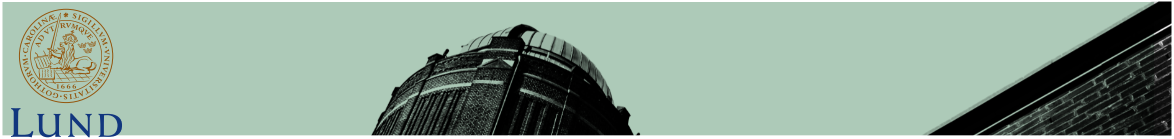

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# Speeding up SM Scattering Amplitudes Using Chirality Flow


**DOKTORANDDAG 7 DECEMBER 2022 - ANDREW LIFSON**


**BASED ON HEP-PH:2003.05877 (EPJC), HEP-PH:2011.10075 (EPJC), AND HEP-PH:2203.13618 (EPJC)**


**IN COLLABORATION WITH JOAKIM ALNEFJORD, SIMON PLÄTZER, CHRISTIAN REUSCHLE, MALIN SJÖDAHL, AND ZENNY WETTERSTEN**


# A Quick Introduction to Me



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- Australian living in Sweden
- Final-year PhD student (defend April 26th)
- Into running, football (both Australian and world types), cycling, golf, travelling
- Pre-covid: organised joint theory/experimental drinks
  - Who will organise them again now?

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# Interesting, Cool, or Useful Things in my PhD

- Strong involvement in unions including chair of NDR
  - Friends outside department
  - Leadership and organisational skills (both real and for CV)
  - Better understanding of university organisation
- Five month stay in UC Louvain
  - Learned new skills (Python, Fortran, MadGraph)
  - New contacts for future collaboration, referee letters etc.
- Took several personal development courses at university
  - How to finish on time workshop
  - Career outside academia
  - Career control for researchers
  - List of options available at https://www.staff.lu.se/employment/professional-and-careers-development/career-development-academic-staff/career-development-doctoral-students

# And now onto the physics...

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# Scattering Amplitudes Recap

- In any detector we count events, $N_{events} = \sigma I$
  - $\sigma =$ Cross section, defined by the type of interaction
  - $I =$ Intensity, parameter of experiment
- Cross-section is:

$$\sigma(a+b \to x) = \underbrace{d\phi_n}_{\text{kinematics}} \times \underbrace{\overline{|\mathcal{M}(a+b \to x)|}^2}_{\text{squared scattering amplitude}}$$

### How to Calculate Scattering Amplitude $\mathcal{M}(a+b \to x)$?

(Usually) use Feynman diagrams
Use and exploit symmetries in theory

# Factoring out Symmetries

- In general: {incoming particles} $\rightarrow$ {outgoing particles}
- Use crossing symmetry to work with {0} $\rightarrow$ {outgoing particles}
- Symmetry groups and kinematics (Lorentz symmetry) factorise

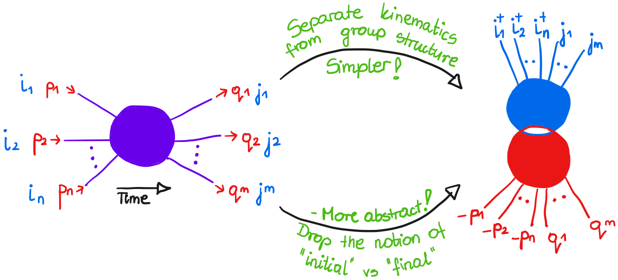


Figure: By Mattias Sjö and Ewa Kwasniewicz

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# Symmetries in Physics

- Always on the lookout for **symmetries**
  $\rightarrow$ conserved quantities: **quantum numbers**
- The most important symmetry groups: Lorentz (Poincaré) group, SU($n$)

## Crash course in Lorentz group

- Particles transform in different ways when boosted or rotated
- Algebra of Lorentz group $\equiv so(3,1)_{\mathbb{C}} \cong su(2) \oplus su(2)$
- Representations of Lorentz group
  - $(0,0)$ scalar particles
  - $(\frac{1}{2},0)$ left-chiral and $(0,\frac{1}{2})$ right-chiral Weyl (2-component) spinors.
  - $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$, Dirac (4-component) spinors.
  - $(\frac{1}{2},\frac{1}{2})$ vectors, e.g. photons, gluons

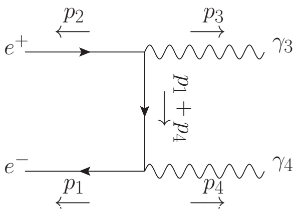
# Connecting Lorentz Group to an Amplitude



## How to Calculate a Process

Feynman diagrams are a proxy of a real scattering process
Each leg, vertex, etc. $\equiv$ a mathematical expression
Different Lorentz reps $\equiv$ different line-types $\equiv$ different expressions

e.g.




