Cosmological phase transition thermodynamics: pushing perturbation theory to its limits

Tuomas V. I. Tenkanen

NORDITA, KTH and Stockholm University
T.D. Lee Institute/Shanghai Jiao Tong University

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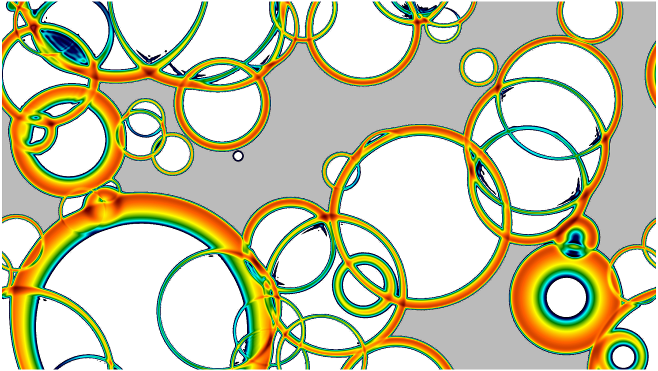
E-mail: tuomas.tenkanen@su.se

Motivation

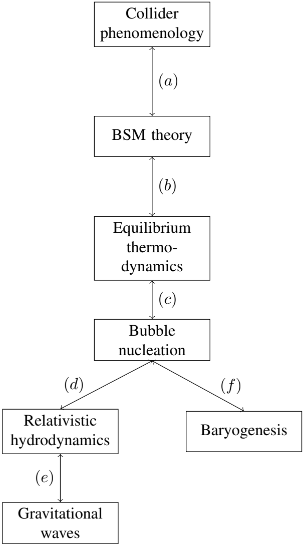
- Thermal history of the electroweak symmetry breaking is interesting! (baryogenesis) $\rightarrow$ Standard Model has a crossover transition.
- First order Electroweak phase transition: playground for beyond the SM physics near the EW scale, with relatively light fields and strong enough couplings to Higgs $\rightarrow$ collider probes, gravitational wave signals?
- Our understanding of cosmological phase transitions and their GW signatures are impaired by theoretical uncertainties, due to insufficiencies in the computation of thermodynamics.

1st order cosmological phase transition (fig. from David J. Weir)

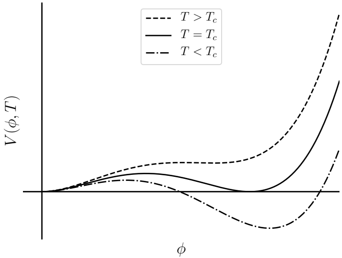


Cosmological GW background: huge discovery potential, to be probed by future space-based GW observatories.

Pipeline: EWPT in BSM theories.



Thermodynamics in perturbation theory: effective potential



$$V_{\text{eff}} \simeq \frac{1}{2}(-m^2 + \# g^2 T^2)\phi^2 + \frac{1}{4!}g^2\phi^4 + \# T\phi^3 + \ldots$$

- Symmetric minimum at high T, broken minimum at $T = 0$, degenerate at T_c, barrier $\rightarrow$ 1st order phase transition.
- Formal power counting: $m^2 \sim (gT)^2$, $g^2/4\pi \ll 1$ (weak coupling).

Perturbative expansion at $T = 0$

The coupling expansion of the effective potential

$$V_{\text{eff}}^{T=0} = \underbrace{A_2 g^2}_{\text{tree-level}} + \underbrace{A_4 g^4}_{\text{1-loop}} + \underbrace{A_6 g^6}_{\text{2-loop}} + \dots \; . \tag{1}$$

The running of couplings takes the form

$$\frac{dg^2}{d \log \mu} = \underbrace{B_4 g^4}_{\text{1-loop}} + \underbrace{B_6 g^6}_{\text{2-loop}} + \dots \; , \tag{2}$$

The one-loop running of $A_2 g^2$ is an $\mathcal{O}(g^4)$ effect and is cancelled exactly by explicit logarithms in $A_4 g^4$, i.e. at one-loop level

$$\frac{dV_{\text{eff}}^{T=0}}{d \log \mu} = \underbrace{\left(A_2 B_4 + \frac{dA_4}{d \log \mu} \right)}_{\text{cancels}} g^4 + \mathcal{O}(g^6). \tag{3}$$

Renormalisation scale invariance: RG improved effective potential.

