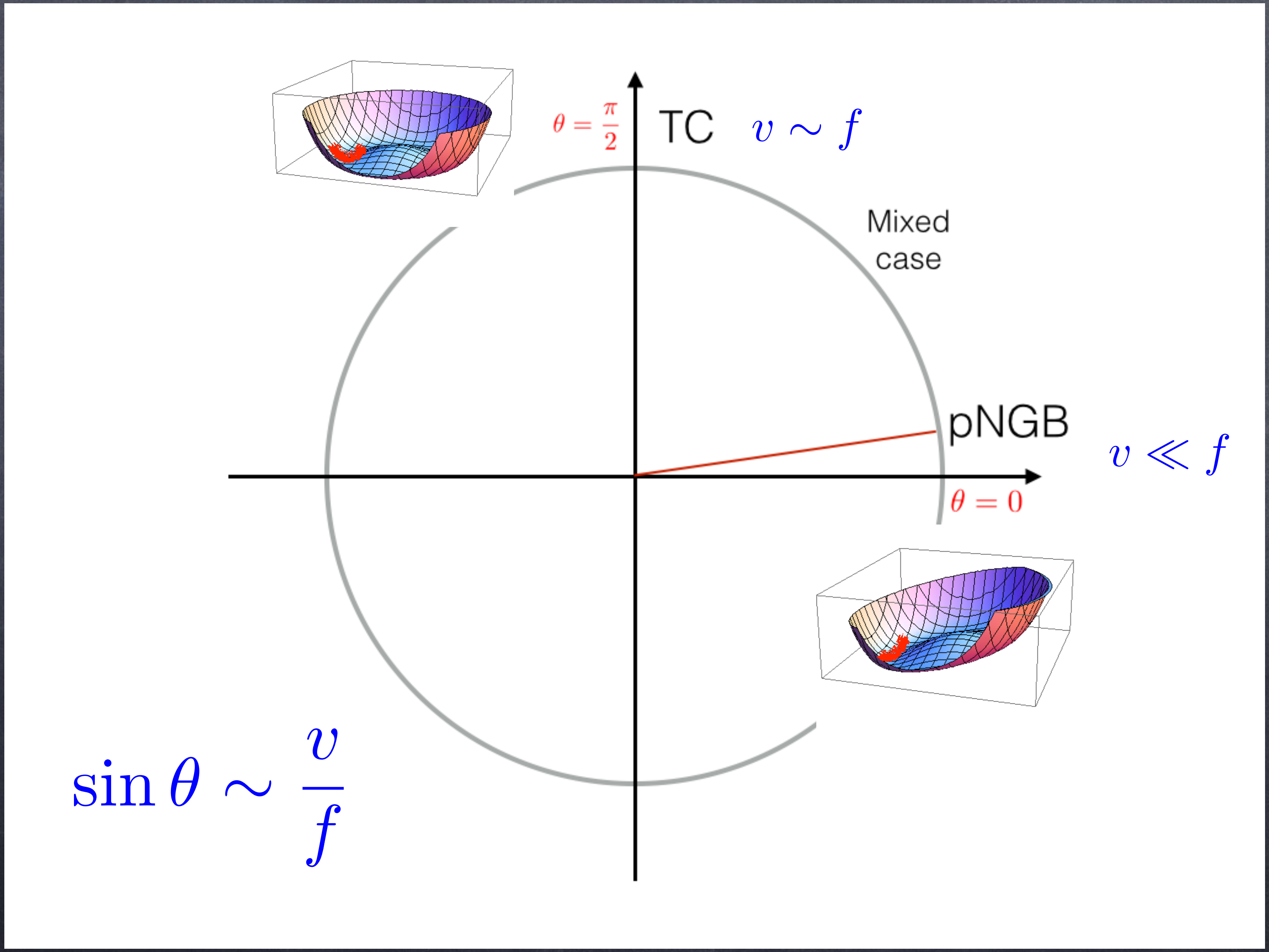
- the Higgs is a pseudo-Goldstone (pNGB)

σ sigma

Compositeness, and the Higgs boson



Compositeness, and the Higgs boson



Compositeness, and the Higgs boson

QCD template:

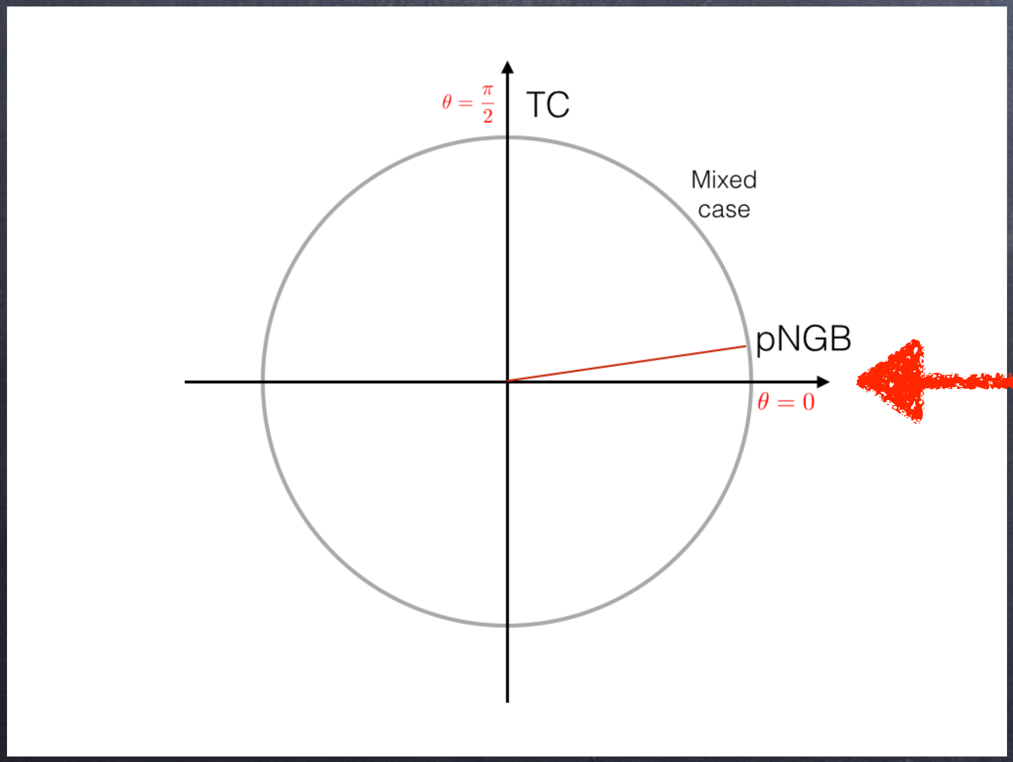
		QCD	TC	pNGB	
			$f = v$	$\frac{v}{f} \sim 0.2$	
	f	130 MeV	246 GeV	1.2 TeV	
pions ->	pNGBs	135 MeV	255 GeV	1.3 TeV	<- the Higgs?
	sigma	500 MeV	950 GeV	4.7 TeV	
	rho	775 MeV	1.5 TeV	7 TeV	
	proton	938 MeV	1.8 TeV	9 TeV	

Anatomy of the potential

Higgs mass in the small theta limit:

$$m_h \sim yf\sin\theta \sim yv_{SM}$$

Naturally in the right ballpark,
without fine tuning!



