Detecting Gravitational Waves with the LHC

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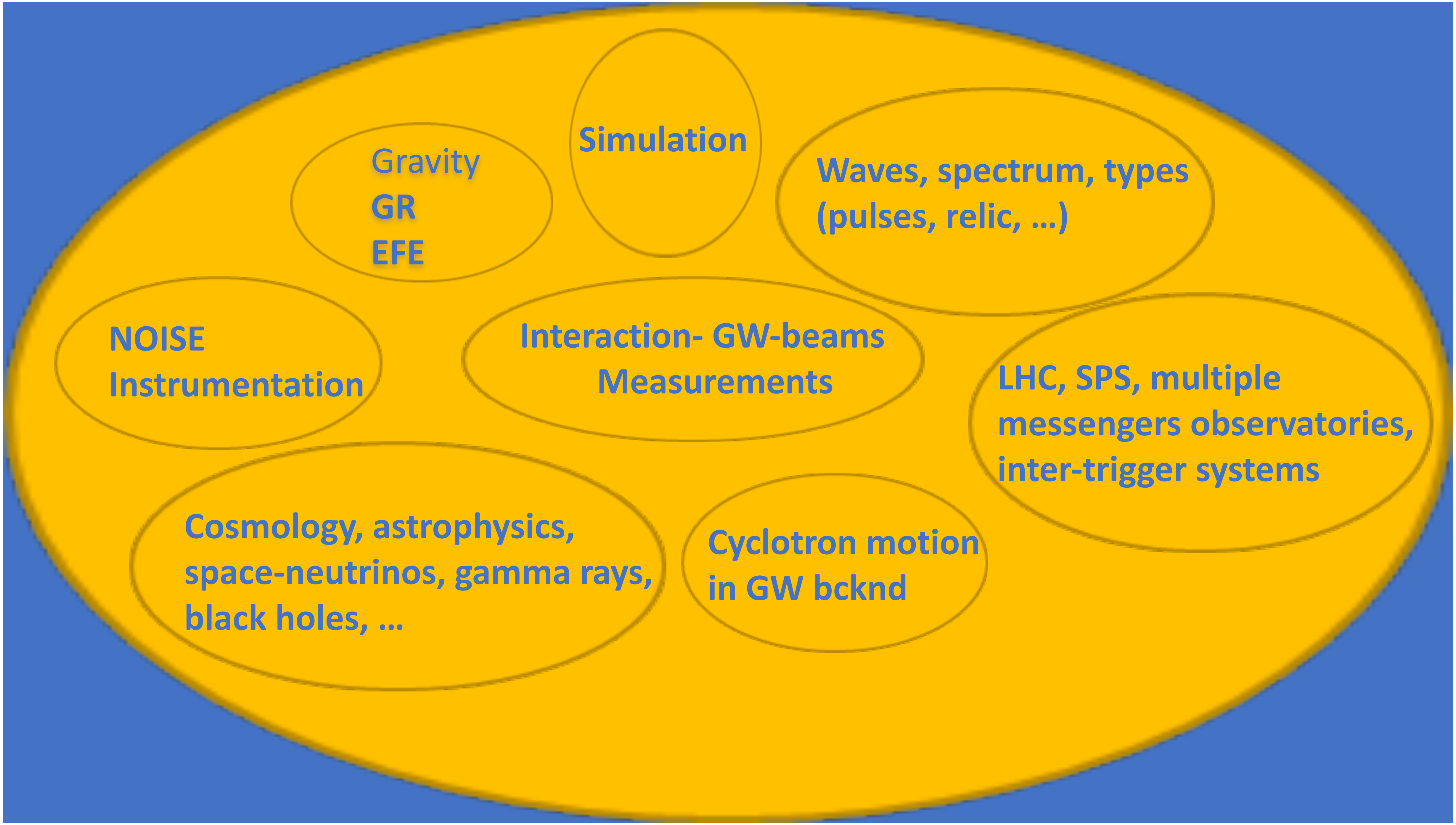
Outlook

- ➢Introduction
- ➢Maxwell's Waves
- ➢Newton-Gauss gravity
- ➢General Relativity Field Equations
- ➢Einstein's Gravitational Waves
- ➢Hulse-Taylor pulsar and orbit decay
- ➢GWs spectrum
- ➢Direct observation of GWs by LIGO
- ➢Cyclotron motion (one particle) in a GW background
- ➢Interaction of a GW with a relativistic circular beam in a storage ring
- ➢LHC Instrumentation, orbit reconstruction, event definition, trigger and storing
- ➢ Highlights ARIES APEC 2021 workshop on "Storage Rings and GWs"
- ➢Conclusion


Gravity
GR
EFE
Simulation
Waves, spectrum, types
(pulses, relic, …)
NOISE
Instrumentation
Interaction- GW-beams
Measurements
LHC, SPS, multiple
messengers observatories,
inter-trigger systems
Cosmology, astrophysics,
space-neutrinos, gamma rays,
black holes, …
Cyclotron motion
in GW bcknd

Maxwell’s Wave Prototype

1) Generation,
2) Propagation, and,
3) Detection

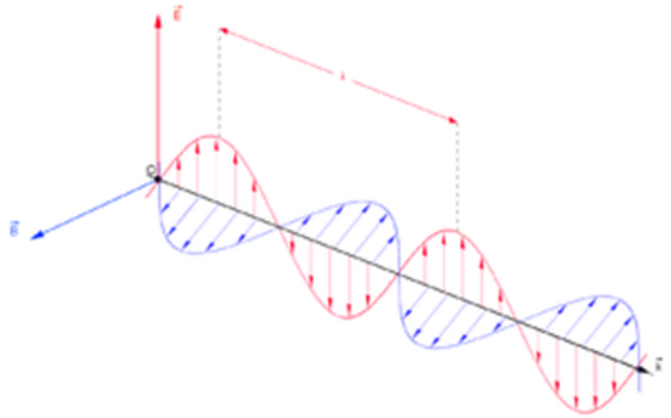
of a dynamic disturbance (change from equilibrium) of one or more quantities, described by a wave equation, that
transfers energy, momentum, angular momentum, and,
information about the generating process. And, it does not get tired!

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$ Gauss's law

$$\nabla \cdot \mathbf{B} = 0$$ Gauss's law for magnetism

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ Faraday's law of induction

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$ Ampère's law



Vacuum Maxwell’s equations

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

$$\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

E and **B** coupled Waves
EM - Wave

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} = 0$$

$$\frac{1}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} = 0$$

Compact notation for the wave equation using the D'Alembert operator (u = u(t, x, y, z)):

$$\Box u = 0,$$

$$\Box = \partial^\mu \partial_\mu = g^{\mu\nu}\partial_\nu \partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$
$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \ .$$

With inverse signature between the time (00) and space (ii) components of the Minkowski metric tensor
-> Time and space became spacetime, $\mathrm{diag}(+, - , \ - , \ -)$

$$\begin{cases} g_{00} = 1 & \\ g_{11} = g_{22} = g_{33} = -1 & \\ g_{\mu\nu} = 0 & \text{for } \mu \neq \nu. \end{cases}$$

In one dimension, with initial conditions

$$f(x) = u(x, 0) \ \text{ and } \ g(x) = \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0},$$

the solution is given by

$$u(x,t) = \frac{f(x-ct) + f(x+ct)}{2} + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\, ds$$

Covariant formulation of classical electromagnetism

$$x^\alpha = (ct, \mathbf{x}) = (ct, x, y, z) \ .$$
$$u^\alpha = \gamma(c, \mathbf{u}), \quad \gamma = \frac{dt}{d\tau}$$
$$p^\alpha = (E/c, \mathbf{p}) = m_0 u^\alpha$$
$$\partial^\nu = \frac{\partial}{\partial x_\nu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right),$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$J^\alpha = (c\rho, \mathbf{j}) \ . \qquad A^\alpha = (\phi/c, \mathbf{A}) \ . \qquad F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \ .$$

$$\boxed{\partial_\alpha F^{\alpha\beta} = \mu_0 J^\beta, \qquad \partial_\alpha\left(\tfrac{1}{2}\epsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}\right) = 0}$$

Maxwell's equations Lorentz covariant representation

$$\boxed{\partial^\nu \partial_\nu F^{\alpha\beta} \stackrel{\text{def}}{=} \partial^2 F^{\alpha\beta} \stackrel{\text{def}}{=} \frac{1}{c^2}\frac{\partial^2 F^{\alpha\beta}}{\partial t^2} - \nabla^2 F^{\alpha\beta} = 0,}$$

EM - Wave

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad F^{\mu\nu} \stackrel{\text{def}}{=} \eta^{\mu\alpha}\, F_{\alpha\beta}\, \eta^{\beta\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}. \qquad \gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{v_x^2+v_y^2+v_z^2}{c^2}}}$$

$$T^{\alpha\beta} = \begin{pmatrix} \epsilon_0 E^2/2 + B^2/2\mu_0 & S_x/c & S_y/c & S_z/c \\ S_x/c & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y/c & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z/c & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{pmatrix}, \qquad T^{\alpha\beta} = \frac{1}{\mu_0}\left(\eta^{\alpha\nu} F_{\nu\gamma} F^{\gamma\beta} + \frac{1}{4}\eta^{\alpha\beta} F_{\gamma\nu} F^{\gamma\nu}\right)$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \qquad \sigma_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2\right)\delta_{ij}\,.$$

$$\frac{\mathrm{d}p^\alpha}{\mathrm{d}\tau} = qF^{\alpha\beta}U_\beta \qquad p^\alpha = (p_0, p_1, p_2, p_3) = (\gamma mc, p_x, p_y, p_z)\,, \quad U_\beta = (U_0, U_1, U_2, U_3) = \gamma\,(c, -v_x, -v_y, -v_z)\,,$$

Newton-Gauss Gravity

Gauss’s gravity is mathematically similar to Gauss's electrostatics, one of Maxwell's equations,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

Gauss's law for gravity has the same mathematical relation to Newton's law of gravity as Gauss's law for electrostatics bears to Coulomb's law. This is because both Newton's law and Coulomb's law describe inverse-square interaction in a 3-dimensional space. Gauss’s gravitational field **g** (also called gravitational acceleration) is a vector field, **F**=m**g**, and,

$$\nabla \cdot \mathbf{g} = -4\pi G\rho, \quad G = 6{,}674\,30(15) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$$

With the gravitational potential given by

$$\mathbf{g} = -\nabla\phi.$$

and,

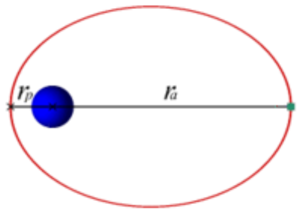
$$\nabla^2\phi = 4\pi G\rho.$$

In radially symmetric systems, the gravitational potential is a function of only one variable, r,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = 4\pi G\rho(r)$$

$$\mathbf{g}(\mathbf{r}) = -\mathbf{e}_\mathbf{r}\frac{\partial\phi}{\partial r}.$$

$$\mathbf{g}(\mathbf{r}) = -GM\frac{\mathbf{e}_\mathbf{r}}{r^2}$$

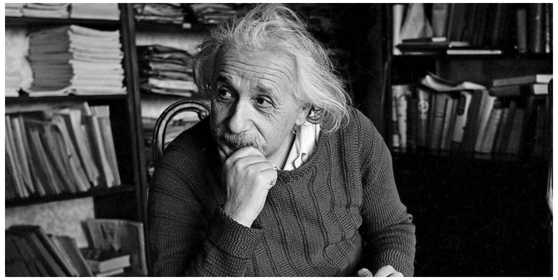


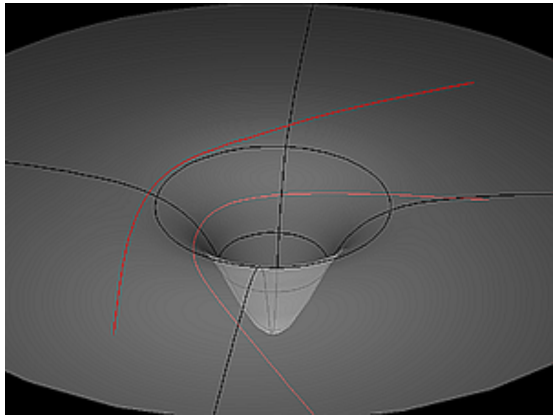
Gauss’s is a weak gravitational field and slow motion approximation.

