Cosmological implications of a U(1) extension of the Standard Model

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Work done in collaboration with the ELTE Phenomenology Group, in particular with Sho Iwamoto and Zsolt Szép, under the supervision of Zoltán Trócsányi.

Introduction to the Super-Weak Model

Shortcomings of the Standard Models

Two Standard Models $\to$ $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$ and ΛCDM

Particle physics	Cosmology
Neutrino masses & oscillation	Dark energy
Muon $g-2$ (?)	Dark matter
Electroweak vacuum stability	Inflation
Baryon asymmetry	Hubble tension
...	...

Not always obvious how to separate the issues

Extending the Standard Model

A way to solve some of the issues is to extend the Standard Model gauge group:

Super-weak gauge group: $G_{SW} = \underbrace{SU(3)_c \otimes SU(2)_L \otimes U(1)_y}_{G_{SM}} \otimes U(1)_z$

Why an extra U(1)?

- Phenomenologically the simplest choice $\longrightarrow$ Avoid having many new parameters

What is the goal of the model?

- Check if a simple model is capable of explaining a large number of shortcomings of the SM simultaneously.
- Positive or negative answers are both exciting!

Super-Weak Model Spectrum and Charges

We extend the spectrum of the Standard Model with

- $N_{1,2,3}$ $\rightarrow$ 3 right-handed neutrinos charged only under $\mathrm{U}(1)_z$,
- Z' $\rightarrow$ the massive gauge boson of $\mathrm{U}(1)_z$,
- χ $\rightarrow$ complex scalar charged only under $\mathrm{U}(1)_z$.

The lightest sterile neutrino N_1 is the dark matter candidate.

- $N_{2,3}$ are considered to be heavy $\rightarrow$ relevant for the effective potential & leptogenesis

Charge assignment for $\mathrm{U}(1)_z$ has to be anomaly-free.

- The condition can be satisfied in many ways.
- In our assignment all particles are charged under $\mathrm{U}(1)_z$

Super-Weak Model Parameters

1. Gauge coupling, g_z
   - In order to avoid various constraints, $\boxed{\mathcal{O}(g_z/g_{Z^0}) \ll 1}$.
2. Vacuum expectation value of χ singlet, w_0
   - For DM study we will use the mass of Z' instead. It is assumed that $\boxed{M_{Z'} \ll M_Z.}$
3. $\mathrm{U}(1)_y \otimes \mathrm{U}(1)_z$ gauge mixing parameter, η
   - Its value can be determined from RGE, at relevant scales $0 \leq \eta < 1$, but we use $\boxed{\eta = 0}$ for simplicity (no qualitative difference).
4. Neutrino masses, N_i
   - We assume N_1 to be light $\boxed{M_1 = \mathcal{O}(10)\ \mathrm{MeV},}$ while $\boxed{M_{2,3} \sim \Lambda_{\mathrm{EW}}}$.
5. Singlet scalar mass, M_s
   - Interesting for the effective potential, $\boxed{M_s > M_h}$.

Super-Weak Model Summary

Points to remember for the talk:

1. New interaction with strength much weaker than that of the weak interaction.
2. Light new physics at MeV scale
   - Light new mediator Z' of mass 20 MeV $\lesssim M_{Z'} < m_\pi$
   - Lightest RHN is $M_1 = \mathcal{O}(10)$ MeV
3. Additional singlet scalar with nonzero vacuum expectation value

Dark Matter production

Portals and mechanisms

Portal: a weak interaction connecting the Standard Model and the sterile particles.

