Using Quantum Annealing to Fold and Design HP Lattice Proteins

Lucas Knuthson¹ Centre for Environmental and Climate Science (CEC), Lund University **Nordic Lattice 2024 in Lund**

With: Anders Irbäck, Sandipan Mohanty (Jülich), Carsten Peterson

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Quantum Annealing (D-Wave)

QPU Hybrid

HP Lattice Proteins

Folding [Irbäck et al. 2022, PhysRevResearch.4.043013]

Mapping and Energy Function Results: Hybrid Results: QPU Conclusion

Design [Irbäck et al. 2024, PhysRevResearch.6.013162]

Mapping and Energy Function Results: Hybrid Results: QPU Conclusion

If I have time: Some QAOA

Quantum Computing

Replace bits $\{0,1\}^n$ with qubits $|state\rangle = \bigotimes_i^n (\alpha_i |0\rangle + \beta_i |1\rangle)$

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Quantum annealing: $|f\rangle = \mathcal{T} \exp(\int_0^T H(t)dt) |start\rangle$, $H(t) = H_P(t/T) + H_D(1 - t/T)$

 ${\cal H}_{\cal P}$ is the problem Hamiltonian, and ${\cal H}_{\cal D}$ driver in which ground state we start in

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 H_p is the problem Hamiltonian, and H_D driver in which ground state we start in

Guaranteed to find the ground state if $T >> \max \frac{\langle gs(t) | \dot{H} | e(t) \rangle}{(E_{gs}(t) - E_e(t))^2}$

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Hybrid quantum-classical solver: Subproblems sent as queries to the QPU

HP Lattice Proteins [Lau, Dill 1989]

Simplified protein model with two types of amino acids, H and P. Hs interact, and Ps do not. $E_{\rm HP}=-N_{\rm HH}$



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For 2D lattice, all sequences with unique ground state structures are known for chains with \leq 30 amino acids

Despite the simplicity, both folding (given a sequence find its ground state structure) and design (given a structure find a sequence that folds into that structure) are computationally difficult



Folding HP Lattice Proteins: Questions We Wanted to Answer

Previous attempts used a turn-based encoding. Non-local interactions became difficult to implement. (short chains 6-9 amino acids)



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Can we find a simpler mapping which is quadratic?

We wanted to test the mapping on actual hardware

Can we find a simpler mapping which is quadratic?

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Choose a lattice and enumerate the beads in the sequence

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Energy function takes the form $E = E_{HP} + \sum_{i=1}^{3} \lambda_i E_i$, with E_i being constraints to ensure a valid chain

(b)







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 $E_3 = \sum_{1 \le f < N} \sum_s \sigma_s^f \sum_{|s'-s| > 1} \sigma_{s'}^{f+1}$



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Implement on hardware: Hybrid minimize $E = E_{HP} + \sum_{i=1}^{3} \lambda_i E_i$

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100 runs for each sequence (10^2 lattice, runtime 4 s) 100% hit rate



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For comparison, classical SA with explicit chains and spins (run time >4 s)



$$E = E_{\rm HP} + \sum_{i=1}^{3} \lambda_i E_i$$

Hybrid solver was insensitive to changes, no fine tuning needed



We also tried two longer sequences (without exact results) that have been extensively studied with classical methods

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The lowest known energies were recovered with high probability, once the runtime was high enough



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Limited to short sequences

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Longest sequence was 14-beads on a 4^2 grid


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The hybrid outperforms our SA approaches

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The QPU is less impressive, with a roughly exponential decrease in hit rate with increasing system size

Design of HP Lattice Proteins: Questions We Wish to Answer

Can we find a protocol to design HP lattice proteins using QA?

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What is the source for the exponential decay?

Goal: find HP sequences that fold to a given target structure. Requires search in both sequence and structure spaces.

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Two-step procedure:

- 1 Seek sequences with minimum energy in the target structure
- 2 Test whether or nor the optimized sequence fold to the intended structure (using the folding mapping)

Step 1 is done at fixed number of Hs. Otherwise, the all-H sequence is a trivial solution.

Design: Sequence Optimization

The connectivity matrix of a target structure, w_{ij} tells whether amino acid *i* and *j* are in contact ($w_{ij} = -1$) or not ($w_{ij} = 0$)



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Find minimum-energy sequences for a given composition, $N_{\rm H}$, by minimizing $E = \sum_{ij} w_{ij}s_is_j + \lambda \left(\sum_i s_i - N_{\rm H}\right)^2$



Design: Sequence Optimization with Hybrid

Target structures with N = 30, 50, 64. a few different $N_{\rm H}$ values for each structure 100% success rate in all instances



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For every target, at least one sequence found with the intended structure as its unique ground state.



