Phase Transitions and stochastic gravitational waves backgrounds

Lund school 2021

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Plan of the lesson

review of the Standard Model and open problems

First order phase transitions in the early Universe

electroweak phase transition

dark phase transitions

Very high energy phase transitions

QCD phase transitions, mention

The main aim to this lecture is too provide a general picture of the topic.
In one hour I cannot pretend to provide a complete lecture on the subject but I hope I will provide you a general idea and understanding
for future deeper studies

References and bibliography

“Lectures on Landau theory of Phase Transitions”
P. Ousted
https://site.physics.georgetown.edu/~pdo7/ps_files/landau.pdf

Caprini et al, arXiv:1007.1218, arXiv:0901.1661, astro-ph/0603476

A. Addazi, A. Marciano, R. Pasechnick et al, arXiv:1607.08057 , arXiv:1703.03248 ,arXiv:1705.08346, arXiv:1712.03798 , arXiv:1812.07376 , arXiv:1909.09740, arXiv:2003.13244, arXiv:2009.10327, arXiv:1811.09074

M. Sasaki et al, arXiv:1801.05235; B. Carr arXiv:2110.02821; M. Khlopov arXiv:0801.0116

Y. Aldabergenov, A. Addazi, S. Ketov, arXiv:2006.16641 , arXiv:2008.10476

Primordial Black Holes: their genesis

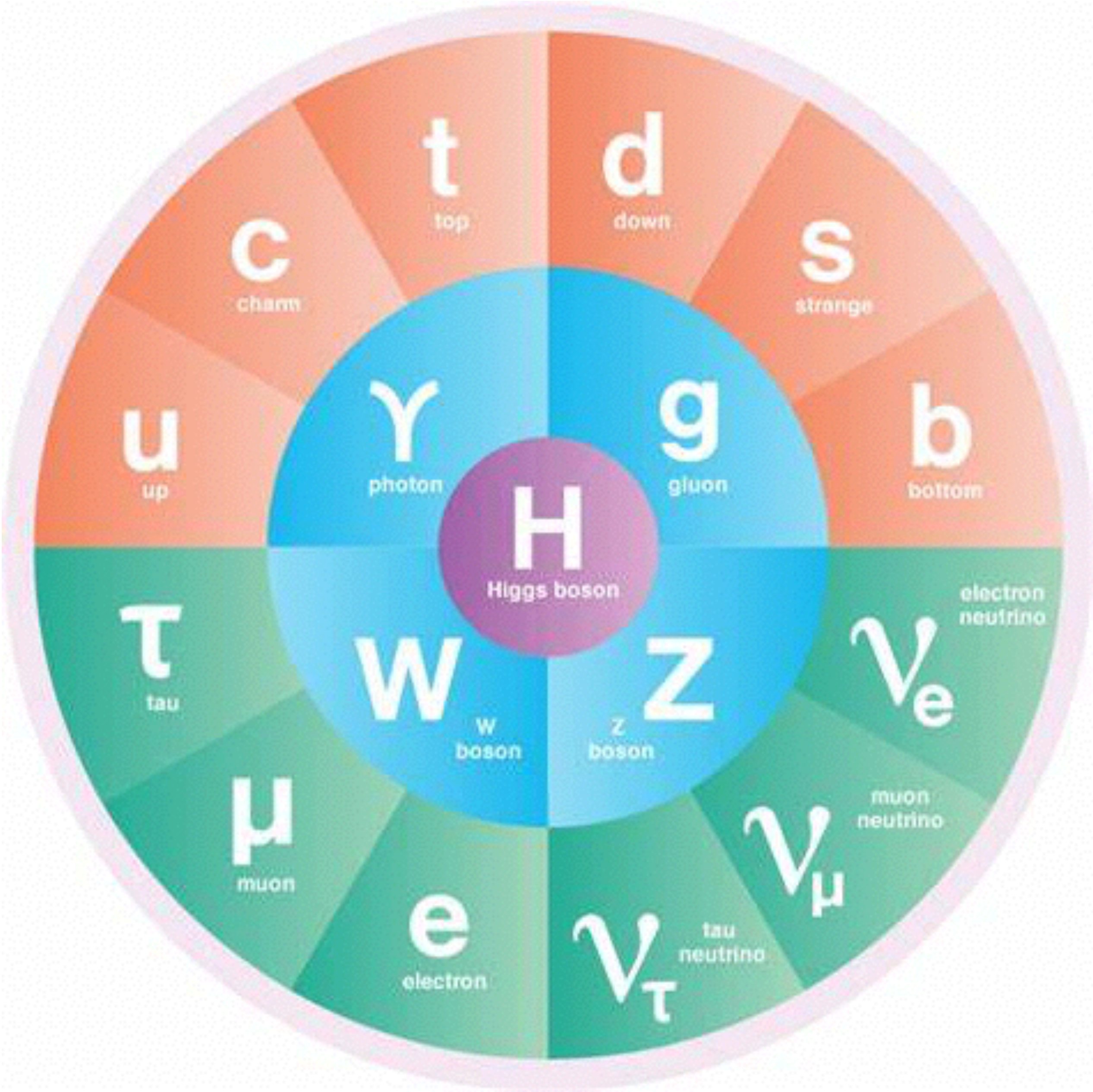
Primordial Black Holes and inflation

Primordial Black Holes and gravitational waves

Primordial Exotic Compact Objects?

Cosmological probes of
High scale supersymmetry and supergravity

Standard Model of particle physics



Standard Model features

A simple theory for the electroweak and strong interactions

it encodes parity violation V-A (Lee & Yang, nobel prize)

It is based on Lorentz invariance, micro-causality, unitarity and CPT symmetry, a simple combination of gauge groups SU(3) x SU(2) x U(1)

Flavor changing neutral currents are suppressed, compatible with all current data from electroweak precision tests as well as high energy colliders

All known particles are coherently organized in three families

The theory is renormalizable at quantum level

Quantum gauge Anomalies automatically cancelled each others while global chiral anomaly into neutal pion decay into two gammas

Misery of the Standard Model

Dark Matter

Hierarchies

Neutrino mass

Strong CP problem

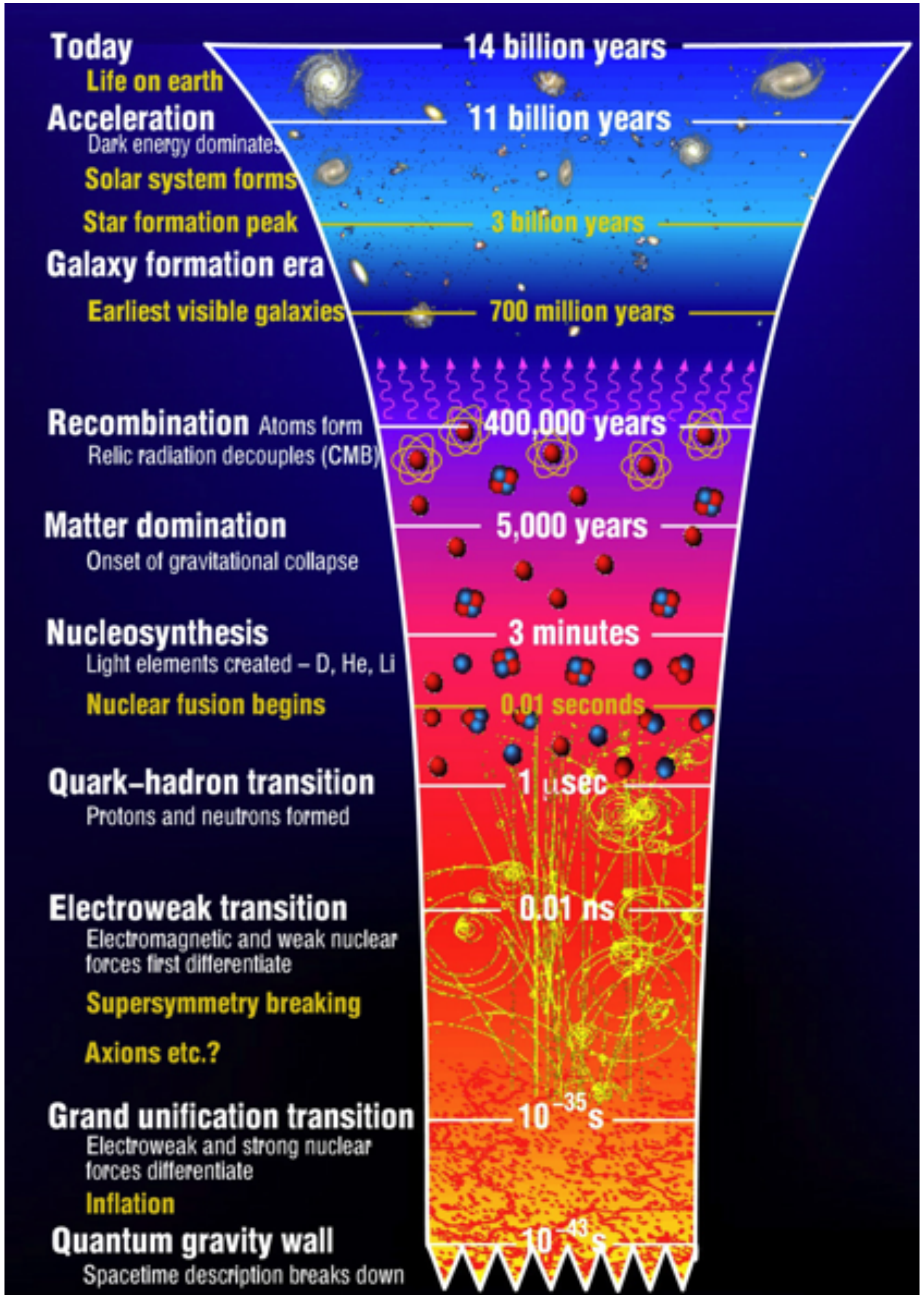
Baryogenesis
and
Matter/Antimatter asymmetry

gravity? Gravitons?

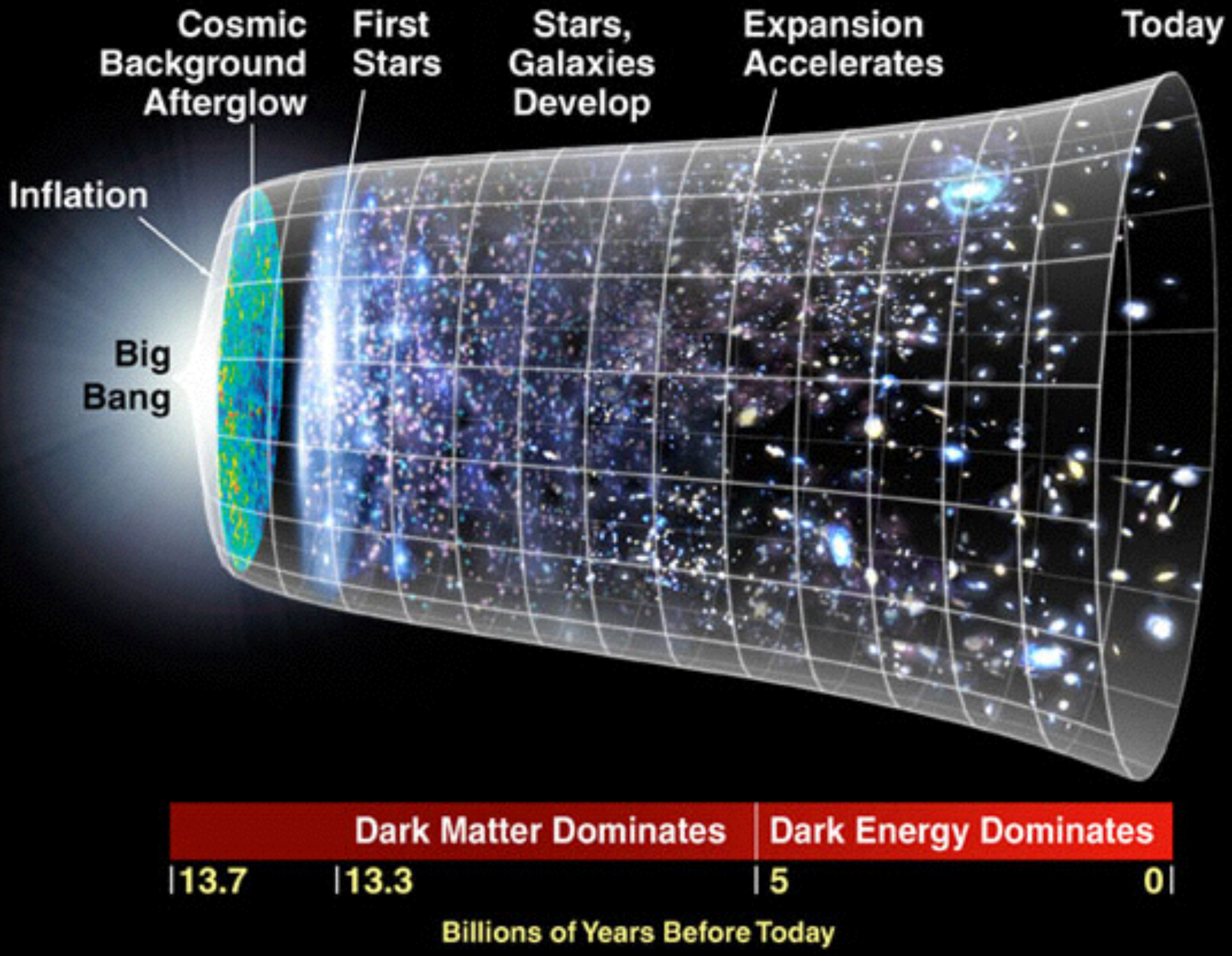
Unifications? Family structures

Higgs discovery posses an
important question:
what is the Nature of the
electroweak phase transition?