GR-EFE generalize to gravity in extreme situations in which there are new predicted phenomena, such as gravitational waves, that transport energy as gravitational radiation, a form of radiant energy, similar to electromagnetic radiation.

GR-EFE

$$\boxed{G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}} \qquad \kappa = \frac{8\pi G}{c^4} \approx 2.077 \times 10^{-43}\ \mathrm{N}^{-1},$$





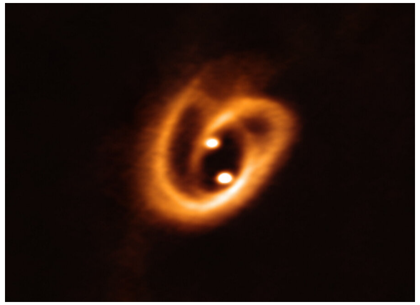
$$R = g^{\mu\nu} R_{\mu\nu}$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}.$$

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}{}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}{}_{\mu\sigma} + \Gamma^{\rho}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\sigma} - \Gamma^{\rho}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\sigma}$$

$$\Gamma^{i}{}_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) = \frac{1}{2} g^{im} \left(g_{mk,l} + g_{ml,k} - g_{kl,m} \right),$$

$$A_{\nu;\rho\sigma} - A_{\nu;\sigma\rho} = A_\beta R^{\beta}{}_{\nu\rho\sigma},$$



$$U_f(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{mc^2 r^3}$$

$$F_f(r) = -\frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{3GML^2}{mc^2 r^4}$$

$$\boxed{\frac{d^2 x^\mu}{ds^2} + \Gamma^{\mu}{}_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0,}$$

The problem of the particle motion is
the same as of the geodesic line equation
in the curved or flat spacetime
for which the metric solves the EFE.

Rate of orbital decay of binary stars

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{64}{5} \frac{G^3}{c^5} \frac{(m_1 m_2)(m_1 + m_2)}{r^3},$$

Time to merger

$$t = \frac{5}{256} \frac{c^5}{G^3} \frac{r^4}{(m_1 m_2)(m_1 + m_2)}.$$

Einstein's waves

Weak-field approximation of the EFE,
linearized gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

luminosity -> gravitosity

pp-wave spacetimes, or pp-waves for short, are an important family of exact solutions of Einstein's field equation.
pp stands for plane-fronted waves with parallel propagation,

$u = u(ct - z)$, $v = v(ct + z)$

$$ds^2 = H(u,x,y)\,du^2 + 2\,du\,dv + dx^2 + dy^2$$

$$H_{xx} + H_{yy} = 0$$

$$H(u,x,y) = a(u)\,(x^2 - y^2) + 2\,b(u)\,xy + c(u)\,(x^2 + y^2)$$

a(u) and b(u) are the wave profiles of the two linearly independent polarization modes of gravitational radiation which may be present, while c(u) is the wave profile of any nongravitational radiation. If c(u) = 0, H(u,x,y) represents the vacuum plane waves, which are often called plane gravitational waves.

$$\begin{aligned}\Delta\tau &= \int_P \frac{1}{c}\sqrt{\eta_{\mu\nu}\,dx^\mu\,dx^\nu} \\ &= \int_P \sqrt{dt^2 - \frac{dx^2}{c^2} - \frac{dy^2}{c^2} - \frac{dz^2}{c^2}} \\ &= \int \sqrt{1 - \frac{1}{c^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}\,dt \\ &= \int \sqrt{1 - \frac{v(t)^2}{c^2}}\,dt = \int \frac{dt}{\gamma(t)},\end{aligned}$$

The rotating disk I, SR

$$(T,\ r\cos(\omega T),\ r\sin(\omega T),\ 0)$$

$$dy = r\omega\cos(\omega T)\,dT \qquad dx = -r\omega\sin(\omega T)\,dT$$

$$d\tau = \sqrt{dT^2 - \left(\frac{r\omega}{c}\right)^2 \sin^2(\omega T)\,dT^2 - \left(\frac{r\omega}{c}\right)^2 \cos^2(\omega T)\,dT^2} = dT\sqrt{1 - \left(\frac{r\omega}{c}\right)^2}.$$

The rotating disk II, GR

$$r = \sqrt{x^2 + y^2} \qquad \theta = \arctan\left(\frac{y}{x}\right) - \omega t.$$

$$d\tau = \sqrt{\left[1 - \left(\frac{r\omega}{c}\right)^2\right]dt^2 - \frac{dr^2}{c^2} - \frac{r^2\,d\theta^2}{c^2} - \frac{dz^2}{c^2} - 2\frac{r^2\omega\,dt\,d\theta}{c^2}}.$$

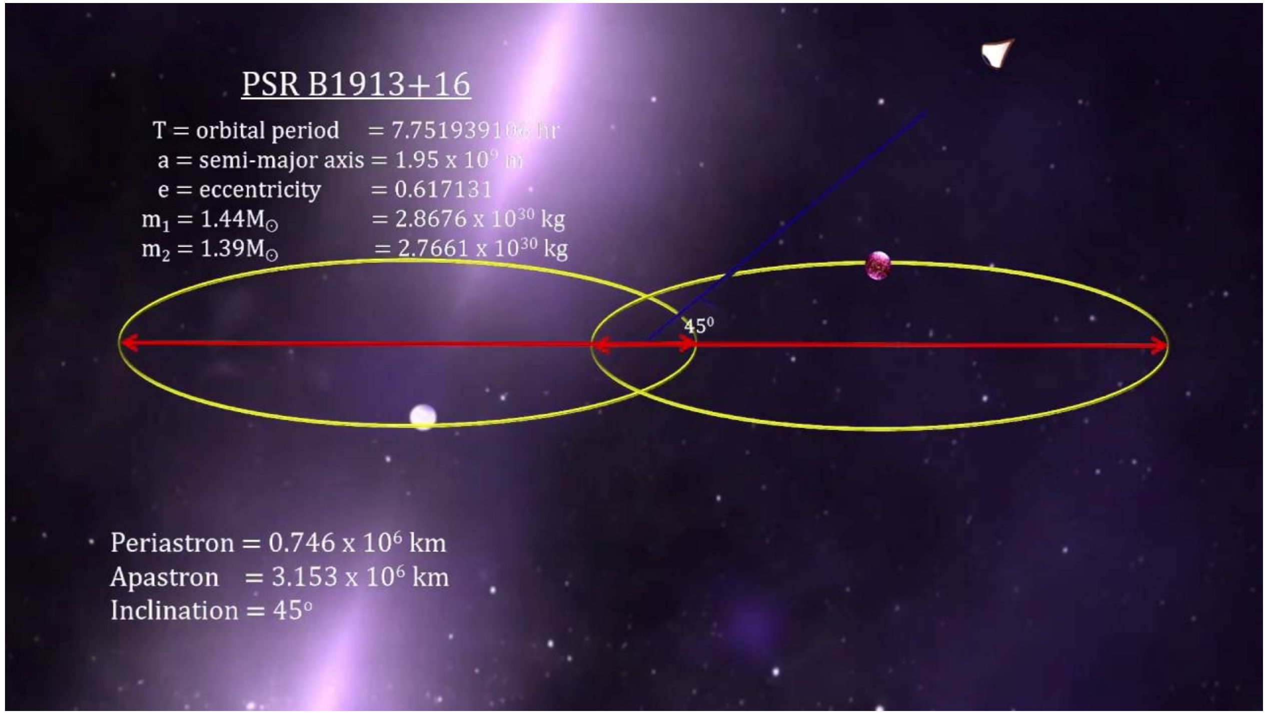
Hulse-Taylor binary: First pulsar in a binary stars system and orbit decay observation

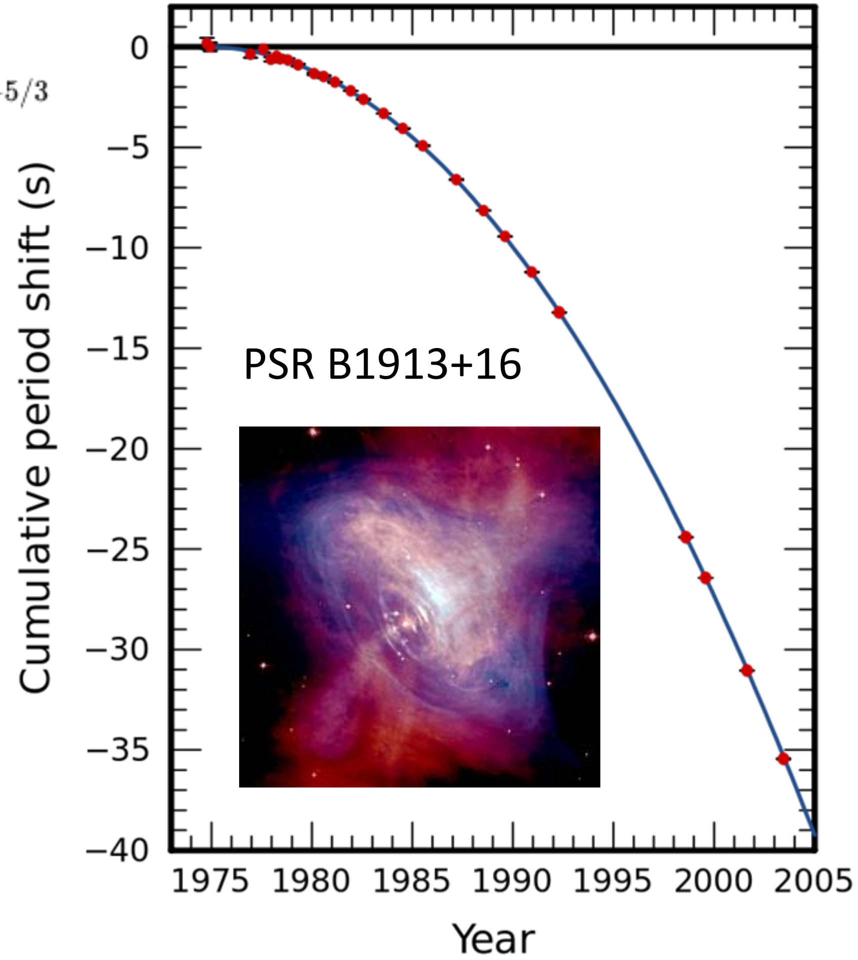
$$-\left\langle \frac{dE}{dt} \right\rangle = \frac{32G^4 m_1^2 m_2^2 (m_1+m_2)}{5c^5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

