Dark Phase Transition and Gravitational Wave of Strongly Coupled Hidden Sectors

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Motivations and what we do

- (Dark) composite dynamics: non perturbative physics, dynamical symmetry breaking, UV completion, naturalness
- (Dark) composite dynamics face challenges to be explored both theoretically and via experiments
- We unify first principle lattice simulations and gravitational wave astronomy to constrain the dark sector

What composes the strongly coupled sector?

- Dark Yang-Mills theories
- Pure gluons $\Rightarrow$ confinement-deconfinement phase transition
- Gluons + Fermions
  - Fermions in fundamental representation $\Rightarrow$ chiral phase transition
  - Fermions in adjoint representation $\Rightarrow$ confinement phase transition
  - Fermions in 2-index symmetric representation $\Rightarrow$ confinement phase transition

How to describe the strongly coupled sector?

- Pure gluons
  - Polyakov loop model (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)
  - Matrix Model (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
  - Holographic QCD model
- Gluons + Fermions
  - Polyakov loop improved Nambu-Jona-Lasinio model (Reichert, Sannino, Z-W W and Zhang, arXiv:2109.11552.)
  - linear sigma model
  - Polyakov Quark Meson model
  - Polyakov loop improved linear sigma model

Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory.
- In a local $SU(N)$ gauge theory, a global center symmetry $Z(N)$ is used to distinguish confinment phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the thermal Wilson line:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) \, \mathrm{d}\tau \right]$$

  The symbol $\mathcal{P}$ denotes path ordering and A_4 is the Euclidean temporal component of the gauge field
- The thermal Wilson line transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- The Polyakov loop as an order parameter for the $Z(N)$ symmetry is defined as the trace of the thermal Wilson line:

$$\ell(\vec{x}) = \frac{1}{N}\mathrm{Tr}_c[\mathbf{L}],$$

  where Tr_c denotes the trace in the colour space.
- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \to e^{i\phi}\ell, \qquad \phi = \frac{2\pi k}{N}, \qquad j = 0, 1, \cdots, (N-1)$$

- The expectation value of ℓ i.e. $< \ell >$ has the important property:

$$\langle \ell \rangle = 0 \quad (T < T_c)\,; \qquad \langle \ell \rangle > 0 \quad (T > T_c)$$

- At very high temperature, the vacua exhibit a $N-$fold degeneracy:

$$\langle \ell \rangle = \exp\left(i\frac{2\pi j}{N}\right)\ell_0, \qquad j = 0, 1, \cdots, (N-1)$$

  where ℓ_0 is defined to be real and $\ell_0 \to 1$ as $T \to \infty$

Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by

$$V_{\rm PLM}^{(\rm poly)} = T^4 \left(-\frac{b_2(T)}{2}|\ell|^2 + b_4|\ell|^4 - b_3\left(\ell^N + \ell^{*N}\right) \right)$$

$$\text{where} \quad b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 + a_4\left(\frac{T_0}{T}\right)^4$$

- For the $SU(3)$ case, there is also an alternative logarithmic form

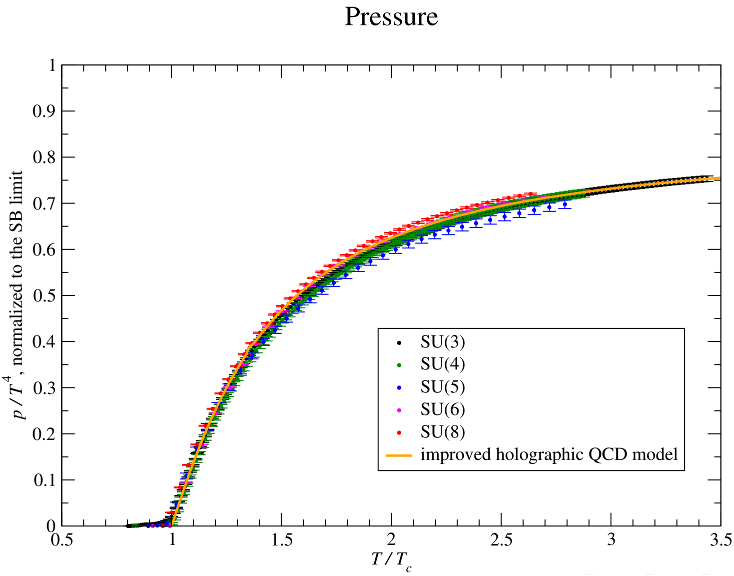
$$V_{\rm PLM}^{(\rm 3log)} = T^4 \left(-\frac{a(T)}{2}|\ell|^2 + b(T)\ln\left(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4\right) \right)$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3, \quad b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