The Higgs need to become a massless Goldstone to join the other 3 in a full multiplet of the unbroken SU(2)xU(1) symmetry

Higgs:
pNGB vs. sigma

- Mass is param. lighter that the compositeness scale
- The couplings to SM states are naturally close to SM
- Tuning the tilt in the potential!

- Mass is expected to be heavy (close to the rho)
- The couplings to SM states are Unknown!
- No tuning is necessary!

pNGB Composite Higgses: which model?

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU(2)\times SU(2)}}\ (\mathbf{r}_{\mathrm{SU(2)\times U(1)}})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$	[11]
SU(3) $\times$ U(1)	SU(2) $\times$ U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[29, 47, 64]
SU(4)	$[\mathrm{SU}(2)]^2 \times$ U(1)	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	[66]
SO(7)	SO(5) $\times$ U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$	–
Sp(6)	Sp(4) $\times$ SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SU(5)	SU(4) $\times$ U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SO(9)	SO(5) $\times$ SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$	[34]
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[32]
SU(4) $\times$ U(1)	SU(3) $\times$ U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3\cdot\mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3\cdot(\mathbf{1},\mathbf{1})$	[30, 47]
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$	[36]

Table 1: Symmetry breaking patterns $\mathcal{G} \to \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension N_G of the coset, while the fifth contains the representations of the GB's under $\mathcal{H}$ and SO(4) $\cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ (or simply $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ if there is no custodial symmetry). In case of more than two SU(2)'s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.

Bellazzini, Csaki, Serra 1401.2457

The FCD approach

G.C., F.Sannino
1402.0233

- Define a confining gauge group (G_{TC})
- Add in N fermions charged under the confining group G_{TC}
- Assign SM quantum numbers to the fermions (thus providing embedding in the global symmetry)
- Couple them to SM fermions (see tomorrow)

The FCD approach

	G_{TC}	G_F	SM
ψ	R_{TC}	R_F	R_{SM}

R_{TC} is real: G_F = $SU(N_\psi)$ $\quad\langle\psi^i\psi^j\rangle\quad$ $SU(N_\psi)\to SO(N_\psi)$

pseudo-real: G_F = $SU(2N_\psi)$ $\quad\langle\psi^i\psi^j\rangle\quad$ $SU(2N_\psi)\to Sp(2N_\psi)$

complex: G_F = $SU(N_\psi)^2$ $\quad\langle\bar{\psi}^i\psi^j\rangle\quad$ $SU(N_\psi)^2\to SU(N_\psi)$

The FCD approach

coset	GTC	TF	Higgs doublets	pNGBs	
SU(4)/Sp(4)	Sp(2N)	fund	1	5	Minimal!
SU(5)/SO(5)	SU(4)	6	1	14	Dugan, Georgi, Kaplan 1985!!!
SU(4)xSU(4)/SU(4)	SU(N)	fund	2	15	G.C., T.Ma 1508.07014
SU(6)/Sp(6)	Sp(2N)	fund	2	14	G.C., M.Lespinasse in prep.

A minimal case

T.Ryttov, F.Sannino 0809.0713
Galloway, Evans, Luty, Tacchi 1001.1361

$$G_{\mathrm{TC}} = SU(2) \qquad \begin{pmatrix} U \\ D \end{pmatrix}$$

2 Dirac doublets

$$\psi^1 = U_L \quad \psi^2 = D_L \quad \psi^3 = (i\sigma^2)_{\mathrm{TC}} U_R^C \quad \psi^4 = (i\sigma^2)_{\mathrm{TC}} D_R^C$$

	$SU(2)_{\mathrm{TC}}$	$SU(4)_{\psi}$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$	$\square$	$\square$	**2**	0
ψ^3	$\square$		**1**	-1/2
ψ^4	$\square$		**1**	1/2

$\Big\}$ $SU(2)_R$ doublet

A minimal case

T.Ryttov, F.Sannino 0809.0713
Galloway, Evans, Luty, Tacchi 1001.1361

	$SU(2)_{\rm TC}$	$SU(4)_\psi$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix}$	$\square$	$\square$	**2**	0
ψ^3	$\square$		**1**	-1/2
ψ^4	$\square$		**1**	1/2

The EW symmetry
is embedded in the global
flavour symmetry
SU(4) !

Generators of SU(4) corresponding to $SU(2)_L \times SU(2)_R$

$$S^{1,2,3} = \frac{1}{2}\begin{pmatrix}\sigma_i & 0\\ 0 & 0\end{pmatrix}, \qquad S^{4,5,6} = \frac{1}{2}\begin{pmatrix}0 & 0\\ 0 & -\sigma_i^T\end{pmatrix},$$

A minimal case

T.Ryttov, F.Sannino 0809.0713
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	$SU(2)_{\mathrm{TC}}$	$SU(4)_{\psi}$	$SU(2)_L$	$U(1)_Y$
$\begin{pmatrix}\psi^1 \\ \psi^2\end{pmatrix}$	$\square$	$\square$	**2**	0
ψ^3	$\square$		**1**	-1/2
ψ^4	$\square$		**1**	1/2

This theory is
Asymptotic Free
and confines in the IR!

Antisymmetric matrix

$$\langle \psi^i \psi^j \rangle = \Sigma_0$$

Under the global symmetry SU(4):

$$\Sigma_0 \to U \cdot \Sigma_0 \cdot U^T$$

Un-broken subgroup defined by:

$$S \cdot \Sigma_0 + \Sigma_0 \cdot S^T = 0$$

SU(4) -> Sp(4) !!!

A minimal case

Anti-symmetric

$$\langle \psi^i \psi^j \rangle = 6_{\mathrm{SU}(4)} \to 5_{\mathrm{Sp}(4)} \oplus 1_{\mathrm{Sp}(4)}$$

Sp(4) ~ SO(5) contains a SO(4) subgroup:
identify with custodial symmetry!

Pions: $5_{\mathrm{Sp}(4)} \to (2,2) \oplus (1,1)$

$$\Sigma_0 = \begin{pmatrix} (i\sigma^2) & 0 \\ 0 & -(i\sigma^2) \end{pmatrix}$$

Preserves the EW generators.