- We use the Z' to mediate between the SM and dark sectors $\rightarrow$ vector boson portal
- Other known portals are scalar (or Higgs) portal, and neutrino portal

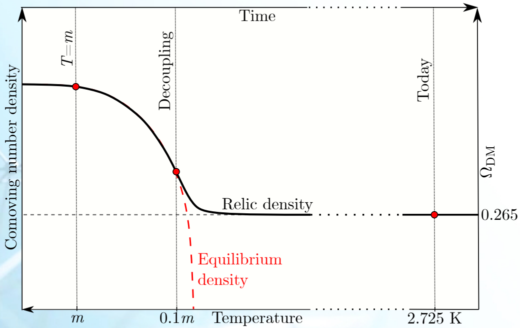
Mechanism: determined by the relevant process which establishes the DM density

- Freeze-out: annihilations of DM to SM particles play a central role
- Freeze-in: decays into DM particles play a central role
- For scalar DM, large self interactions can also lead to interesting and distinct phenomenology

Freeze-out Mechanism

Freeze-out mechanism for a particle with mass m:

1. The particle species was in equilibrium at high temperatures ($T > m$),
2. Decoupling is a result of scattering processes becoming slow compared to Hubble expansion,
3. Decoupling happens at temperatures comparable to the mass of the particle, $T_{\text{dec}} \simeq 0.1m$.



Thermally Averaged Rates

Describe interaction rates at finite temperature with assuming Maxwell-Boltzmann distributions for all species (approximation, not necessary)

Decay rate

Decaying particle mass: M

$$z = M/T$$

$$\langle\Gamma\rangle = \Gamma \frac{K_1(z)}{K_2(z)}$$

Monotone increasing function of z.

$$\max(\langle\Gamma\rangle) = \lim_{z\to\infty}\langle\Gamma\rangle = \Gamma$$

Cross section

Incoming/Outgoing particle mass: $m_{\text{in/out}}$

$$\mu = \max(m_{\text{in}}, m_{\text{out}})$$

$$\langle\sigma v_{\text{Møl}}\rangle \propto \int_{4\mu^2}^{\infty} \mathrm{d}s\, \sigma(s)(s - 4m_{\text{in}}^2)\sqrt{s}K_1\left(\frac{\sqrt{s}}{T}\right)$$

Resonance can dominate the integral.

Decoupling: $\langle\sigma v_{\text{Møl}}\rangle(T \ll \mu) \to 0.$

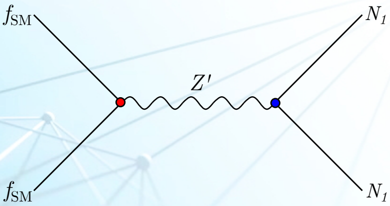
Dark Matter production: Resonant freeze-out

Freeze-out in the Super-Weak Model: Processes

We consider $M_1 = \mathcal{O}(10)$ MeV $\longrightarrow$ decoupling happens at $T_{\text{dec}} = \mathcal{O}(1)$ MeV.

At this temperature range electrons and SM neutrinos are abundant, negligible amounts of heavier fermions.

$$N_1 N_1 \to f_{\text{SM}} f_{\text{SM}} : \quad \sigma_{\text{t}} \propto g_z^4 \sqrt{1 - \frac{4M_1^2}{s}} \frac{s}{(s - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}$$



Resonant Amplification

In the freeze-out mechanism increasing the interaction rate decreases the relic density.

- But large couplings are ruled out by experiments!
- Need another way out: increase $\langle \sigma v_{\text{Møl}} \rangle$ by exploiting resonance $(2M_1 \lesssim M_{Z'})$

$$\text{Resonance:} \quad \langle \sigma v_{\text{Møl}} \rangle = (\dots) \int_{4M_1^2}^{\infty} \mathrm{d}s \underbrace{\frac{(\dots)}{(s - M_{Z'}^2)^2 + M_{Z'}^2 \Gamma_{Z'}^2}}_{\text{strongly peaked around } s = M_{Z'}^2} \times K_1 \left(\frac{\sqrt{s}}{T} \right)$$

