Improving Bayesian parameter estimation of QCD matter with the latest LHC data

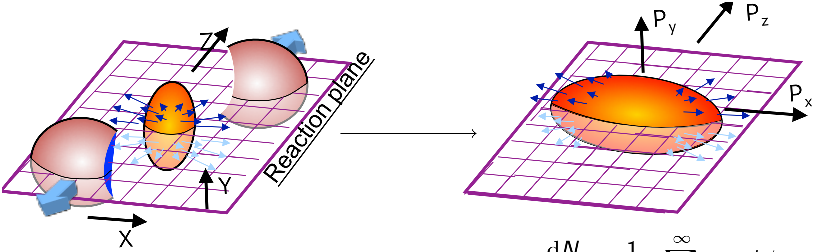
Jasper Parkkila, Anna Önnerstad, Dong Jo Kim
Based on PRC 104 (2021) 054904, arXiv:2111.08145

Science Coffee, Lund University
10. May 2022

UNIVERSITY OF JYVÄSKYLÄ
HELSINKI INSTITUTE OF PHYSICS
CERN
CoE QM

Anisotropic Flow

Initial geometry fluctuations → Transport $\delta_\mu T^{\mu\nu} = 0$ → final-state particles



$$\varepsilon_n e^{in\Phi_n} \equiv -\frac{\langle r^n e^{in(\phi-\Phi_n)}\rangle}{\langle r^n\rangle}, n \geq 2.$$

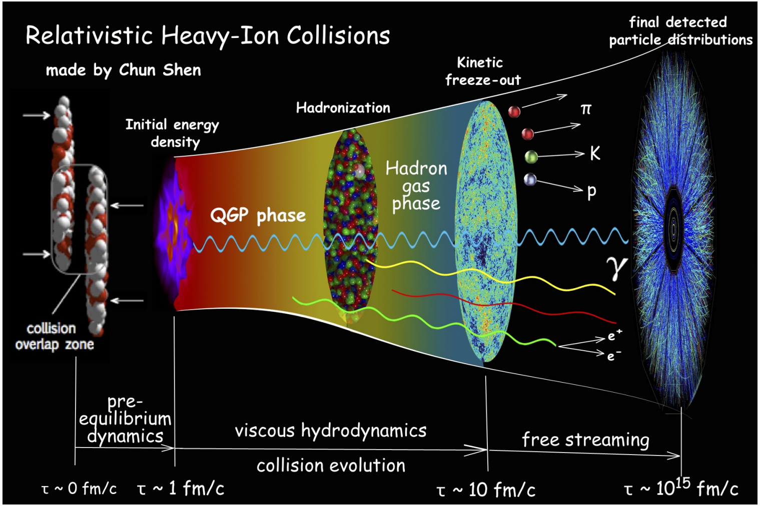
(theory only - initial state models)

$$\frac{dN}{d\phi} \propto \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \underbrace{\langle e^{in\phi}\rangle}_{V_n=v_n e^{in\psi_n}} e^{-in\phi},$$

(experiments, theory - hydro+hadronization models)

- Collectivity as a probe to the properties of the medium – transport properties such as $\eta/s(T)$, $\zeta/s(T)$

The different stages of Heavy-Ion collisions

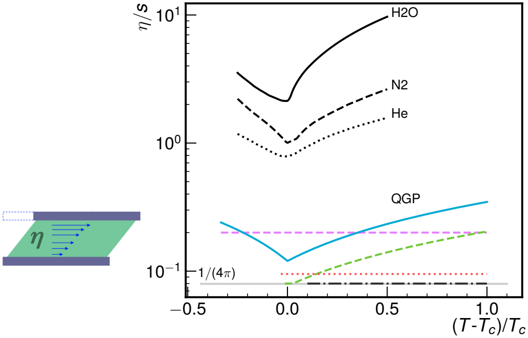


TRENTo+VISH(2+1D) + UrQMD in heavy-ion collisions

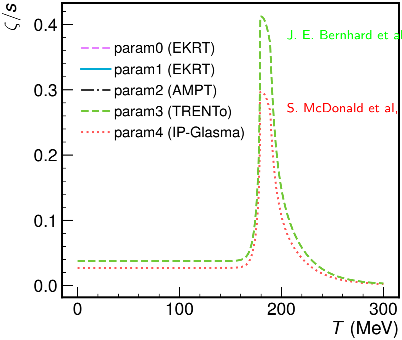
Credits to Chun Shen

Transport properties in Heavy-ion collisions

Shear Viscosity(η)