Expansion parameter at high-T

At high T loop expansion and coupling expansion misalign, quadratic term: $\frac{1}{2}(-m^2 + \#g^2 T^2)\phi^2$

and perturbation theory requires *resummations* near critical temperature: $-m^2 \to m^2_{\text{eff}} = -m^2 + \#g^2 T^2$.

The effective expansion parameter for light bosons is

$$\frac{g^2}{e^{E/T} - 1} \approx \frac{g^2 T}{E} \leq \frac{g^2 T}{m_{\text{eff}}} \sim g$$

for $m_{\text{eff}} \sim gT$. Physically: Bose enhancement, or high occupancy of infrared bosonic modes.

For "*ultrasoft*" $m_{\text{eff}} \lesssim g^2 T$, the perturbative expansion breaks down:
Light bosons are nonperturbative at finite temperature (Linde's IR problem): lattice simulations required.

Leftover scale dependence at one-loop at high-T

$$V_{\text{eff}}^{\text{1-loop}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{CW}} + V_T + V_{\text{daisy}} \tag{4}$$

$$= \underbrace{V_{\text{tree}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{soft}}^{\text{resummed}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{hard}}}_{\mathcal{O}(\phi^2 g^2)+\mathcal{O}(\phi^4 g^4)} , \tag{5}$$

Quadratic piece (that determines T_c!) is strongly scale dependent at one-loop

$$V_{\text{eff}} \simeq \underbrace{\frac{1}{2}(\underbrace{-\mu^2}_{\text{tree-level}} + \underbrace{\#\lambda T^2}_{\text{1-loop}})}_{\mathcal{O}(g^2)\text{, runs at }\mathcal{O}(g^4)} \phi^2 + \underbrace{\text{(2-loop)}}_{\mathcal{O}(g^4)\ T^2\text{ log-terms}} \phi^2 \ldots$$

Full $\mathcal{O}(g^4)$ – and cancellation of RG-scale – requires 2-loop thermal masses!

Perturbative expansion at high-T

At high temperature, the enhancement of IR bosonic modes modifies the coupling expansion of the effective potential,

$$V_{\text{eff}}^{\text{high-}T} = \underbrace{a_2 g^2}_{\text{tree-level and 1-loop}} + \underbrace{a_3 g^3}_{\text{resummed 1-loop}} + \underbrace{a_4 g^4}_{\text{1-loop, (resummed) 2-loop}} + \underbrace{a_5 g^5}_{\text{resummed 3-loop}} + \dots \quad (6)$$

Odd powers of g arise, and loop orders are mixed in the expansion coefficients.

One-loop running links the coefficients a_i with a_{i+2} (and higher loop running with a_{i+4} and so forth),

$$\frac{dV_{\text{eff}}^{\text{high-}T}}{d\log\mu} = \underbrace{\left(a_2 B_4 + \frac{da_4}{d\log\mu}\right)}_{\text{cancels}} g^4 + \underbrace{\left(\frac{3}{2} a_3 B_4 + \frac{da_5}{d\log\mu}\right)}_{\text{cancels}} g^5 + \mathcal{O}(g^6). \quad (7)$$

Perturbative expansion: summary

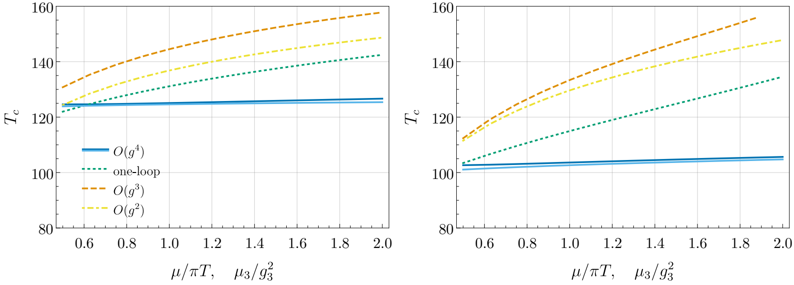
- The one-loop approximation to the thermal effective potential is incomplete at $\mathcal{O}(g^4)$. This causes residual μ-dependence, and leads to numerical inaccuracies.
- At high-T, first possible cancellation of μ requires 2-loop computation (and accuracy that matches that of $T = 0$, requires 3-loop computation of resummed, IR contributions).
- This detail is overlooked by almost all recent literature on gravitational waves from cosmic phase transition (that resort to one-loop level computation)!