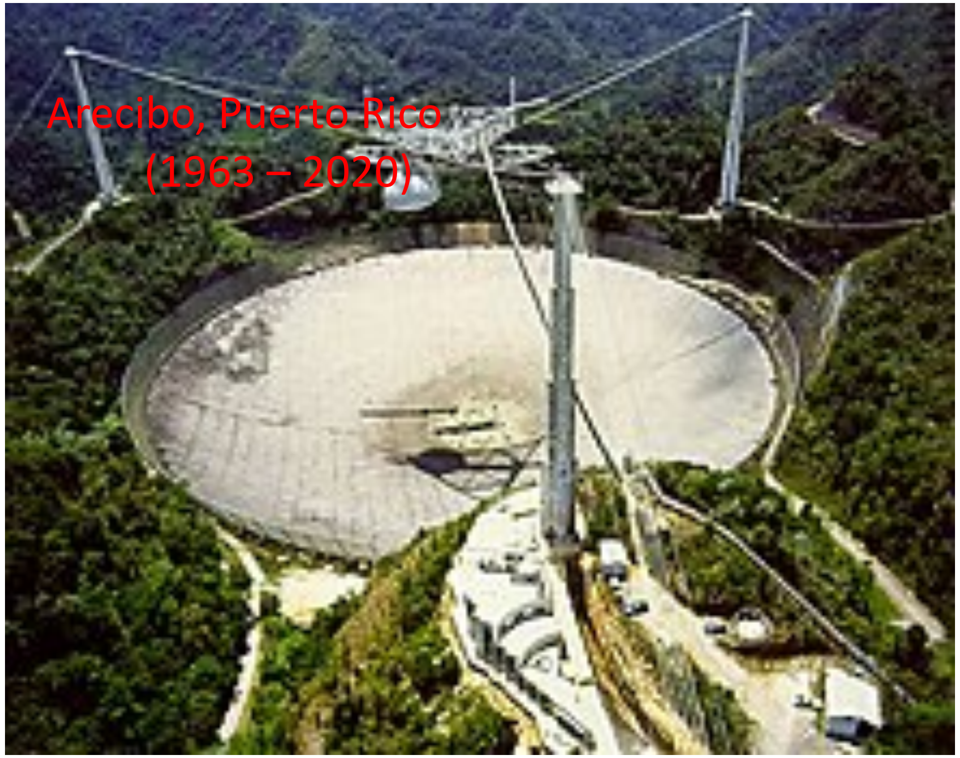
$$-\left\langle \frac{dL_z}{dt} \right\rangle = \frac{32G^{7/2} m_1^2 m_2^2 \sqrt{m_1+m_2}}{5c^5 a^{7/2} (1-e^2)^2} \left(1 + \frac{7}{8}e^2\right)$$

$$-\left\langle \frac{dP_b}{dt} \right\rangle = \frac{192\pi G^{5/3} m_1 m_2 (m_1+m_2)^{-1/3}}{5c^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right) \left(\frac{P_b}{2\pi}\right)^{-5/3}$$

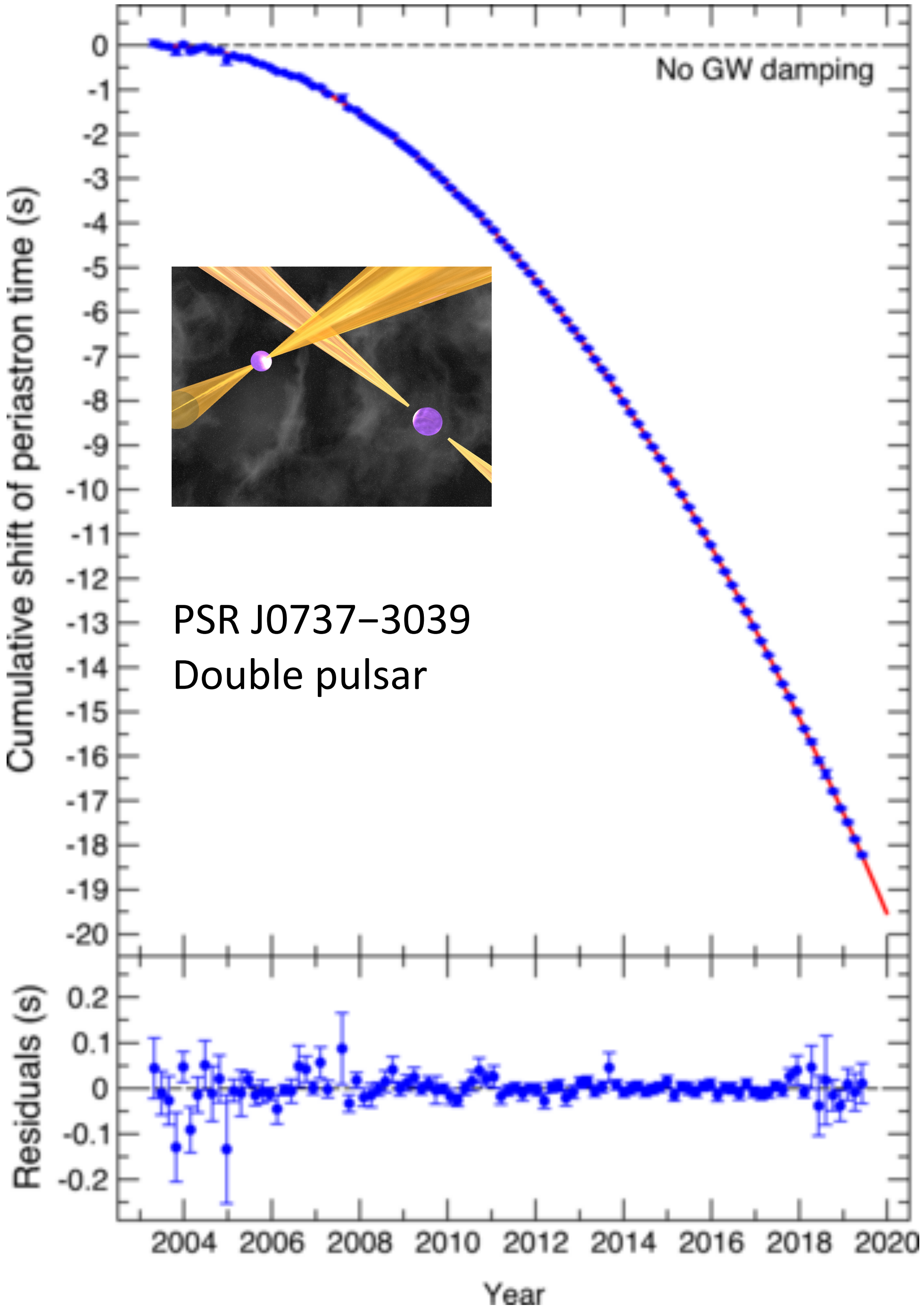
The orbit has decayed since the binary system was initially discovered (1974), in precise agreement with the loss of energy due to gravitational waves



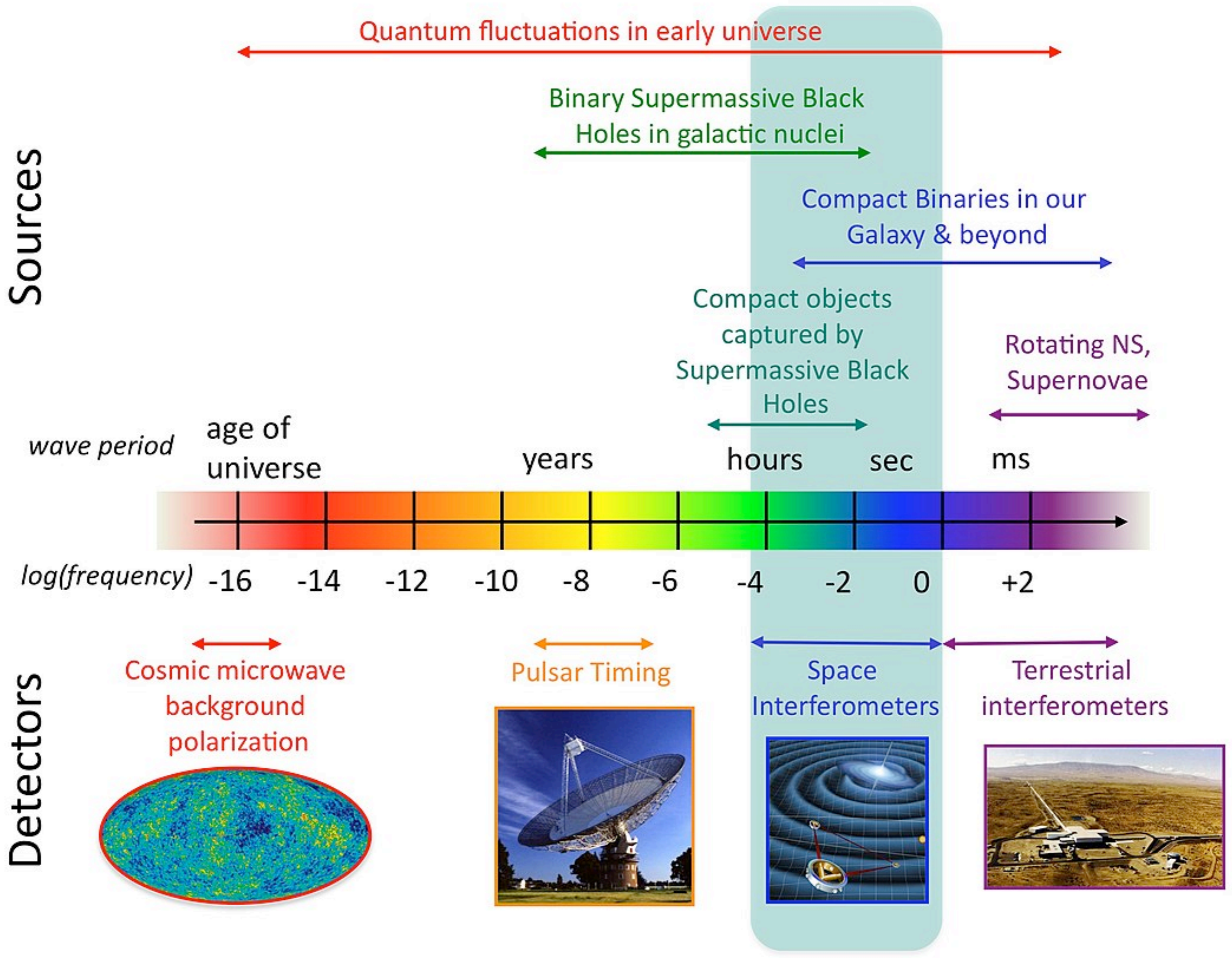


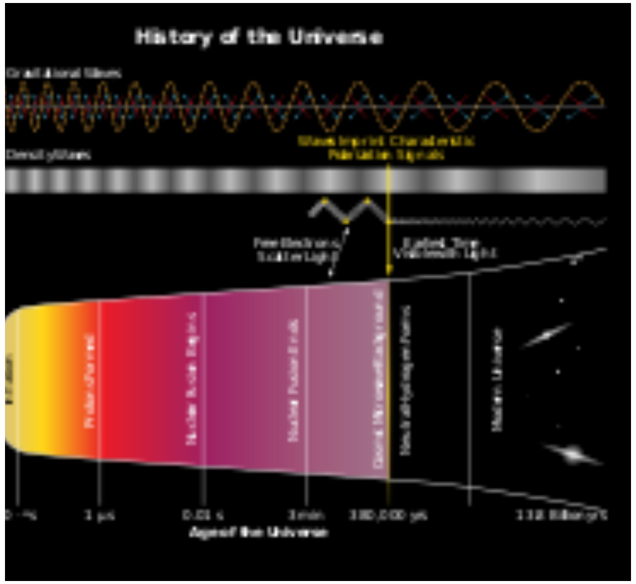

Arecibo, Puerto Rico
(1963 – 2020)


FAST PRC (2016 -)