- The a_i, b_i coefficients in $V_{\text{PLM}}^{(\text{poly})}$ and $V_{\text{PLM}}^{(\text{3log})}$ are determined by fitting the lattice results

Fitting the Coefficients Using the Lattice Results: I

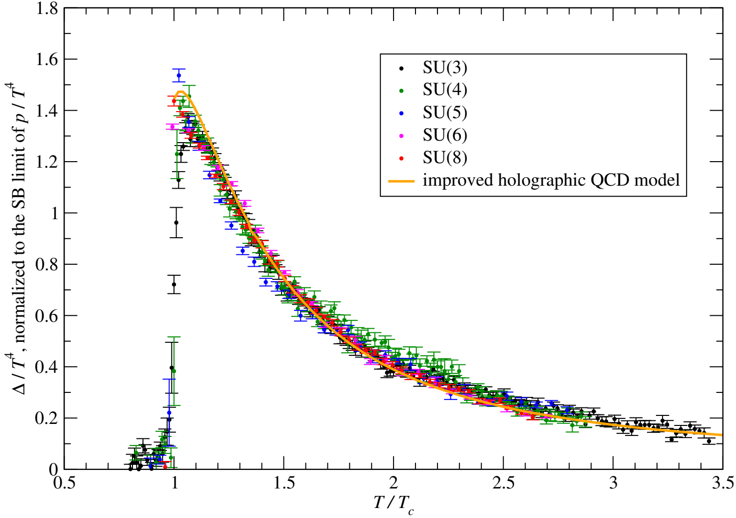
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001

Trace of the energy-momentum tensor



Fitting the Coefficients Using the Lattice Results: III

Table: The parameters for the best-fit points.

N	3	$3\ \log$	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

Sample of PLM Potential

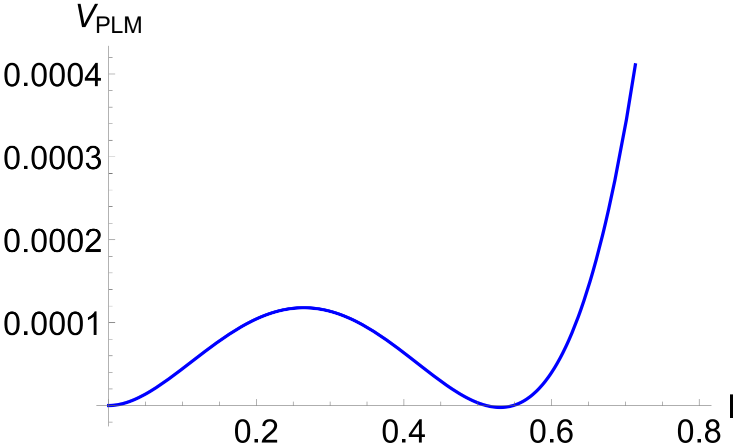


Figure: PLM potential at $T = T_c$

Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model

- The PNJL model can be used to describe phase-transition dynamics in dark gauge-fermion sectors (beyond pure gluon case)
- The finite-temperature grand potential of the PNJL models can be generically written as

$$V_{\text{PNJL}} = V_{\text{PLM}}[\ell, \ell^*] + V_{\text{cond}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right] + V_{\text{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$$

- $V_{\text{PLM}}[\ell, \ell^*]$ is the Polyakov loop model potential (discussed above)
- $V_{\text{cond}}\left[\langle\bar{\psi}\psi\rangle\right]$ represents the condensate energy
- $V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right]$ denotes the fermion zero-point energy
- The medium potential $V_{\text{medium}}\left[\langle\bar{\psi}\psi\rangle, \ell, \ell^*\right]$ encodes the interactions between the chiral and gauge sector which arises from an integration over the quark fields coupled to a background gauge field