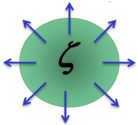


Bulk Viscosity(ζ)



J. E. Bernhard et al., Phys. Rev. C 94 024907 (2016)

S. McDonald et al, Phys. Rev. C 95, 064913 (2017)



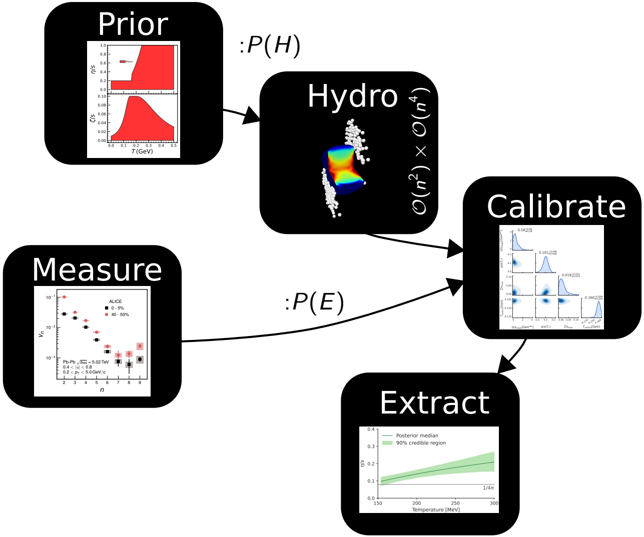
ALICE, JHEP05 (2020) 085

$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\mathrm{slope}}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\mathrm{curve}}}$$

$$(\zeta/s)(T) = \frac{(\zeta/s)_{\mathrm{max}}}{1+\left(\frac{T-(\zeta/s)_{T_{\mathrm{peak}}}}{(\zeta/s)_{\mathrm{width}}}\right)^2}$$

$$\boxed{T^{hyd}_{\mu\nu} = T^{ideal}_{\mu\nu} - \eta\sigma_{\mu\nu} - \zeta\Pi\Delta_{\mu\nu} + \Pi^{(2)}_{\mu\nu}, \qquad \delta_\mu T^{\mu\nu} = 0}$$

Bayesian parameter estimation



Prior

$:P(H)$

Hydro

$\mathcal{O}(n^2) \times \mathcal{O}(n^4)$

Calibrate

Measure

$:P(E)$

Extract

Bayes' theorem:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

$$\mathrm{P(E)} = \sum_{i=1}^{n} P(E|H_i)P(H_i)$$

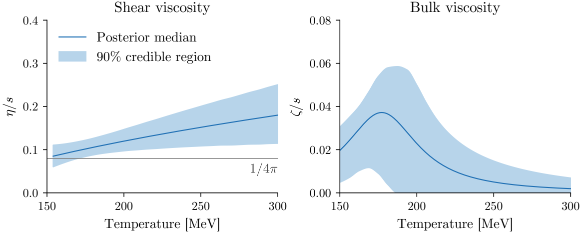
- Find optimal set of model parameters that best reproduce the experimental data
- Utilize constraints, such as flow observables, to help narrow down the $\eta/s(T)$ and such.

Testing a single set of parameters requires $\mathcal{O}(10^4)$ hydro events, and evaluating eight different parameters five times each requires $5^8 \times 10^4 \approx 10^9$ hydro events.

That's roughly 10^5 CPU years!

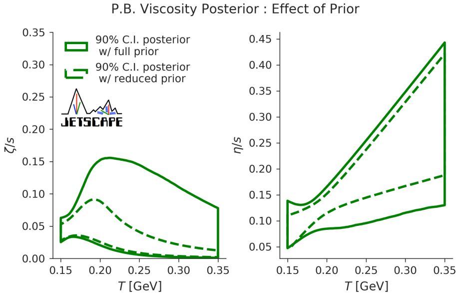
Bayesian parameter estimation: Previous work

Duke-hydro TRENTo+VISH(2+1D)+UrQMD



Steffen A. Bass *et. al*, Nature Physics (2019)

JETSCAPE TRENTo+MUSIC+SMASH



JETSCAPE Collaboration, PRC **103** (2021) 054904

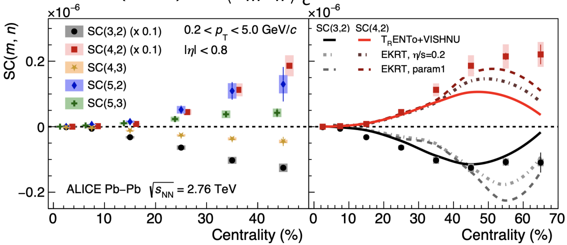
- Low to moderate temperature dependence on $\eta/s(T)$
- Moderate magnitude of $\zeta/s(T)$ ($\sim 0.1\times$ w.r.t PRL. **94**, 072305 (2005))
- Large uncertainty for both $\eta/s(T)$ and $\zeta/s(T)$.
- Subsequent studies with still limited observables:
  - J. Auvinen *et al.* PRC. **102**, 044911 (2020)
  - G. Nijs *et al.* PRL. **126**, 202301 (2021)

Uncertainties need to and can be further improved.