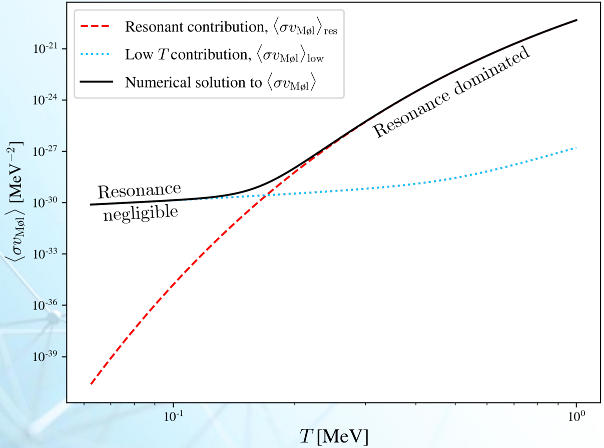
→ Recall that $T_{\text{dec}} \approx 0.1 M_1$, then at the resonance $s = M_{Z'}^2$ the Bessel function is $K_1(10 M_{Z'}/M_1)$

→ The Bessel function is exponentially small if its argument is large

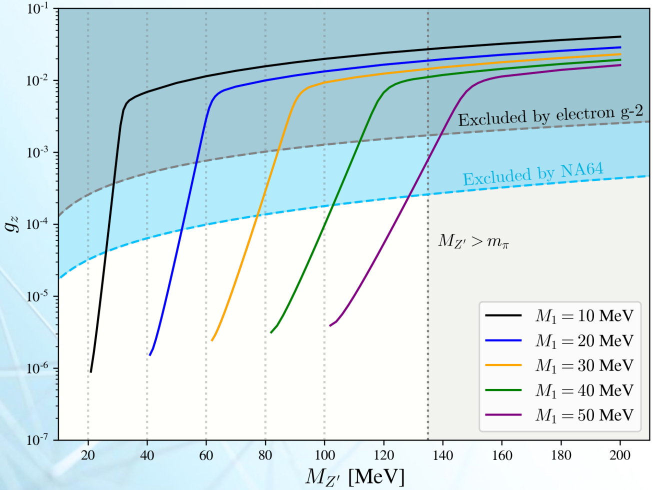
⟶ need $M_{Z'} \approx M_1$, i.e., resonance.

Resonant Amplification: Example

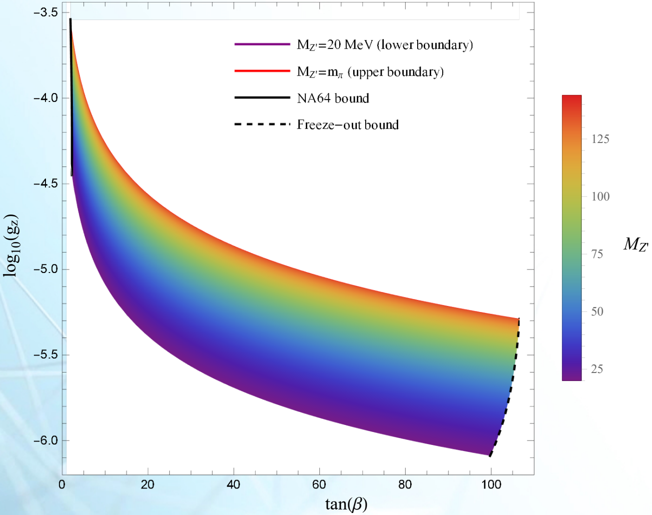
Example calculated within the super-weak model for $M_1 = 10$ MeV and $M_{Z'} = 30$ MeV.



Freeze-out in the Super-Weak Model I.



Freeze-out in the Super-Weak Model II.



Phase transitions and effective potentials

Introduction

- We wish to study leptogenesis $\rightarrow$ quite complicated theory with many moving parts
  - Strength of CP violation in given model
  - Thermal masses and cross sections/decay rates
  - Relevant temperature range and value of VEVs
- Good understanding requires FTQFT calculations and a proper analysis of the effective potential
- Few problems with the effective potential
  1. Becomes complex for small background field values $\rightarrow$ Should be real
  2. Infrared divergence due to massless Goldstones $\rightarrow$ Should be free of IR divergence
  3. The potential is concave $\rightarrow$ Should be convex
- How to consistently deal with these issues?