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Previously studied sequences with N = 64 and $N_{\rm H} = 42$, the ground state is degenerate.

Design: Sequence Optimization with the QPU

The success rate decays rapidly



HP Lattice Proteins with QA

Design: Sequence Optimization with the QPU

The success rate decays rapidly

Potential error sources include (i) thermal noise, (ii) chain breaks, (iii) finite annealing time, (iv) control errors



Design: Sequence Optimization with the QPU

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Potential error sources include (i) thermal noise, (ii) chain breaks, (iii) finite annealing time, (iv) control errors

Right shows the data without chain breaks and large energy gaps



Design: Sequence Optimization with the QPU - finite time

(a) Pure QPU hit rate against annealing time

(b) Success rates when integrating the time-dependent Schrödinger

equation for different systems using a fixed annealing time



Design: Sequence Optimization with the QPU - finite time

(a) Pure QPU hit rate against annealing time

(b) Success rates when integrating the time-dependent Schrödinger

equation for different systems using a fixed annealing time

No indication that finite annealing time explains the decay in success rate



Therefore, if you give a problem, $\sum_i h_i \sigma_z^i + \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$ with h_i s and/or J_{ij} s outside this range, we rescale

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Remember the chains? Several strongly coupled qubits represent one logical qubit.

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Remember the chains? Several strongly coupled qubits represent one logical qubit. The chain coupling strength is often the largest J_{ij} in the reformed problem

Design: Sequence Optimization with the QPU - control errors

Imperfect implementation if the problem Hamiltonian $\tilde{H}_P = \sum_i (h_i + \delta h_i) \sigma_z^i + \sum_{ij} (J_{ij} + \delta J_{ij}) \sigma_i^z \sigma_j^z$

Assume δh_i , δJ_{ij} independent and Gaussian with std devs σ_h and σ_J

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Assume δh_i , δJ_{ij} independent and Gaussian with std devs σ_h and σ_J D-Wave: $\sigma_h = x \max|h_i|$ and $\sigma_J = x \max|J_{ij}|$, x = 0.015



The hybrid quantum-classical method swiftly and consistently solves the lattice protein folding and design problems for system sizes that are non-trivial with classical methods.

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The pure QPU results are less impressive. Control errors may have a significant impact on the success rate

Acknowledgements: All QA computations were done on the D-Wave Advantage System at the Jülich Supercomputing Centre, Germany.

QA: $|f\rangle = \mathcal{T} \exp(\int_0^T H(t)dt) |start\rangle$, QAOA: use $\mathcal{T} \exp(\int_0^T H(t)dt) \approx \exp(-i\gamma_k H_P) \exp(-i\beta_k H_D) \cdots \exp(-i\gamma_1 H_P) \exp(-i\beta_1 H_D)$ when $k \to \infty$

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run $|f\rangle = exp(-i\gamma_k H_P) \exp(-i\beta_k H_D) \cdots \exp(-i\gamma_1 H_P) \exp(-i\beta_1 H_D) |start\rangle$ and optimize $\vec{\gamma}$ and $\vec{\beta}$ classically

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Free to choose $|start\rangle$ and H_D (which causes transitions)

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In collaboration with Leif Gellersen and Stefan Prestel

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In collaboration with Leif Gellersen and Stefan Prestel

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$$E_1 = \sum_f \left(\sum_s \sigma_s^f - 1 \right)^2, \quad E_2 = \frac{1}{2} \sum_{f_1 \neq f_2} \sum_s \sigma_s^{f_1} \sigma_s^{f_2}$$

$$E_3 = \sum_{1 \le f < N} \sum_s \sigma_s^f \sum_{|s'-s|>1} \sigma_{s'}^{f+1}$$

Each term in the above Hamiltonian gets its own gate $(\exp(-i\alpha_i H_P))$. That is a lot of gates.

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Make a graph with spins as nodes and the quadratic terms in E_i s as edges

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Remember that H_D causes transition. We choose an H_D that only move between sets of spins with no edges in common.

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QAOA: Folding [Manuscript in preparation]

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There is such a Hamiltonian called the MIS-mixer

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If we combine this with divide-and-conquer methods we can do large proteins with few qubits and gates