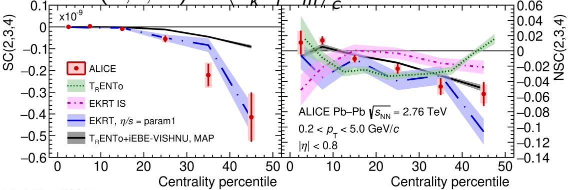
Only low harmonic v_n was used, including a limited set of mostly 2.76 TeV observables.

Improving results with higher harmonics and more precision- Symmetric Cumulants

$$SC(m,n) = \langle v_m^2 v_n^2 \rangle_c$$



$$SC(k,l,m) = \langle v_k^2 v_l^2 v_m^2 \rangle_c$$



ALI-DER-479271

Jyväskylä+TUM+NBI in ALICE

Phys. Rev. Lett. 117 (2016) 182301
Phys. Rev. C 97 no. 2, (2018) 024906

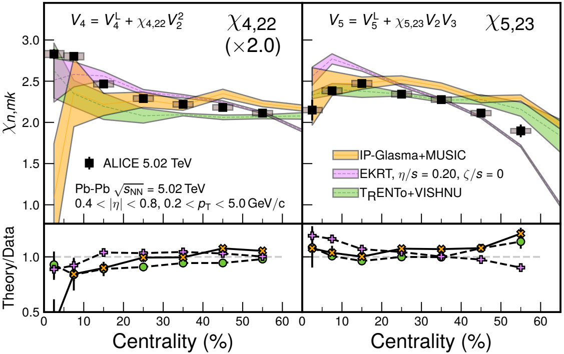
- Accessing the temperature dependence of $\eta/s(T)$

Phys. Rev. Lett. 127 (2021) 092302

- $\eta/s(T)$ and accessing $\zeta/s(T)$

- Thanks to Peter Christiansen (IRC) and Alice Ohlson (ARC+IRC)
- Standard Candle → Symmetric Cumulant (feat. P.Christiansen)

Improving results with higher harmonics and more precision - Non-linear flow modes



JHEP05 **085** (2020), J. Parkkila, D. J Kim (Jyväskylä)

- Higher order v_n's (n>3) were studied → non-linear dependence on lower orders
- Characterised by the non-linear flow mode coefficients, $\chi_{n,mk}$
- Better sensitivity to $\eta/s(T)$.

Our arsenal of observables - stochastic approach

- Together various flow observables cover the sensitivity for all components of transport properties.

Name	Symbol	Measure	Sensitivity-stochastic approach
Flow coefficients	v_n	System expansion and anisotropy of the flow	Average $\langle\eta/s\rangle$ and $\zeta/s(T)$ peak
(Normalized) Symmetric cumulants	$\mathrm{(N)SC}(k,l,m)$	Correlations between magnitudes of flow harmonics	$\eta/s(T)$ temperature dependence
Linear and non-linear contributions	$v_{n,\mathrm{L}},\ v_{n,mk}$	Magnitude of the linear and non-linear contributions	$\eta/s(T)$ and initial conditions, not used
Non-linear flow mode coefficients	$\chi_{n,mk}$	Quantification of the non-linear response	η/s at the freeze-out
Symmetry-plane correlations	$\rho_{n,mk}$	Correlations between the directions of flow harmonics	$\eta/s(T)$

Thanks to excellent ALICE papers over years:

- Phys.Rev.Lett. 117 (2016) 182301, Phys.Lett. B773 (2017) 68, Phys.Rev. C 97 (2018) 024906, JHEP05 (2020) 085, Phys.Lett. B818 (2021) 136354, Phys.Rev.Lett. 127 (2021) 092302 - **flow**
- Phys.Rev.Lett. 106 (2011) 032301, Phys.Rev.C 88 (2013) 044910, Phys.Lett. B772 (2017) 567-577, Phys.Rev.C 101, 044907 (2020) - N_{ch} **and** $\langle p_{\mathrm{T}}\rangle$