Numerical example: SM plus singlet

- Couple real scalar S to the SM with the Higgs portal $\frac{1}{2}a_2 H^\dagger H\, S^2$ and singlet self-interaction $\frac{1}{4}b_4 S^4$.
- Fix (A): $m_S = 160$ GeV, $a_2 = 1.1$ and $b_4 = 0.45$
- and (B): $m_S = 160$ GeV, $a_2 = 1.4$ and $b_4 = 1.4$.
- These two benchmark points have strong two-step phase transitions.

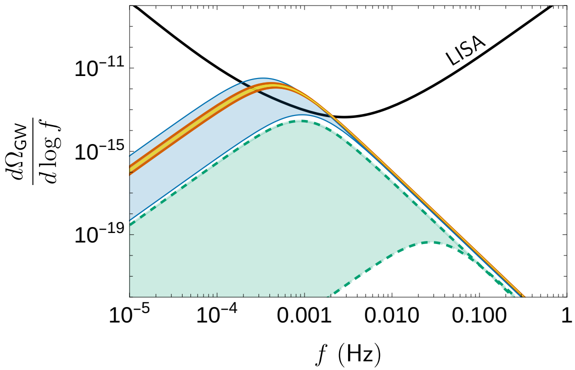
Critical temperature as function of RG scale μ

Left: BM-(A), right: BM-(B).



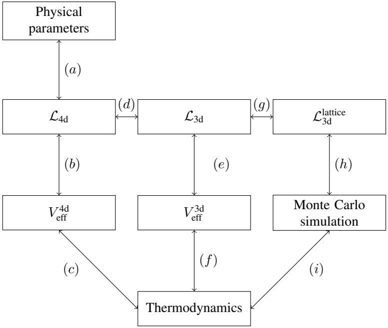
- Conclusion: *equilibrium* quantities such as T_c are in control, only if computed consistently up to $\mathcal{O}(g^4)$.

Multiple orders-of-magnitude uncertainty in the peak GW amplitude



Green: one-loop.
Blue: $\mathcal{O}(g^4)$ for equilibrium thermodynamics.
Yellow (estimate!): $\mathcal{O}(g^4)$ for bubble nucleation rate.

How do I resum thee? (weak coupling g: $\mathcal{O}(g^n)$)



- "4d approach" or 1-loop V_{eff} with daisy resummation: (a)-(b)-(c): $\mathcal{O}(g^3)$
- Perturbative (dimensionally reduced) 3d EFT approach: (a)-(d)-(e)-(f): $\mathcal{O}(g^4)$ (2-loop) or $\mathcal{O}(g^5)$ (3-loop).
- Non-perturbative 3d EFT approach: (a)-(d)-(g)-(h)-(i): $\mathcal{O}(g^6)$ and captures non-perturbative IR physics.

Generic models: 3d EFT in Mathematica within seconds

DRalgo: a package for effective field theory approach for thermal phase transitions

[2205.08815]: `https://github.com/DR-algo/DRalgo`

Pushing perturbation theory to its limits

The pressure (p) admits a schematic expansion

$$p \simeq T^4 \left(a + bg^2 + cg^3 + dg^4 + eg^5 \right) + \mathcal{O}(g^6 T^4), \qquad (8)$$

where a-e represent constants. Higher orders require non-perturbative lattice simulations, so the $\mathcal{O}(g^5 T^4)$ piece is **the final order computable in perturbation theory.** Its computation involves the 3-loop effective potential within the 3d EFT.

Challenge: compute all thermodynamic quantities to maximal order in perturbation theory.

Low order bubble nucleation rate is the most dominant source of uncertainty.