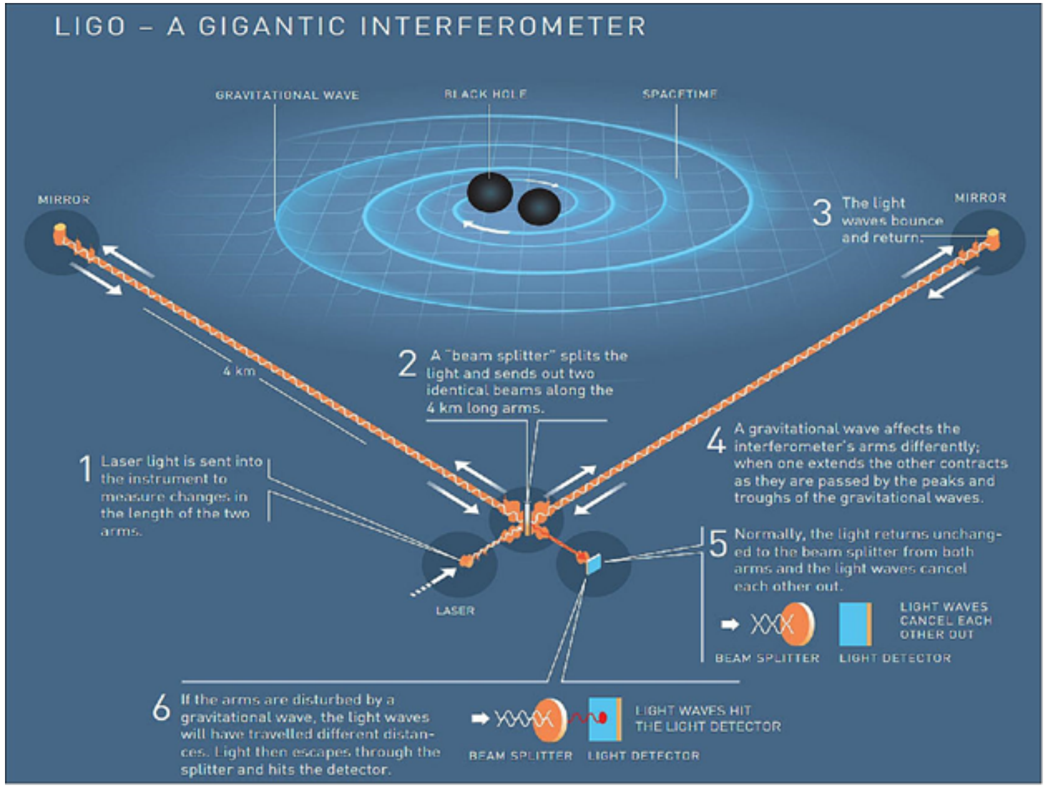

No GW damping
PSR J0737−3039
Double pulsar
Cumulative shift of periastron time (s)
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
-13
-14
-15
-16
-17
-18
-19
-20
Residuals (s)
0.2
0.1
0
-0.1
-0.2
2004
2006
2008
2010
2012
2014
2016
2018
2020
Year

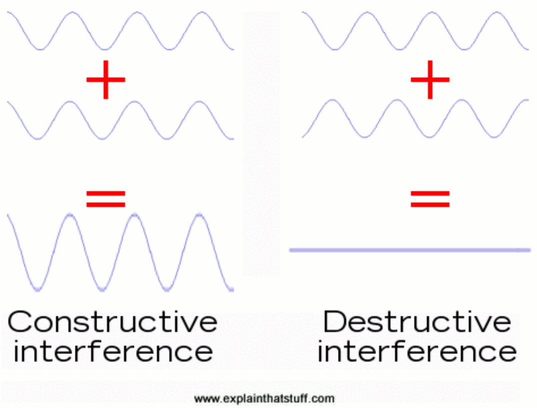
The Gravitational Wave Spectrum

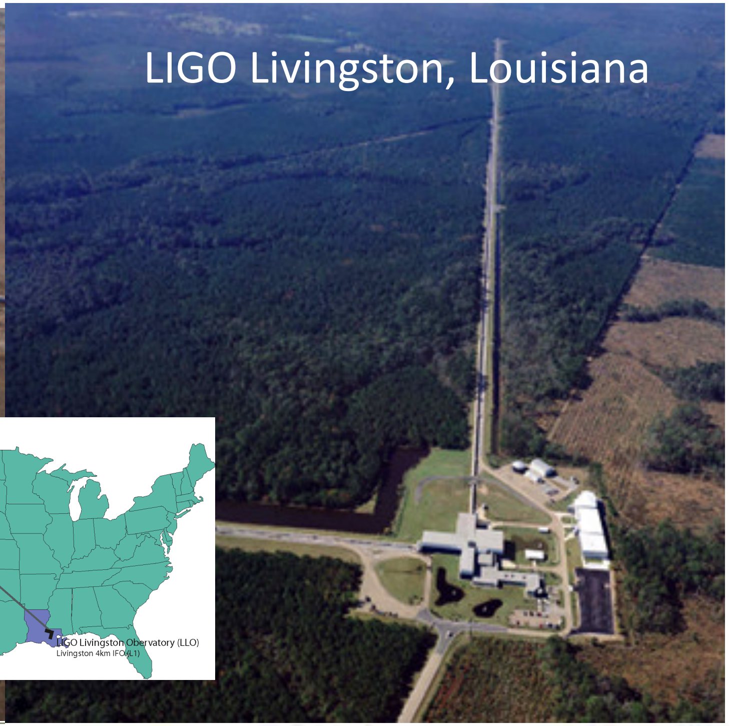


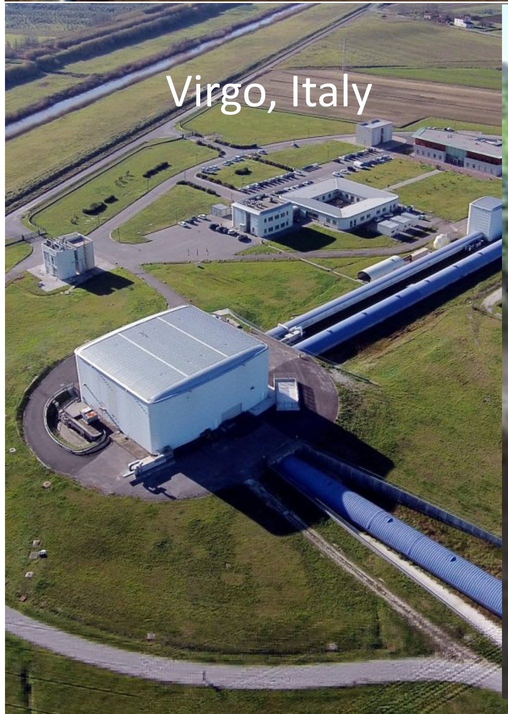


GWs laser interferometers observatory's network





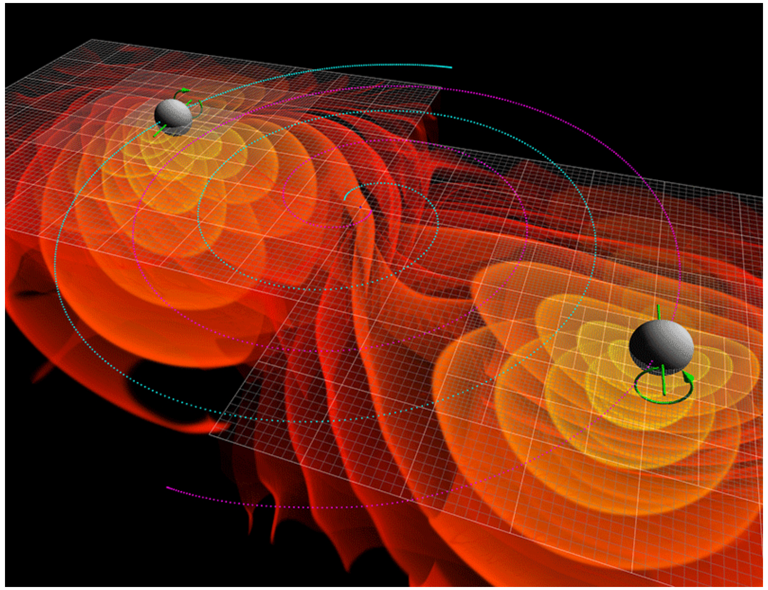


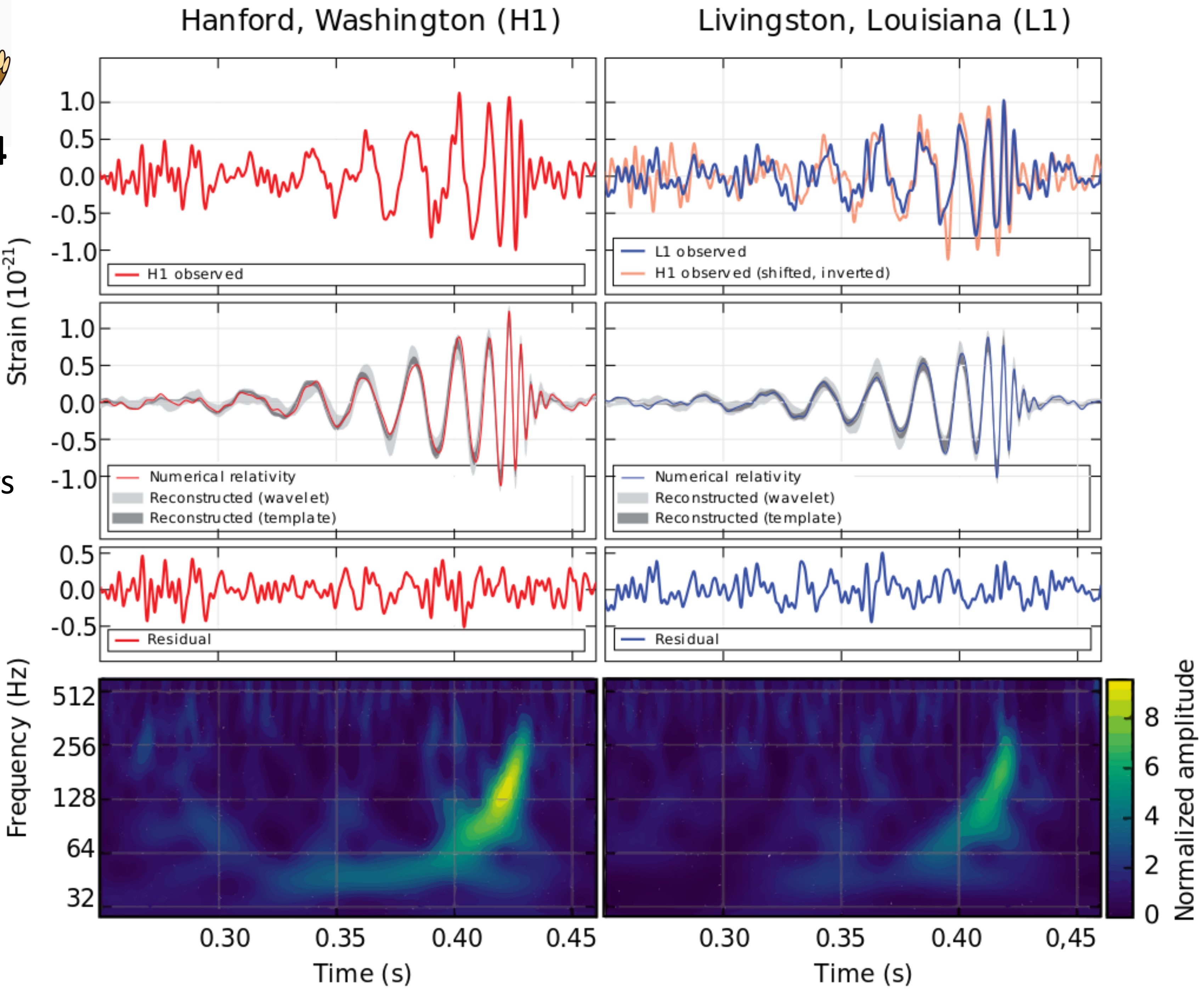


First observation of GWs and merger of a pair of black holes GW150914

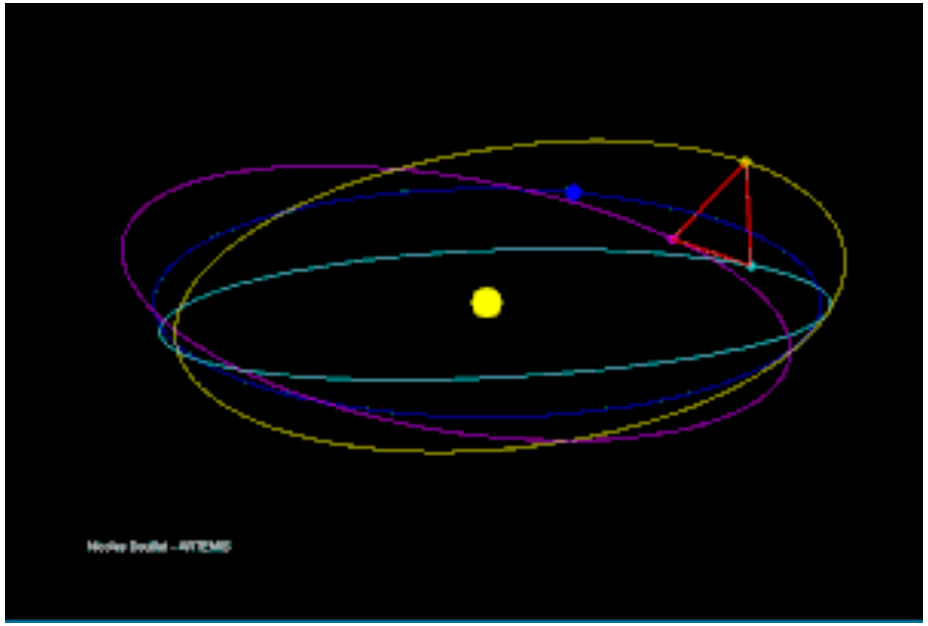
Long list of new GWs observations since 2015