$$\sim \underbrace{[\bar{u}(p_1)\gamma^\mu (p_1^\nu + p_4^\nu)\gamma_\nu\gamma^\rho v(p_2)]\,\epsilon_\rho(p_3)\epsilon_\mu(p_2)}_{\text{A mathematical expression we have simplify and square}}$$

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# How to Calculate? The Original Method

Square $4 \times 4$ matrix, take trace

Very slow, not computer efficient

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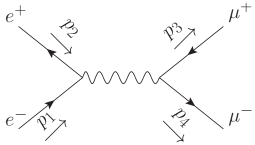
# How to Calculate? The Original Method

Square $4 \times 4$ matrix, take trace
Very slow, not computer efficient

- Keep all particles unpolarised
- Obtain amplitude as matrix



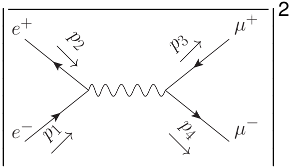

$$\sim [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)]$$

# How to Calculate? The Original Method



Square $4 \times 4$ matrix, take trace
Very slow, not computer efficient

- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal

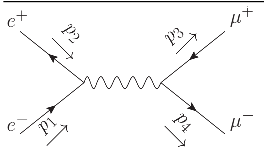



$$\sim \sum_{r,s,t,w} [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)]$$
$$\times [\bar{u}_s(p_1)\gamma^\nu v_r(p_2)][\bar{v}_w(p_3)\gamma_\nu u_t(p_4)]$$

# How to Calculate? The Original Method

Square $4 \times 4$ matrix, take trace
Very slow, not computer efficient

- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal
- Move components around



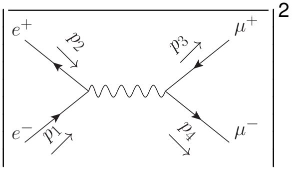

$$\left| \; \cdot \; \right|^2 \sim \sum_{r,s,t,w} [\bar{v}_r(p_2)\gamma^\mu u_s(p_1)][\bar{u}_t(p_4)\gamma_\mu v_w(p_3)] \times [\bar{u}_s(p_1)\gamma^\nu v_r(p_2)][\bar{v}_w(p_3)\gamma_\nu u_t(p_4)]$$

$$\sim \sum_{r,s,t,w} [\gamma^\nu v_r(p_2)\bar{v}_r(p_2)\gamma^\mu u_s(p_1)\bar{u}_s(p_1)] \times [\gamma_\nu u_t(p_4)\bar{u}_t(p_4)\gamma_\mu v_w(p_3)\bar{v}_w(p_3)]$$

# How to Calculate? The Original Method

Square $4 \times 4$ matrix, take trace
Very slow, not computer efficient

- Keep all particles unpolarised
- Obtain amplitude as matrix
- Square the matrix amplitude
  - Spin states are orthogonal
- Move components around
- Use spin sums
- Take trace of fermionic structure
- Requires identities of $\gamma^\mu$
- Simplify




$$\sim \mathrm{Tr}\left[\gamma^\nu(\not{p}_2 - m_e)\gamma^\mu(\not{p}_1 + m_e)\right]$$
$$\times\, \mathrm{Tr}\left[\gamma_\nu(\not{p}_4 + m_\mu)\gamma_\mu(\not{p}_3 + m_\mu)\right]$$

$$\mathrm{Tr}\left[\gamma^{\mu_1}\gamma^{\mu_2}\right] = 4g^{\mu_1\mu_2}$$
$$\mathrm{Tr}\left[\gamma^{\mu_1}\ldots\gamma^{\mu_4}\right] = 4(g^{\mu_1\mu_2}g^{\mu_3\mu_4} - g^{\mu_1\mu_3}g^{\mu_2\mu_4} + g^{\mu_1\mu_4}g^{\mu_3\mu_2})$$
$$\mathrm{Tr}\left[\gamma^{\mu_1}\ldots\gamma^{\mu_{2n+1}}\right] = 0$$

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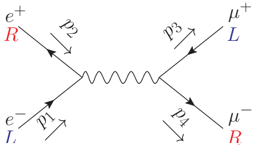
# How to Calculate? Spinor-Helicity

Give each particle a defined helicity $\Rightarrow$ amplitude now a number!