Our arsenal of observables

Duke (2019)

2.76 TeV

- PID[1] mult. and N_{ch}
- Transverse energy E_{T}
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- $\delta p_{\mathrm{T}} / \langle p_{\mathrm{T}} \rangle$
- v_2 to v_4

5.02 TeV

- N_{ch}
- v_2 to v_4

[1] Jyväskylä (2021)

5.02 TeV

- PID[2] mult. and N_{ch}
- v_2 to v_7
- NSC(3,2) to NSC(4,3)
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- $\chi_{4,22}$ to $\chi_{6,mk}$

[2] Jyväskylä (2022)

2.76 TeV

- N_{ch}
- NSC(3,2), NSC(4,2)
- NSC(2,3,4), NSC(2,3,5)
- PID[1] $\langle p_{\mathrm{T}} \rangle$
- v_2 to v_4
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$

5.02 TeV

- PID[2] mult. and N_{ch}
- NSC(3,2) to NSC(4,3)
- PID $\langle p_{\mathrm{T}} \rangle$
- v_2 to v_7
- $\chi_{4,22}$ to $\chi_{6,mk}$
- $\rho_{4,22}$ to $\rho_{6,mk}$

All reference data based on ALICE measurements.

Red: Missing from other group(Duke etc)
Blue: New since our PRC.
Orange: Not used in our studies.

---

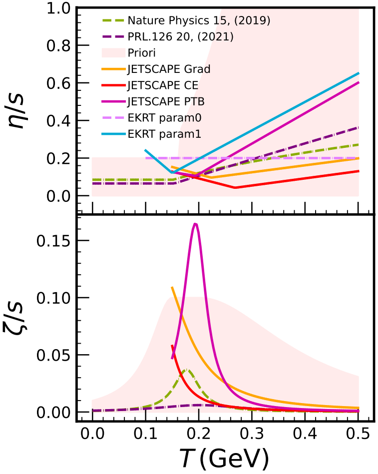
[1] $\pi^{\pm}$, $\mathrm{K}^{\pm}$ and $\mathrm{p}^{\pm}$

[2] $\mathrm{p}^{\pm}$

[1]. J.E. Parkkila, A. Onnerstad, D.J. Kim, PRC **104** (2021) 054904

[2]. J.E. Parkkila, A. Onnerstad, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, arXiv:2111.08145

Analysis steps and priori



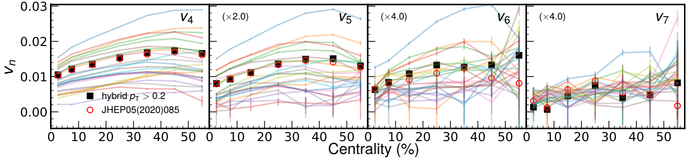
PRC 104 (2021) 054904

① Choose prior parameter range based on results from 2019

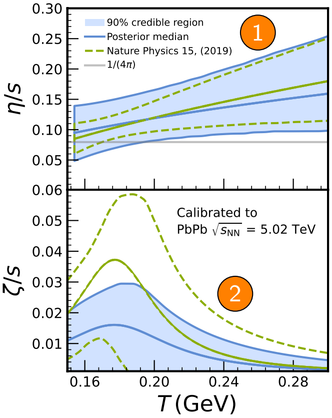
② Run hydro T

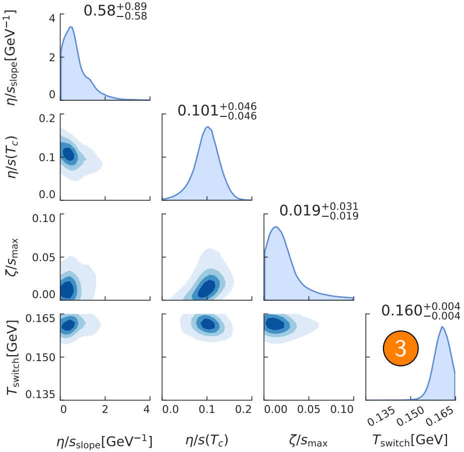
③ Calculate observables using our experimental framework

④ Train emulator and setup/run Bayesian analysis



$\eta/s(T)$ and $\zeta/s(T)$ - Jyväskylä (2021) (5.02 TeV only)