A minimal case

Broken SU(4) generators

$$X^1 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \quad X^2 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad X^3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix},$$
$$X^4 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad X^5 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$X^1,\ X^2,\ X^3,\ X^4$ Higgs doublet X^5 singlet

$$\Sigma = e^{\frac{i}{2f}\sum_i X^i \pi^i} \cdot \Sigma_0 \cdot e^{\frac{i}{2f}\sum_i X^{i\,T} \pi^i} = U \cdot \Sigma_0 \cdot U^T = U^2 \cdot \Sigma_0$$

Let's give a VEV to the Higgs:

$\langle \pi^4 \rangle = v$ $\qquad \Sigma_0' = e^{i\frac{v}{f}X^4} \cdot \Sigma_0$ $\qquad$ New EW breaking vacuum

A minimal case

$$e^{i\frac{v}{f}X^4} = \left(\cos\frac{v}{2\sqrt{2}f} + i2\sqrt{2}X^4\sin\frac{v}{2\sqrt{2}f}\right)$$

$$= \left(\cos\theta + i2\sqrt{2}X^4\sin\theta\right) \qquad \theta = \frac{v}{2\sqrt{2}f}$$

Defines a rotation in the SU(4) space! To study the theory in the new vacuum, it is enough to apply this rotation to the strong sector! $\longrightarrow$ Spurions, however, are not rotated.

$\downarrow$

Mis-alignment!

A minimal case

In the Unitary gauge:

$$\Sigma = e^{\frac{i}{f}(hY^4+\eta Y^5)} \cdot \Sigma_0 = \left[\cos\frac{x}{f}\,1 + \frac{i}{x}\sin\frac{x}{f}\;(hY^4 + \eta Y^5)\right]\cdot \Sigma_0' \,,$$

broken gen. in new vacuum

Chiral Lagrangian:

$$\begin{aligned} f^2\,\mathrm{Tr}(D_\mu\Sigma)^\dagger D^\mu\Sigma \;=\; & \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu\eta)^2 \\ & + \frac{1}{48f^2}\left[-(h\partial_\mu\eta - \eta\partial_\mu h)^2\right] + \mathcal{O}(f^{-3}) \\ & + \left(2g^2W^+_\mu W^{-\mu} + (g^2+g'^2)Z_\mu Z^\mu\right)\left[f^2 s_\theta^2 + \frac{s_{2\theta}f}{2\sqrt{2}}h\left(1-\frac{1}{12f^2}(h^2+\eta^2)\right)\right. \\ & \left. + \frac{1}{8}(c_{2\theta}h^2 - s_\theta^2\eta^2)\left(1-\frac{1}{24f^2}(h^2+\eta^2)\right) + \mathcal{O}(f^{-3})\right] \,. \end{aligned} \tag{25}$$

A minimal case

$$
\begin{aligned}
f^2\,\mathrm{Tr}(D_\mu\Sigma)^\dagger D^\mu\Sigma \;=\;& \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \eta)^2 \\
&+\frac{1}{48f^2}\left[-(h\partial_\mu\eta - \eta\partial_\mu h)^2\right] + \mathcal{O}(f^{-3}) \\
&+\underline{\left(2g^2 W_\mu^+ W^{-\mu} + (g^2+g'^2)Z_\mu Z^\mu\right)}\left[f^2 s_\theta^2 + \frac{s_{2\theta} f}{2\sqrt{2}} h\left(1-\frac{1}{12f^2}(h^2+\eta^2)\right)\right. \\
&\left.+\frac{1}{8}(c_{2\theta}h^2 - s_\theta^2\eta^2)\left(1-\frac{1}{24f^2}(h^2+\eta^2)\right) + \mathcal{O}(f^{-3})\right]\,. \qquad (25)
\end{aligned}
$$

$$m_W^2 = 2g^2 f^2 \sin^2\theta = \frac{g^2 v^2}{4}$$

$$g_{hWW} = \sqrt{2}g^2 f \sin\theta\cos\theta = g m_W \;\cos\theta$$

- Decoupling: SM approached for small theta
- Deviations w.r.t. SM predictions due to theta!
- There are no linear couplings of the singlet!

A minimal case

Frigerio, Pomarol, Riva, Urbano
1204.2808

$$
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&+\left(2g^2W^+_\mu W^{-\mu}+(g^2+g'^2)Z_\mu Z^\mu\right)\Bigg[f^2 s_\theta^2+\frac{s_{2\theta}f}{2\sqrt{2}}h\left(1-\frac{1}{12f^2}(h^2+\eta^2)\right)\\
&+\frac{1}{8}(c_{2\theta}h^2-s_\theta^2\eta^2)\left(1-\frac{1}{24f^2}(h^2+\eta^2)\right)+\mathcal{O}(f^{-3})\Bigg]\,. \qquad (25)
\end{aligned}
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No linear couplings in the chiral Lagrangian:
Tempting to think of the singlet as DM!

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No linear couplings in the chiral Lagrangian:
Tempting to think of the singlet as DM!

The DM candidate has derivative couplings with the Higgs boson: weakened DD constraints!

Balkin, Ruhdorfer, Salvioni, Weiler
1809.09106

A minimal case

Frigerio, Pomarol, Riva, Urbano
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&+\frac{1}{48 f^2}\left[\underline{-(h\partial_\mu\eta-\eta\partial_\mu h)^2}\right]+\mathcal{O}(f^{-3}) \\
&+\left(2g^2W^+_\mu W^{-\mu}+(g^2+g'^2)Z_\mu Z^\mu\right)\left[\cancel{f^2 s^2_\theta}+\cancel{\frac{s_{2\theta} f}{2\sqrt{2}}}h\left(1-\frac{1}{12f^2}(h^2+\eta^2)\right)\right. \\
&\left.+\frac{1}{8}(\overset{=1}{c_{2\theta}}h^2-\cancel{s^2_\theta\eta^2})\left(1-\frac{1}{24f^2}(h^2+\eta^2)\right)+\mathcal{O}(f^{-3})\right]\,. \qquad (25)
\end{aligned}
$$

$$h \to v + H$$

No linear couplings in the chiral Lagrangian: Tempting to think of the singlet as DM!

The DM candidate has derivative couplings with the Higgs boson: weakened DD constraints!

Basis dependent!!!