Cosmological history of the Universe. Primordial Plasma state crosses phase transitions

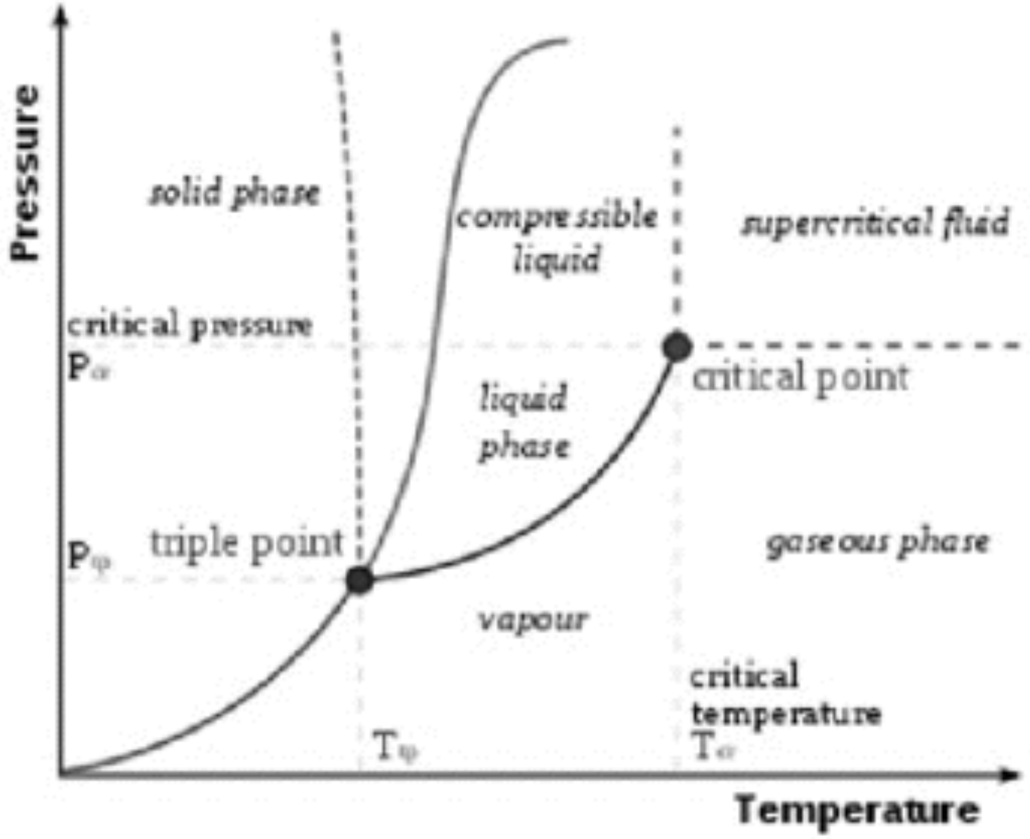


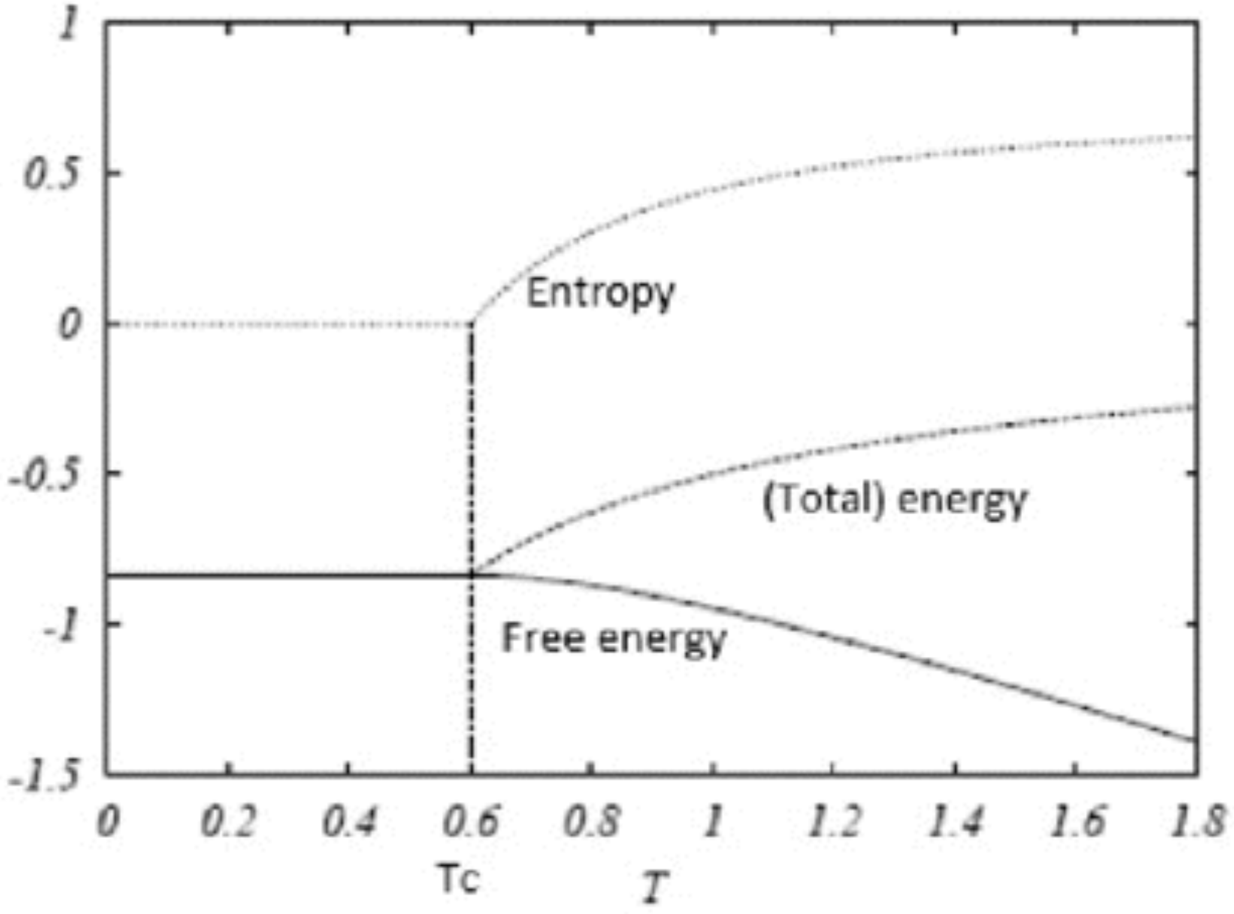
THE EXPANDING UNIVERSE: A CAPSULE HISTORY



Phase transitions

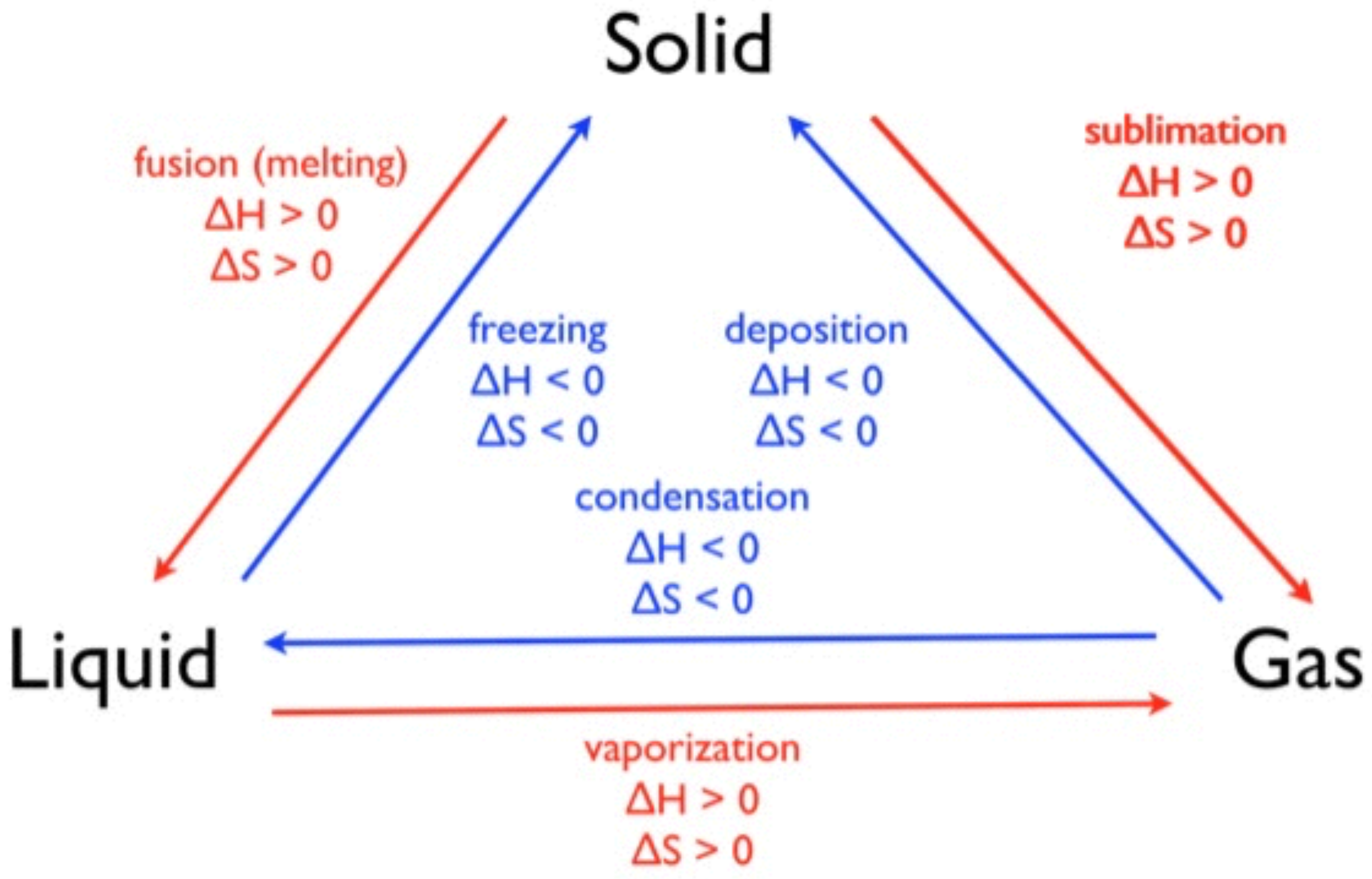
First and second order phase transitions





Transition Temperature Tc (critical temperature)

The different phase transitions



Solid

fusion (melting)
$\Delta H > 0$
$\Delta S > 0$

sublimation
$\Delta H > 0$
$\Delta S > 0$

freezing
$\Delta H < 0$
$\Delta S < 0$

deposition
$\Delta H < 0$
$\Delta S < 0$

condensation
$\Delta H < 0$
$\Delta S < 0$

Liquid

Gas

vaporization
$\Delta H > 0$
$\Delta S > 0$

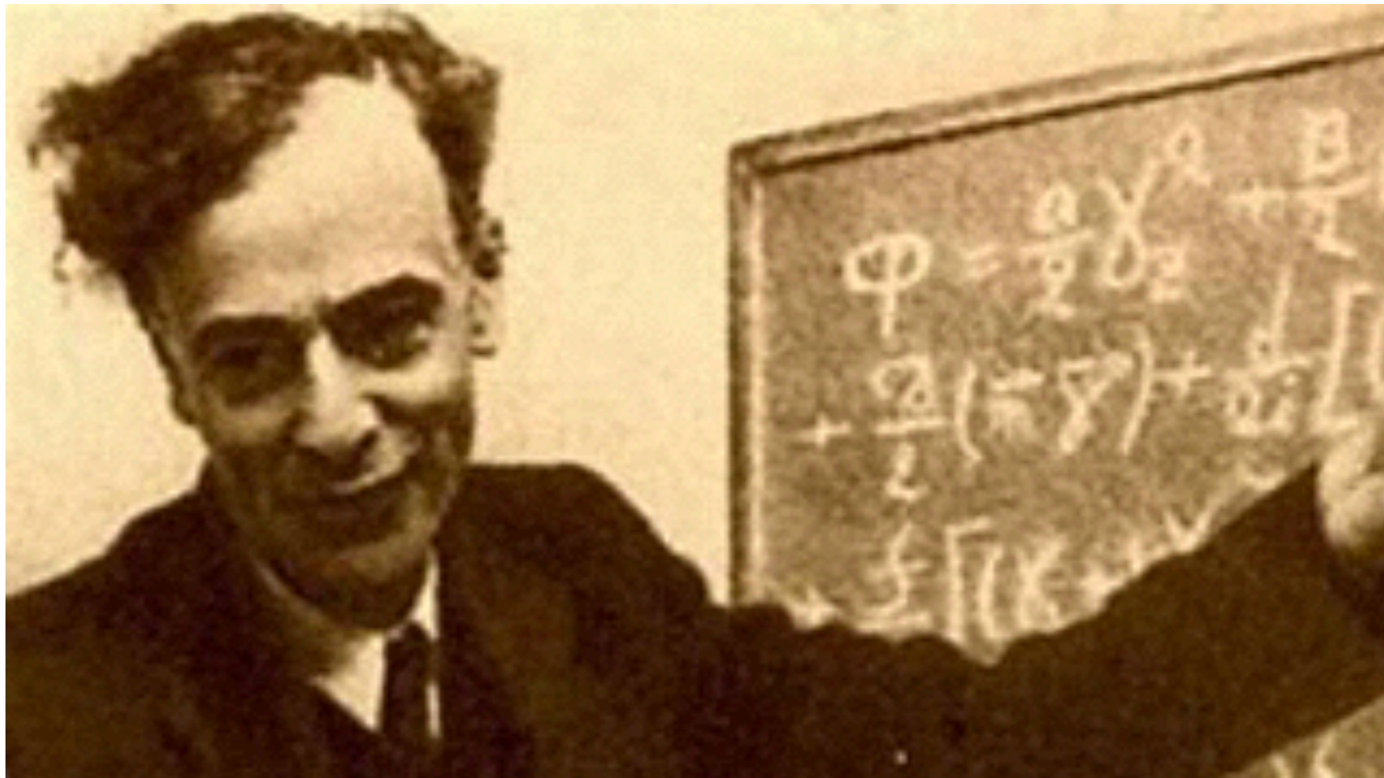
A more general concept

ferromagnetism

superconductivity

topological phases

scalar field theory…



Lev Landau, Landau's theory of phase transitions developed in 1940

“A phase transition occurs when the equilibrium state of a system changes qualitatively as a function of externally imposed constraints.