- Spinors, polarisation vectors in terms of left-chiral $|p]$, $[p|$ and right-chiral $|p\rangle$, $\langle p|$
- $\tau^\mu \equiv$ Pauli matrices




$$\sim \langle p_2|\bar{\tau}^\mu|p_1]\langle p_4|\bar{\tau}_\mu|p_3]$$
$$= [p_1|\tau^\mu|p_2\rangle\langle p_4|\bar{\tau}_\mu|p_3]$$
$$= \langle p_4 p_2\rangle[p_1 p_3]$$

## Spinor Inner Products

Do maths to get spinor inner products $\langle ij\rangle$, $[ij] \sim \sqrt{2p_i \cdot p_j}$
Easy to square, computer efficient

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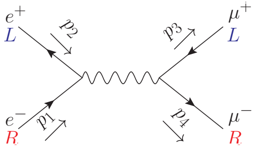


$$\sim [p_2|\tau^\mu|p_1\rangle\langle p_4|\bar{\tau}_\mu|p_3]$$
$$= \langle p_4 p_1\rangle[p_2 p_3]$$

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Do maths to get spinor inner products $\langle ij\rangle, [ij] \sim \sqrt{2p_i \cdot p_j}$
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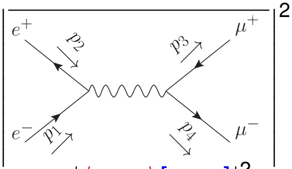


$$\sim |\langle p_4 p_2\rangle [p_1 p_3]|^2 + |\langle p_4 p_1\rangle [p_2 p_3]|^2$$

## Spinor Inner Products

Do maths to get spinor inner products $\langle ij\rangle$, $[ij] \sim \sqrt{2p_i \cdot p_j}$

Easy to square, computer efficient

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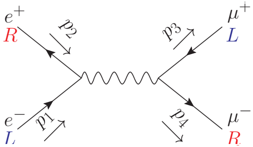
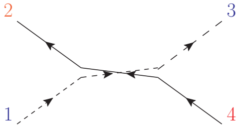
# How to Calculate: Chirality-Flow

## Chirality-Flow: Our New Method

Assign flow lines instead of $|p]$, $[p|$, $|p\rangle$, $\langle p|$ etc.
Join lines consistently, then read off numbers $\langle ij\rangle$, $[ij] \sim \sqrt{2p_i \cdot p_j}$ to square




$\sim$




$\equiv \langle p_4 p_2\rangle [p_1 p_3]$

## Inner products now represented by connected lines

$\langle ij\rangle = -\langle ji\rangle =$ 


$[ij] = -[ji] =$ 

# How to Calculate: Chirality-Flow

## Chirality-Flow: Our New Method

Assign flow lines instead of $|p]$, $[p|$, $|p\rangle$, $\langle p|$ etc.
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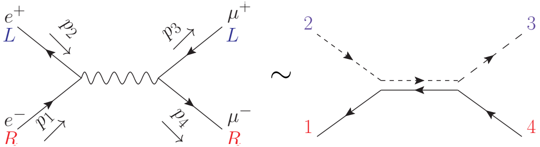



$\equiv \langle p_4 p_1\rangle [p_2 p_3]$

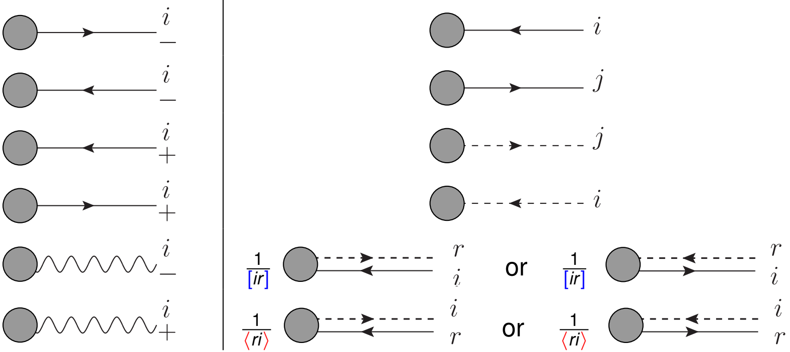
## Inner products now represented by connected lines

$\langle ij\rangle = -\langle ji\rangle =$ 


$[ij] = -[ji] =$ 

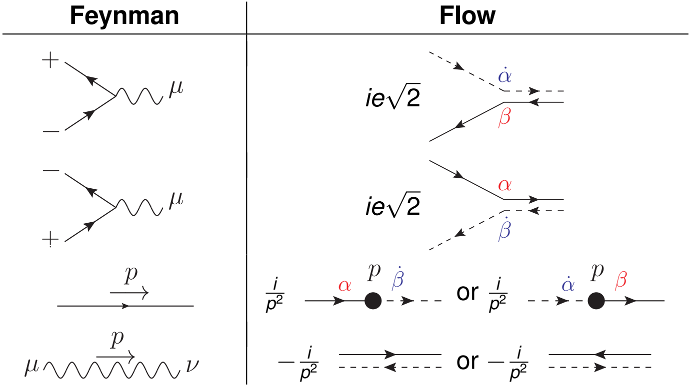
# The Massless QED Flow Rules: External Particles

| Species | Feynman | Flow |
|---|---|---|
| $\bar{u}^{-}(p_i)$ | | |
| $v^{-}(p_j)$ | | |
| $v^{+}(p_j)$ | | |
| $\bar{u}^{+}(p_i)$ | | |
| $\epsilon^{\mu}_{-}(p_i, r)$ | | |
| $\epsilon^{\mu}_{+}(p_i, r)$ | | |