Balkin, Ruhdorfer, Salvioni, Weiler
1809.09106

A minimal case

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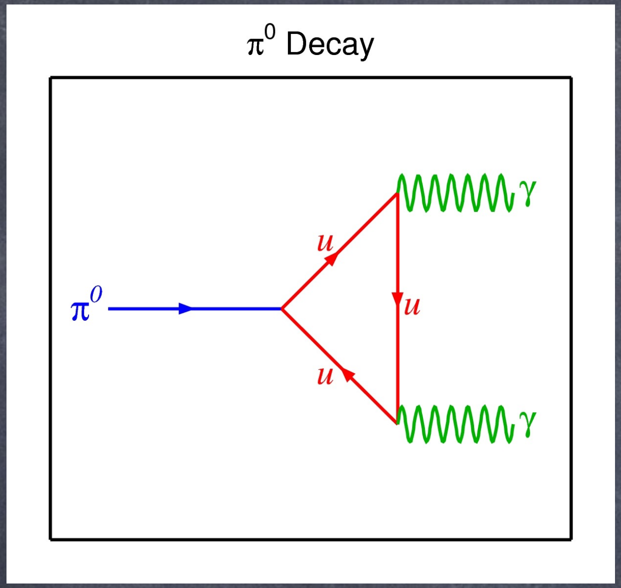
\end{aligned}
$$

No linear couplings in the chiral Lagrangian:
Tempting to think of the singlet as DM!

Defining the Higgs h in the misaligned basis,
no linear derivative couplings of the DM candidate
to the Higgs exist!

WZW matters!



In QCD, coupling of the pions to EW gauge bosons are generated by (global) anomalies!

$$\mathcal{L}_{WZW} = \frac{d_\psi}{64\pi^2}\frac{\eta}{f}\left(g^2 W_{\mu\nu}\tilde{W}^{\mu\nu} - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$$

$d_\psi = 2$

Dimension of TC rep

- Predictive power!
- Coupling to 2 photons vanishes!

A minimal case

TC limit: $\theta = \frac{\pi}{2}$

$$
\begin{aligned}
f^2\ \mathrm{Tr}(D_\mu\Sigma)^\dagger D^\mu\Sigma \ = \ & \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \eta)^2 \\
& + \frac{1}{48 f^2}\left[-(h\partial_\mu\eta - \eta\partial_\mu h)^2\right] + \mathcal{O}(f^{-3}) \\
& + \left(2g^2 W_\mu^+ W^{-\mu} + (g^2+g'^2) Z_\mu Z^\mu\right)\Bigg[f^2 s_\theta^2 + \cancel{\frac{s_{2\theta} f}{2\sqrt{2}} h \left(1 - \frac{1}{12 f^2}(h^2+\eta^2)\right)} \\
& + \frac{1}{8}\left(\overset{-1}{c_{2\theta}} h^2 - \overset{1}{s_\theta^2}\eta^2\right)\left(1 - \frac{1}{24 f^2}(h^2+\eta^2)\right) + \mathcal{O}(f^{-3})\Bigg] \ . \qquad (25)
\end{aligned}
$$

$$
\mathcal{L}_{\mathrm{WZW}} = \cancel{\frac{d_\psi \cos\theta}{64\pi^2}\frac{\eta}{f}}\left(g^2\ W_{\mu\nu}\tilde{W}^{\mu\nu} - g'^2\ B_{\mu\nu}\tilde{B}^{\mu\nu}\right)
$$

Rhyttov, Sannino
0809.0713

In the TC limit, Sp(4) ⊂ $U(1)_{em} \times U(1)_{DM}$

$\phi = \frac{h + i\eta}{\sqrt{2}}$ is charged under the unbroken $U(1)_{DM}$, and thus stable (TIMP).

The potential

$$V = -C_t {y'_t}^2 f^4 \sin^2\theta - 4C_m f^4 \cos\theta$$

$$\frac{\partial}{\partial\theta} V = 2(-C_t {y'_t}^2 \cos\theta + 2C_m) f^4 \sin\theta$$

$$\cos\theta = \frac{2C_m}{C_t {y'_t}^2} \sim 1$$

Fine tuning!

No tuning!

$$m_h^2 = -\frac{C_t {y'_t}^2 f^2}{4} \cos 2\theta + \frac{C_m f^2}{2} \cos\theta = \frac{C_t {y'_t}^2 f^2}{4} \sin^2\theta = \frac{C_t m_{\rm top}^2}{4}$$

$$C_t \sim 2$$

gives the correct Higgs mass!

$$m_\eta = \frac{m_h}{\sin\theta}$$

Composite Dark Matter

- Some pNGBs may be stable due to residual unbroken global symmetries
- Stable techni-baryons may give rise to asymmetric DM

S.Nussinov
Phys.Lett. B165, 55 (1985)

A composite 2HDM

$SU(3)_{\mathrm{HC}}$

G.C., T.Ma
1508.07014

	SU(N)	SU(2)$_L$	U(1)$_Y$
$\psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	$\square$	**2**	0
$\psi_R = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$	$\square$	**1** **1**	1/2 -1/2

$$SU(4) \times SU(4) \to SU(4)$$

Triplet

Complex bi-doublet
(2HDM)

$$\Pi = \frac{1}{2} \begin{pmatrix} \sigma_i \Delta^i + s/\sqrt{2} & -i\Phi_H \\ i\Phi_H^\dagger & \sigma_i N^i - s/\sqrt{2} \end{pmatrix}$$

SU(2)R Triplet

A composite 2HDM

$SU(3)_{\rm HC}$

G.C., T.Ma
1508.07014

Is it there a parity stabilising the pions?

$$\Sigma = e^{\frac{i}{f}\Pi} \qquad \Sigma \to P \cdot \Sigma^T \cdot P \qquad P = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$$

$$\left.\begin{array}{l} s \to s \\ H_1 \to H_1 \end{array}\right\}$$ Mimics the minimal case

$$\left.\begin{array}{l} H_2 \to -H_2 \\ \Delta \to -\Delta \\ N \to -N \end{array}\right\}$$ Dark Sector!

A composite 2HDM

G.C., T.Ma
1508.07014

$$\Pi = \frac{1}{2}\begin{pmatrix} \sigma_i\Delta^i + s/\sqrt{2} & -i\Phi_H \\ i\Phi_H^\dagger & \sigma_i N^i - s/\sqrt{2} \end{pmatrix}$$

$$\langle\Phi_H\rangle = \langle H_1 + iH_2\rangle = \begin{pmatrix} ve^{i\beta} & 0 \\ 0 & ve^{i\beta} \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \cos\theta\ \mathbf{1} & e^{i\beta}\sin\theta\ \mathbf{1} \\ -e^{i\beta}\sin\theta\ \mathbf{1} & \cos\theta\ \mathbf{1} \end{pmatrix}$$

Beta can be removed by an SU(4) rotation:

$$\Omega_\beta = \mathrm{Exp}\left[-i\frac{\beta}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right] = \begin{pmatrix} e^{-i\beta/2} & 0 \\ 0 & e^{i\beta/2} \end{pmatrix}$$

Beta = relative phase of the two T-quarks!