These constraints could be temperature, pressure, magnetic field, concentration, degree of cross-linking, or any number of other physical quantities.

A transition as a function of temperature, but note that the idea is, of course, more general than that…”

P. Olstead George Town U.

order parameters to understand deformations in a broken symmetry state: this often goes by the name of generalized elasticity, and incorporates elasticity of solids, sound waves in fluids, magnetization in ferromagnetic materials etc.

Landau theory is a mean field theory: the system is assumed to be described by a single macroscopic state.

Landau free energy functionals to calculate observable quantities

Qualitative nature of phase transitions, such as the order of the phase transition, is altered by fluctuation effects and the coupling of different degrees of freedom.

Landau’s theory

$$\mathcal{Z} = \sum_{\mu} e^{-\mathcal{H}[\mu]/k_B T},$$

$$\tilde{F} = F_0(T) + F_L(T, \psi),$$

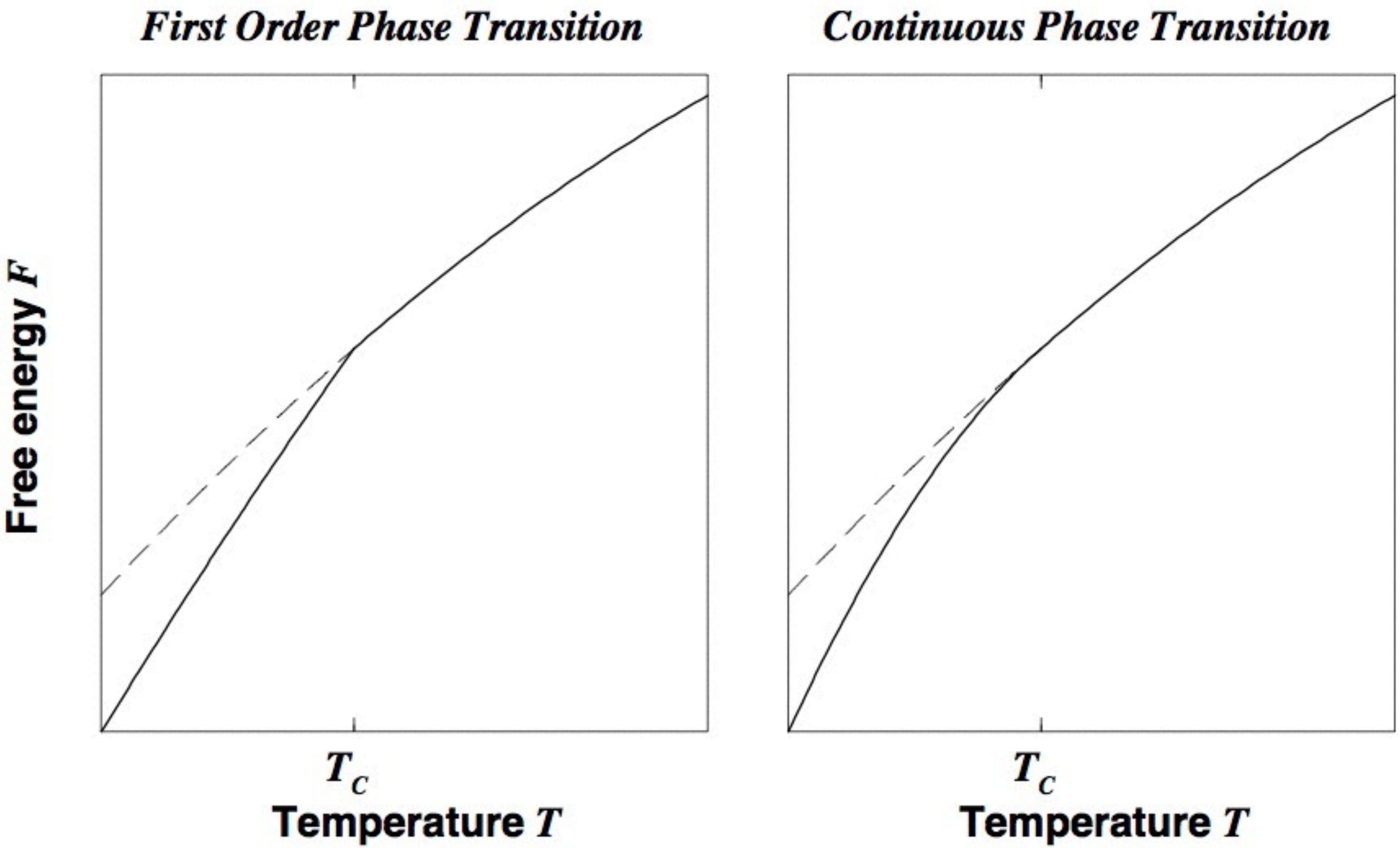
$$F_L[T,\psi] = \int dV \left[\tfrac{1}{2} a_0 (T - T_*) \psi^2 + \ldots\right]$$

$$e^{-F/k_B T} \simeq e^{-F_0/k_B T} \int \mathcal{D}\psi \, e^{-F_L[T,\psi]/k_B T},$$

$$e^{-F/k_B T} \simeq e^{-F_0/k_B T} e^{-\min_{\{\psi\}} F_L[T,\psi]/k_B T}.$$

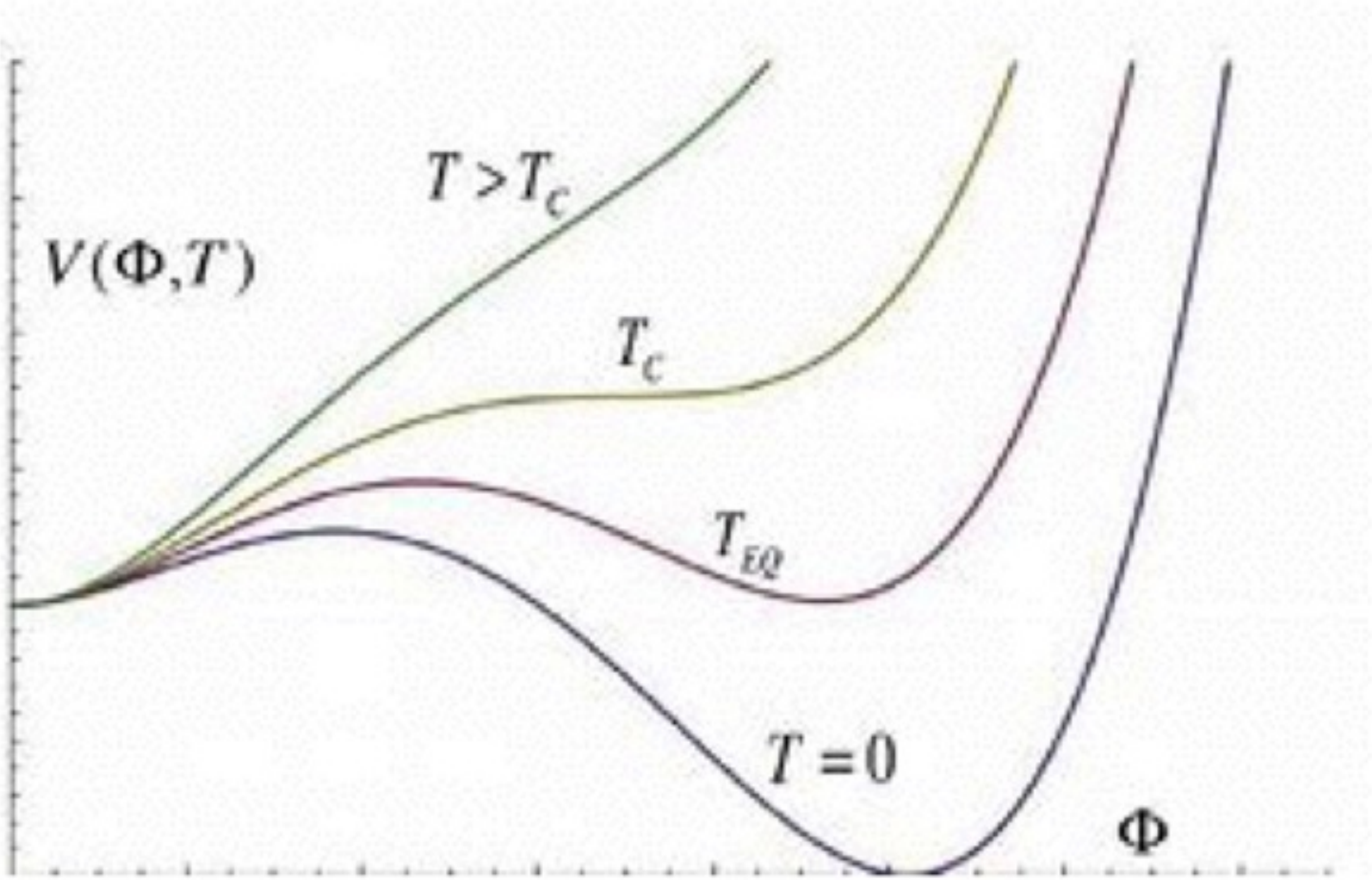
Discontinuity in First derivative

Discontinuity in higher derivatives

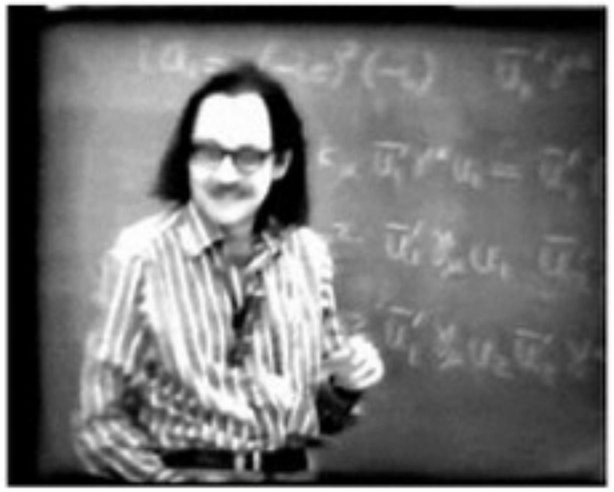


Phase transitions in field theory:
a quantum field can change its
phase state,
for example a scalar field
In this sense the scalar field is considered
as an order parameter of the system.
In primordial bath would depend on temperature
as well