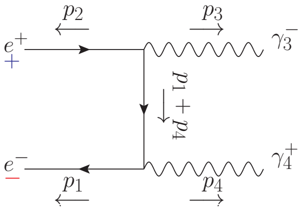

Left-chiral $\equiv$ dotted lines right-chiral $\equiv$ solid lines

# The QED Flow Rules: Vertices and Propagators




Left-chiral ≡ dotted lines    right-chiral ≡ solid lines

# An Illuminating Example: $e^+e^- \to \gamma\gamma$




**Spinor helicity:**

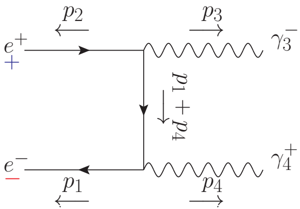
$$\sim \langle p_1|\bar\tau^\mu \underbrace{\left(|p_1]\langle p_1| + |p_4]\langle p_4|\right)}_{\not{p}_1+\not{p}_4} \bar\tau^\nu|p_2] \underbrace{\frac{\langle r_3|\bar\tau_\nu|p_3]}{\langle r_3 3\rangle}}_{\epsilon_3^-} \underbrace{\frac{[r_4|\tau_\mu|p_4\rangle}{[4 r_4]}}_{\epsilon_4^+}$$

$$= \frac{\left(\langle p_1|\bar\tau^\mu|p_1] + \langle p_1|\bar\tau^\mu|p_4]\right)[r_4|\tau_\mu|p_4\rangle \; \left(\langle p_1|\bar\tau^\nu|p_2] + \langle p_4|\bar\tau^\nu|p_2]\right)[p_3|\tau_\nu|r_3\rangle}{\langle r_3 3\rangle[4 r_4]}$$

$$= \underbrace{\frac{\langle 1 r_4\rangle\left([41]\langle 13\rangle + [44]\langle 43\rangle\right)[r_3 2]}{\langle r_3 3\rangle[4 r_4]}}_{\text{Fierz identities like } \langle i|\bar\tau^\mu|j][k|\tau_\mu|l\rangle = \langle il\rangle[kj]} = \underbrace{\frac{\langle 1 r_4\rangle[41]\langle 13\rangle[r_3 2]}{\langle r_3 3\rangle[4 r_4]}}_{[ii]=0}$$

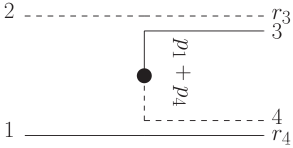
# An Illuminating Example: $e^+e^- \to \gamma\gamma$






**Chirality flow:**

$$\sim \frac{1}{\langle r_3 3\rangle [4 r_4]}$$



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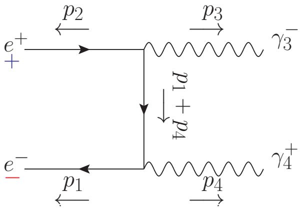


**Chirality flow:**

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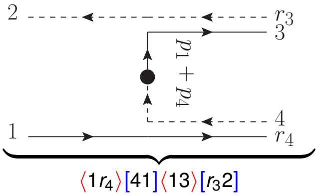

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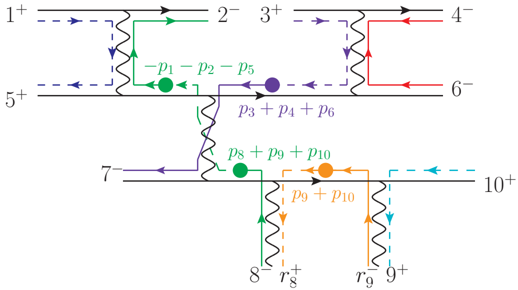
# A complicated QED Example




Compare to:

- Standard QFT:
  $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_{12}})$,
  $2 \times \mathrm{Tr}(\gamma^{\mu_1} \cdots \gamma^{\mu_4})$,
  $2 \times$ photon spin sum
- Standard spinor-helicity:
  5 charge conjugation/Fierz
  $+$ rearranging

$$= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{s_{1\,2}\;s_{3\,4}\;s_{8\,9\,10}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{s_{1\,2\,5}\;s_{3\,4\,6}\;s_{8\,9\,10}\;s_{9\,10}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8 r_8]\langle r_9 9\rangle}}_{\text{polarization vectors}} \quad [15]\langle 64\rangle[10\;9]$$

$$\times \left(\langle r_9 9\rangle[9 r_8] + \langle r_9 10\rangle[10 r_8]\right)\left(\underbrace{[33]}_{0}\langle 37\rangle + [34]\langle 47\rangle + [36]\langle 67\rangle\right)$$

$$\times \left(-\langle 89\rangle[91]\langle 12\rangle - \langle 89\rangle[95]\langle 52\rangle - \langle 8\,10\rangle[10\;1]\langle 12\rangle - \langle 8\,10\rangle[10\;5]\langle 52\rangle\right)$$