The PNJL model Lagrangian

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\mathrm{PNJL}} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{\mathrm{4F}} + \mathcal{L}_{\mathrm{6F}} + \mathcal{L}_k$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{\mathrm{4F}}$ is the four-quark interaction which reads:

$$\mathcal{L}_{\mathrm{4F}} = G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma^5\lambda^a\psi)^2], \qquad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{\mathrm{6F}}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{\mathrm{6F}} = G_D[\det(\bar{\psi}_{Li}\psi_{Rj}) + \det(\bar{\psi}_{Ri}\psi_{Lj})]$$

The Condensate Energy of PNJL Model

- In $\mathcal{L}_{\rm 4F}$, the condensate energy then comes from the combination

$$(\bar{\psi}\lambda^0\psi)^2 + (\bar{\psi}\lambda^3\psi)^2 + (\bar{\psi}\lambda^8\psi)^2 = 2(\bar{u}u)^2 + 2(\bar{d}d)^2 + 2(\bar{s}s)^2$$

- We use the trick is to rewrite $(\bar{u}u)^2$ as

$$\begin{aligned}(\bar{u}u)^2 &= \left[(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle\right]^2 = (\bar{u}u - \langle\bar{u}u\rangle)^2 + 2\langle\bar{u}u\rangle\,(\bar{u}u - \langle\bar{u}u\rangle) + \langle\bar{u}u\rangle^2 \\ &\simeq -\langle\bar{u}u\rangle^2 + 2\langle\bar{u}u\rangle\bar{u}u\,,\end{aligned}$$

  where the $(\bar{u}u - \langle\bar{u}u\rangle)^2$ term is dropped in the spirit of the mean-field approximation.
- The $2\langle\bar{u}u\rangle\bar{u}u$ term contributes to the constituent quark mass of u
- The $-\langle\bar{u}u\rangle^2$ term leads to a contribution to the condensate energy
- Similar procedures can be applied to $(\bar{d}d)^2$ and $(\bar{s}s)^2$, and to $\mathcal{L}_{\rm 6F}$ leading to the total condensate energy:

$$V_{\rm cond} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3\,, \qquad \sigma \equiv \langle\bar{u}u\rangle = \langle\bar{d}d\rangle = \langle\bar{s}s\rangle = \frac{1}{3}\langle\bar{\psi}\psi\rangle$$

The Constituent Quark Mass and Zero Point Energy: I

- The $2\langle \bar{u}u \rangle \bar{u}u$ term represents a mass correction by the mean-field interaction and contributes to the constituent quark mass of u
- Together with additional mass corrections from the KMT interaction, the constituent quark mass is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\text{zero}}\left[\langle \bar{\psi}\psi \rangle\right] = -\dim(\mathrm{R})\, 2N_f \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p\,, \qquad E_p = \sqrt{\vec{p}^{\,2} + M^2}$$

  E_p is the energy of a free quark with constituent mass M and three-momentum $\vec{p}$

- The above momentum integration does not converge and we need to introduce a regularization for the momentum integration. Here we choose a sharp three-momentum cutoff Λ, which enters the expression for observables and is thus also a parameter of the theory.

The Constituent Quark Mass and Zero Point Energy: II

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}\left[\langle\bar{\psi}\psi\rangle\right] = -\frac{\dim(\mathrm{R})N_f\Lambda^4}{8\pi^2}\left[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right],$$

in which $\xi \equiv \frac{M}{\Lambda}$.

Medium Potential: Finite Temperature Contribution

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\mathrm{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \left(\ln \left[1 + e^{-\beta(E-\mu)} \right] + \ln \left[1 + e^{-\beta(E+\mu)} \right] \right)$$

$$\rightarrow -2T \sum_{u,d,s} \int \frac{d^3p}{(2\pi)^3} \, \mathrm{Tr}_c \left\{ \left(\ln \left[1 + \mathbf{L}\, e^{-\beta(E-\mu)} \right] + \ln \left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)} \right] \right) \right\}$$

- $\mathbf{L}$ is the thermal Wilson line:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) \, \mathrm{d}\tau \right]$$

Bubble Nucleation: Generic Discussion

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

- The three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r \, r^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\rho}{\mathrm{d}r} \right)^2 + V_{\text{eff}}(\rho, T) \right] ,$$

where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$

Bubble Nucleation: Confinement Phase Transition

- Confinement phase transition occurs for pure gluon case as well as including adjoint fermions
- Since ℓ is dimensionless while ρ in above has mass dimension one, we rewrite the scalar field as $\rho = \ell\, T$ and convert the radius into a dimensionless quantity $r' = r\, T$:

$$S_3(T) = 4\pi T \int_0^\infty \mathrm{d}r'\, r'^2 \left[\frac{1}{2} \left(\frac{\mathrm{d}\ell}{\mathrm{d}r'} \right)^2 + V'_{\text{eff}}(\ell, T) \right] ,$$

which has the same form as the above generic equation.