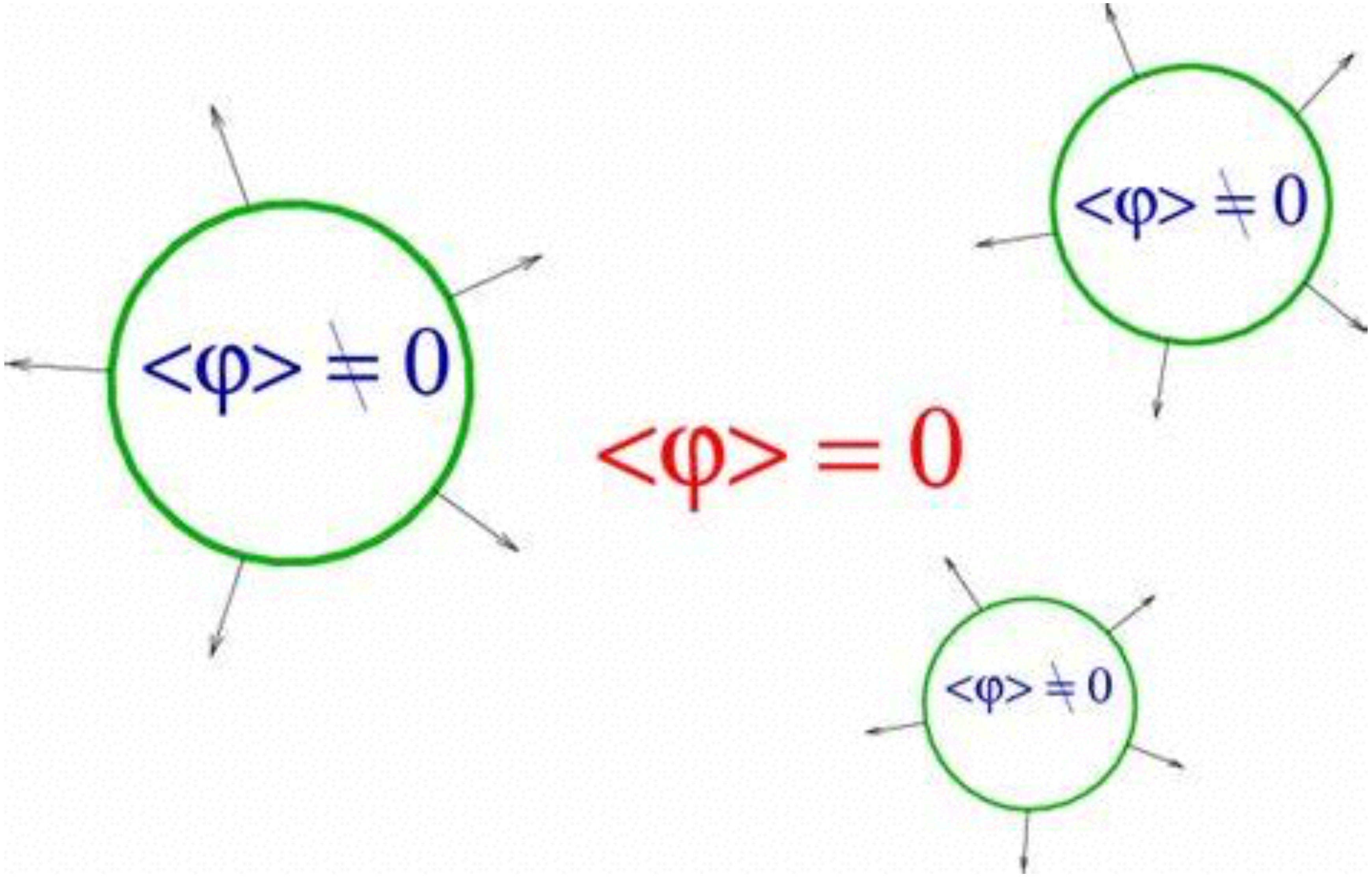
First Order Phase transitions



Coleman De Luccia instantons

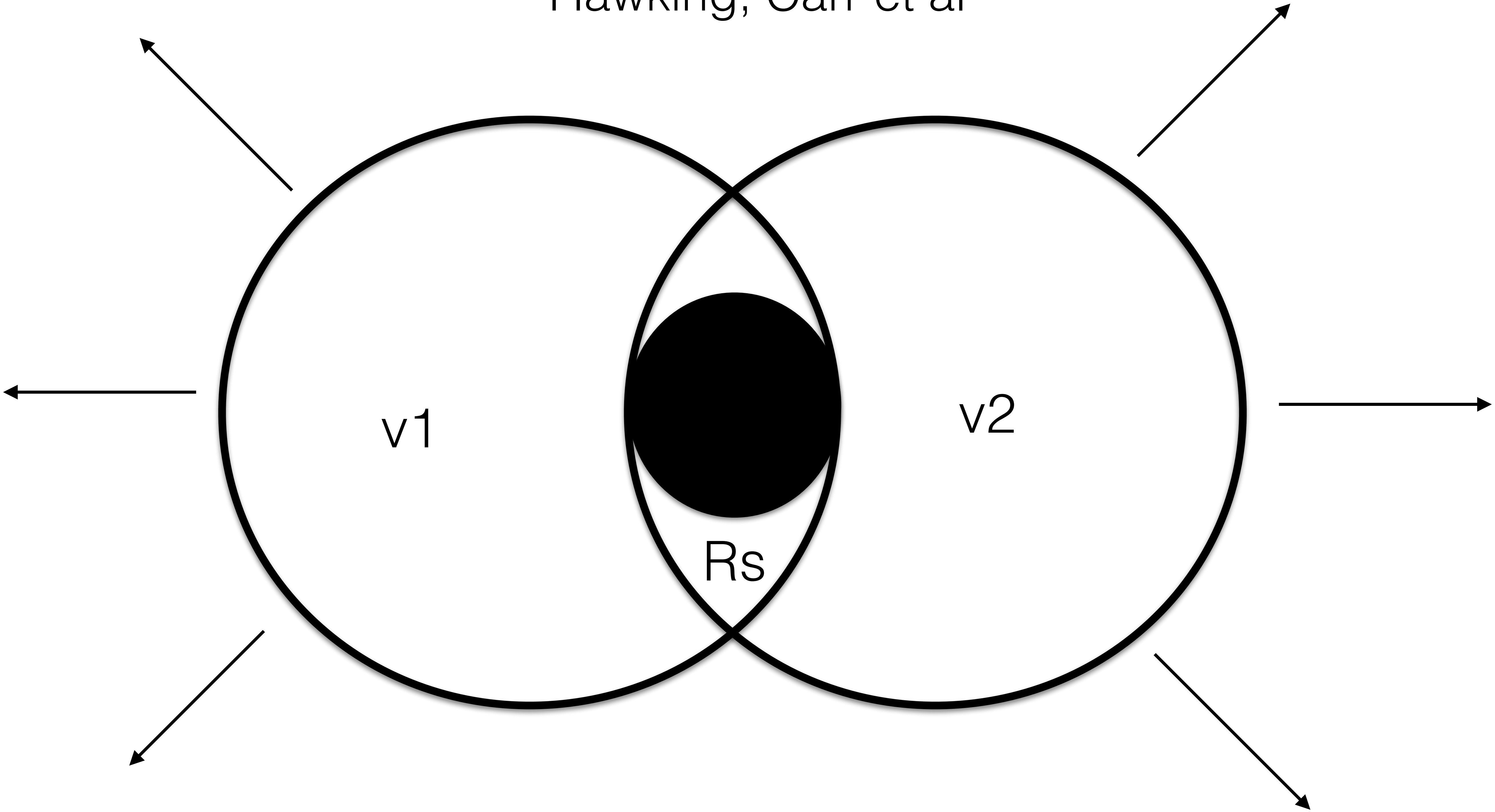


Tunneling from false to true minimum and materialization of bubbles



Primordial Black Holes from first order phase transitions

Hawking, Carr et al



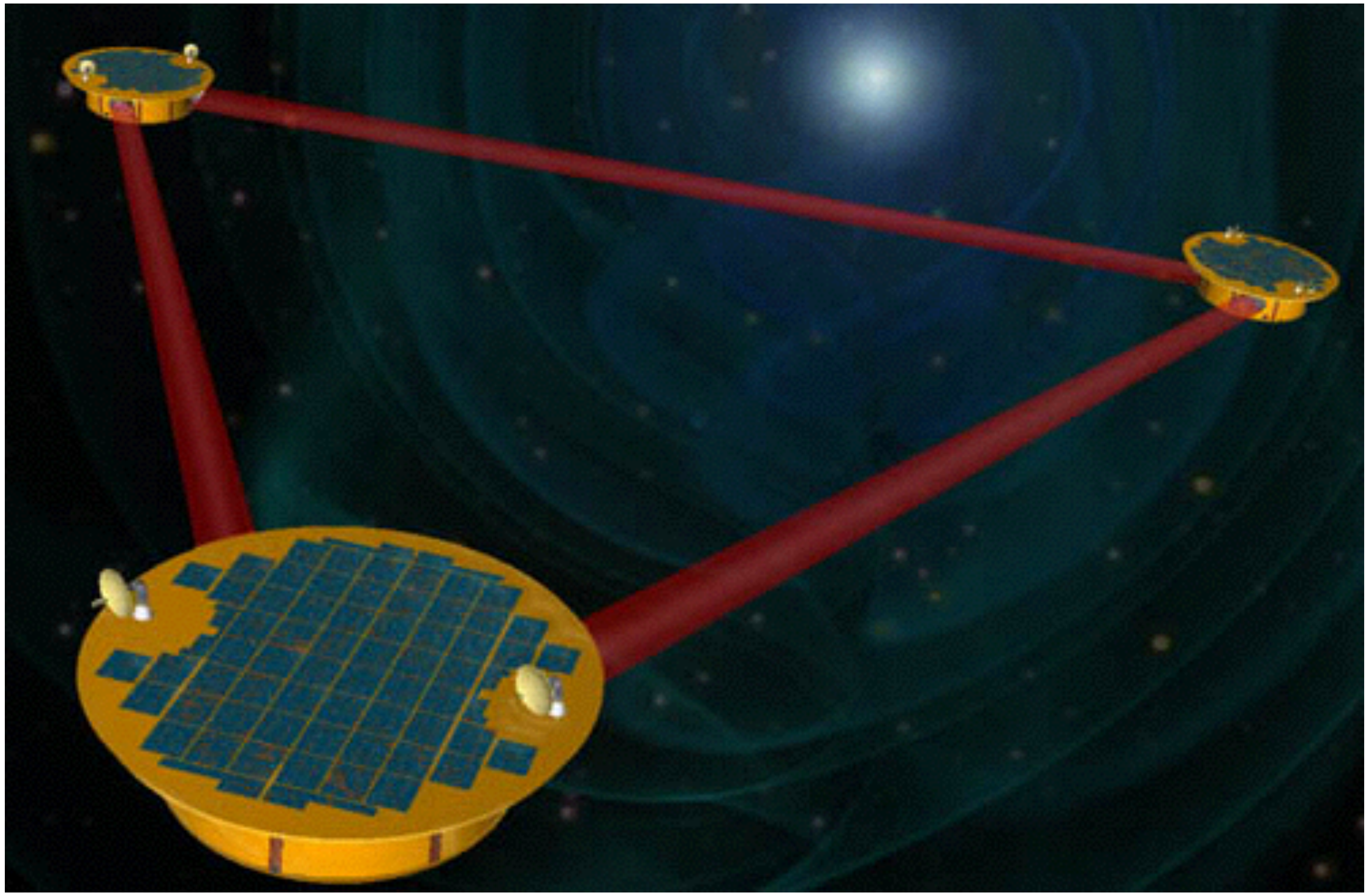
GW from FOPTS

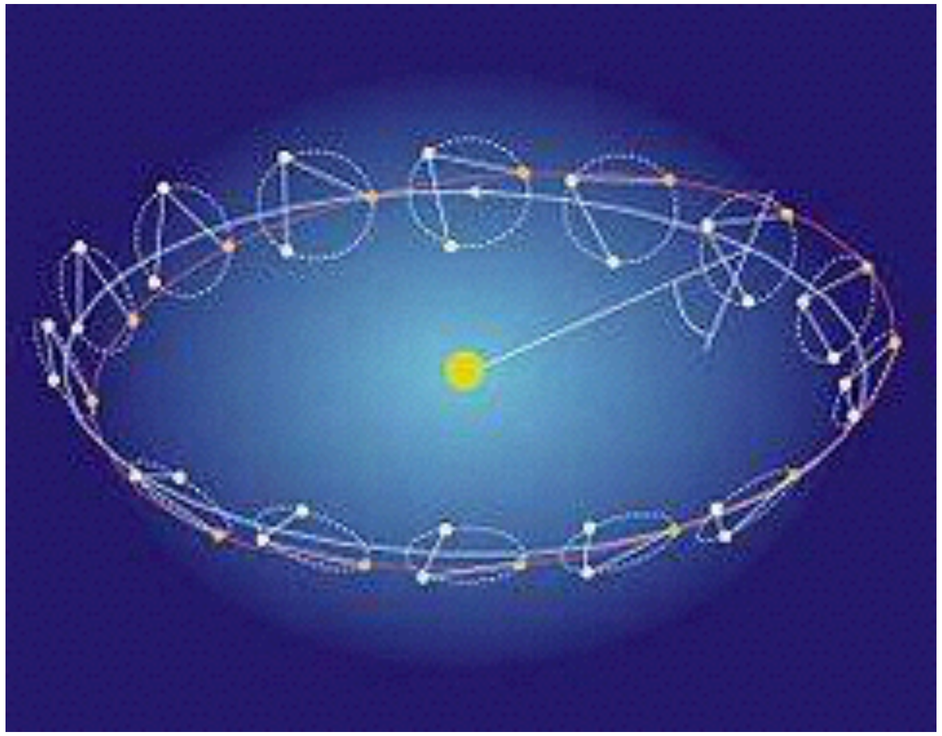
Universe expansion, temperature decreases down to a critical value: phase transition

The PT depends on the particle physics models: it depends from couplings of the scalar field with other fields and its self-interaction potential

if PT is of the first order, it can produce a GW signal *(Hogan '83, Witten '84, Hogan '86... (Turner et al '92, Kosowsky et al '92, Kosowsky and Turner '93, Kamionkowski et al '94, Kosowsky et al '02, Dolgov et al '02...)*