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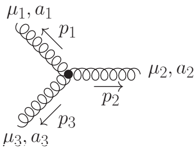
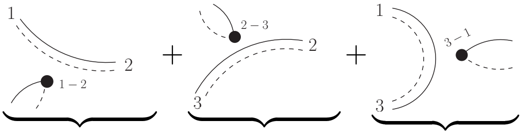
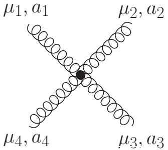
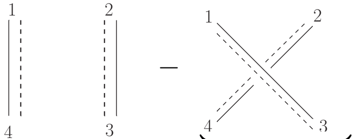
# The Non-abelian Massless QCD Flow Vertices

| Feynman | Flow |
|---|---|
|  | $-\frac{g_s f^{abc}}{2}\left( g_{12}(p_1-p_2)_3 + g_{23}(p_2-p_3)_1 + g_{13}(p_3-p_1)_2 \right)$  |
|  | $ig_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3} \left[ g_{14}g_{23} - g_{13}g_{24} \right]$  |



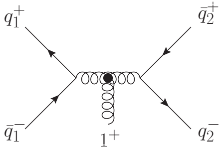




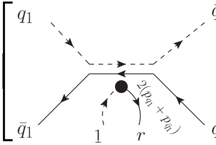


Arrow directions only consistently set within full diagram

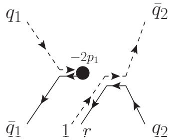
# QCD Example: $q_1\bar{q}_1 \to q_2\bar{q}_2 g$



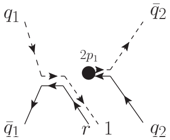

$$= \frac{i g_s^3}{2 s_{q_1\bar{q}_1} s_{q_2\bar{q}_2} \langle r1 \rangle} \Bigg[$$




$+$




$+$




$$\Bigg]$$

$$[\cdots] \equiv \Big\{ 2[q_1\bar{q}_2]\langle q_2\bar{q}_1\rangle\big([1q_1]\langle q_1 r\rangle + [1\bar{q}_1]\langle 1r\rangle\big) - 2[q_1 1]\langle 1\bar{q}_1\rangle\langle q_2 r\rangle[1\bar{q}_2] + 2[q_1 1]\langle r\bar{q}_1\rangle\langle q_2 1\rangle[1q_2] \Big\}$$

# Automation of Chirality Flow: Why and How?



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**Why automate?**

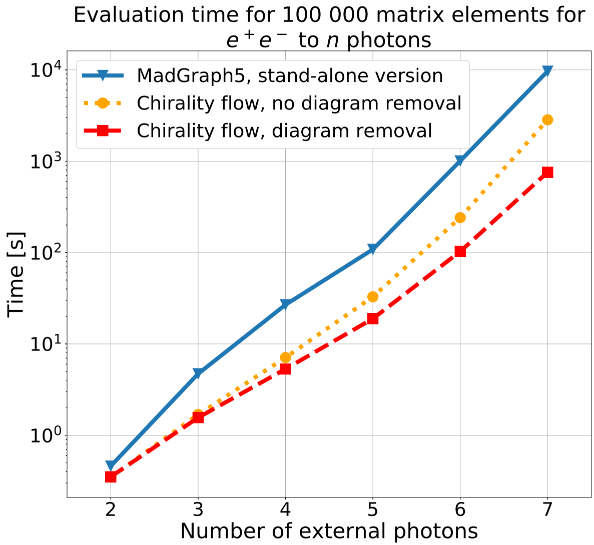
- Real calculations (almost) never done by pen and paper anymore
- Further validation
- Most codes (e.g. MadGraph5_aMC@NLO) brute force matrix multiplication, we remove the need for it
- Can we make faster simulations?

**How to automate?**

- First test case: make minimal changes to massless QED in MadGraph5_aMC@NLO
- Any difference in speed from our changes ⇒ sound conclusions

# Our Main Result (hep-ph:2203.13618)





# Conclusions and Outlook

**Shown today:**

- Chirality flow is shortest route from Feynman diagram to complex number
  - Further simplifies the spinor helicity formalism
  - Calculations often performed in a single step, particularly for massless diagrams
- Fully simplifies tree-level, massless-QED and QCD Feynman diagrams
- Can be automised for faster massless QED calculations

**Not shown today but still valid:**

- Full standard model at tree level understood (see backup slides, papers)