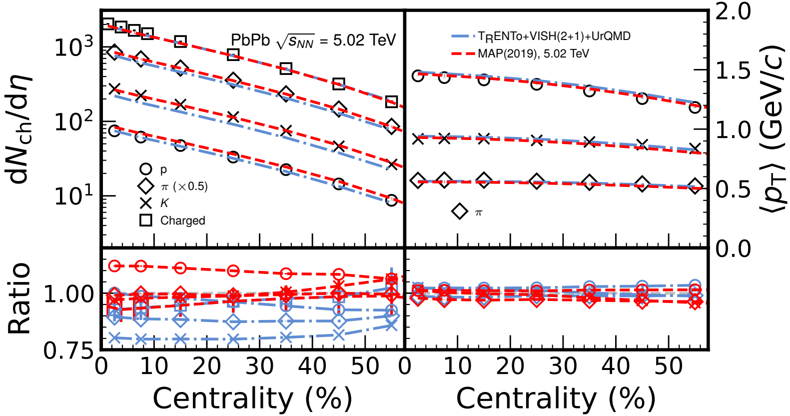
① Similar $\eta/s(T)$ to Duke (2019)

② Lower $\zeta/s(T)$ – much lower to previous calculations

③ Higher switching temperature T_{switch} (vs. Duke 152 MeV)

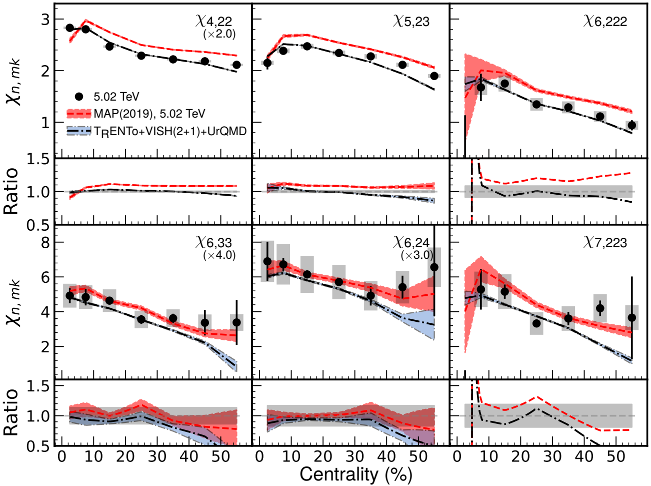
Additional observables have reduced $\zeta/s(T)$. However, one collision energy only limits the potential of the additions.

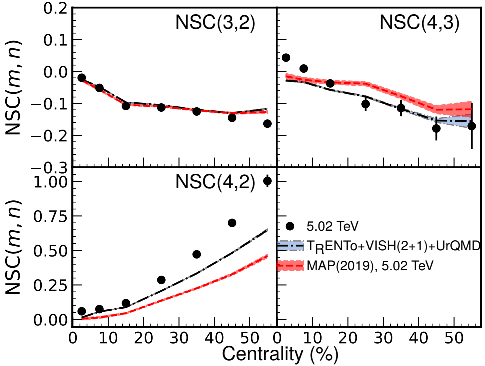
PID multiplicity and $\langle p_{\mathrm{T}} \rangle$ - Jyväskylä (2021) (5.02 TeV only)



- Comparison between our MAP and Duke (2019)
- Good agreement with the measured charged particle yield
- Improved estimate of the proton production
- Disagreeing $\pi^{\pm}$ and $\mathrm{K}^{\pm}$

$\chi_{n,mk}$ and NSC(m,n) - Jyväskylä (2021) (5.02 TeV only)

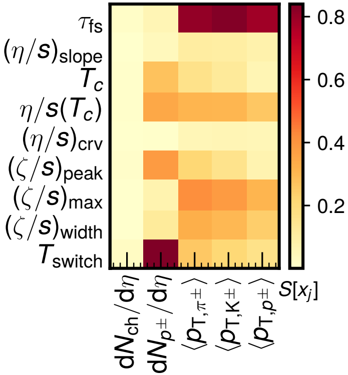


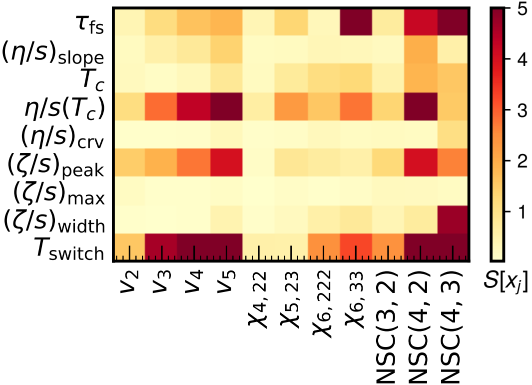


- Improved estimates on $\chi_{n,mk}$ and NSC! See PRC **104** (2021) 054904 for all graphs.