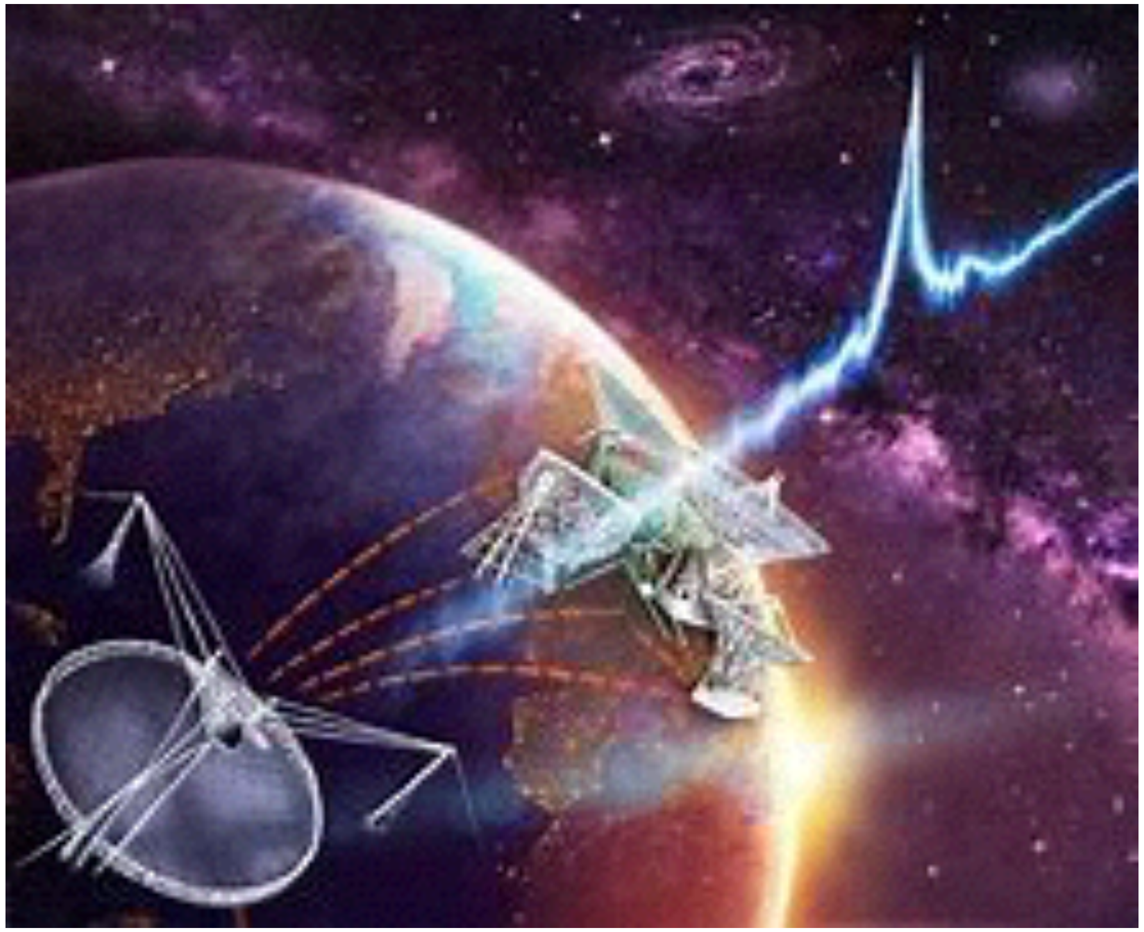
LISA interferometers: mHz window

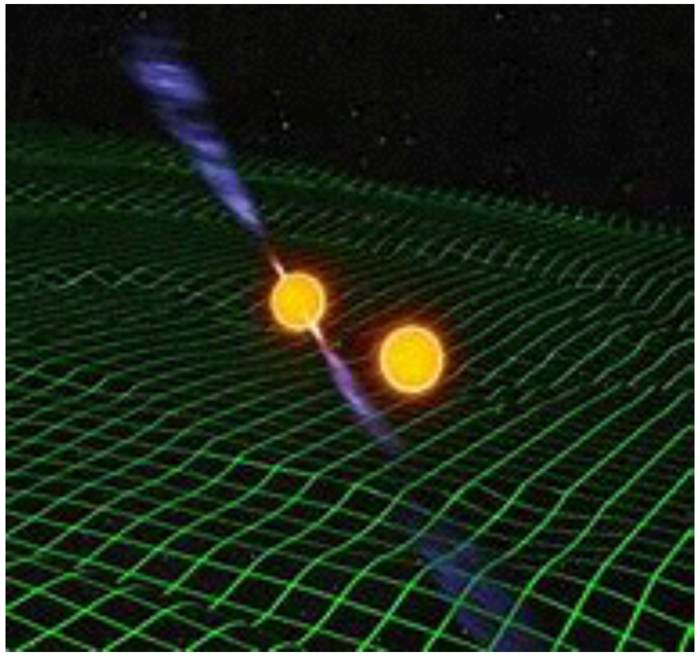




LIGO/VIRGO

FAST, SKA, NANOGrav





GW contributions

Collisions of bubbles

Magnetohydrodynamical (MHD) turbulence

sound shock waves

Key parameters for the phase transition

Transition rate for bubble enucleation $\Gamma(t) = A(t)e^{-S(t)}$

$A(t) \sim \mathcal{M}^4$, where $\mathcal{M} \sim T$

$S(t) \approx S_3/T$

Euclidean action

$$S_3 = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

Classical EoM for bubble profile

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r}\frac{d\phi_b}{dr} - \frac{\partial V}{\partial \phi_b} = 0, \quad \text{with} \quad \left.\frac{d\phi_b}{dr}\right|_{r=0} = 0 \ \text{ and } \ \phi_b|_{r=\infty} = 0$$

$R_b \sim v_b\beta^{-1}$ where v_b is the velocity of the bubble wall.

inverse characteristic time for bubble ennucleation, where t* is time where the transition is complete

$$\beta \equiv - \left.\frac{dS}{dt}\right|_{t_*} \approx \left.\frac{1}{\Gamma}\frac{d\Gamma}{dt}\right|_{t_*}$$

Adiabatic expansion of the universe: $dT/dt = -TH$

$$\frac{\beta}{H_*} = T_* \left.\frac{dS}{dT}\right|_{T_*} = T_* \left.\frac{d}{dT}\left(\frac{S_3}{T}\right)\right|_{T_*}$$

$$\frac{\Gamma}{H^4} \sim O(1) \quad \rightarrow S = -4 \ln \frac{T_*}{m_{Pl}}$$

T* is when the probability for horizon space-time volume is O(1)

latent heat injected in plasma during the transition

$$\epsilon = -\Delta V - T\Delta s = (-\Delta V + T\partial V/\partial T)_{T_*}$$

$$\alpha = \frac{1}{\rho_\gamma}\left[V_i - V_f - \frac{T_n}{4}\left(\frac{\partial V_i}{\partial T} - \frac{\partial V_f}{\partial T}\right)\right],$$

$$\rho_\gamma = g_* \frac{\pi^2}{30} T_n^4$$

Estimation of GW spectrum from bubble collisions

$$P_{GW} = \frac{G}{5}\langle(\dddot{Q}_{ij}^{TT})^2\rangle$$

$$\dddot{Q}_{ij}^{TT} \sim \frac{\text{mass of system in motion} \times (\text{size of system})^2}{(\text{time scale of system})^3} \sim \frac{\text{kinetic energy}}{\text{time scale of system}}$$

$$P_{GW} \sim G\dot{E}_{kin}^2.$$

$$\rho_{GW*} = E_{GW}/(v_b^3\beta^{-3})$$

$$E_{GW} = P_{GW}\beta^{-1}.$$

$$\Omega_{GW*} = \frac{\rho_{GW*}}{\rho_{tot*}} \sim \left(\frac{H_*}{\beta}\right)^2 \kappa^2 \frac{\alpha^2}{(1+\alpha)^2} v_b^3$$

rhot *=(alpha+1)rho rad

More precisely for GW peak:

$$\Omega_{\rm coll}\, h^2(f_{\rm coll}) \simeq 1.1\times 10^{-6}\kappa^2 \left[\frac{H_*}{\beta}\right]^2 \left[\frac{\alpha}{1+\alpha}\right]^2 \left[\frac{v_b^3}{0.24+v_b^3}\right] \left[\frac{100}{g_*}\right]^{1/3}$$

$$f_{\rm coll} \simeq 5.2\times 10^{-3}{\rm mHz}\left[\frac{\beta}{H_*}\right]\left[\frac{T_*}{100{\rm GeV}}\right]\left[\frac{g_*}{100}\right]^{1/6}$$

Sound Waves

$$h^2\Omega_{\rm sw}(f) = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w \, S_{\rm sw}(f) \, ,$$

$$S_{\rm sw}(f) = (f/f_{\rm sw})^3 \left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2} \, .$$

$$f_{\rm sw} = 1.9 \times 10^{-2} \, {\rm mHz} \, \frac{1}{v_w} \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100 \, {\rm GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \, .$$

Turbulence

$$h^2\Omega_{\rm turb}(f) = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)\,,$$

$$S_{\rm turb}(f) = \frac{(f/f_{\rm turb})^3}{[1+(f/f_{\rm turb})]^{\frac{11}{3}}\,(1+8\pi f/h_*)}\,.$$

$$f_{\rm turb} = 2.7 \times 10^{-2}\,{\rm mHz}\,\frac{1}{v_w}\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\,.$$

Runnaway and non-runnaway bubbles, bubble speed

Runnaway in plasma:all terms included

Non-Runnaway in plasma: sound and turbulence terms dominate on others

Runnaway in vacuum: collisions dominate

Two simple ways
for having a first order phase transitions

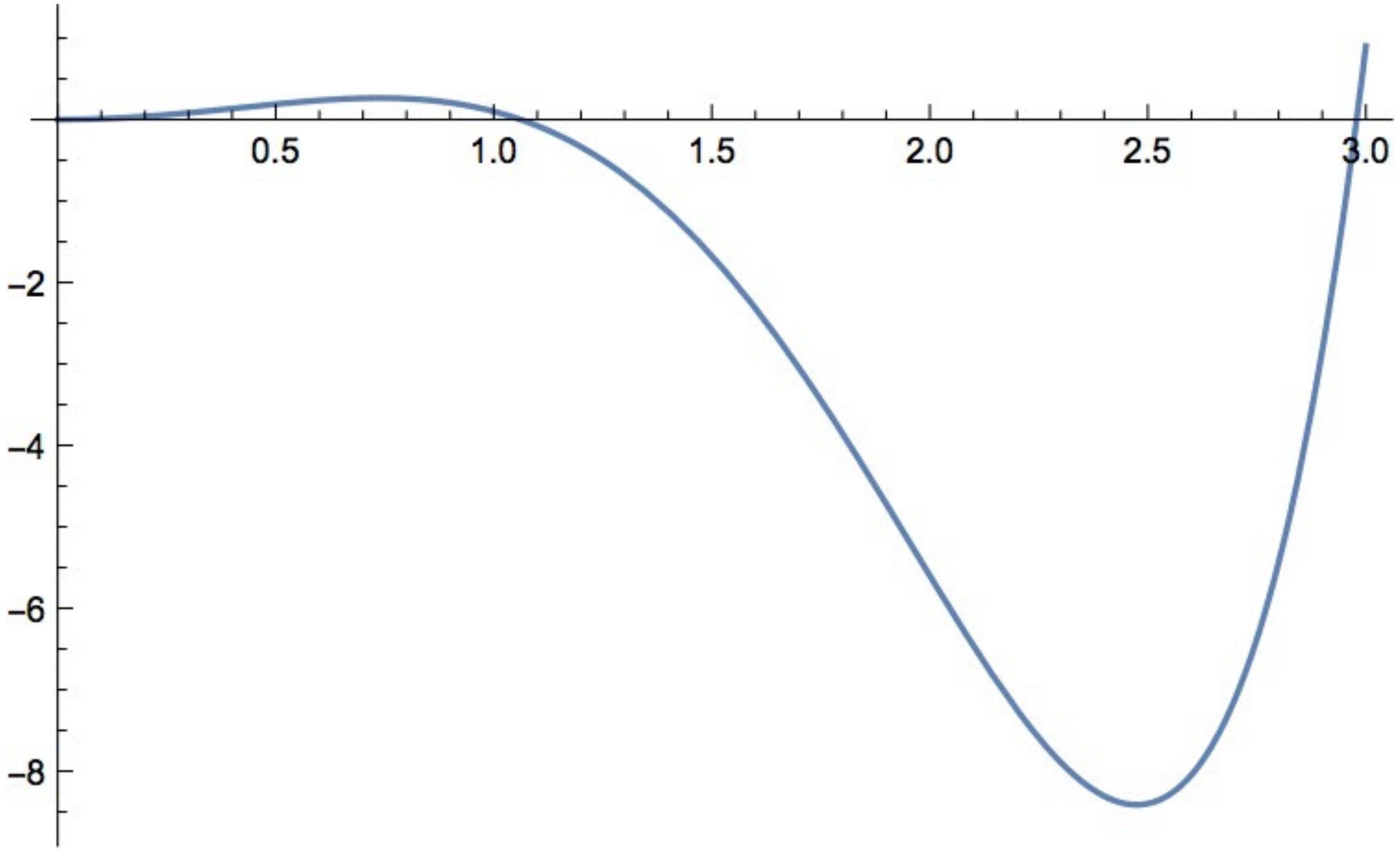
Two scalar fields:
example Higgs coupled with a scalar singlet
or a twin Higgs

Higher order self-interaction terms: for examples dimension 6 extra operators

Higher order self-interaction terms: for examples dimension 6 extra operators

$$V_{\text{tree}}(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4 + \frac{\kappa}{8\Lambda^2} h^6$$

Higher order self-interaction terms: for examples dimension 6 extra operators



Thermal corrections,
thermal field theory,
mass and vertices corrections
to the potential depending by
the couplings with other field

Example: Majoron model

$$U(1)_L \to \mathbb{Z}_2,$$

$$\begin{aligned}\mathcal{V}_0(\Phi,\sigma) = {} & \mu_\Phi^2 \Phi^\dagger\Phi + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \mu_\sigma^2\sigma^\dagger\sigma + \lambda_\sigma(\sigma^\dagger\sigma)^2 \\ & + \lambda_{\Phi\sigma}\Phi^\dagger\Phi\sigma^\dagger\sigma + \left(\frac{1}{2}\mu_b^2\sigma^2 + \text{h.c.}\right),\end{aligned}$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} G + iG' \\ \phi_h + h + i\eta \end{pmatrix}, \qquad \sigma = \frac{1}{\sqrt{2}}(\phi_\sigma + \sigma_R + i\sigma_I),$$

$$\mathcal{L}_{\text{Yuk}}^{\text{Inverse}} = Y_\nu \bar{L} H \nu^c + M\nu^c S + \mu SS + \text{h.c.},$$

μ is also a 3×3 symmetric matrix.

$$m_\nu^{\text{Inverse}} = \frac{v_h^2}{2} Y_\nu^T M^{T^{-1}} \mu M^{-1} Y_\nu \ .$$

Thermal corrections

$$V_{\rm eff}(T) = V_0 + V_{\rm CW}^{(1)} + \Delta V(T) + V_{\rm ct}\,,$$

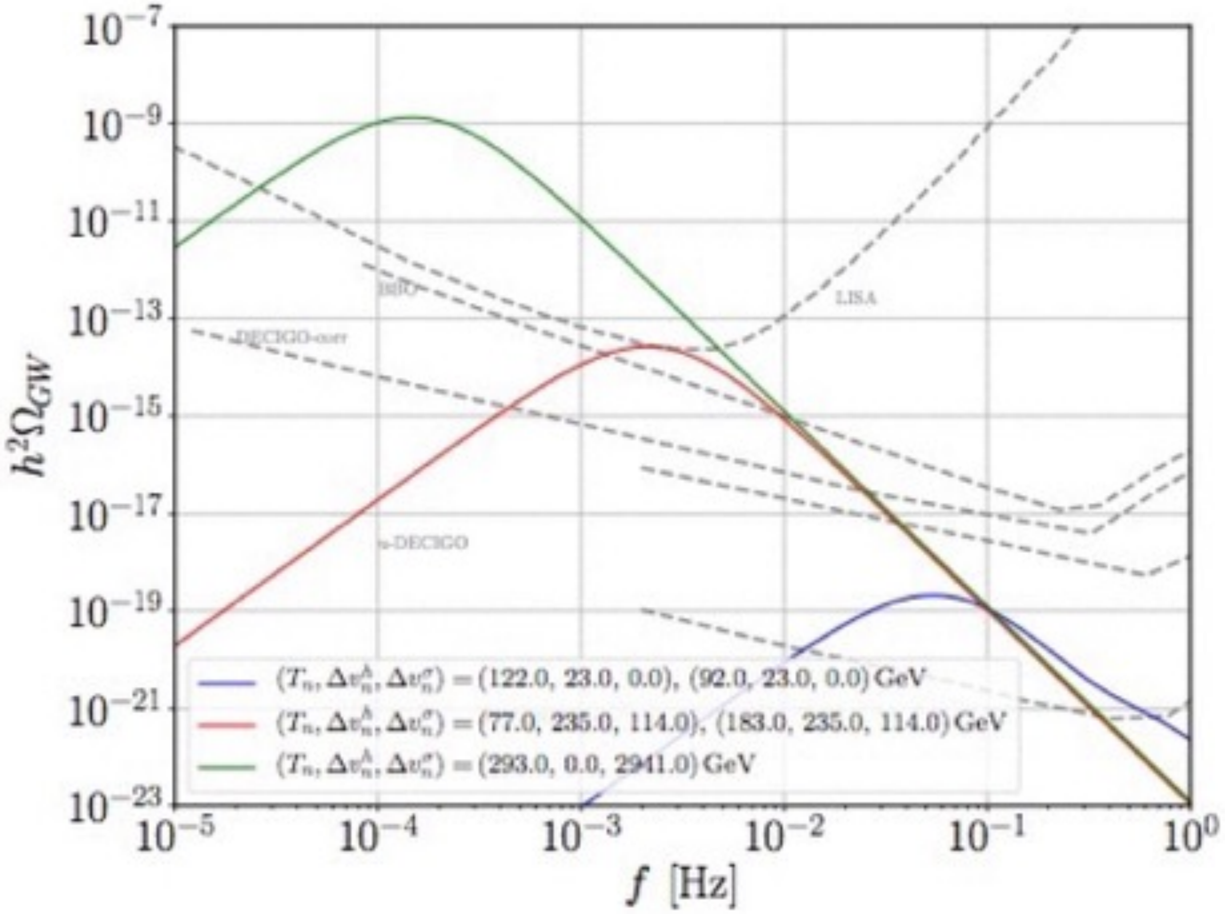
$$\mu_\alpha^2(T) = \mu_\alpha^2 + c_\alpha T^2\,,$$

$$c_h = \frac{3}{16}g^2 + \frac{1}{16}g'^2 + \frac{1}{2}\lambda_\Phi + \frac{1}{12}\lambda_{\Phi\sigma} + \frac{1}{4}(y_t^2 + y_b^2 + y_c^2 + y_s^2 + y_u^2 + y_d^2) + \frac{1}{12}(y_\tau^2 + y_\mu^2 + y_e^2)\,,$$