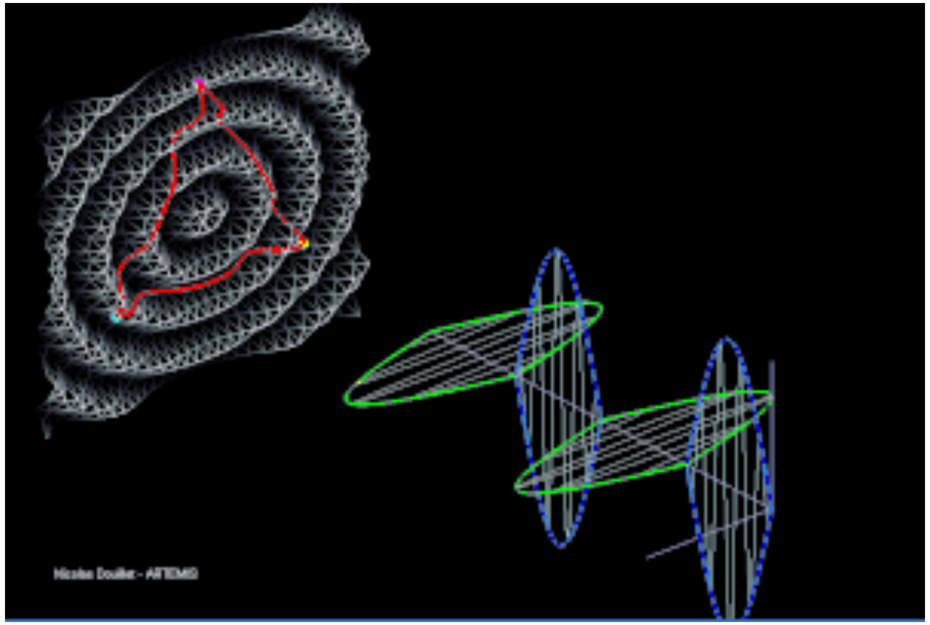
Multi-messenger Astronomy era (neutrinos, GWs, gamma rays, cosmic rays) and coincidence triggers

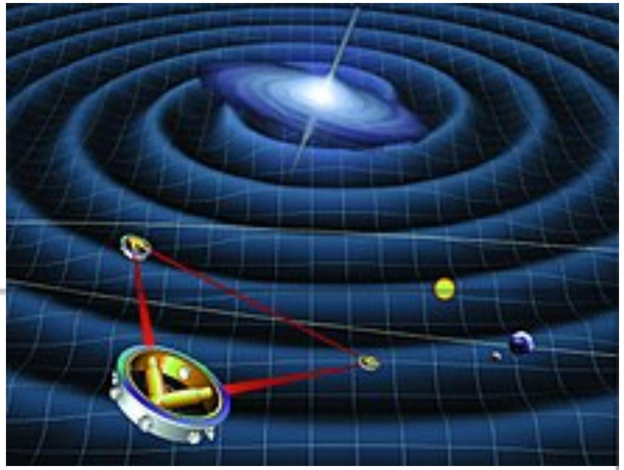


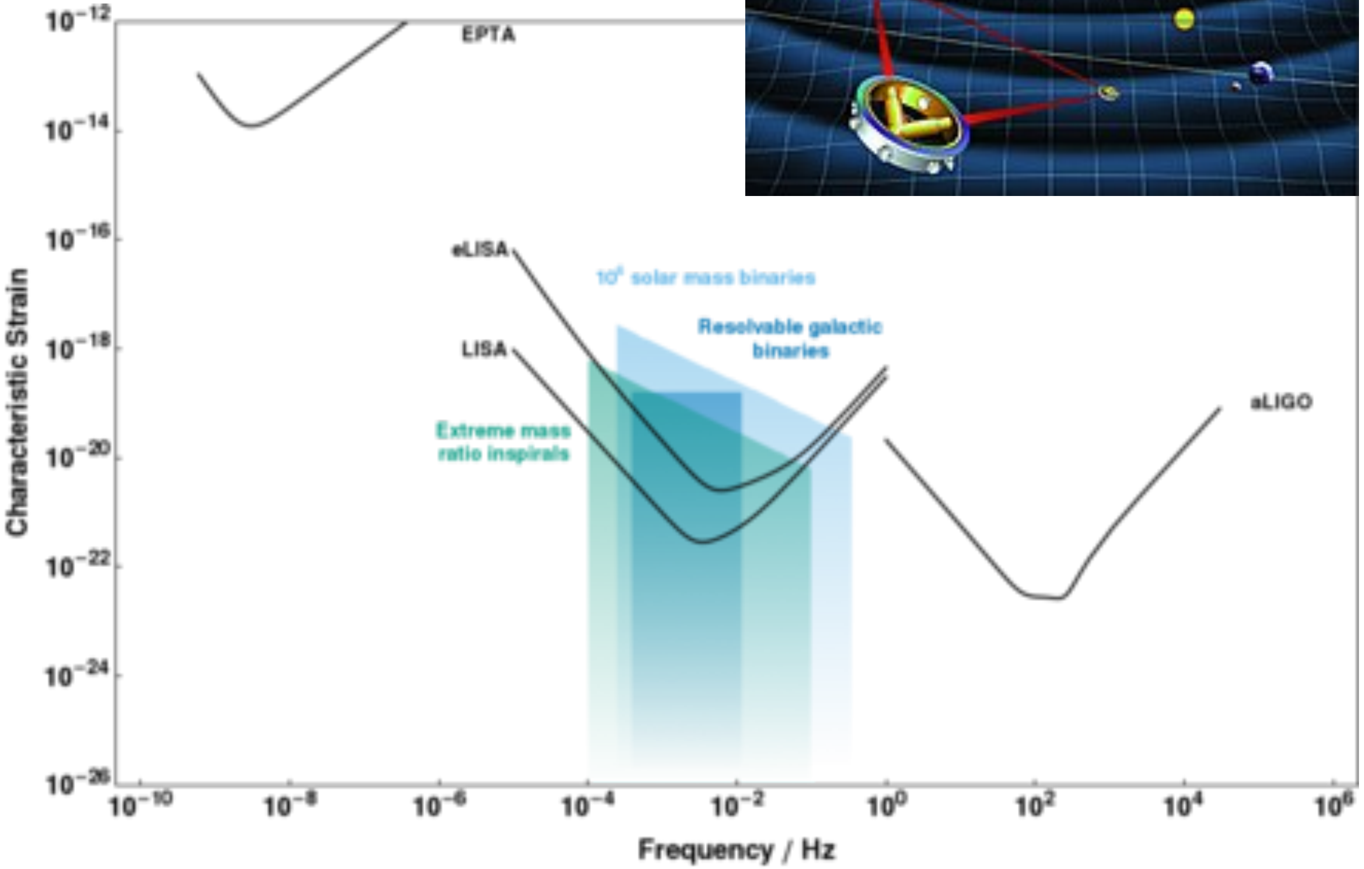


Future of present past: Laser Interferometer Space Antenna (LISA – Planned launch 2037), ESA.









Cyclotron motion in a GW background (Jan van Holten, NIKEF/99-019)

- Need to solve the Equation of motion of a relativistic charged particle in GW and EM background fields
$$\ddot{x}^\mu + \Gamma^\mu_{\lambda\nu}\dot{x}^\lambda\dot{x}^\nu = \frac{q}{m}F^\mu_\nu\dot{x}^\nu$$
- Simple case: constant magnetic field with non-zero comp. $F_{xy} = -F_{yx} = \mathrm{B}$, in the same direction as the GWs, z
- pp standard GWs
H(u,x,y) = a(u) $(x^2 - y^2) + 2\, b(u)\, x\, y$
- Light cone coords. u = ct – z, v = ct + z, to get the ODEs
x'' – k y' + a(u) x + b(u) y = 0
y'' + k x' – a(u) y + b(u) x = 0
- x' = dx/du and k given by k = $\frac{qB}{\mathrm{mc}\gamma}$, and,

$$\frac{1}{\gamma^2} = \frac{1-\frac{v^2}{c^2}}{1-\frac{v_z^2}{c^2}} + \mathrm{H}$$

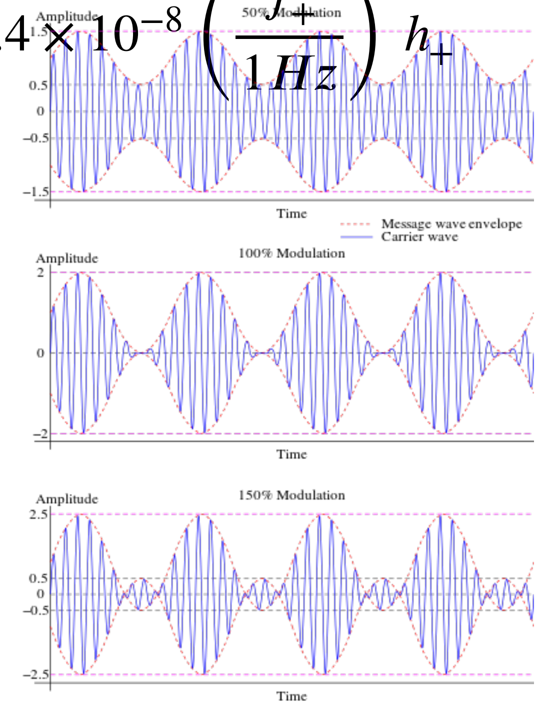
$$\frac{dt}{} \quad \sqrt{1 - H\,\gamma^2}$$

- H = 0 -> $\frac{dt}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma$,
- x(u) = A cos(ku) + B sin(ku)
- y(u) = B cos(ku) – A sin(ku)
- In polar co-ordinates: ρ = $\sqrt{A^2 + B^2} = constant$,
- $\theta = \omega_0\, t = \frac{qBt}{\mathrm{m}\gamma}$
- v = $\rho\,\omega_0$, $\gamma = \sqrt{1 + (q\rho B/mc)^2}$, γ >> 1, k = $\frac{1}{\rho}$
- No GW, v_0 planar velocity, γ = $\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}$
- GW changes v_0 to $v = v_0 + \delta v$
- We look for the change in the kinetic energy due to the GW
- With $\delta v_z \ll \gamma^2 c$, $H \approx 2\,\frac{v_0\,\delta v}{c^2}$, $v_0\,\delta v = \rho_0^2\omega_O\delta\omega = v_0^2\frac{\delta\omega}{\omega_0}$
- H = $\delta\left(\frac{v^2}{C^2}\right) \approx 2\,\frac{\delta\omega}{\omega_0}$, the change in kinetic energy is given

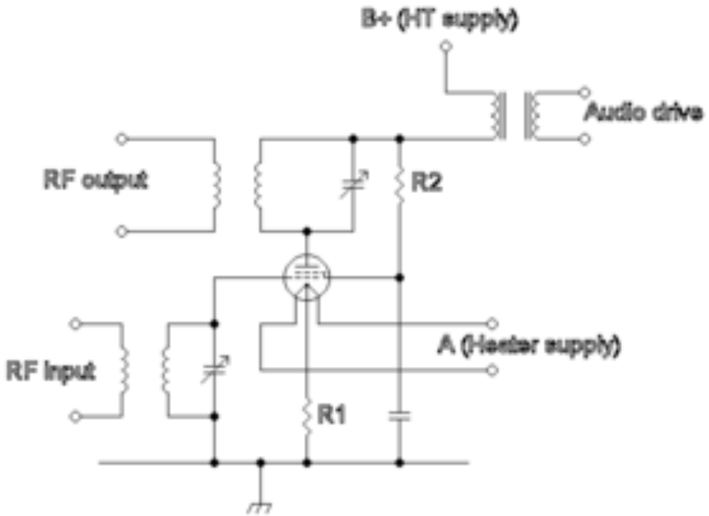
Example: GW with $\omega_+ = 2\pi f_+$ and , $b(u)$=0.