Sensitivity of the constraints to parameters - Jyväskylä (2021) (5.02 TeV only)

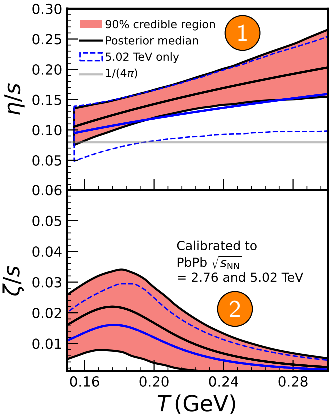
Sensitivity of the observables: $S[x_j] = \Delta/\delta$, where $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$.

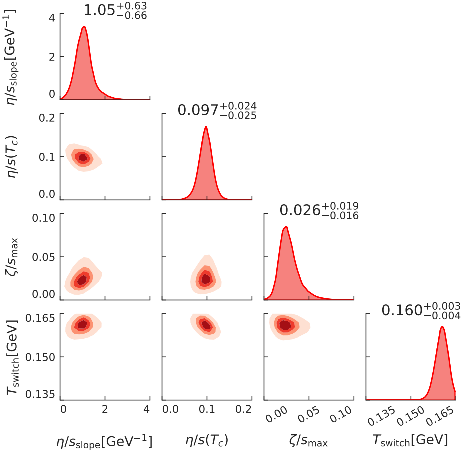




- Symmetric cumulants the most sensitive observables followed by v_n and $\chi_{n,mk}$.

$\eta/s(T)$ and $\zeta/s(T)$ - Jyväskylä (2022), 2.76 + 5.02 TeV





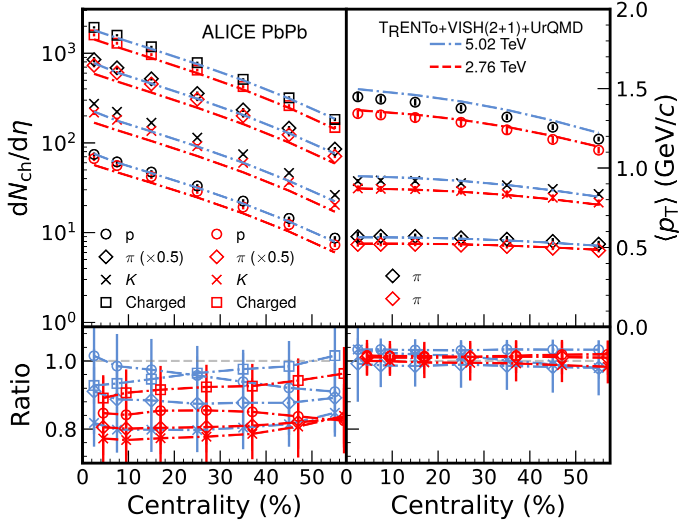
① Significantly improved $\eta/s(T)$ uncertainty

② Non-zero $\zeta/s(T)$

③ Overall better convergence for parameter components

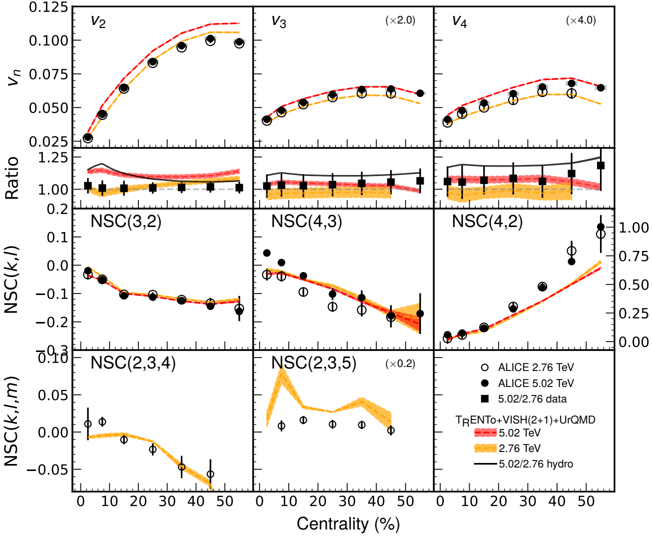
- Together with two collision energies and added observables, the uncertainty has reduced!