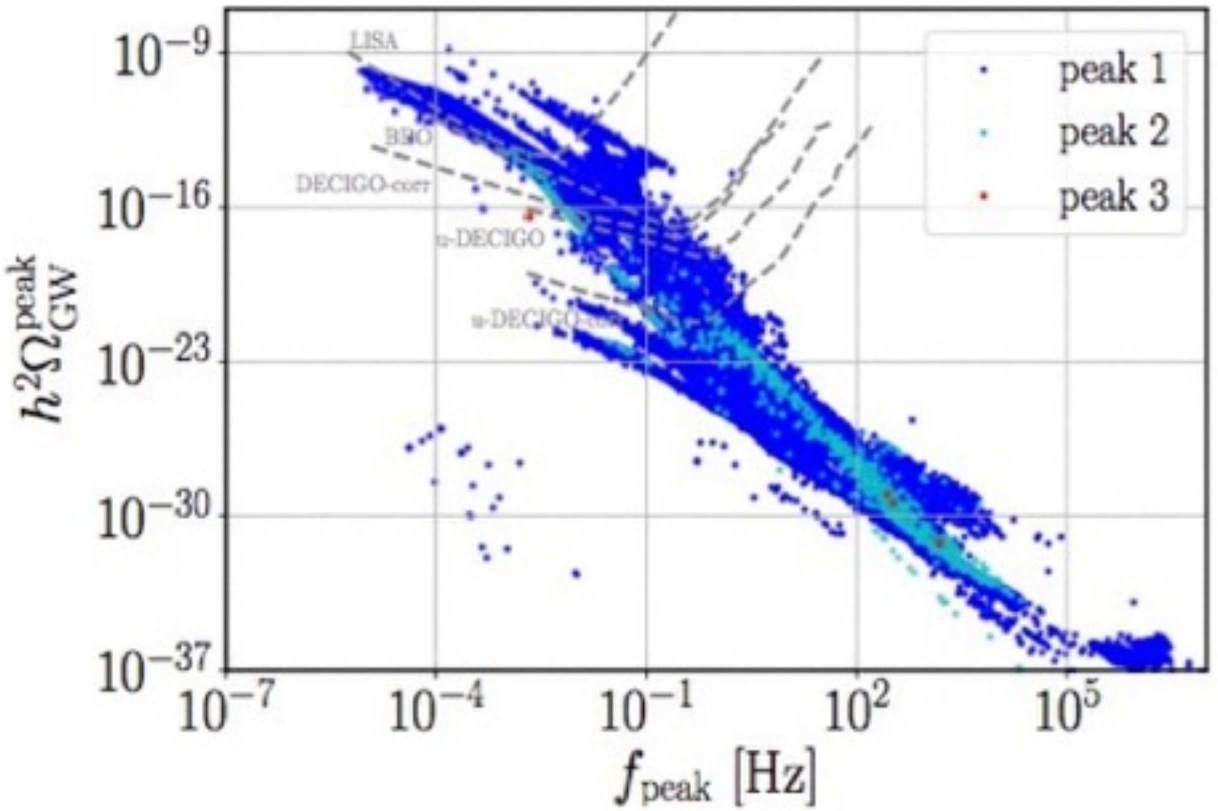
$$c_\sigma = \frac{1}{3}\lambda_\sigma + \frac{1}{6}\lambda_{\Phi\sigma}\,,$$

$$\begin{aligned} m_{W_L}^2(\phi_h;T) &= m_W^2(\phi_h) + \frac{11}{6}g^2T^2\,,\\ m_{Z_L,A_L}^2(\phi_h;T) &= \frac{1}{2}m_Z^2(\phi_h) + \frac{11}{12}(g^2+g'^2)T^2 \pm \mathcal{D}\,, \end{aligned}$$

$$\mathcal{D}^2 = \Big(\frac{1}{2}m_Z^2(\phi_h) + \frac{11}{12}(g^2+g'^2)T^2\Big)^2 - \frac{11}{12}g^2g'^2T^2\Big(\phi_h^2 + \frac{11}{3}T^2\Big)\,.$$



Addazi et al PLB



Dark Phase transitions

Violent Majoron:
decoupled by the Higgs
and phase transitions around KeV-MeV
(Addazi, Cai, Marciano 2017)

$$V_1^{(5)} = \frac{\lambda_1}{\Lambda}\sigma^5 + \frac{\lambda_2}{\Lambda}\sigma^*\sigma^4 + \frac{\lambda_3}{\Lambda}(\sigma^*)^2\sigma^3 + h.c\,,$$

$$V_2^{(5)}(\sigma, H) = \frac{\beta_1}{\Lambda}(H^\dagger H)^2\sigma + \frac{\beta_2}{\Lambda}(H^\dagger H)\sigma^2\sigma^*$$

$$+ \frac{\beta_3}{\Lambda}(H^\dagger H)\sigma^3 + h.c.\,,$$

$$V_1^{(6)}(\sigma) = \frac{\gamma_1}{\Lambda^2}\sigma^6 + \frac{\gamma_2}{\Lambda^2}\sigma^*\sigma^5 + \frac{\gamma_3}{\Lambda^2}(\sigma^*)^2\sigma^4$$

$$+ \frac{\gamma_4}{\Lambda^2}(\sigma^*)^3\sigma^3 + h.c.\,,$$

$$V_2^{(6)}(\sigma, H) = \frac{\delta_1}{\Lambda^2}(H^\dagger H)^2\sigma^2 + \frac{\delta_2}{\Lambda^2}(H^\dagger H)^2\sigma^*\sigma$$

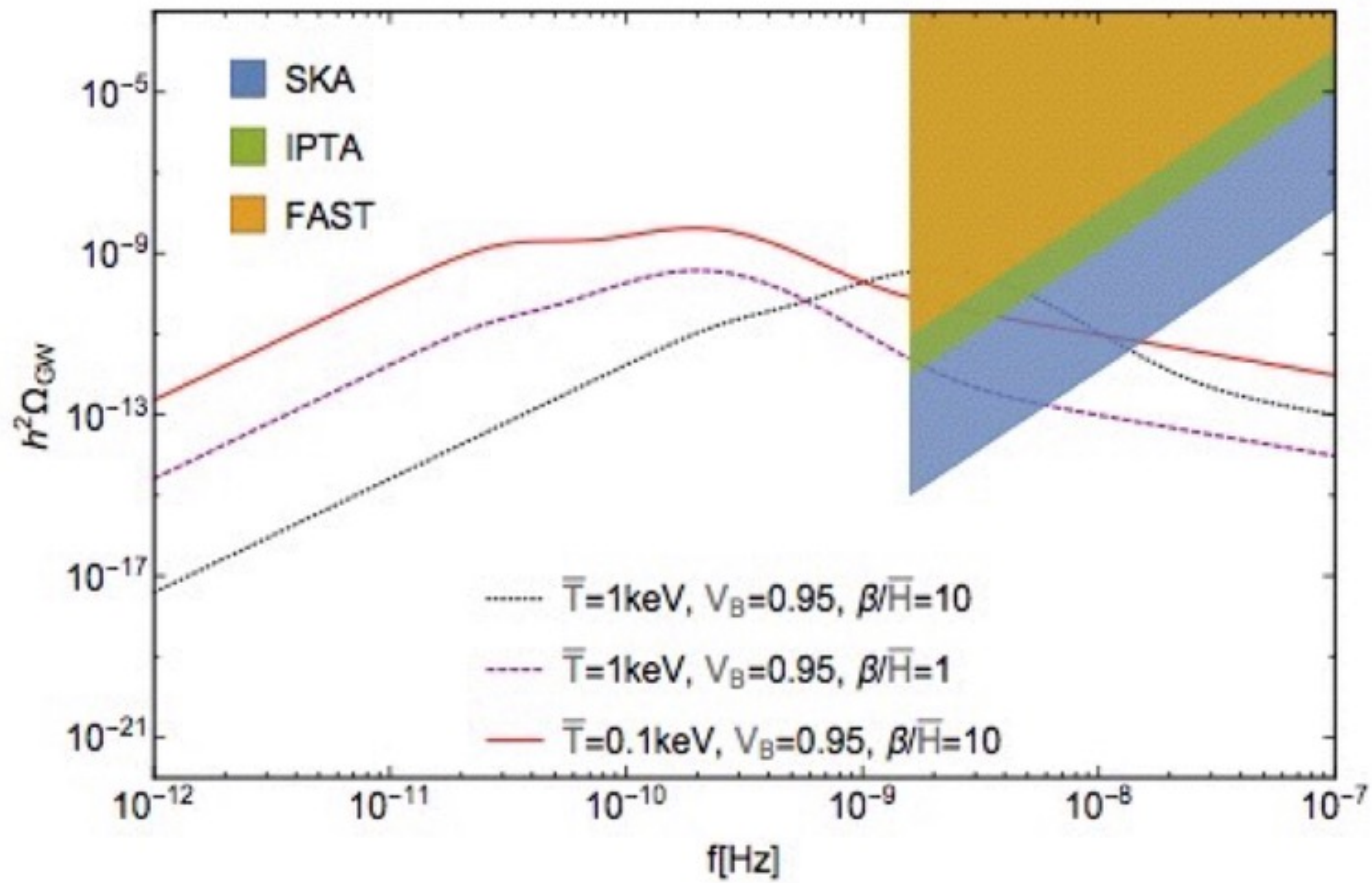
$$+ \frac{\delta_3}{\Lambda^2}(H^\dagger H)\sigma^3\sigma^* + \frac{\delta_4}{\Lambda^2}(H^\dagger H)(\sigma\sigma^*)^2$$

$$V_{\text{eff}}(\sigma, T) \simeq C T^2 (\sigma^\dagger \sigma) + V(\sigma, H)\,,$$

$$C = \frac{1}{4}\Big(\frac{m_\sigma^2}{v'^2} + \frac{m_H^2}{v^2} + h_L^2 + h_R^2 - 24K\Big)\,,$$

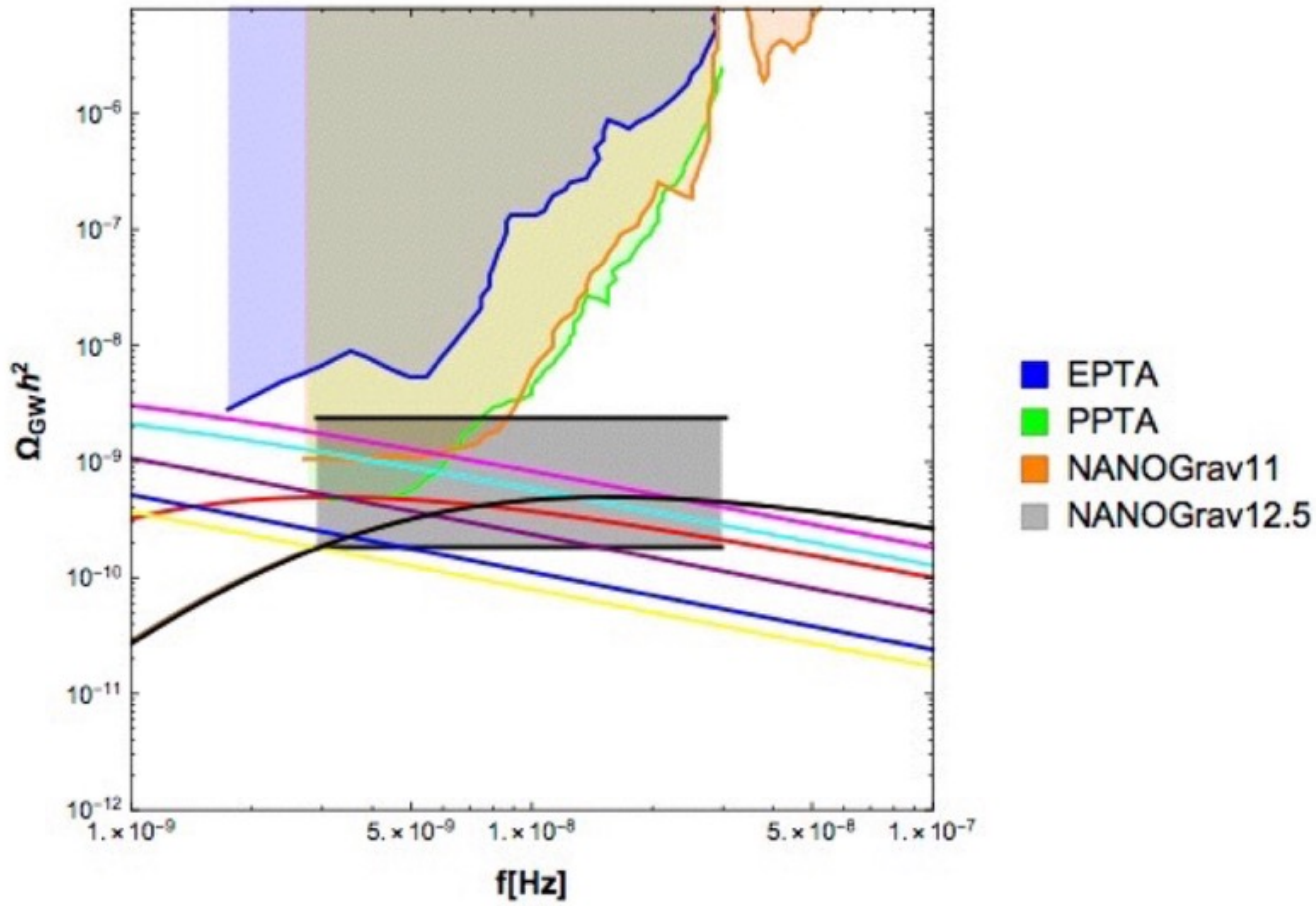
$$K = K^{(5)} = (\lambda_2 + \lambda_3)\frac{v'}{\Lambda} + \beta_2 \frac{v'}{\Lambda}\,,$$

$$K = K^{(6)} = \frac{1}{\Lambda^2}[(\delta_2 + \delta_3 + \gamma_2 + \gamma_3 + \gamma_4)v'^2 + (\delta_2 + \delta_3)v^2]\,.$$