- a(u) = $\dfrac{\omega_+^2}{2C^2} h_+ \cos\left(\omega_+ t - \alpha\right)$
- H = $\dfrac{\rho_0^2\omega_+^2}{2c^2} h_+ \cos 2\omega_0 \mathrm{t} \cos\left(\omega_+ t - \alpha\right) = \dfrac{\rho_0^2\omega_+^2}{4c^2} h_+[\cos\left(\left(\omega_+ + 2\omega_0\right)t - \alpha\right) + \cos\left(\left(\omega_+ - 2\omega_0\right)t - \alpha\right)]$
- For the LHC, $\rho_0 = 4.2$ Km, H = $\delta\left(\dfrac{v^2}{C^2}\right) \leq 0.4 \times 10^{-8} \left(\dfrac{f_+}{1\,Hz}\right)^2 h_+$

(Hz)	H/
500	
1.5	1
1.5	
1.5	



Amplitude modulation due to vertical GW



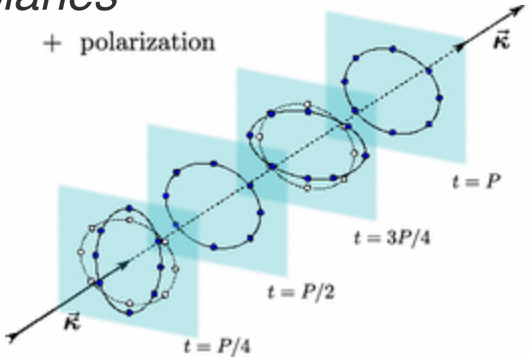
- Open question (not in Jan's paper, need research): longitudinal and ***frequency modulation*** effects. Easier to measure, FM is always cleaner.

Effect of a GW on a rotating ring (LHC beam) (D. Zer-Zion, CERN-EP/98-63, Astroparticle Physics 2000 (pp. 239-243))

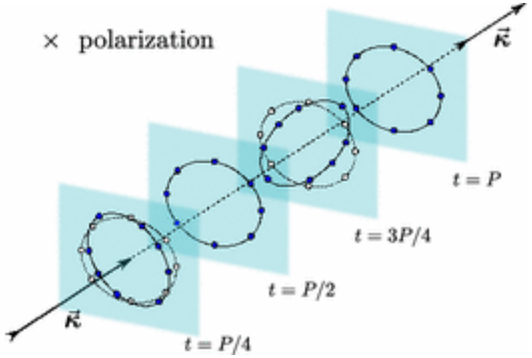
Geometry of a collider (LHC) beam

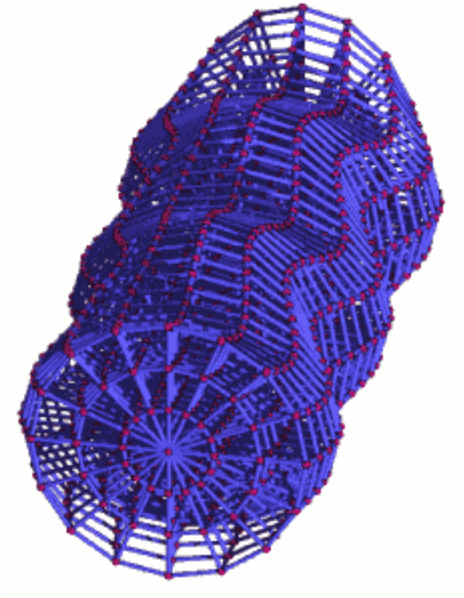
- c = 1, change polar co-ords. from lab to a system of polar co-ords. attached to a bunch in the beam
  $\rho_0 = \rho,\ z_0 = \mathrm{z},\ \ t_o = \mathrm{t},\ \theta_0 = \theta + \omega t$
- Apply the transformation to a spacetime line element in the lab
  $$ds^2 = -\,dt_0^2 + d\rho_0^2 + \rho_0^2\,d\theta_0^2 + dz_0^2$$
  $$ds^2 = -\left(1-\omega^2\rho^2\right)dt^2 + d\rho^2 + \rho^2\,d\theta^2 + 2\omega\rho^2\,d\theta\,dt + dz^2 = -\,dt'^2 + dl'^2$$
  $$dt' = \frac{dt}{\gamma} - \omega\rho^2\gamma\,d\theta,\ \ \gamma = \frac{1}{\sqrt{1-\omega^2\rho^2}}$$
  $$dl'^2 = d\rho^2 + \rho^2\gamma^2\,d\theta^2 + dz^2$$
- $\dfrac{d\theta}{dt} = -\,\omega\ \rightarrow\ \ 1 + \rho^2\omega^2\gamma^2 = \gamma^2,\ dt = \gamma^{-1}\,dt'$ (particle at rest in the lab as seen from the ring)
- Calculate spatial Gauss curvature using Riemann and Ricci tensors (N. Rosen, PR, 1943) $\approx\ -\dfrac{3\gamma^4}{\rho^2}$
- The beam can locally be flat (clock hypothesis) but, being an extended object, with (negative) curvature it is subject to the GW *at large*: every bunch is considered a particle in a rotating ring and a GW affects the bunch wrt the nominal orbit of the collider which is a (forced) geodesic line.

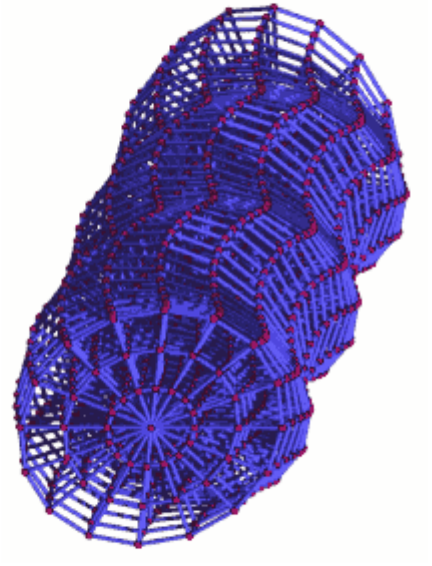
Gravitational wave traveling towards you through different planes



Gravitational wave – elliptically polarized







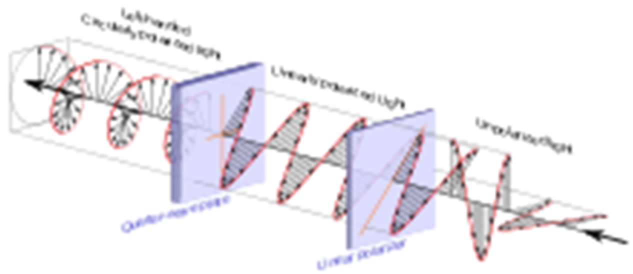
Markus Pössel, "The wave nature of simple gravitational waves" in: *Einstein Online* **Band 02** (2006), 02-1008
https://www.einstein-online.info/en/spotlight/gw_waves/

EM waves study areas:

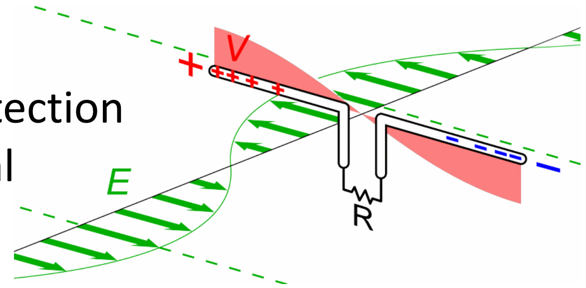
- Transmission
- Absorption
- Reflection
- Refraction
- Diffraction
- Interference
- Polarization
- Dispersion
- Transition to QM, photons, gravitons, QED

Extended GW astrophysics

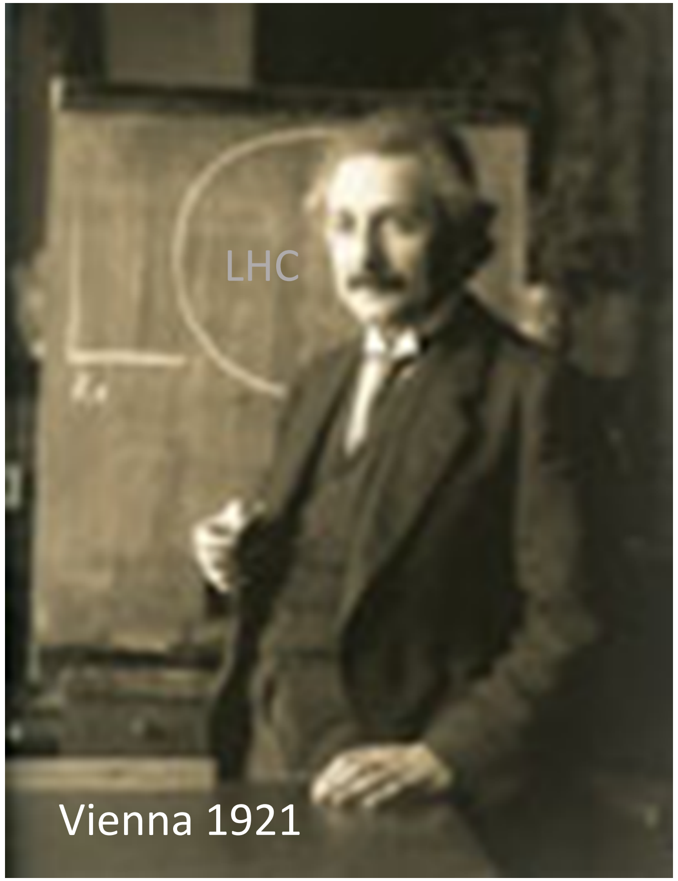
Amplitude, h
Frequency, f
Wavelength, λ
Speed, c
$f\,\lambda = c$



Generation <> Detection
Time reversal



Interaction of a GW with the LHC beam I



- Charge and mass densities are constant
- EM fields impose curvature on the rotating ring and GWs cause local deviations from the steady state well correlated all around the ring
- Consider a monochromatic («monogravtic») GW

$$h_{xx} = -\,h_{yy} = h_{+}e^{-i\omega_{g}(t\,-\,z)}$$
$$h_{xy} = \quad h_{yx} = h_{\times}e^{-i\omega_{g}(t\,-\,z)}$$