- The bubble profile (instanton solution) is obtained by solving the E.O.M. of the $S_3(T)$

$$\frac{\mathrm{d}^2\ell(r')}{\mathrm{d}r'^2} + \frac{2}{r'} \frac{\mathrm{d}\ell(r')}{\mathrm{d}r'} - \frac{\partial V'_{\text{eff}}(\ell, T)}{\partial \ell} = 0$$

- The boundary conditions (deconfinement $\rightarrow$ confinement) are

$$\frac{\mathrm{d}\ell(r'=0, T)}{\mathrm{d}r'} = 0\, , \qquad \lim_{r'\to 0} \ell(r', T) = 0$$

- We used the method of overshooting/undershooting (`Python` package)

Bubble Profile of Confinement Phase Transition

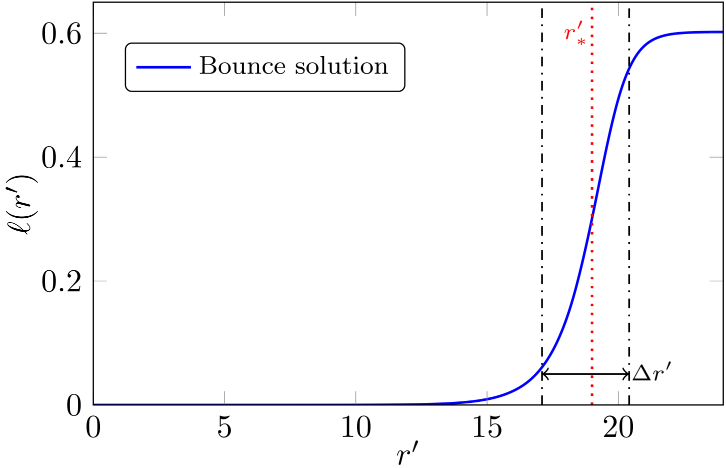


Figure: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$), the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$), the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Bubble Nucleation: Chiral Phase Transition

- Chiral phase transition occurs for case including fermions in fundamental representation
- Since $\bar{\sigma}$ is not a fundamental field, we have to include its wave-function renormalization Z_σ with $Z_\sigma^{-1} = -\frac{\mathrm{d}\Gamma_{\sigma\sigma}(q^0,\mathbf{q},\bar{\sigma})}{\mathrm{d}\mathbf{q}^2}\Big|_{q^0=0,\mathbf{q}^2=0}$

  M. Reichert, F. Sannino, Z. W. Wang and C. Zhang, arXiv:2109.11552.
- The three-dimensional Euclidean action is slightly modified to:

$$S_3(T) = 4\pi \int_0^\infty \mathrm{d}r\, r^2 \left[\frac{Z_\sigma^{-1}}{2} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} \right)^2 + V_{\text{eff}}(\bar{\sigma}, T) \right]$$

- The bubble profile is obtained by solving the E.O.M. of the action:

$$\frac{\mathrm{d}^2\bar{\sigma}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} - \frac{1}{2}\frac{\partial \log Z_\sigma}{\partial \bar{\sigma}} \left(\frac{\mathrm{d}\bar{\sigma}}{\mathrm{d}r} \right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{\mathrm{d}\bar{\sigma}(r=0,T)}{\mathrm{d}r} = 0\,, \qquad \lim_{r\to\infty} \bar{\sigma}(r,T) = 0$$

Gravitational Wave Parameters: Inverse Duration Time

- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use percolation temperature T_p: the temperature at which the probability to have the false vacuum is about 0.7.
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*) e^{\beta(t-t_*)}$$

- The inverse duration time then follows as

$$\beta = -\frac{\mathrm{d}}{\mathrm{d}t} \frac{S_3(T)}{T} \bigg|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T \frac{\mathrm{d}}{\mathrm{d}T} \frac{S_3(T)}{T} \bigg|_{T=T_*},$$

where we used that $\mathrm{d}T/\mathrm{d}t = -H(T)T$.