PID multiplicity and $\langle p_\mathrm{T} \rangle$ - Jyväskylä (2022), 2.76 + 5.02 TeV



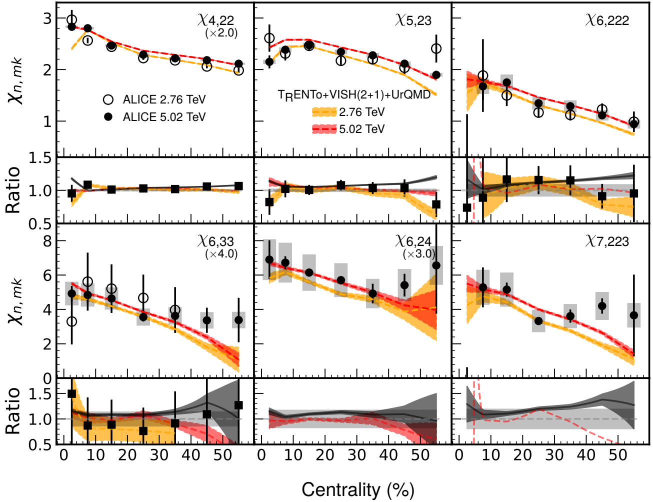
- Agreement for charged particle yield in 2.76 TeV and 5.02 TeV
- 10–20% difference for PID multiplicity
- Qualitative agreement for $\langle p_\mathrm{T} \rangle_{\pi,\mathrm{K}}$

v_n and NSC(n,m) - Jyväskylä (2022), 2.76 + 5.02 TeV



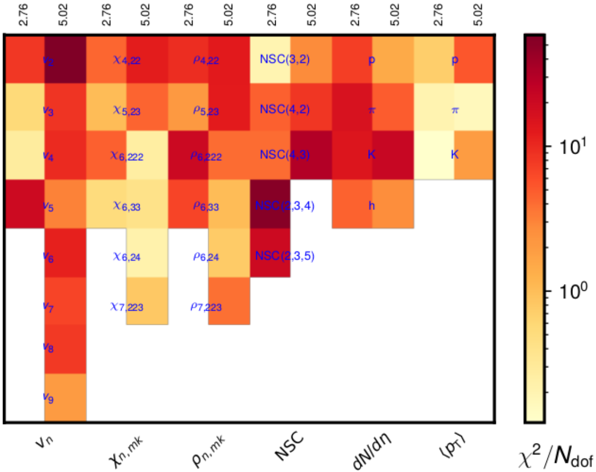
- Good agreement for 2.76 TeV v_n, overestimated v_n for 5.02 TeV by $\sim$ 10%
- Magnitude and centrality dependence of NSC well captured. Further improved estimate for NSC(4,2).
- Good agreement for NSC(2,3,4). NSC(2,3,5) overestimated.

$\chi_{n,mk}$ - Jyväskylä (2022), 2.76 + 5.02 TeV



- Qualitative agreement in both beam energies for all mode coupling coefficients.
- See arXiv:2111.08145 for all graphs.

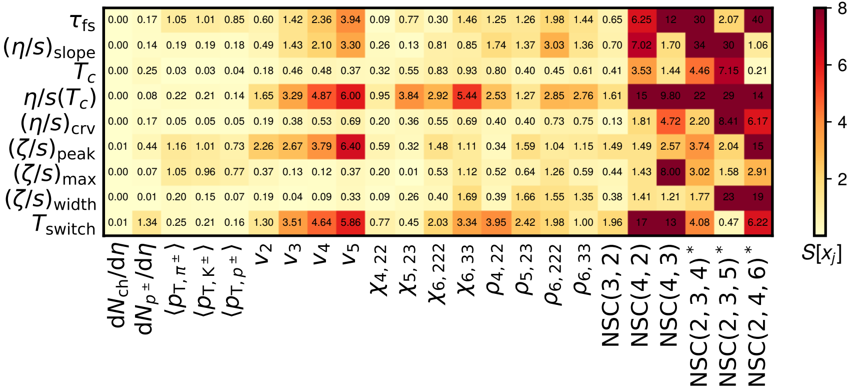
Remaining Concerns?



- Higher energy description worse for all observables except for:
  - v_5
  - $\chi_{6,222}$
  - charged particle multiplicity
  - $\rho_{6,222}$ and $\rho_{6,33}$
- Concerns
  - overestimated v_n for 5.02 TeV by $\sim 10\%$
  - still underestimated NSC(4,2)
  - overestimated NSC(2,3,5)
  - PID multiplicity (especially $\pi^{\pm}$)

Sensitivity of the constraints to parameters

Sensitivity of the observables: $S[x_j] = \Delta/\delta$, where $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$.