A composite 2HDM

G.C., T.Ma
1508.07014

$$\mathcal{L}_{\rm Yuk} = -f\,(\bar{q}_L^\alpha t_R)\,\left[\mathrm{Tr}[P_{1,\alpha}(y_{t1}\Sigma + y_{t2}\Sigma^\dagger)] + (i\sigma_2)_{\alpha\beta}\mathrm{Tr}[P_2^\beta(y_{t3}\Sigma + y_{t4}\Sigma^\dagger)]\right] + h.c.$$

4 "Yukawa" couplings!

$$Y_t = \frac{y_{t1} - y_{t2} - (y_{t3} - y_{t4})}{2\sqrt{2}}, \quad Y_D = \frac{y_{t1} - y_{t2} + (y_{t3} - y_{t4})}{2\sqrt{2}},$$
$$Y_T = \frac{y_{t1} + y_{t2} + (y_{t3} + y_{t4})}{2\sqrt{2}}, \quad Y_0 = \frac{y_{t1} + y_{t2} - (y_{t3} + y_{t4})}{2\sqrt{2}}.$$

$$V_{\rm top}(\theta) = -C_t f^4 \left[8|Y_t|^2\sin^2\theta + \right.$$

Potential for theta

$$2\sqrt{2}|Y_t|^2\sin(2\theta)\,\frac{h_1}{f} +$$

Set to zero by phase-shift

$$+4\sqrt{2}\,\mathrm{Im}(Y_D^* Y_t)\sin\theta\,\frac{h_2}{f}$$

Custodial violating VEVs!!!

$$+2\sqrt{2}\,\mathrm{Re}(Y_D^* Y_t)\sin(2\theta)\,\frac{A_0}{f}$$

$$\left. + 4\,\mathrm{Im}(Y_T^* Y_t)\sin^2\theta\,\frac{N_0 + \Delta_0}{f} + \ldots\right]$$

A composite 2HDM

G.C., T.Ma
1508.07014

$$\mathcal{L}_{\rm Yuk} = -f\,(\bar{q}_L^{\alpha} t_R)\,\Big[{\rm Tr}[P_{1,\alpha}(\underline{y_{t1}}\Sigma + \underline{y_{t2}}\Sigma^\dagger)] + (i\sigma_2)_{\alpha\beta}{\rm Tr}[P_2^{\beta}(\underline{y_{t3}}\Sigma + \underline{y_{t4}}\Sigma^\dagger)]\Big] + h.c.$$

4 "Yukawa" couplings!

$$Y_t = \frac{y_{t1} - y_{t2} - (y_{t3} - y_{t4})}{2\sqrt{2}}\,, \qquad Y_D = \frac{y_{t1} - y_{t2} + (y_{t3} - y_{t4})}{2\sqrt{2}}\,,$$
$$Y_T = \frac{y_{t1} + y_{t2} + (y_{t3} + y_{t4})}{2\sqrt{2}}\,, \qquad Y_0 = \frac{y_{t1} + y_{t2} - (y_{t3} + y_{t4})}{2\sqrt{2}}\,.$$

$$V_{\rm top}(\theta) = -C_t f^4 \Big[8|Y_t|^2 \sin^2\theta +$$

Potential for theta

$$2\sqrt{2}|Y_t|^2 \sin(2\theta)\,\frac{h_1}{f} +$$

DM parity!

Set to zero by phase-shift

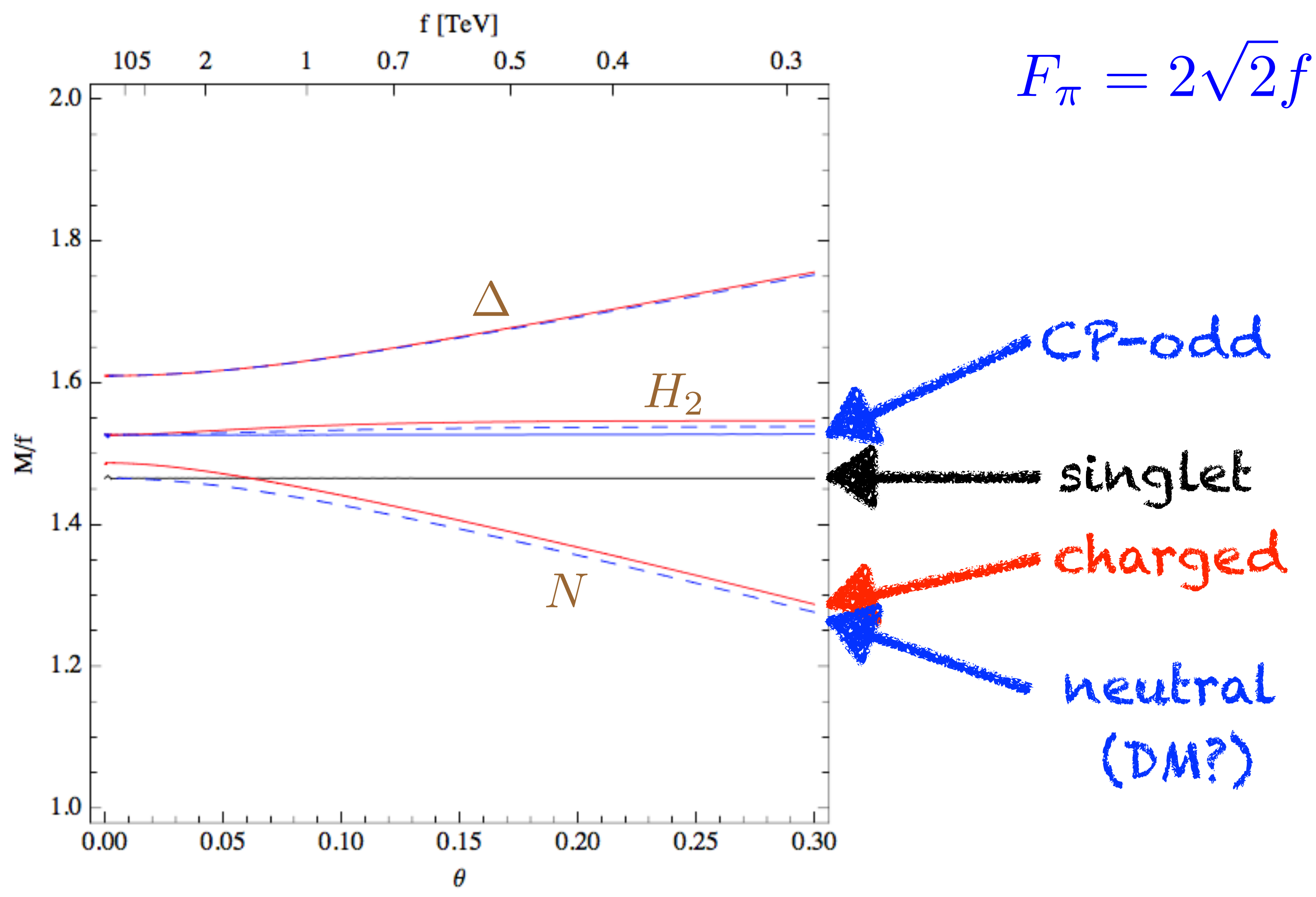
$$+4\sqrt{2}\,{\rm Im}(\cancel{Y_D^*}Y_t)\sin\theta\,\frac{h_2}{f}$$