Problem details

1. Complexity problem
   - Due to the scalar fields $\rightarrow$ their effect is generally tiny, so leaving the imaginary part does not introduce large errors
   - Alternatively one can resum the Goldstone propagators such that the resummed mass squared is positive
2. Infrared divergence
   - The effective potential itself at one loop is finite
   - The second derivative is not $\rightarrow$ curvature mass
   - Renormalization condition with the pole mass $\rightarrow$ this cancels the IR divergence
3. Convexity problem
   - From theoretical arguments the effective potential should be convex
   - Does not have to hold at any perturbative order, only for the full potential
   - Lattice (non-perturbative) seems to give a convex potential when extrapolated to $a \rightarrow 0$
   - FRG (non-perturbative) gives a convex potential at $k \rightarrow 0$

Building the effective potential in SM

- The effective potential can be expanded perturbatively ($\mu^2 < 0\,,\ \lambda > 0$)

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3(x) + i\phi_4(x) \\ v + h(x) + i\phi_2(x) \end{pmatrix}$$

Tree level: $V_{\text{cl}}(v) = \frac{\mu^2}{2} v^2 + \frac{\lambda}{4} v^4$

One-loop: $V_{\text{eff}}^{(1)}(v) = \sum_i n_i V_{\text{CW}}\big(m_i^2(v)\big)$

- Coleman-Weinberg potential in dimensional regularization (Q=regularization scale)

$$V_{\text{CW}}(m_i^2(v)) = \frac{m_i^4(v)}{64\pi^2} \left(\log\left(\frac{m_i^2(v)}{Q^2}\right) - c_i\right)\,, \quad c_{\text{gb}} = \frac{5}{6}\,,\ c_{\text{f,s}} = \frac{3}{2}$$

- SM masses → highlighted ones can be negative since $\mu^2 < 0$

$$m_W^2 = \frac{g_L^2 v^2}{4}\,,\ m_Z^2 = \frac{(g_L^2 + g_Y^2) v^2}{4}\,,\ m_t^2 = \frac{y_t^2 v^2}{2}\,,\ m_h^2 = \mu^2 + 3\lambda v^2\,,\ m_G^2 = \mu^2 + \lambda v^2$$

Optimized perturbation theory approach

- The root of the problem is $\mu^2 < 0 \to$ Introduce a shifted mass parameter $m^2 > 0$

$$\mathcal{L} \supset \mathcal{V}_{\text{OPT}}(\phi) = m^2|\phi|^2 + \lambda|\phi^4| + \boxed{(\mu^2 - m^2)|\phi|^2}$$

[arXiv:hep-ph/9803226]

- Important to keep in mind:
  - Treat the last term as an interaction or finite part of counter-term
  - Tree level masses defined above are now shifted as $\mu^2 \to m^2$

$$V_{\text{OPT}}^{[1]}(v;\mu^2,m^2) = V_{\text{cl}}(v;m^2) + \underbrace{V_{\text{eff}}^{(1)}(v;m^2) + \boxed{\frac{\mu^2 - m^2}{2}v^2}}_{\text{1-loop corrections}} \qquad (1)$$

- The classical potential and the last term combines into $V_{\text{cl}}(v;\mu^2)$ thus the tree level result is preserved

Parametrization conditions

- Need physical conditions $\rightarrow$ fix the values of parameters $\{\mu^2, m^2, \lambda\}$

Condition 1: $$\left.\frac{\partial V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial v}\right|_{v=v_0} = 0 \qquad \leftarrow$$ Position of minimum