Gravitational Wave Parameters: Strength Parameter I

- We define the strength parameter α from the trace of the energy-momentum tensor θ weighted by the enthalpy

$$\alpha = \frac{1}{3}\frac{\Delta\theta}{w_+} = \frac{1}{3}\frac{\Delta e \; - \; 3\Delta p}{w_+}\,, \qquad \Delta X = X^{(+)} - X^{(-)}, \text{ for } X = (\theta, e, p)$$

  $(+)$ denotes the meta-stable phase (outside of the bubble) while $(-)$ denotes the stable phase (inside of the bubble).

- The relations between enthalpy w, pressure p, and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}\,, \qquad e = \frac{\partial p}{\partial \ln T} - p\,,$$

  which are extracted from the effective potential with

$$p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

Gravitational Wave Parameters: Strength Parameter II

- α is thus given by

$$\alpha = \frac{1}{3} \frac{4\Delta V_{\text{eff}} - T\frac{\partial \Delta V_{\text{eff}}}{\partial T}}{-T\frac{\partial V_{\text{eff}}^{(+)}}{\partial T}} \, ,$$

- For confinement phase transition: $\alpha \approx 1/3$ (ΔV_{eff} is negligible since $e_+ \gg p_+$ and $e_- \sim p_- \sim 0$ in PLM potential)
- For chiral phase transition: we find smaller values, $\alpha \sim \mathcal{O}(10^{-2})$, due to the fact that more relativistic d.o.f.s participate in the phase transition
- Relativistic SM d.o.f.s do not contribute to our definition of α since they are fully decoupled from the phase transition but these d.o.f.s will play a role to dilute the GW signals

GW parameters α, β and PNJL observables

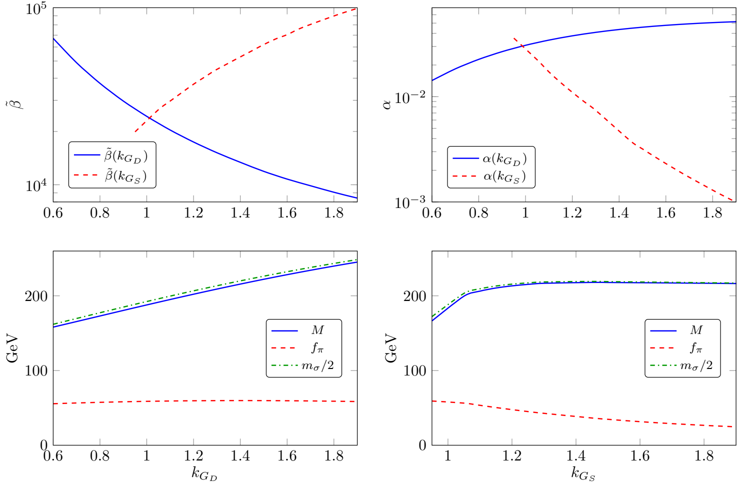


Figure: The GW parameters $\tilde{\beta}$, α with the observables M, f_π, and m_σ as a function of $G_S = k_{G_S} \cdot 4.6\,\text{GeV}^{-2}$ and $G_D = k_{G_D} \cdot (-743\,\text{GeV}^{-5})$. We use $T_c = 100\,\text{GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S,\text{crit}} = 0.882$, no chiral symmetry breaking occurs.

Gravitational-wave spectrum

- The contributions from bubble collision and turbulence in the plasma are subleading compared with sound waves
- The GW spectrum from sound waves is given by

$$h^2\Omega_{\text{GW}}(f) = h^2\Omega_{\text{GW}}^{\text{peak}}\left(\frac{f}{f_{\text{peak}}}\right)^3\left[\frac{4}{7}+\frac{3}{7}\left(\frac{f}{f_{\text{peak}}}\right)^2\right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\text{peak}} \simeq 1.9\cdot 10^{-5}\,\text{Hz}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T}{100\,\text{GeV}}\right)\left(\frac{\tilde{\beta}}{v_w}\right)$$

- The peak amplitude

$$h^2\Omega_{\text{GW}}^{\text{peak}} \simeq 2.65\cdot 10^{-6}\left(\frac{v_w}{\tilde{\beta}}\right)\left(\frac{\kappa_{sw}\,\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Omega_{\text{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the visible SM matter which does not participate in the phase transition.