Custodial violating VEVs!!!

$$+2\sqrt{2}\,{\rm Re}(\cancel{Y_D^*}Y_t)\sin(2\theta)\,\frac{A_0}{f}$$

$$+\,4\,{\rm Im}(\cancel{Y_T^*}Y_t)\sin^2\theta\,\frac{N_0+\Delta_0}{f} + \ldots \Big]$$

A composite 2HDM: spectrum

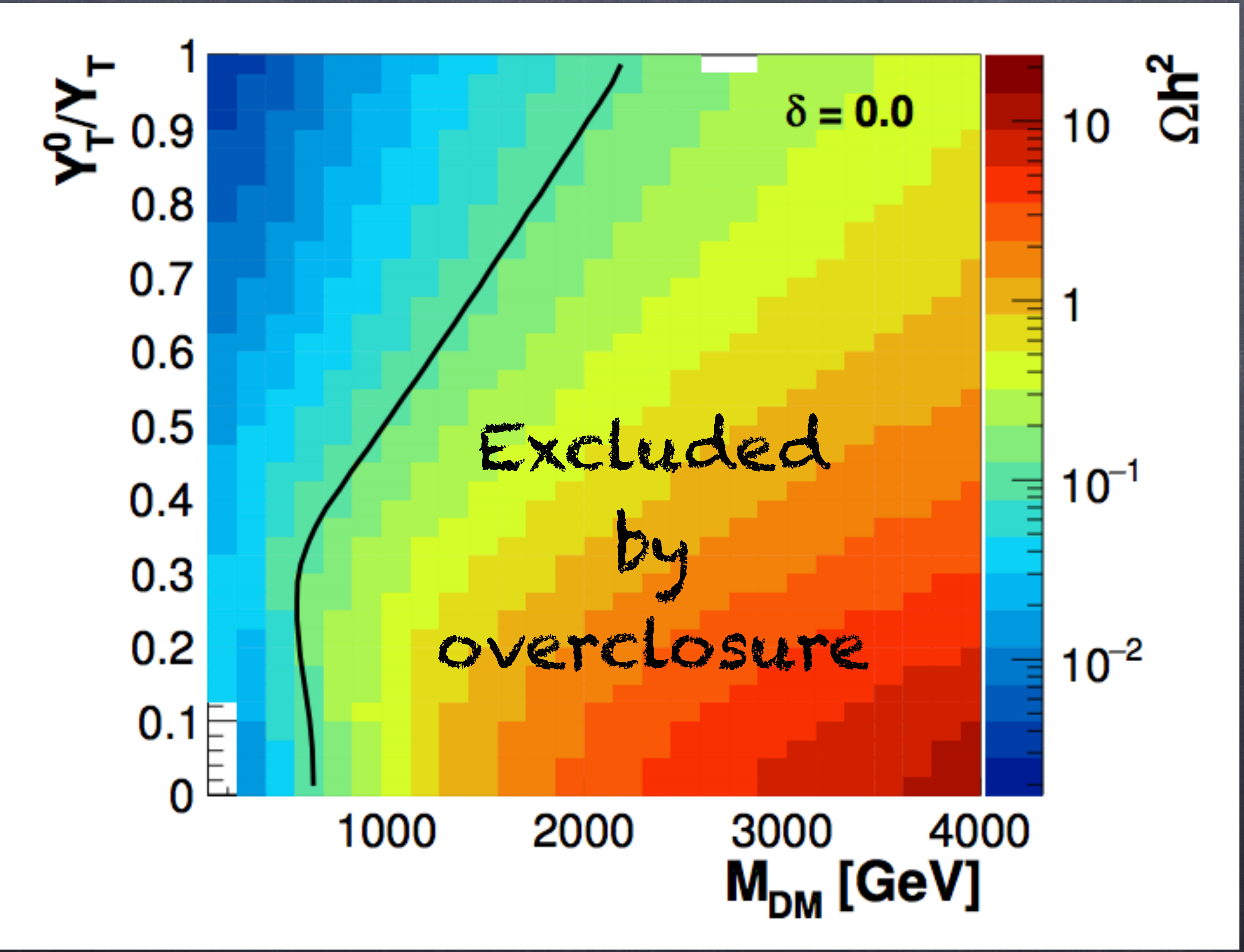


$$F_\pi = 2\sqrt{2}f$$

A composite 2HDM: Dark-Matter

G.C., T.Ma, Y.Wu, B.Zhang
1703.06903

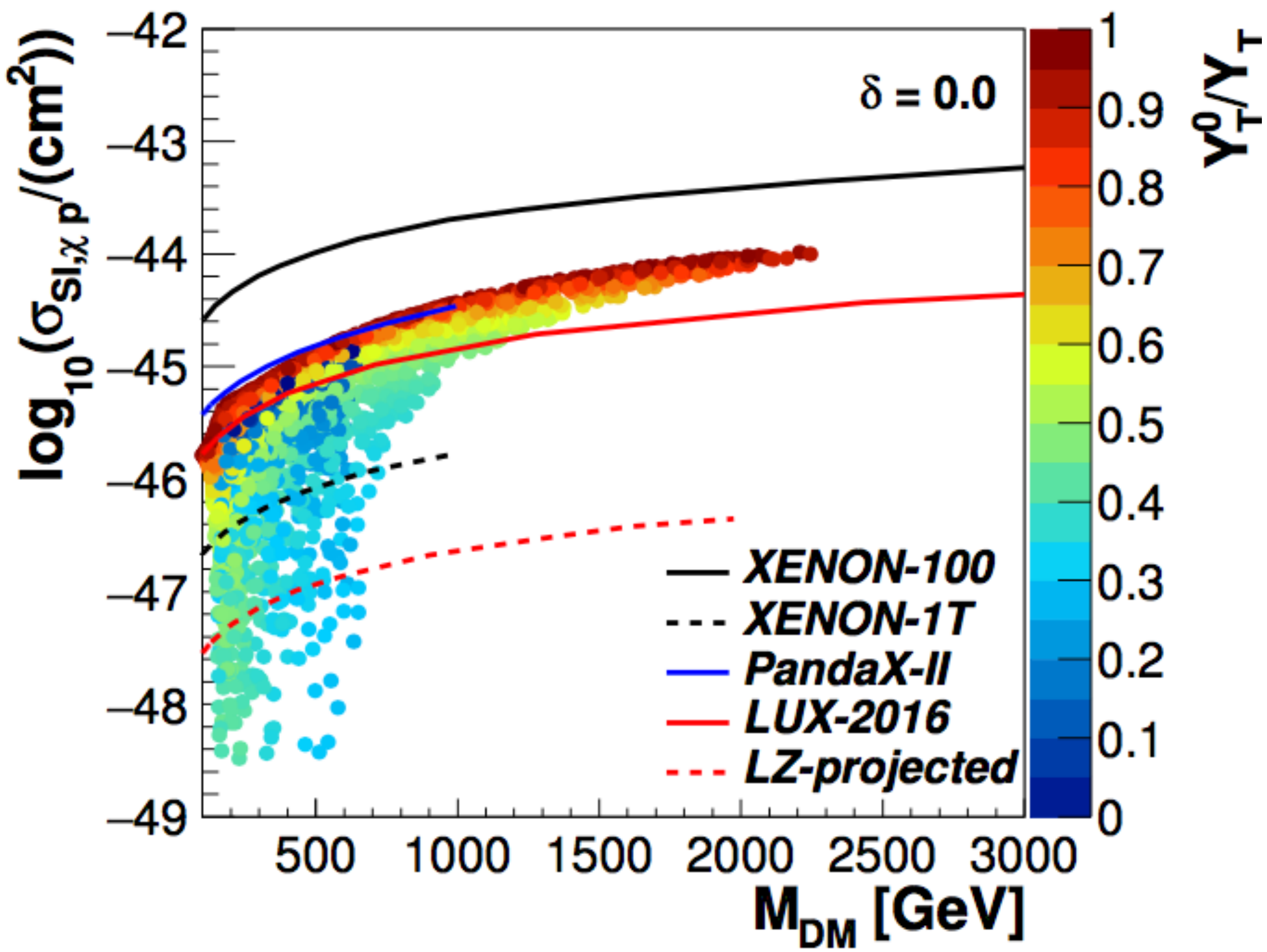
Relic abundance:



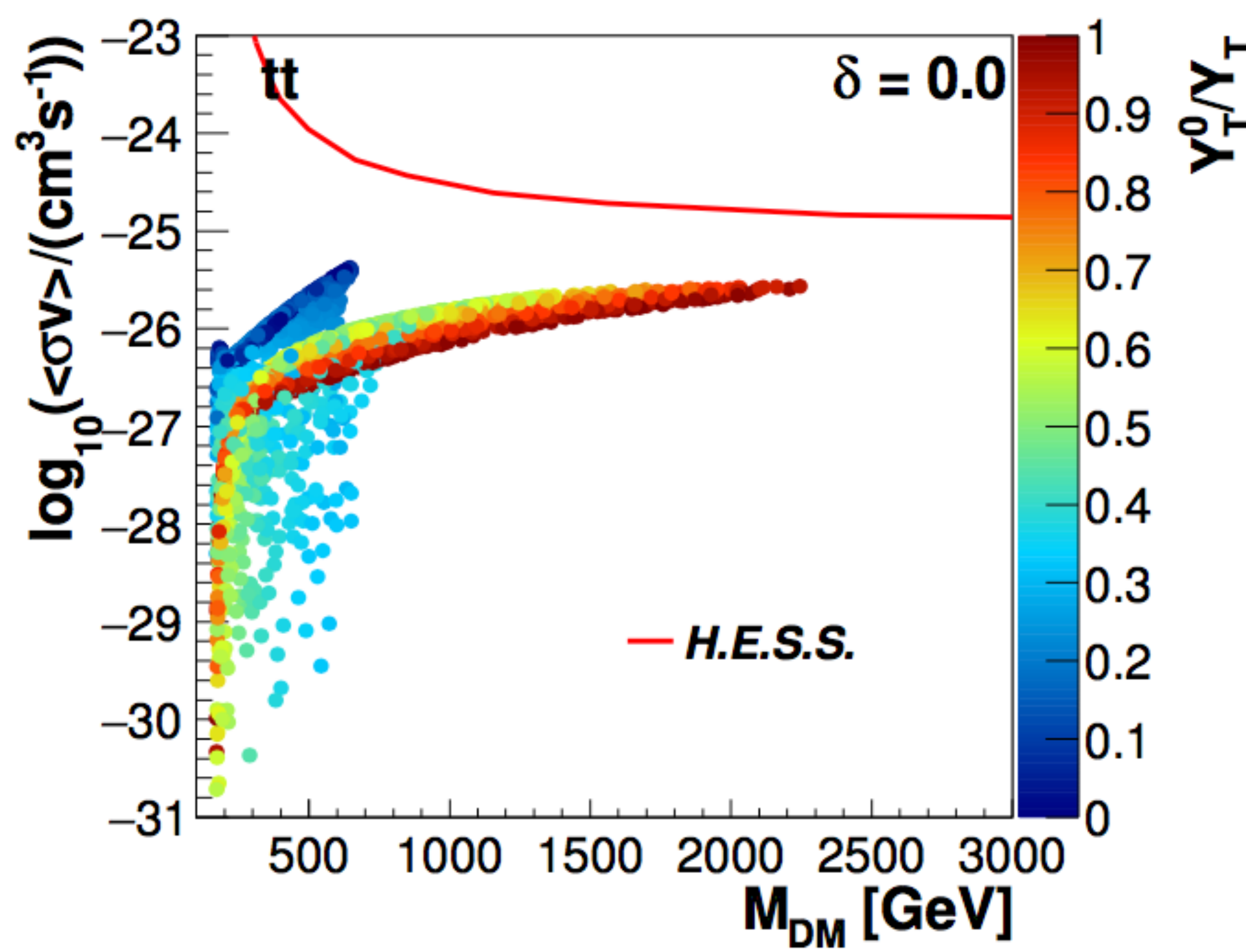
A composite 2HDM: Dark-Matter

G.C., T.Ma, Y.Wu, B.Zhang
1703.06903

Direct Detection

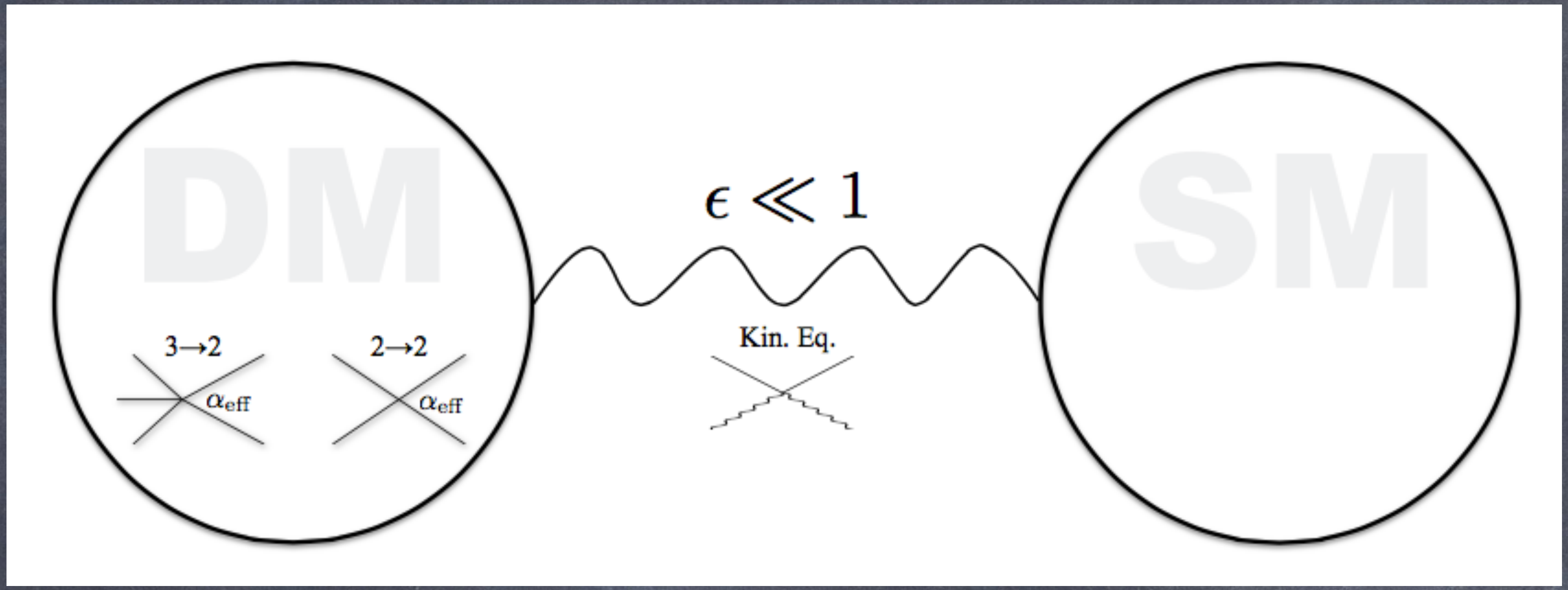


Indirect Detection



The SIMP miracle

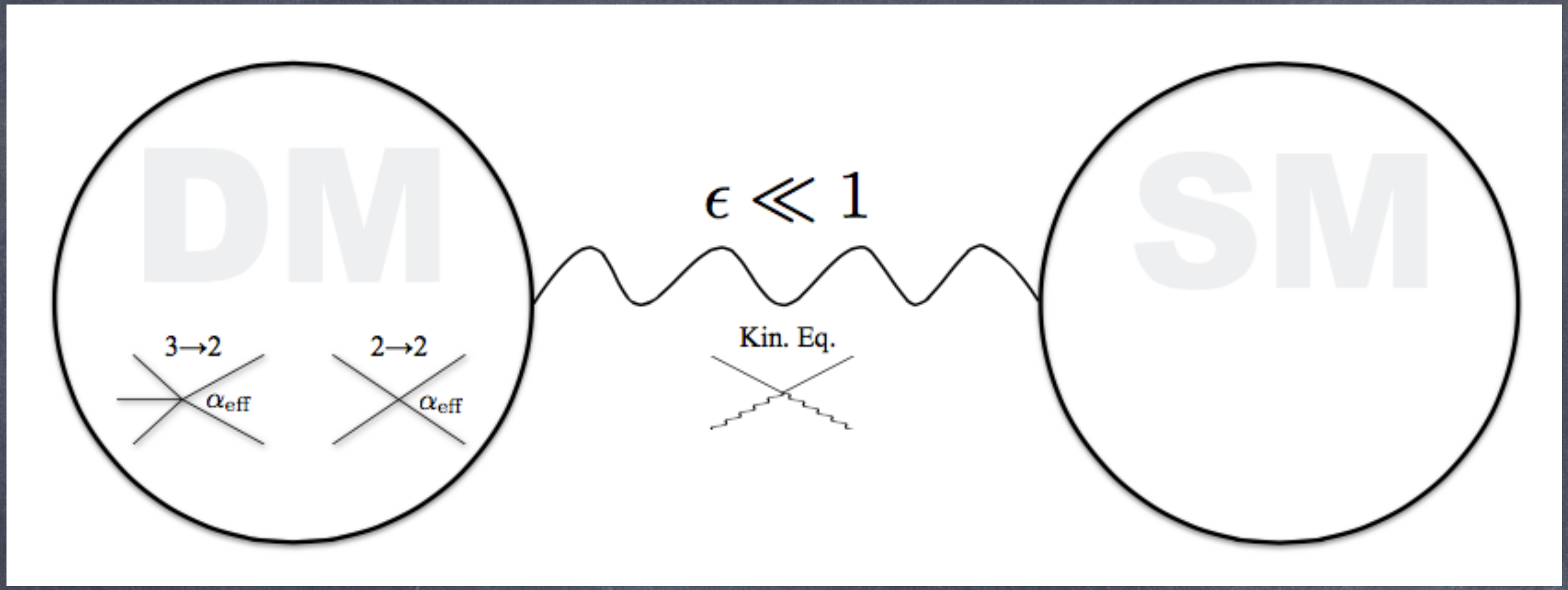
Hochberg, Kuflik, Volanski, Wacker
1402.5143



- 3 -> 2 drives freeze-out, while epsilons keeps equilibrium with the SM
- 2 -> 2 provides self-interactions to explain struture formation

The SIMP miracle

Hochberg, Kuflik, Volanski, Wacker
1402.5143



Hochberg, Kuflik, Murayama, Volanski, Wacker 1411.3727

- Natural set-up: composite theories!
- 3 -> 2 from WZW anomaly interactions
- 2 -> 2 is normal pion scattering

SU(2) with
2 Dirac doublets!

Not-so SIMPle miracle

Hansen, Langæble, Sannino
1507.514301590

- 2 -> 2 is a LO process, while 3 -> 2 (WZW) NLO!
- Consistent NLO chiral expansion needed.

NLO band from counter-terms

Perturbative upper bound

Relic density

Upper bound from bullet cluster