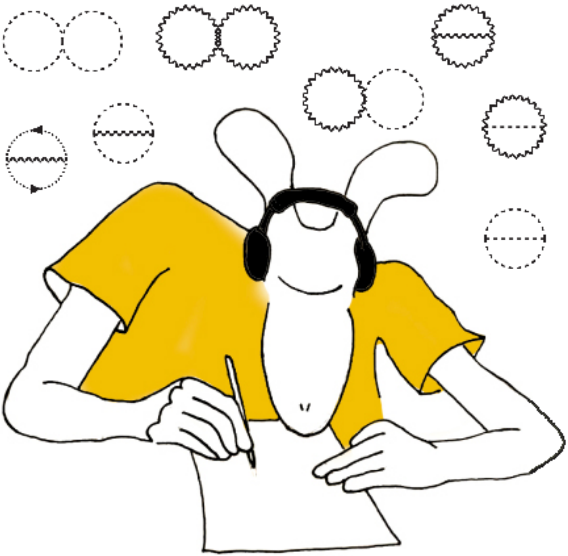
Recent developments in [2104.11804], [2108.04377], [2112.05472], [2112.08912].

Also the bubble wall speed should be derived as a function of BSM model parameters.

Summary

- Reliable theoretical predictions for LISA inevitably require bringing the study of cosmological phase transitions into the domain of precision cosmology.
- Dimensional reduction and use of 3d EFT is systematic way to organise thermal resummations and attack the IR problem and slow convergence at high-T.
- Challenge: determination of cosmological phase transition thermodynamics up to the maximal order in perturbation theory $\to$ sets a gold-standard which cannot be exceeded without non-perturbative lattice simulations.
- Lattice simulations cannot replace perturbation theory in scans over large parameter spaces of BSM theories.

Thanks!



SM + triplet scalar Σ^a: collider phenomenology

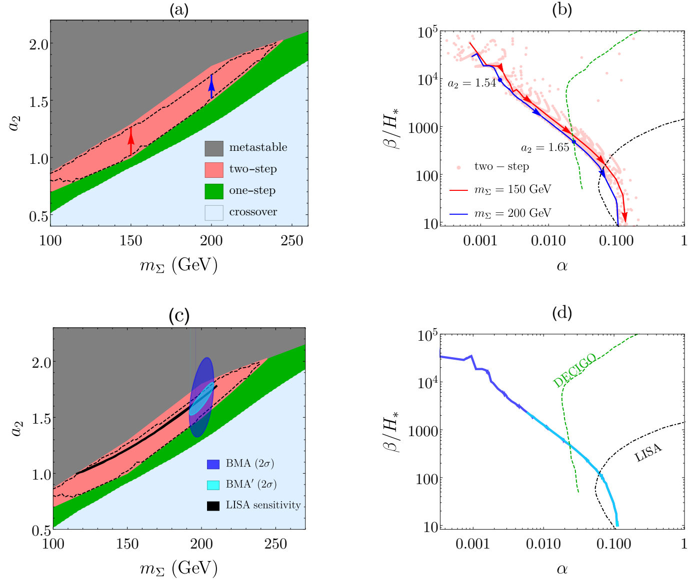
- Portal to Higgs by coupling a_2: $\mathcal{L}_{\text{SM}} + \mathcal{L}_{\Sigma} + \frac{1}{2} a_2 H^\dagger H (\Sigma^a \Sigma^a)$
- Possible pheno targets for future colliders: triplet mass, deviation to Higgs to digamma decay rate, branching fraction $\Sigma^0 \to ZZ$.

Envision a future measurement:

$$
\begin{aligned}
m_\Sigma &= (\ldots) \pm (\ldots) \\
\delta_{\gamma\gamma} &= (\ldots) \pm (\ldots) \\
\text{BR}(\Sigma^0 \to ZZ) &= (\ldots) \pm (\ldots)
\end{aligned}
$$

Relate to a_2 (usual $T = 0$ QFT).

pheno $\to (m_\Sigma, a_2) \to (T_*, \alpha, \beta/H_*, v_w) \to$ LISA SNR



Key points

- Need to go beyond 1-loop approximations in perturbation theory.
- A first order transition during the second step could generate a signal accessible to LISA generation detectors.
- Possible GW signal displays a strong sensitivity to the portal coupling between the new scalar and the Higgs boson.