Addazi, Cai, Marciano, PLB

NANOGrav Excess 3.1 sigma



Addazi, Cai, Marciano, arxiv 2009.10327

Dark gauge sectors, dark photons

$$\mathcal{L} = K_s(s) + K_\chi(\chi) - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\varepsilon}{2}F'^{\mu\nu}F^{Y}_{\mu\nu} + U(s,\chi)\,,$$

$$K_s(s) + K_\chi(\chi) = (\mathcal{D}_\mu s^\dagger)(\mathcal{D}^\mu s) + \bar{\chi}(i\gamma_\mu\mathcal{D}_\mu - \mu_\chi)\chi$$

$$U(s,\chi) = V(s) + y' s\bar{\chi}\chi\,,$$

$$V(s) = m_s^2 s^\dagger s + \frac{1}{4}\lambda_S(s^\dagger s)^2 + \frac{1}{\Lambda^2}(s^\dagger s)^3 + \cdots$$

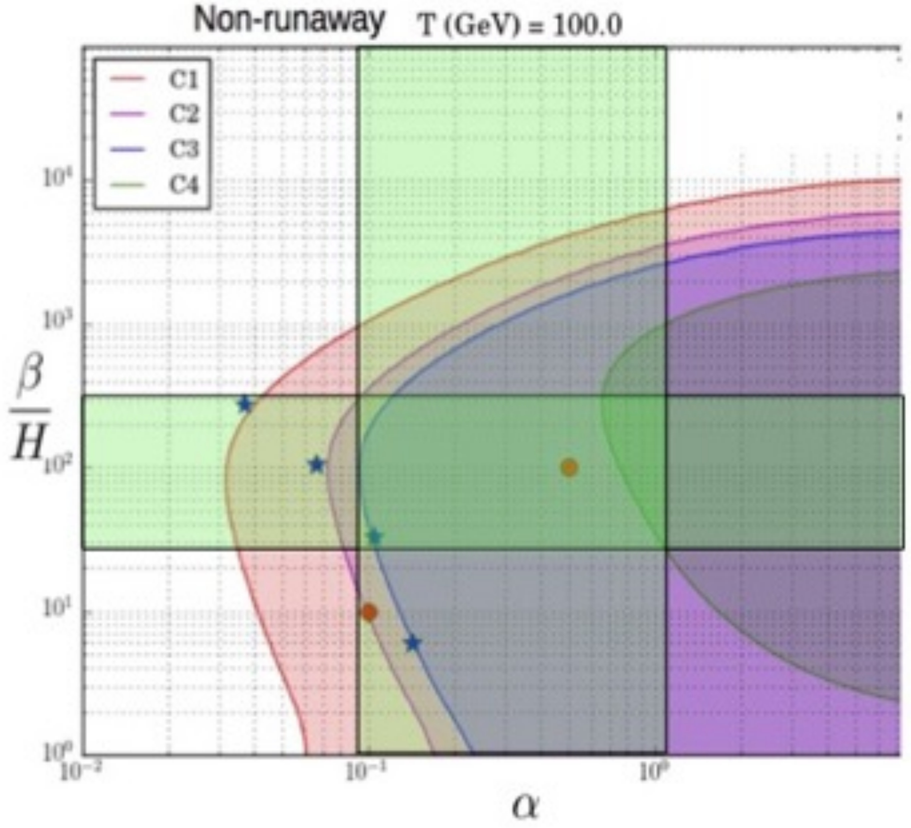
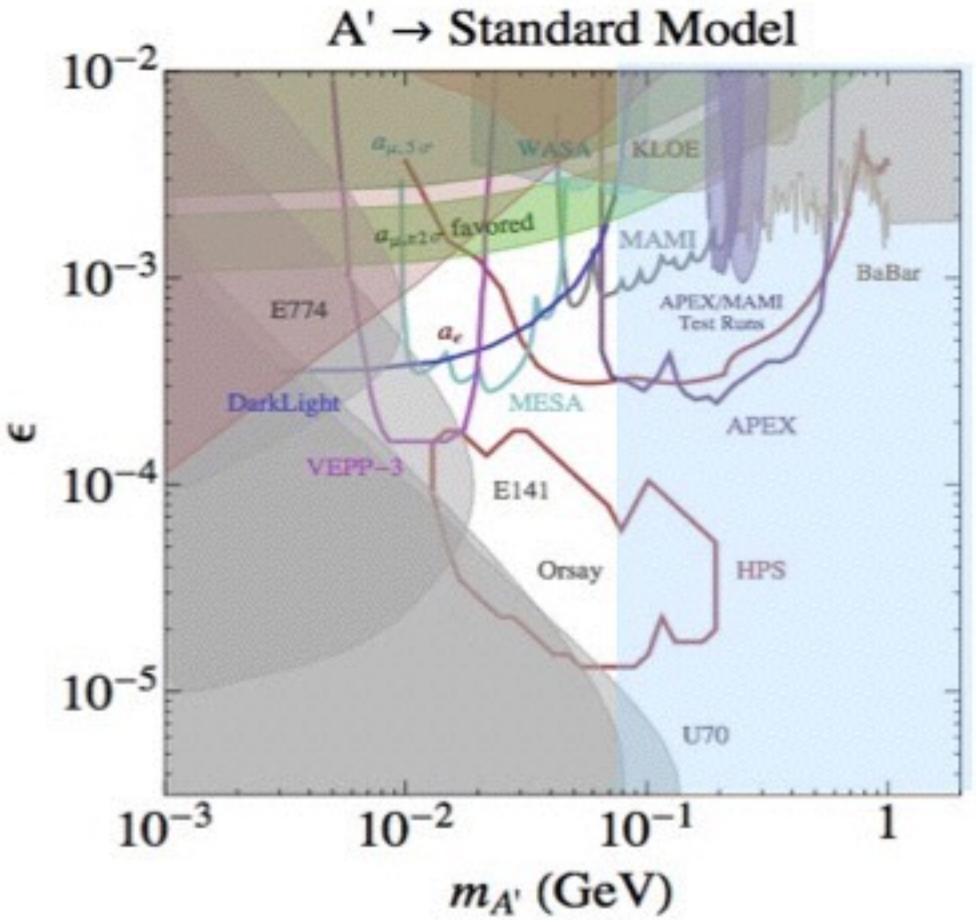
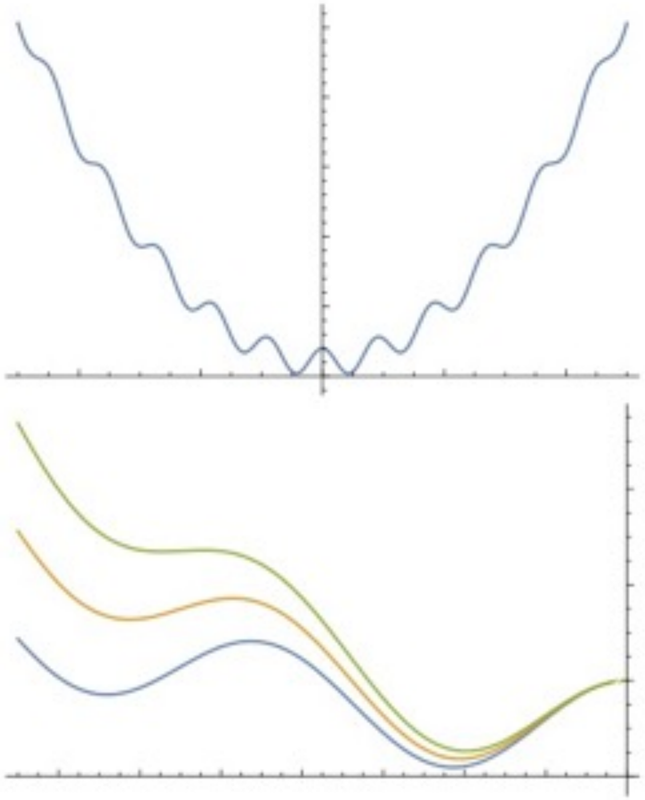
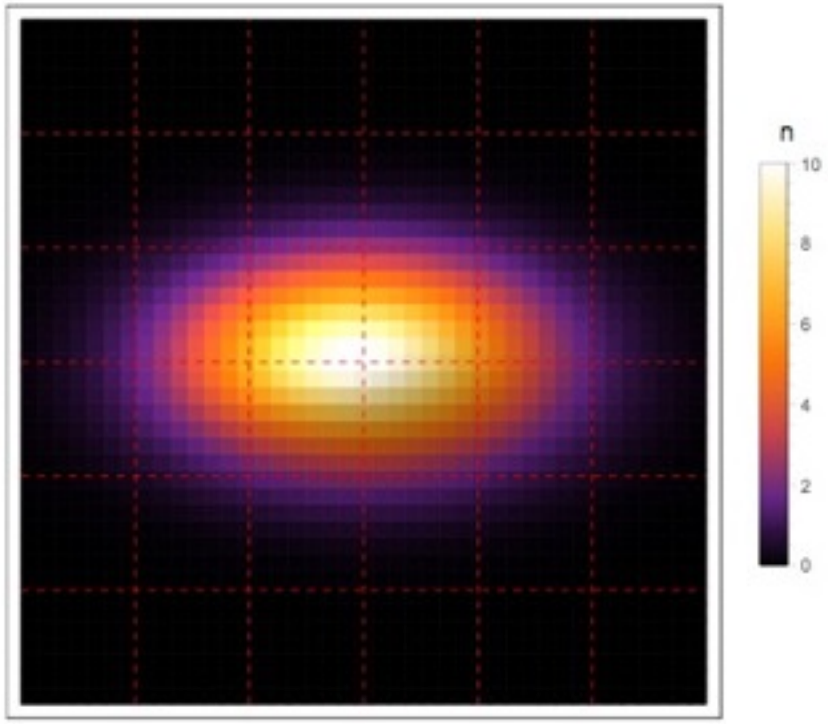


FIG. 2. We show the predicted region for our model in the (α, β) parameters' space. This corresponds to the intersection of the two green regions, and is put in comparison with model independent regions for eLISA, as discussed in [3] assuming a VEV scale 100 GeV.



Very High energy FOPTS





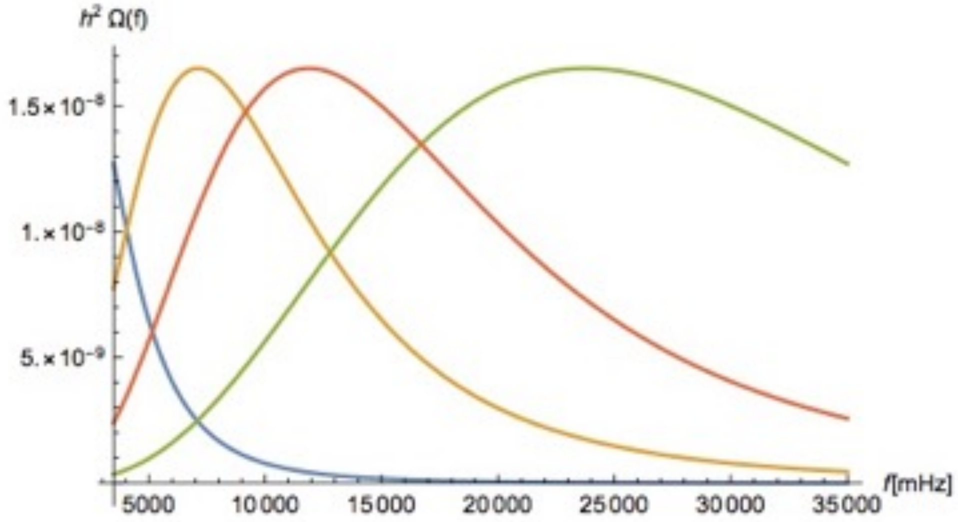


FIG. 1. Examples of non-runaway cases are displayed, with the same value of the parameters $v_w = 0.8$, $\alpha = 0.9$, $g_* \simeq g_{SM}$ and $\beta/H_* = 10$, but with varying FOPT temperature, namely $T_*/(10^8\,\text{GeV}) = \{0.1, 0.3, 0.5, 5\}$, corresponding to the blue, orange, red and green lines, respectively.