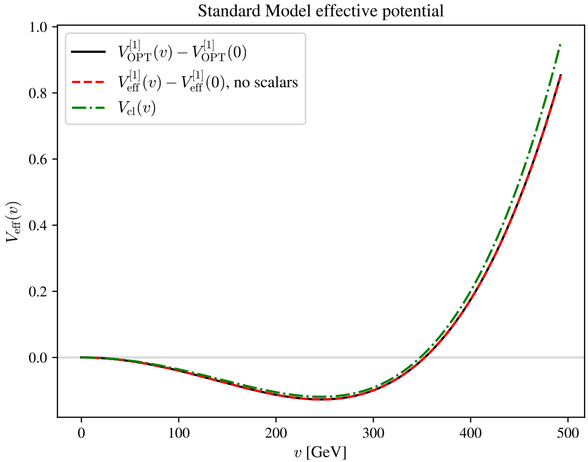
Condition 2: $$\left.\frac{\partial^2 V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial v^2}\right|_{v=v_0} = M_h^2 \qquad \leftarrow$$ Curvature mass is the Higgs

Condition 3: $$\left.\frac{\partial V^{[1]}_{\mathrm{OPT}}(v;\mu^2,m^2)}{\partial m^2}\right|_{v=v_0} = 0 \qquad \leftarrow$$ Principle of minimum sensitivity

- Proof of concept: system is solvable for SM with parameter values

$$m^2 = 69\,094.6\ \mathrm{GeV}^2, \qquad \lambda = 0.12\,861, \qquad \mu^2 = -8\,847.85\ \mathrm{GeV}^2$$

SM effective potential



Scalar potential in singlet extensions

- Adding a new singlet scalar to the potential:

$$V_{\text{cl}}(v, w) = \frac{\mu_h^2}{2} v^2 + \frac{\lambda_h}{4} v^4 + \frac{\mu_s^2}{2} w^2 + \frac{\lambda_s}{4} w^4 + \frac{\lambda_{hs}}{4} v^2 w^2$$

- Mixing λ_{hs} modifies the scalar masses $(m_h^2(v, w)$ and $m_s^2(v, w))$
  - The scalar masses are obtained as the eigenvalues of the Hessian of the classical potential
- 1-loop corrections equivalent to the SM, except the sum now includes new particles
- Using conditions to set the minima $\{v_0, w_0\}$ and curvature masses $\{M_h^2, M_s^2\}$ the potential cannot be parametrized again (complex values)

Generalizing the OPT approach

- Similar approach to SM → introduce shifted mass parameters for both $\mu^2_{h,s} \to m^2_{h,s}$
- The parameter fixing conditions are doubled → can fix 6 out of 7 parameters

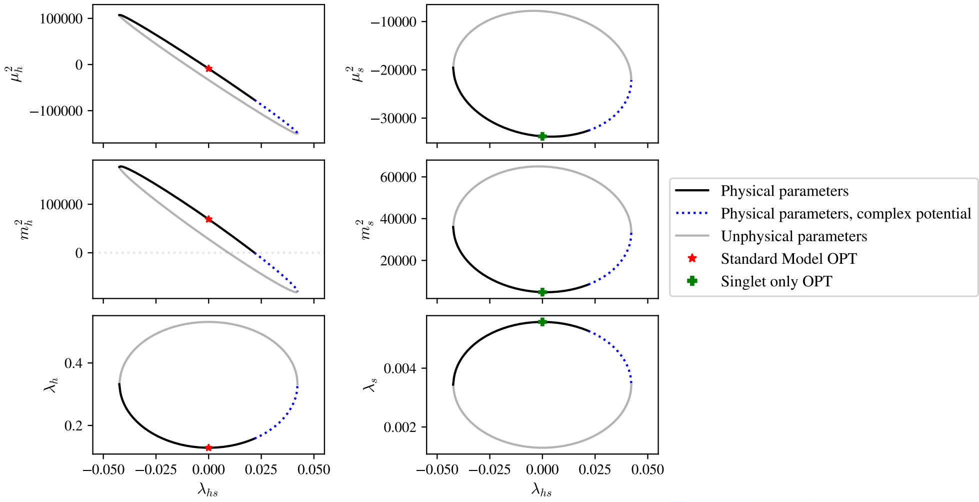
$$\text{Condition 1,2:} \quad \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial v}\right|_{\text{min}} = \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial w}\right|_{\text{min}} = 0$$