$$\Omega_{\text{dark}} = \frac{\rho_{\text{rad,dark}}}{\rho_{\text{rad,tot}}} = \frac{g_{*,\text{dark}}}{g_{*,\text{dark}}+g_{*,\text{SM}}}$$

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{sw} consist of the factor κ_v as well as an additional suppression due to the length of the sound-wave period τ_{sw}

$$\kappa_{\text{sw}} = \sqrt{\tau_{\text{sw}}}\,\kappa_v$$

- τ_{sw} is dimensionless and measured in units of the Hubble time

$$\tau_{\text{sw}} = 1 - 1/\sqrt{1 + 2\frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f}} \Rightarrow \tau_{\text{sw}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta}\,\bar{U}_f} \quad \text{for}\,\beta >> 1$$

where $\bar{U}_f$ is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} \mathrm{d}\xi\,\xi^2 \frac{v(\xi)^2}{1 - v(\xi)^2} \simeq \frac{3}{4}\frac{\alpha}{1+\alpha}\kappa_v$$

- τ_{sw} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w. At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

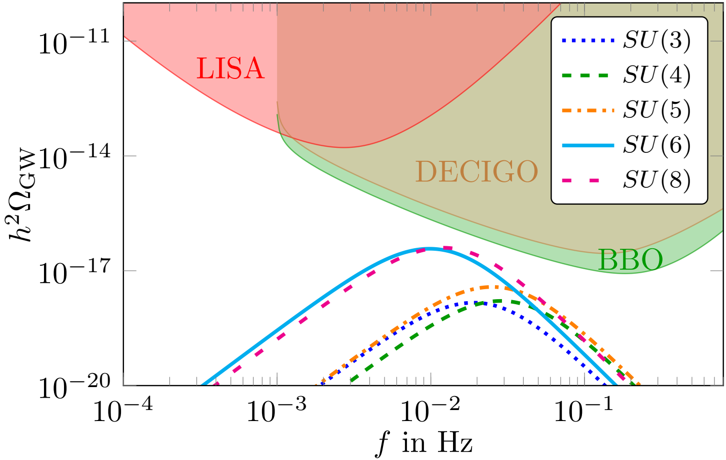


Figure: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with Tc$= 1\,\mathrm{GeV}$.

A Landscape of GW Signatures with Pure Gluon

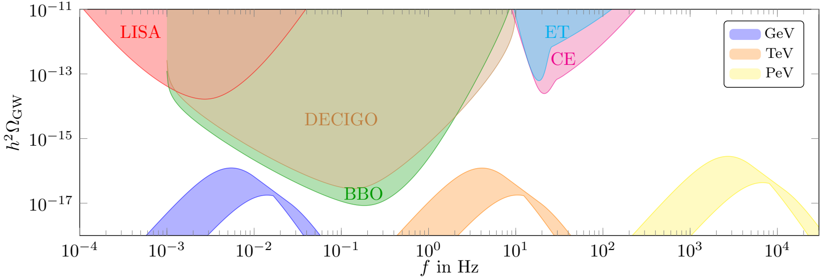


Figure: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Landscape of GW spectrum with three Dirac fermions

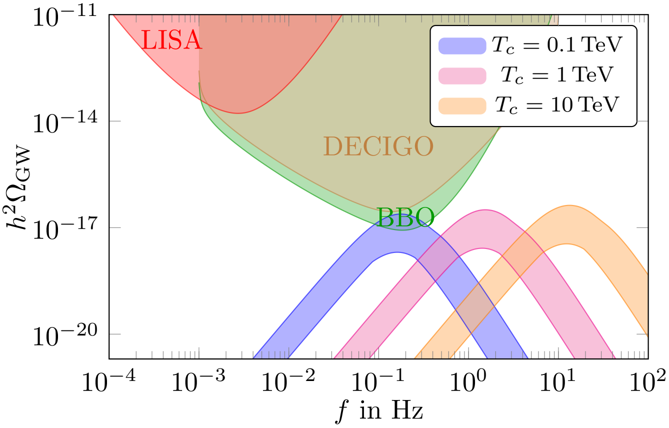


Figure: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures. The dark sector and visible sector are thermalized in the very early universe but decouple sufficiently prior to the CMB epoch (before the electroweak scale $T_{\rm ew}$ and the chiral phase transition should happen even before the decoupling).

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