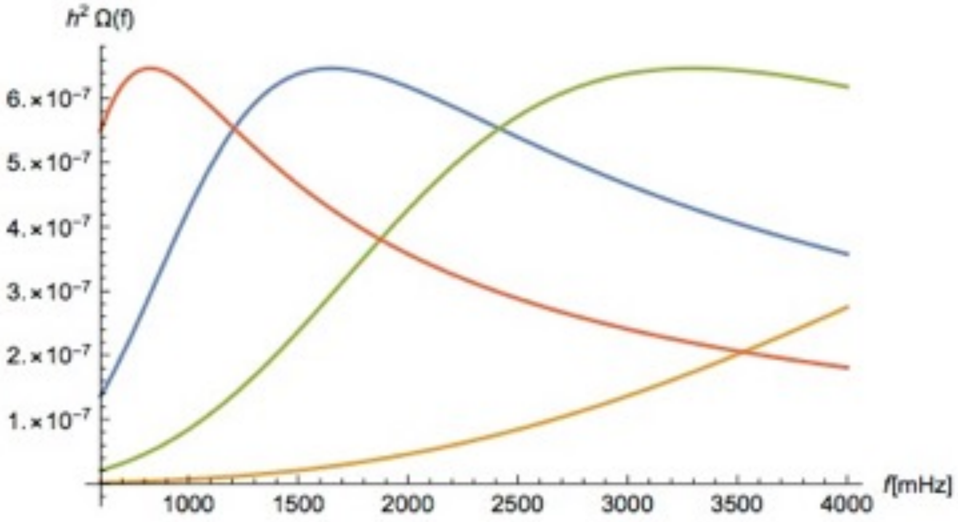
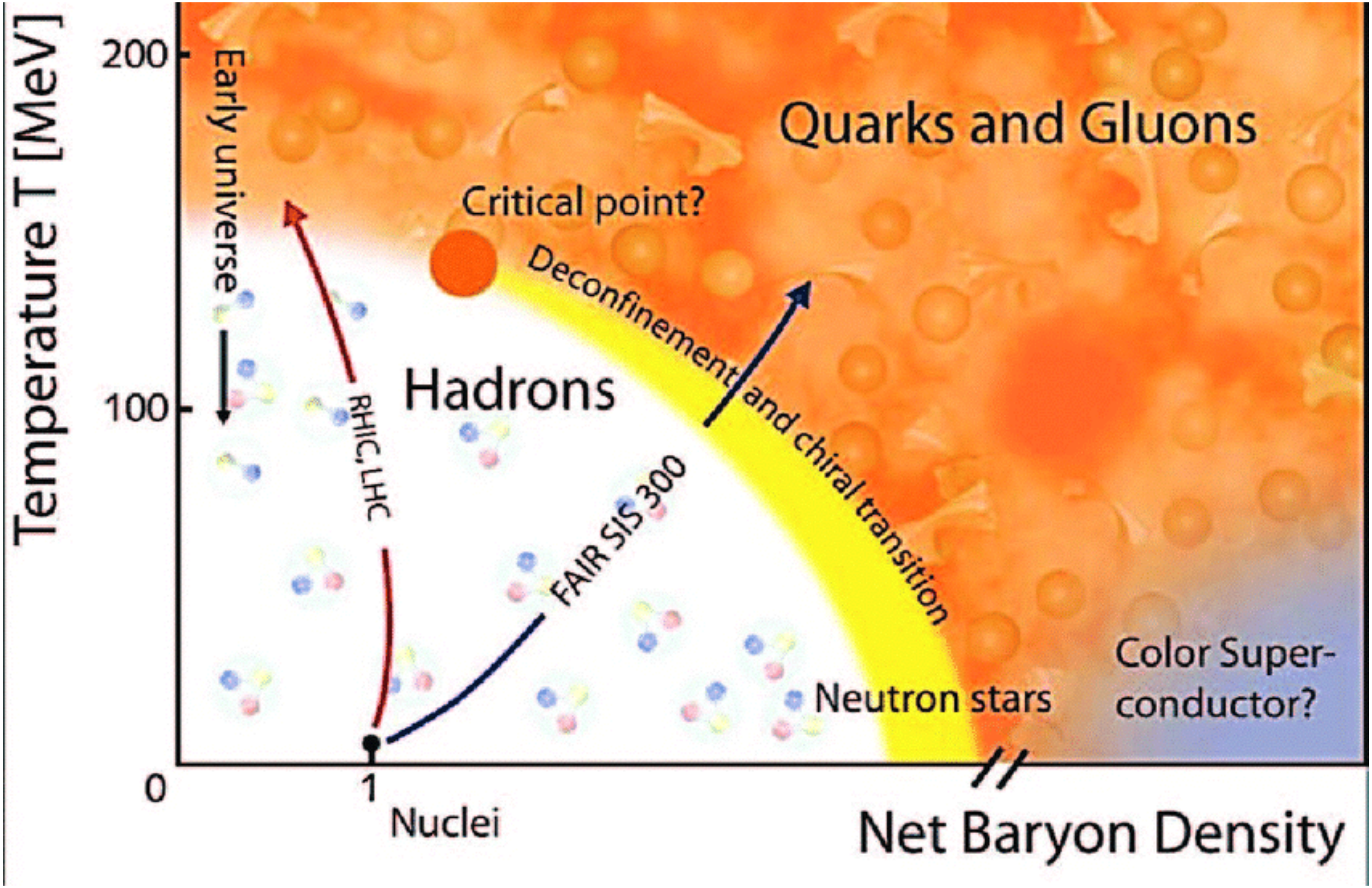
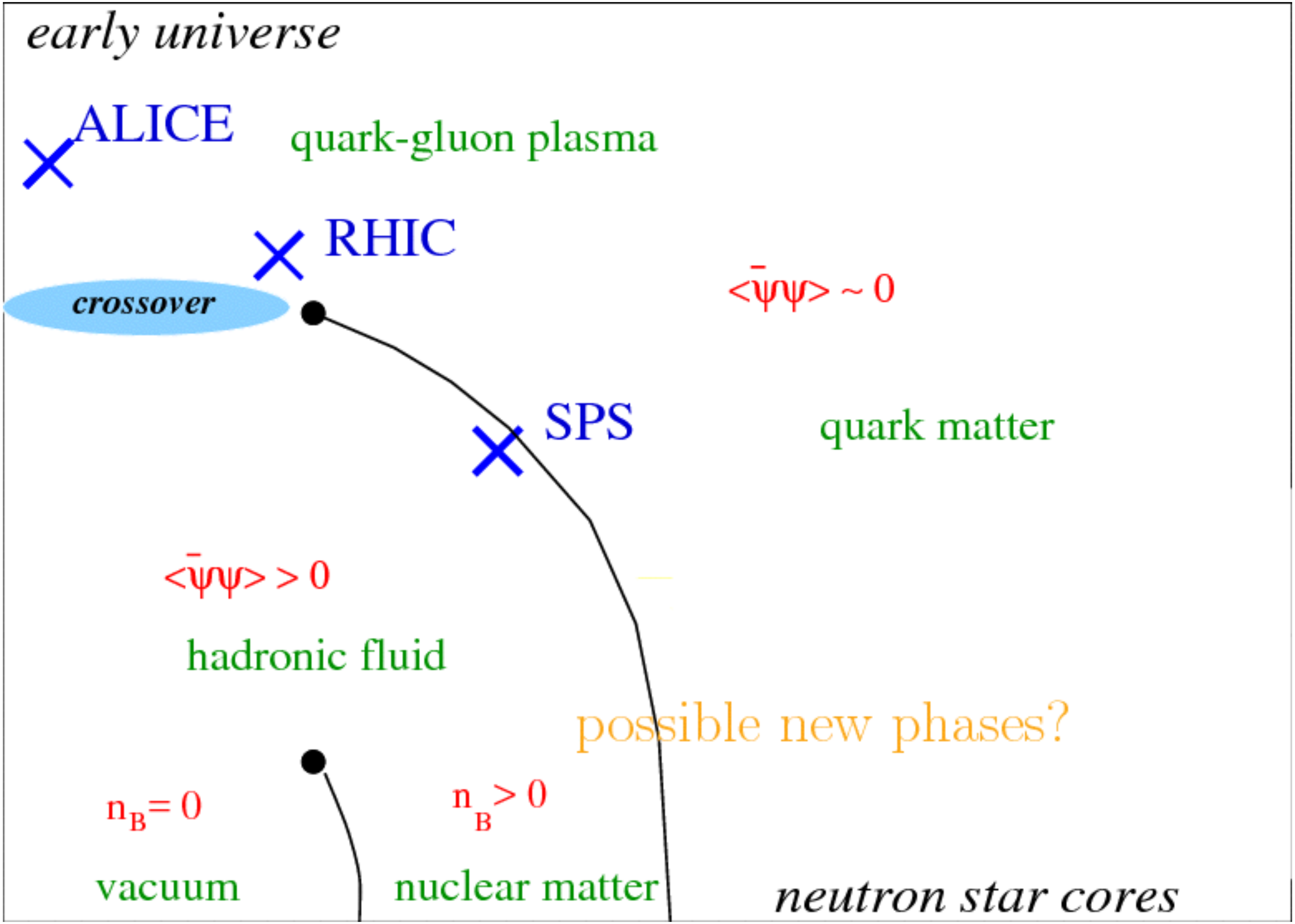


FIG. 2. Examples of runaway cases are displayed, with same $v_w = 1$, $\alpha = 1$, $g_* \simeq g_{\text{SM}}$, $\beta/H_* = 10$ and $T_*/(10^8\,\text{GeV}) = \{0.5, 1, 2, 5\}$ in red, blue, green, orange lines, respectively.

QCD phase transition

QCD phase transitions




early universe
ALICE
quark-gluon plasma
RHIC
crossover
$\langle\bar{\psi}\psi\rangle \sim 0$
SPS
quark matter
$\langle\bar{\psi}\psi\rangle > 0$
hadronic fluid
possible new phases?
$n_B = 0$
$n_B > 0$
vacuum
nuclear matter
neutron star cores

Not clear if of the first order or second order…

Time crystals

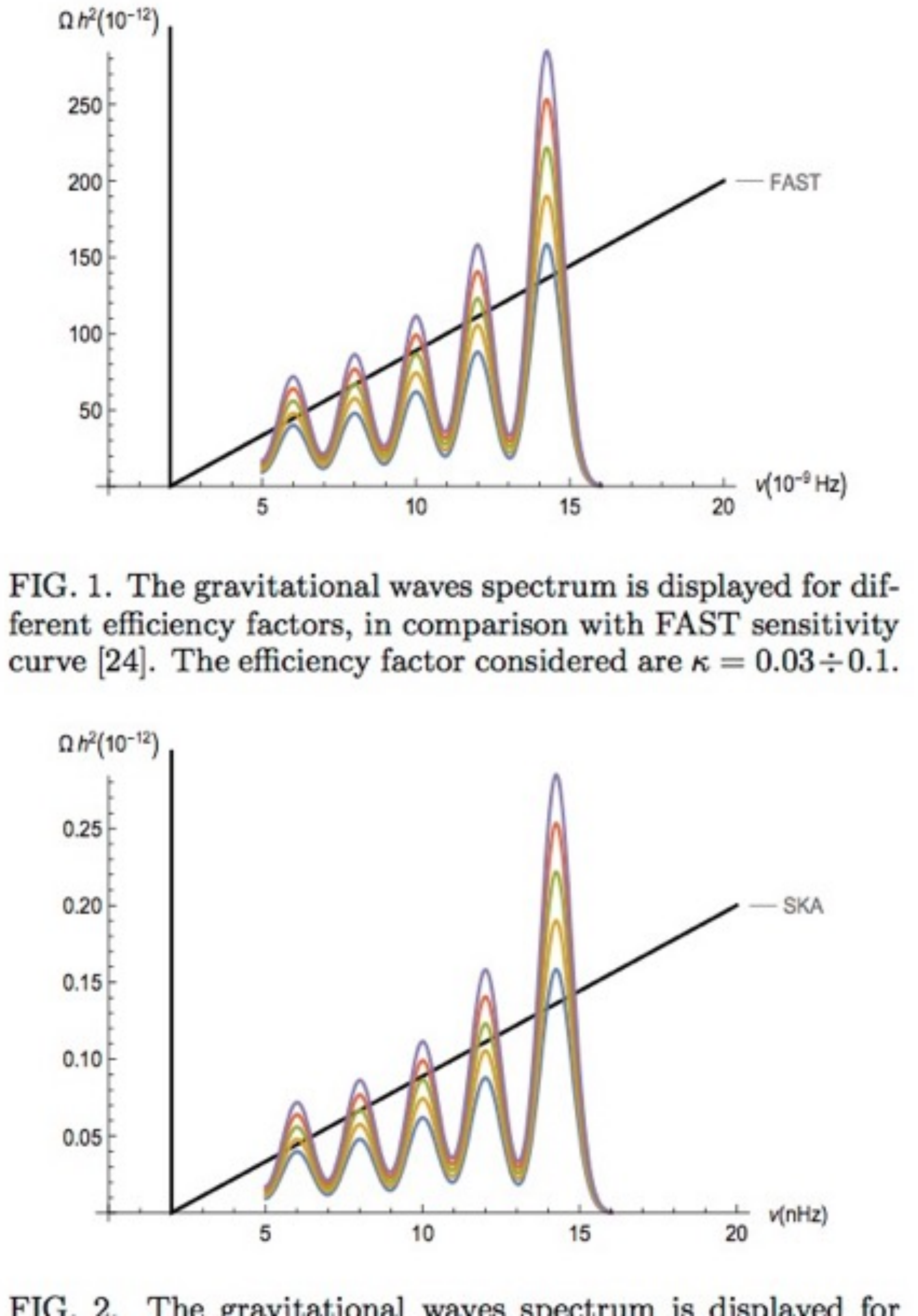


FIG. 1. The gravitational waves spectrum is displayed for different efficiency factors, in comparison with FAST sensitivity curve [24]. The efficiency factor considered are $\kappa = 0.03 \div 0.1$.

FIG. 2. The gravitational waves spectrum is displayed for

Conclusions

GW stochastic background as a possible cosmological probe of new physics beyond the standard model

electroweak phase transition

dark phase transition

QCD phase transition not completely understood

space interferometers, terrestrial interferometers and radio-astronomy for scrutinizing different ranges

Thanks for the attention!