	$dN_{\mathrm{ch}}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{\mathrm{T},\pi^\pm}\rangle$	$\langle p_{\mathrm{T},\mathrm{K}^\pm}\rangle$	$\langle p_{\mathrm{T},p^\pm}\rangle$	v_2	v_3	v_4	v_5	$\chi_{4,22}$	$\chi_{5,23}$	$\chi_{6,222}$	$\chi_{6,33}$	$\rho_{4,22}$	$\rho_{5,23}$	$\rho_{6,222}$	$\rho_{6,33}$	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)	NSC(2, 3, 4)*	NSC(2, 3, 5)*	NSC(2, 4, 6)*
τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
$(\eta/s)_{\mathrm{slope}}$	0.00	0.14	0.19	0.19	0.18	0.49	1.43	2.10	3.30	0.26	0.13	0.81	0.85	1.74	1.37	3.03	1.36	0.70	7.02	1.70	34	30	1.06
T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
$\eta/s(T_c)$	0.00	0.08	0.22	0.21	0.14	1.65	3.29	4.87	6.00	0.95	3.84	2.92	5.44	2.53	1.27	2.85	2.76	1.61	15	9.80	22	29	14
$(\eta/s)_{\mathrm{crv}}$	0.00	0.17	0.05	0.05	0.05	0.19	0.38	0.53	0.69	0.20	0.36	0.55	0.69	0.40	0.40	0.73	0.75	0.13	1.81	4.72	2.20	8.41	6.17
$(\zeta/s)_{\mathrm{peak}}$	0.01	0.44	1.16	1.01	0.73	2.26	2.67	3.79	6.40	0.59	0.32	1.48	1.11	0.34	1.59	1.04	1.15	1.49	1.49	2.57	3.74	2.04	15
$(\zeta/s)_{\mathrm{max}}$	0.00	0.07	1.05	0.96	0.77	0.37	0.13	0.12	0.37	0.20	0.01	0.53	1.12	0.52	0.64	1.26	0.59	0.44	1.43	8.00	3.02	1.58	2.91
$(\zeta/s)_{\mathrm{width}}$	0.00	0.01	0.20	0.15	0.07	0.19	0.04	0.09	0.33	0.09	0.26	0.40	1.69	0.39	1.66	1.55	1.35	0.38	1.41	1.21	1.77	23	19
T_{switch}	0.01	1.34	0.25	0.21	0.16	1.30	3.51	4.64	5.86	0.77	0.45	2.03	3.34	3.95	2.42	1.98	1.00	1.96	17	13	4.08	0.47	6.22

- Symmetric cumulants, especially NSC(n,m,k) among the most sensitive observables followed by v_n and $\chi_{n,mk}$.
- The precision measurements of observables, reflecting mostly non-linear hydrodynamic responses, are crucial.

Summary

Success:

- Higher harmonic orders and non-linear flow observables. → better constraints.
- Improved the overall uncertainty by a factor of two by combining two beam energy data.
- As a bonus, sensitivities of the observables are now quantified
  → precision measurements of observables, reflecting non-linear hydrodynamic responses.

Challenges:

- 10% difference for v_2 (5.02 TeV)
- NSC description improved except for NSC(4,2)
- Remaining discrepancy for PID multiplicity (especially $\pi^{\pm}$)
- Improving the initial state model, with dynamical collision model or subnucleon structure à la IP-Glasma, might help us to improve the results.

Outlook

Data

- RHIC data (AuAu collisions) - Energy and system size dependence
- LHC pPb and pp data - System size dependence
- Use new observables from Jyväskylä + TUM
  - Higher order ($n > 5$) Symmetric cumulants : My thesis work
  - Improved Symmetric Plane Correlation (SPC) : Jyväskylä + TUM, independent from flow magnitude correlations
  - Asymmetric Cumulants (AC) : Cindy Mordasini

Theory

- Improving the initial conditions with
  - EKRT (collaboration with Harri Niemi and Kari Eskola)
  - IP+Glasma (collaboration with Sangyoung Jeon and Heikki Mäntysaari)
- Testing hydro limit of small systems?
- Role of the small system.

Thanks

Thank you for your attention!

Acknowledgments:

- CSC for providing the ∼24 million CPU hours
- Harri Niemi, Kari Eskola, Jonah E. Bernhard, J. Scott Moreland and Steffen A. Bass for their useful comments