**Some examples of ongoing work:**

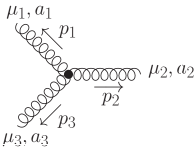
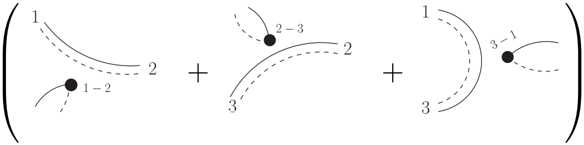
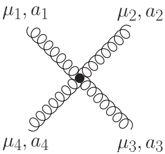
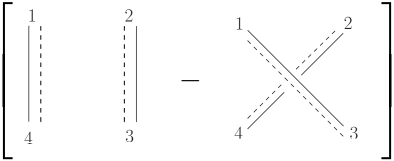
- Automise for rest of standard model (so far QCD implemented, with Emil Boman, Malin Sjödahl, and Adam Warnebring)
- Use to calculate loops (with Simon Plätzer and Malin Sjödahl)

**Other work in this direction**

- Simon Plätzer and Malin Sjödahl used chirality flow as basis for resummation (hep-ph:2204.03258)

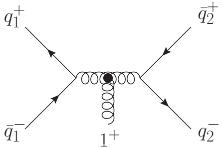
# The Non-abelian Massless QCD Flow Vertices



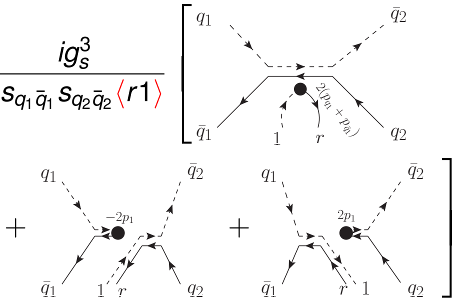
| Feynman | Flow |
| --- | --- |
|  | $-\frac{g_s f^{abc}}{2}$  |
|  | $ig_s^2 \sum_{Z(2,3,4)} f^{a_1 a_2 b} f^{b a_4 a_3}$  |

Arrow directions only consistently set within full diagram
Double line $\equiv g_{\mu\nu}$, momentum dot $\equiv p_\mu$

# QCD Example: $q_1\bar{q}_1 \to q_2\bar{q}_2 g$




$$= \frac{ig_s^3}{2 s_{q_1\bar{q}_1} s_{q_2\bar{q}_2} \langle r 1 \rangle}$$



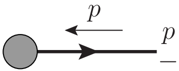
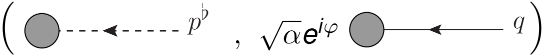
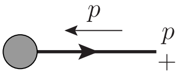
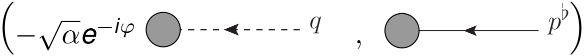
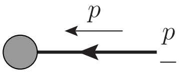
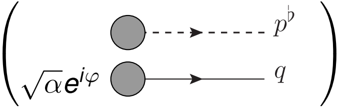
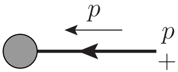
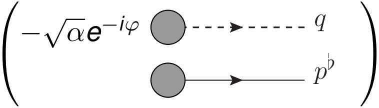

$$[\cdots] \equiv \Big\{ 2[q_1\bar{q}_2]\langle q_2\bar{q}_1\rangle\big([1q_1]\langle q_1 r\rangle + [1\bar{q}_1]\langle 1 r\rangle\big)$$

$$- 2[q_1 1]\langle 1\bar{q}_1\rangle\langle q_2 r\rangle[1\bar{q}_2] + 2[q_1 1]\langle r\bar{q}_1\rangle\langle q_2 1\rangle[1 q_2] \Big\}$$

# Incoming Massive Spinors in Chirality Flow

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad e^{i\varphi}\sqrt{\alpha} = \frac{m}{\langle p^\flat q\rangle} \,, \qquad e^{-i\varphi}\sqrt{\alpha} = \frac{m}{[qp^\flat]}$$

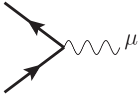
Spin operator $-\dfrac{\Sigma^\mu s_\mu}{2} = \dfrac{\gamma^5 s^\mu \gamma_\mu}{2}, \quad s^\mu = \dfrac{1}{m}(p^{\flat,\mu} - \alpha q^\mu)$

| Spinor | Feynman | Flow |
|---|---|---|
| $\bar{v}^-(p)$ |  |  |
| $\bar{v}^+(p)$ |  |  |
| $u^-(p)$ |  |  |
| $u^+(p)$ |  |  |

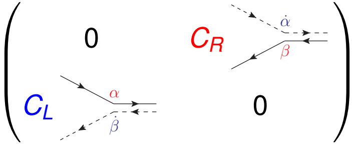
# Some Fermion Flow Rules

$$p^\mu = p^{\flat,\mu} + \alpha q^\mu \,, \quad (p^\flat)^2 = q^2 = 0 \,, \quad \alpha = \frac{p^2}{2p\cdot q} \neq 0$$