- Which give the respective non-zero elements of the Riemann curvature tensor

$$R_{xtxt} = -\,R_{ytyt} = -\,\frac{1}{2}\,h_{xx,tt}$$
$$R_{xtyt} = R_{ytxt} = -\,\frac{1}{2}\,h_{xy,tt}$$

- For the simple case $h_{\times}$= 0, z = 0,

$$\ddot{x} = \;-\,R_{xtjt}x^{j} = -\,\frac{1}{2}\omega_{g}^{2}\,h_{+}\,x\;e^{-i\omega_{g}(t\,-\,z)}$$
$$\ddot{y} = \;-\,R_{ytjt}x^{j} = +\frac{1}{2}\omega_{g}^{2}\,h_{+}y\;\;e^{-i\omega_{g}(t\,-\,z)}$$
$$\ddot{z} = \;0$$

- Driving radial acceleration due to the GW $\ddot{\xi} = \dfrac{x}{\rho_c}\,\ddot{x} + \dfrac{y}{\rho_c}\,\ddot{y} = -\,\dfrac{1}{2}\omega_{g}^{2}\,\rho_{c}\,h_{+}\,cos2\theta\;e^{-i\omega_{g}t}$
- $\ddot{\xi}$ is the relative radial accelaration due to the GW bunch wrt the nominal orbit -> Equation of relative geodesic deviation (see *Weinberg* or *MTW*)

Interaction of a GW with the LHC beam II

- Simple ring with natural frequency of vibration ω_0 and dampint time $\tau_0 \gg 1/\omega_0$
- Equation of motion for a bunch in the beam (see *Goldtein*, *LL*)

$$\ddot{\xi} + \dot{\xi}/\tau_0 + \omega_0^2 \xi = -\frac{1}{2}\omega_g^2\, \rho_c\, h_+\, cos2\theta\, e^{-i\omega_g t}$$

$$\xi = \frac{1}{2}\frac{\omega_g^2\, \rho_c\, h_+\, cos2\theta\, e^{-i\omega_g t}}{\omega_g^2 - \omega_0^2 + i\omega_g/\tau_0} = \alpha_g cos2\theta\, \cos\omega_g t$$

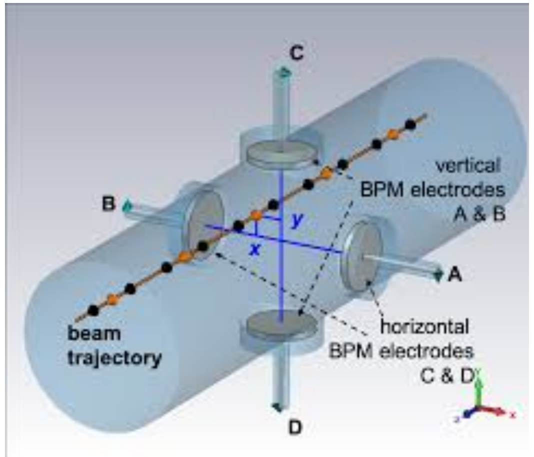
$$\alpha_g = \frac{1}{2}\frac{\omega_g^2\, \rho_c\, h_+}{\omega_g^2 - \omega_0^2 + i\omega_g/\tau_0}$$

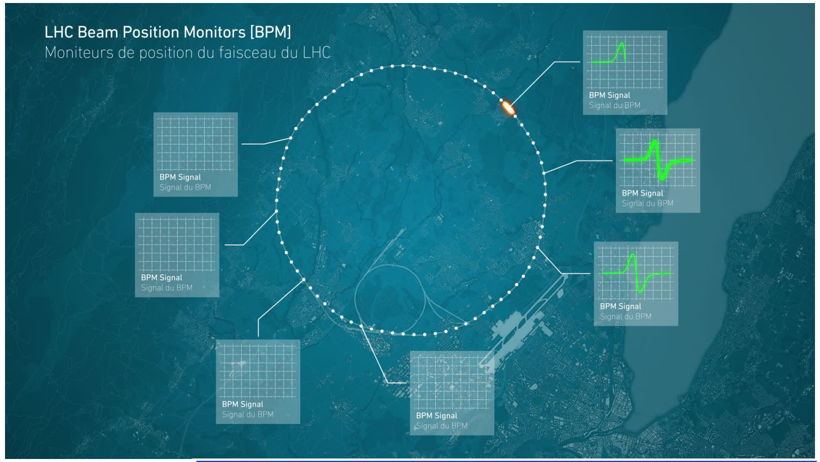
- $\omega_g \approx \omega_0, \quad \alpha_g^{res} = \frac{1}{2}\, Q\, \rho_c\, h_+$
- From the energy absorbed and dissipated by the beam due to the GW (see articles) a cross section can be calculated

$$\sigma_{res} = \pi \rho_c^2 M \omega_g Q$$

- In the lab. : $\xi = \alpha_g \cos\left[2\left(\theta_0 - \omega t_0\right)\right] \cos\omega_g t_0$
- Need research on Q large values to improve the potential for observation
- There is also a change in the arrival time of a bunch. Need research on BTM (*Beam time measurements*).

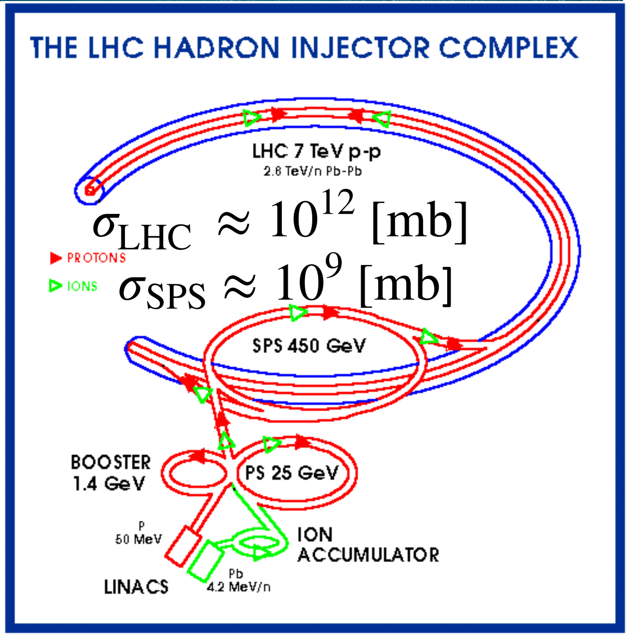
Measurement of the effect





- ➢ Last report indicates LHC BPM resolution of 1 μm
- ➢ Streak cameras (rev by rev observation images of short light pulses) to see the time decay
- ➢ Synchronised BPMs storing the information long enough for short-range logic decisions
- ➢ Trigger system

  LHC 500 pickups used to form a closed orbit measurement
- ➢ Frequency power spectrum studies and Fourier on line
- ➢ Computer simulations MC programs to create wavelets for possible templates and compare them with real data.
- ➢ Add LHC to other observatories (bi-directional) such as gamma rays, LIGO, neutrinos. Be part of the network
- ➢ The all beam reconstruction gives the trigger, the adjacent BPMs outputs the relative motion of adjacent bunches, which is the effect
- ➢ Trigger LHC and SPS coincidences to reduce noise effects and use the fact that the beams rotate in opposite directions (need research on the developement of a spectrometer method)

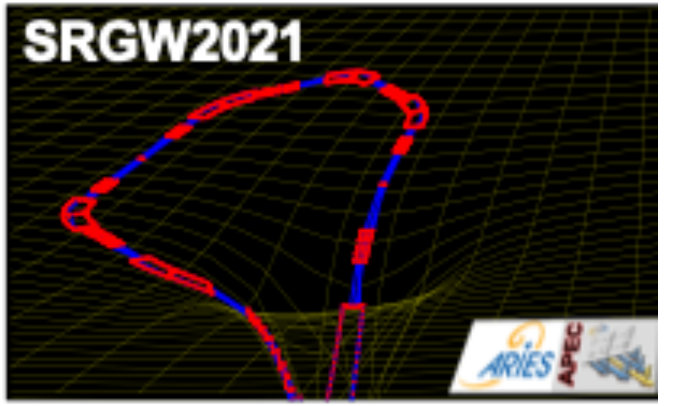


type	C km	E GeV	γ	Parts per bunch 10^{10}	Nr of bunches	k	RF MHz	Bunch Separ m	Bunch length cm	
pp	26.66	7	7461	10.5	2800	11.24	400	7.5	8.1	LHC
p	6.9	450	480	1.2	4160	43.38	200	1.5	30	SPS

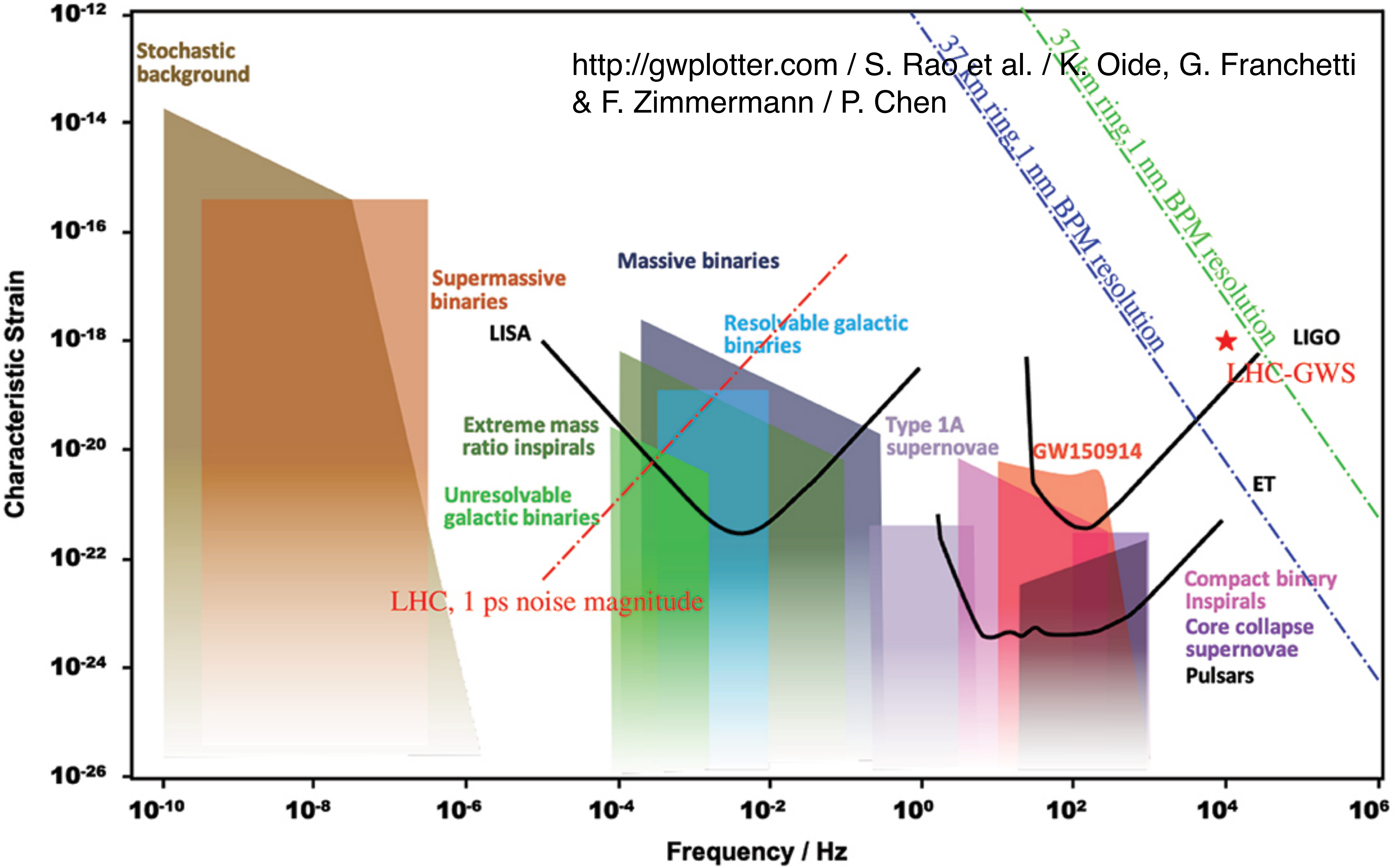
Comment on Noise

- ➢ Noise is more a crucial point in GWs detectors than any other domain (dogma)
- ➢ Signal small, never really seen, but inferred
- ➢ Beams are stable for many hours after injection, acceleration and steering to nominal energy, most noise sources are known, such as
  - Parasitic currents
  - Thermal behaviour
  - Sporadic changes in magnetic fields of magnets
- ➢ Geological sources
- ➢ Deeper insight and research needed