$$\text{Condition 3,4:} \quad \left.\begin{pmatrix} \partial^2_v V^{[1]}_{\text{OPT}} & \partial_v \partial_w V^{[1]}_{\text{OPT}} \\ \partial_w \partial_v V^{[1]}_{\text{OPT}} & \partial^2_w V^{[1]}_{\text{OPT}} \end{pmatrix}\right|_{\text{min}} \to \begin{pmatrix} M^2_h & 0 \\ 0 & M^2_s \end{pmatrix}$$

$$\text{Condition 5,6:} \quad \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial m^2_h}\right|_{\text{min}} = \left.\frac{\partial V^{[1]}_{\text{OPT}}(v,w)}{\partial m^2_s}\right|_{\text{min}} = 0$$

- We choose λ_{hs} as the unfixed parameter in terms of which we investigate the parametrization

Parametrization of the singlet model



Finite temperature corrections

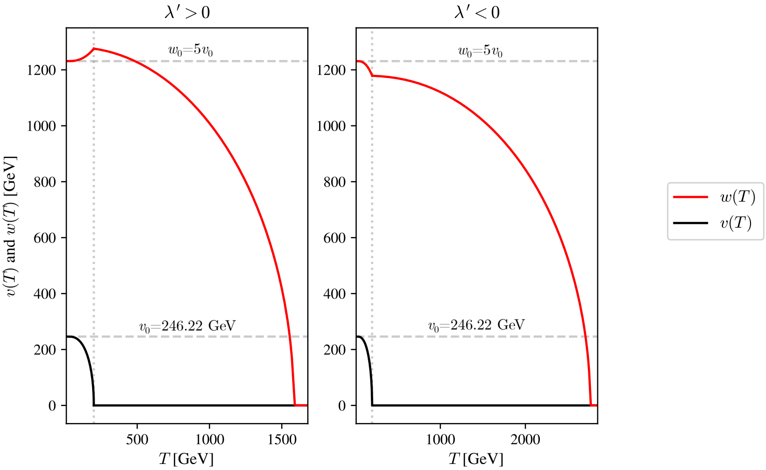
- Finite temperature corrections are added to the 1-loop effective potential

$$V_{\text{eff}}(v,w,T) = V_{\text{cl}}(v,w) + V_{\text{eff}}^{(1)}(v,w) + V_{\text{T}}(v,w,T)$$

$$V_{\text{T}}(v,w,T) = \frac{T^4}{2\pi^2}\sum_i n_i J_{\pm}^{(i)}\left(m_i^2(v,w,T)\right)$$

- We require each potential term to be real otherwise minimization is meaningless
- Both $V_{\text{eff}}^{(1)}$ and V_{T} are real if all $m_i^2 > 0$
- To cure IR divergence $\to$ Daisy resummation of $n=0$ bosonic modes (thermal mass for scalars and longitudinal gauge bosons)

Example phase transition



Conclusions

- The super-weak extension can provide a valid dark matter candidate, the lightest sterile neutrino
- Current experiments allow for both freeze-in and freeze-out scenarios
- We have introduced a method to obtain a fully real effective potential
- The super-weak effective potential can provide a double phase transition of signature $\{0,0\} \to \{0, w_0\} \to \{v_0, w_0\}$ as the Universe cools
- In the near future: check if the temperature interval between phase transitions can provide enough time for leptogenesis

• Thank You for Your Attention! •