Fermion-vector vertex

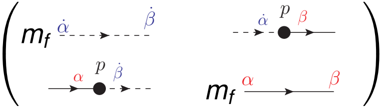



$$= ie(P_L C_L + P_R C_R)\gamma^\mu = ie\sqrt{2}$$




Fermion propagator

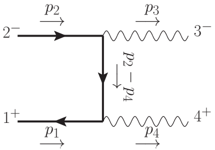
$$\frac{i}{p^2 - m_f^2}\begin{pmatrix} m_f \delta^{\dot\alpha}{}_{\dot\beta} & \sqrt{2}p^{\dot\alpha\beta} \\ \sqrt{2}\bar{p}_{\alpha\dot\beta} & m_f \delta_\alpha{}^\beta \end{pmatrix} = \frac{i}{p^2 - m_f^2}$$




Left and right chiral couplings may differ

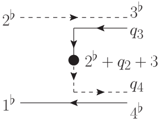
# A Massive *Illuminating* Example

Consider the same diagram of $f_1^+ \bar{f}_2^- \to \gamma_3^+ \gamma_4^-$ as before but include mass $m_f$

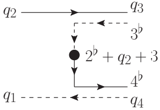



- Obtain 3 new terms
- Simplify with choices of $q_1, q_2, r_3, r_4$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i\rangle}$ , $\quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$

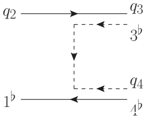
$$= \frac{-2ie^2}{(s_{23}-m_f^2)\langle r_3 3\rangle[4 r_4]} \Bigg\{$$



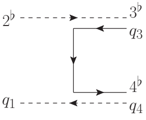

$$- \sqrt{\alpha_1 \alpha_2}\, e^{i(\varphi_2 - \varphi_1)}$$




$$+ m_f \Bigg( \sqrt{\alpha_2}\, e^{i\varphi_2}$$



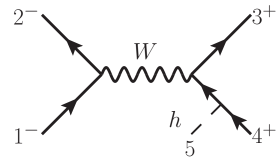

$$- \sqrt{\alpha_1}\, e^{-i\varphi_2}$$



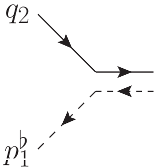

$$\Bigg) \Bigg\}$$

# A Second Massive Example: $f_1\bar{f}_2 \to W \to f_3\bar{f}_4 h_5$

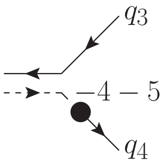
- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \cdots q_5$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i\rangle}\,, \quad e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$
- Scalar has no flow line



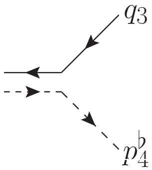

Step 1: Draw fermion lines: $\sim C_{L,12}\sqrt{\alpha_2}e^{i\varphi_2}$




$\times\, C_{L,34}\sqrt{\alpha_3}(-e^{i\varphi_3})\Big[\sqrt{\alpha_4}(-e^{i\varphi_4})$



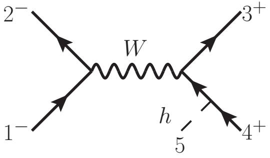

$+\, m_4$



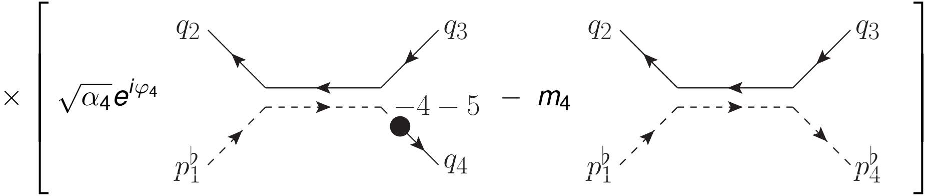

$\Big]$

# A Second Massive Example: $f_1\bar{f}_2 \to W \to f_3\bar{f}_4 h_5$

- W bosons simplifies ($C_R = 0$)
- Simplify with choices of $q_1, \cdots q_5$
- $e^{i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{\langle p_i^\flat q_i \rangle}$ , $e^{-i\varphi_i}\sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^\flat]}$
- Scalar has no flow line




Step 2: Flip arrows and connect: $C_{L,12}C_{L,34}\sqrt{\alpha_2\alpha_3}e^{i(\varphi_2+\varphi_3)}$



# The Standard Model and its Fundamental Particles



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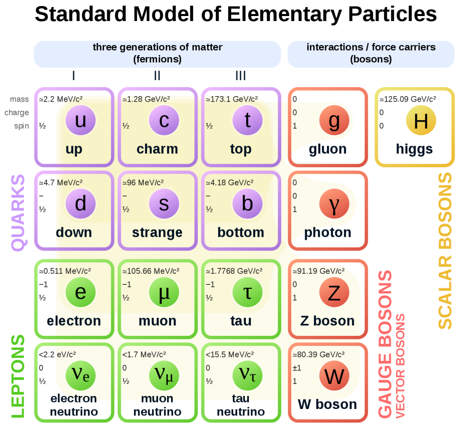