Highlights from the ARIES APEC workshop SRGW2021



- ➢ Workshop on virtual space Feb. 2 to Mar. 18 2021
- ➢ arXiv:2015.00992v1 Storage Rings and GWs: Summary and Outlook (*Ellis, Zimmermann*)
- ➢ Storage-ring experts, accelarator scientists, experimental particle physicists, theoretical physicists, astrophysicits, members of the GW community.
- ➢ 115 participants
- ➢ Highlights:
  - Storage rings sensitivity to tides and large-scale perturbations, earthquakes, noise examples from LEP and LHC (*Wenninger*)
  - Ground vibration (*Deng*)
  - Response of a SR beam to a GW (*Oide*)
  - Detection of millihertz GW using SR (*Rao, Bruggen, Liske*)
  - SR as detectors for relic GW background (*Ivanov, Kobushkin, Wellenzohn*)
  - GW at particle SRs (*D'Agnolo*)
  - Gravitational synchrotron radiation from SRs (*Chen; Jowett*)
  - GW detection in SRF cavity (*Berlin, D'Agnolo, Ellis*)
  - Atom interferometer to search for ultralight dark matter and GW at CERN (*Buchmuller, Ellis)*



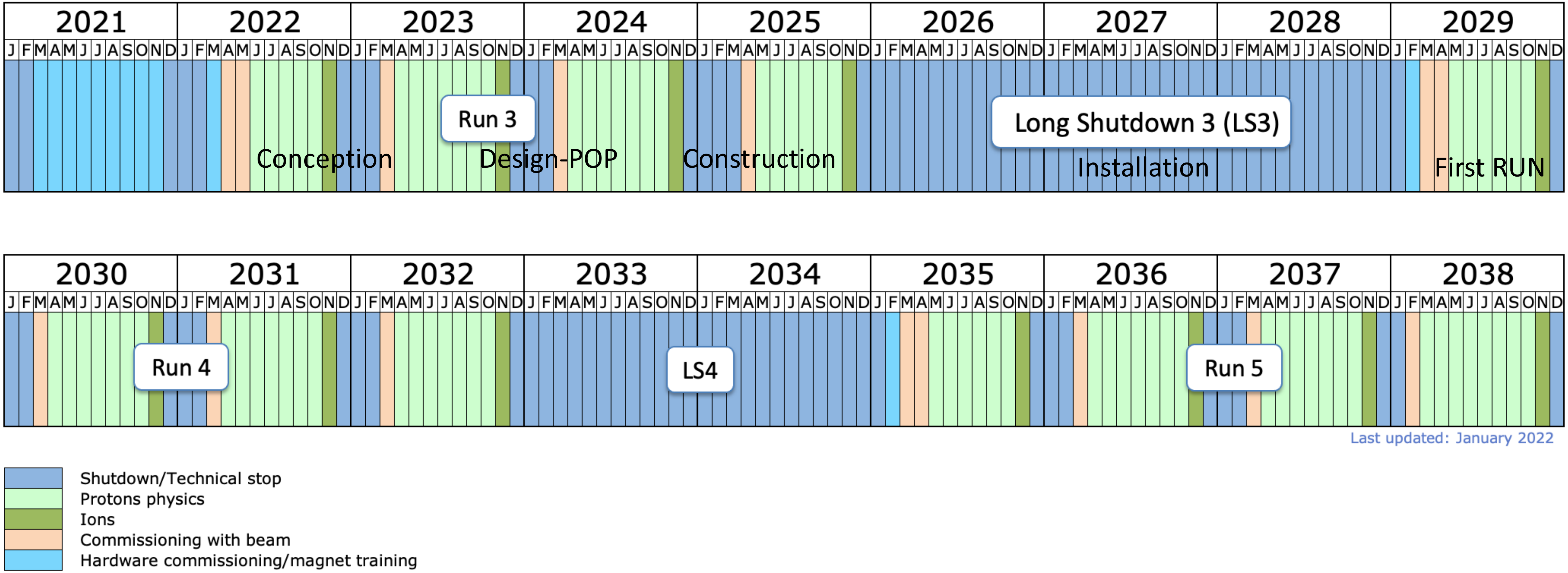
http://gwplotter.com / S. Rao et al. / K. Oide, G. Franchetti & F. Zimmermann / P. Chen
Characteristic Strain
Frequency / Hz
10-12
10-14
10-16
10-18
10-20
10-22
10-24
10-26
10-10
10-8
10-6
10-4
10-2
100
102
104
106
Stochastic background
Supermassive binaries
Massive binaries
LISA
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
LHC, 1 ps noise magnitude
Type 1A supernovae
GW150914
37 km ring 1 nm BPM resolution
37 km ring 1 nm BPM resolution
LIGO
LHC-GWS
ET
Compact binary Inspirals
Core collapse supernovae
Pulsars

Conclusion

«The prospects of using SRs as GWs detectors or sources are fascinating, and the opportunities for using CERN's technical infrastructure to add GW physics to its Physics Beyond Colliders programme are too interesting to be ignored. « (*J. Ellis*)

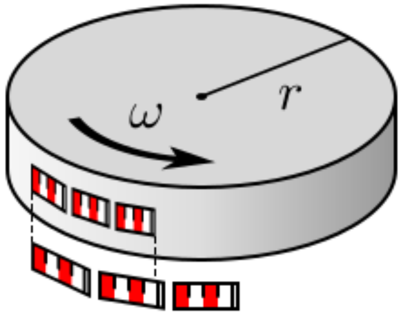
A group, having the BPMs and machine experts working on GWs has still to be formed.

LHC long term Schedule



Historical background (see Øyvind Grøn, Oslo)

Problem: Is an observer in a frame of reference on a ring with constant radial accalaration in hyperbolic motion?

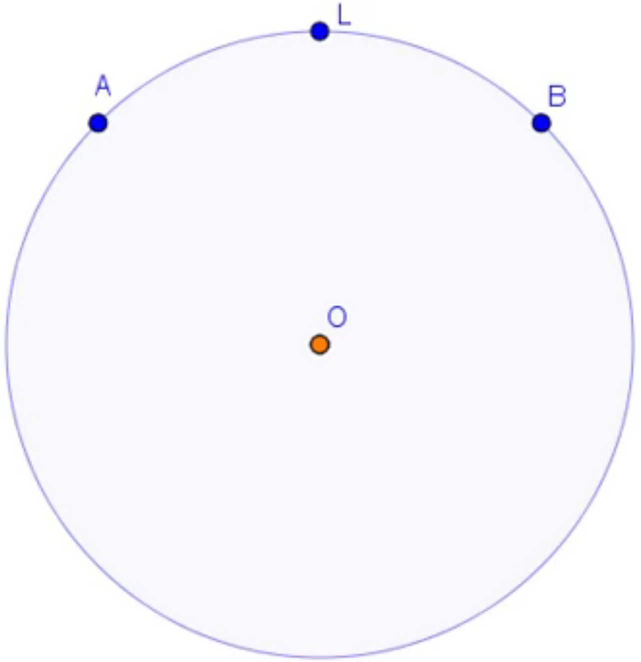


Ehrenfest paradox (1909) :

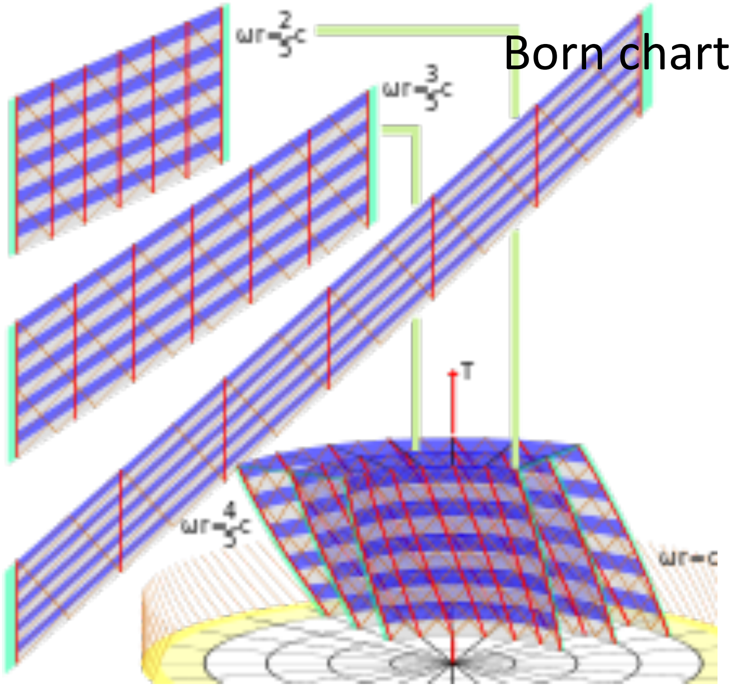
$$\frac{\text{circumference}}{\text{diameter}} = \frac{2\pi R\sqrt{1-(\omega R)^2/c^2}}{2R} = \pi\sqrt{1-(\omega R)^2/c^2}.$$

$r = r'$, $v = \omega r$, $C = 2\pi r > C' = 2\pi r'$ ☹

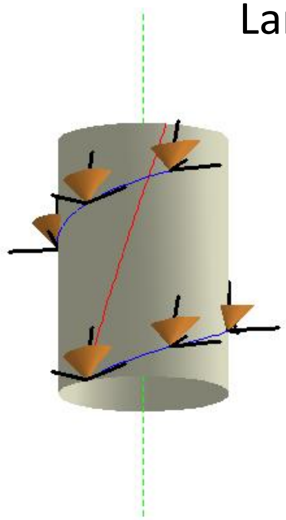
Einstein: The rotating disk is not Euclidean, GR is born (see Gorn @ Oslo).

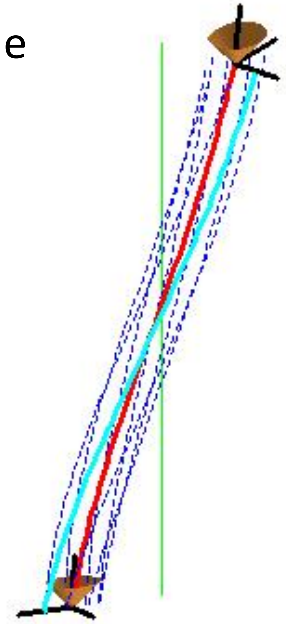


Born chart



Langevin frame





App. Quadrupole formula

In GR, the quadrupole formula describes the rate at which gravitational waves are emitted from a system of masses based on the change of the mass quadrupole moment.

The spatial part of the trace reversed perturbation of the metric, i.e. the gravitational wave, is given by

$$\bar{h}_{ij}(t,r) = \frac{2G}{c^4 r}\ddot{I}_{ij}(t - r/c),$$

with the traceless part of the mass quadrupole moment

$$I_{ij}^T = \int \rho(\mathbf{x}) \left[r_i r_j - \frac{1}{3} r^2 \delta_{ij} \right] d^3 r,$$

The total energy (luminosity -> gravitosity) carried away by gravitational waves is

$$\frac{dE}{dt} = \sum_{ij} \frac{G}{5c^5} \left(\frac{dI_{ij}^T}{dt^3} \right)^2$$