Figure from en.wikipedia.org/wiki/Standard_Model

Slide layout adapted from Marius Utheim's 2018 talk

**Fermions (spin 1/2, Pauli exclusion)**

- Leptons (EM and weakly charged)
- Quarks (EW and colour charged)

**Gauge Bosons (spin 1, B-E statistics)**

- Mediate forces
- Photon = EM
- W, Z = Weak
- Gluon = Strong (QCD)

**Scalar Boson (spin 0, B-E statistics)**

- Higgs (gives mass)

# Lorentz Group Representations

Lorentz group elements: $e^{i(\theta_i J_i + \eta_i K_i)}$ $\quad J_i \equiv$ rotations, $\quad K_i \equiv$ boosts

- Lorentz group generators $\simeq$ 2 copies of su(2) generators
  - $so(3,1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

Group algebra defined by commutator relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} J_k$$

$$N_i^{\pm} = \frac{1}{2}(J_i \pm iK_i) , \quad [N_i^-, N_j^+] = 0 ,$$

$$[N_i^-, N_j^-] = i\epsilon_{ijk} N_k^- , \qquad [N_i^+, N_j^+] = i\epsilon_{ijk} N_k^+$$

- Representations (i.e. realisations of $N_i^{\pm}$)
  - $(0,0)$ scalar particles
  - $(\frac{1}{2},0)$ left-chiral and $(0,\frac{1}{2})$ right-chiral Weyl (2-component) spinors.
  - $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$, Dirac (4-component) spinors.
  - $(\frac{1}{2},\frac{1}{2})$ vectors, e.g. gauge bosons

# How to Calculate? Spinor-Helicity

Give each particle a defined helicity ⇒ amplitude now a number!

Spinors (in chiral basis):

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \qquad u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = \begin{pmatrix} [p| & 0 \end{pmatrix} \qquad \bar{u}^-(p) = \bar{v}^+(p) = \begin{pmatrix} 0 & \langle p| \end{pmatrix}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\tau^\mu \\ \sqrt{2}\bar{\tau}^\mu & 0 \end{pmatrix} \qquad \sqrt{2}\tau^\mu = (1, \vec{\sigma}), \quad \sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma}),$$

- Amplitude written in terms of Lorentz-invariant spinor inner products

$$\langle ij\rangle = -\langle ji\rangle \equiv \langle i||j\rangle \text{ and } [ij] = -[ji] \equiv [i||j]$$

  - These are well known complex numbers, $\langle ij\rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$
- Remove $\tau/\bar{\tau}$ matrices in amplitude with

$$\langle i|\bar{\tau}^\mu|j][k|\tau_\mu|l\rangle = \langle il\rangle[kj] \,, \qquad \langle i|\bar{\tau}^\mu|j] = [j|\tau^\mu|i\rangle$$

# Define Problem

**Kinematic part of amplitude slowed by spin and vector structures**

- Can we still improve on this?
  - Deriving spinor inner products $\langle ij \rangle$, $[kl]$ requires at least 2 steps
    - Re-write every object as spinors
    - Use Fierz identity $\bar{\tau}^{\mu}_{\alpha\dot{\beta}}\tau_{\mu}^{\dot{\alpha}\beta} = \delta_{\alpha}^{\ \beta}\delta^{\dot{\alpha}}_{\ \dot{\beta}}$
  - Not intuitive which inner products we obtain
- In SU(N) use graphical reps for calculations
  - E.g. using the colour-flow method
  - (Also birdtracks etc.)
- Spinor-helicity $\equiv su(2) \oplus su(2)$
  - Can we use graphical reps?

# Creating Chirality Flow: Building Blocks

A flow is a directed line from one object to another

$su(2)$ objects have dotted indices and $su(2)$ objects undotted indices

- First step: Ansatz for spinor inner products (only possible Lorentz invariant)

$$\langle i|^{\alpha}|j\rangle_{\alpha} \equiv \langle ij\rangle = -\langle ji\rangle =$$


$$[i|_{\dot{\beta}}|j]^{\dot{\beta}} \equiv [ij] = -[ji] =$$


- Spinors and Kronecker deltas follow

$$\langle i|^{\alpha} =$$


,

$$|j\rangle_{\alpha} =$$


$$[i|_{\dot{\beta}} =$$


,

$$|j]^{\dot{\beta}} =$$


$$\delta_{\alpha}^{\ \beta} \equiv \mathbb{1}_{\alpha}^{\ \beta} =$$


,

$$\delta^{\dot{\beta}}_{\ \dot{\alpha}} \equiv \mathbb{1}^{\dot{\beta}}_{\ \dot{\alpha}} =$$