## Using Quantum Annealing to Fold and Design HP Lattice Proteins

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With: Anders Irbäck, Sandipan Mohanty (Jülich), Carsten Peterson

## Introduction

Quantum Annealing (D-Wave)
QPU
Hybrid
HP Lattice Proteins
Folding [Irbäck et al. 2022, PhysRevResearch.4.043013]
Mapping and Energy Function
Results: Hybrid
Results: QPU
Conclusion
Design [Irbäck et al. 2024, PhysRevResearch.6.013162]
Mapping and Energy Function
Results: Hybrid
Results: QPU
Conclusion
If I have time: Some QAOA

## Quantum Computing

Replace bits $\{0,1\}^{n}$ with qubits $\mid$ state $\rangle=\bigotimes_{i}^{n}\left(\alpha_{i}|0\rangle+\beta_{i}|1\rangle\right)$

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Guaranteed to find the ground state if $T \gg \max \frac{\langle g s(t)| \dot{H}|e(t)\rangle}{\left(E_{g s}(t)-E_{e}(t)\right)^{2}}$

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Start in the ground state, $|+\rangle^{n}$, of $H_{D}=-\sum_{i} \sigma_{i}^{x}$

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Hybrid quantum-classical solver: Subproblems sent as queries to the QPU

## HP Lattice Proteins [Lau,Dill 1989]

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For 2D lattice, all sequences with unique ground state structures are known for chains with $\leq 30$ amino acids

Despite the simplicity, both folding (given a sequence find its ground state structure) and design (given a structure find a sequence that folds into that structure) are computationally difficult


## Folding HP Lattice Proteins: Questions We Wanted to

 AnswerPrevious attempts used a turn-based encoding. Non-local interactions became difficult to implement. (short chains 6-9 amino acids)


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We wanted to test the mapping on actual hardware

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(b)

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Energy function takes the form $E=E_{H P}+\sum_{i=1}^{3} \lambda_{i} E_{i}$, with $E_{i}$ being constraints to ensure a valid chain

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& E_{1}=\sum_{f}\left(\sum_{s} \sigma_{s}^{f}-1\right)^{2},
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100 runs for each sequence ( $10^{2}$ lattice, runtime 4 s) $100 \%$ hit rate
For comparison, classical SA with explicit chains and spins (run time $>4 \mathrm{~s}$ )


## Folding: Hybrid

$E=E_{\mathrm{HP}}+\sum_{i=1}^{3} \lambda_{i} E_{i}$

Hybrid solver was insensitive to changes, no fine tuning needed


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We also tried two longer sequences (without exact results) that have been extensively studied with classical methods

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The lowest known energies were recovered with high probability, once the runtime was high enough



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Longest sequence was 14 -beads on a $4^{2}$ grid


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The QPU is less impressive, with a roughly exponential decrease in hit rate with increasing system size

## Design of HP Lattice Proteins: Questions We Wish to Answer

Can we find a protocol to design HP lattice proteins using QA?

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What is the source for the exponential decay?

## Design: Procedure

Goal: find HP sequences that fold to a given target structure. Requires search in both sequence and structure spaces.

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Two-step procedure:
1 Seek sequences with minimum energy in the target structure
2 Test whether or nor the optimized sequence fold to the intended structure (using the folding mapping)
Step 1 is done at fixed number of Hs . Otherwise, the all-H sequence is a trivial solution.

## Design: Sequence Optimization

The connectivity matrix of a target structure, $w_{i j}$ tells whether amino acid $i$ and $j$ are in contact $\left(w_{i j}=-1\right)$ or not $\left(w_{i j}=0\right)$


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Find minimum-energy sequences for a given composition, $N_{\mathrm{H}}$, by minimizing $E=\sum_{i j} w_{i j} s_{i} s_{j}+\lambda\left(\sum_{i} s_{i}-N_{\mathrm{H}}\right)^{2}$


## Design: Sequence Optimization with Hybrid

Target structures with $N=30,50,64$.
a few different $N_{\mathrm{H}}$ values for each structure $100 \%$ success rate in all instances




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$E=\sum_{i j} w_{i j} S_{i} S_{j}+\lambda\left(\sum_{i} S_{i}-N_{H}\right)^{2}$

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For every target, at least one sequence found with the intended structure as its unique ground state.


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Previously studied sequences with $N=64$ and $N_{\mathrm{H}}=42$, the ground state is degenerate.

## Design: Sequence Optimization with the QPU

The success rate decays rapidly


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Potential error sources include (i) thermal noise, (ii) chain breaks, (iii) finite annealing time, (iv) control errors


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Right shows the data without chain breaks and large energy gaps



## Design: Sequence Optimization with the QPU - finite time

(a) Pure QPU hit rate against annealing time
(b) Success rates when integrating the time-dependent Schrödinger equation for different systems using a fixed annealing time


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(a) Pure QPU hit rate against annealing time
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No indication that finite annealing time explains the decay in success rate



## Intermezzo: How does the D-Wave work?

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Therefore, if you give a problem, $\sum_{i} h_{i} \sigma_{z}^{i}+\sum_{i j} J_{i j} \sigma_{i}^{z} \sigma_{j}^{z}$ with $h_{i}$ s and/or $J_{i j}$ s outside this range, we rescale

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Remember the chains? Several strongly coupled qubits represent one logical qubit. The chain coupling strength is often the largest $J_{i j}$ in the reformed problem

## Design: Sequence Optimization with the QPU - control errors

Imperfect implementation if the problem Hamiltonian
$\tilde{H}_{P}=\sum_{i}\left(h_{i}+\delta h_{i}\right) \sigma_{z}^{i}+\sum_{i j}\left(J_{i j}+\delta J_{i j}\right) \sigma_{i}^{z} \sigma_{j}^{z}$
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Assume $\delta h_{i}, \delta J_{i j}$ independent and Gaussian with std devs $\sigma_{h}$ and $\sigma_{J}$ D-Wave: $\sigma_{h}=x \max \left|h_{i}\right|$ and $\sigma_{J}=x \max \left|J_{i j}\right|, x=0.015$

Semi-quantitative agreement


## Design: Conclusion

The hybrid quantum-classical method swiftly and consistently solves the lattice protein folding and design problems for system sizes that are non-trivial with classical methods.

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The pure QPU results are less impressive. Control errors may have a significant impact on the success rate

## Acknowledgements

Acknowledgements: All QA computations were done on the D-Wave Advantage System at the Jülich Supercomputing Centre, Germany.

## QAOA: Gate-based optimization

QA: $|f\rangle=\mathcal{T} \exp \left(\int_{0}^{T} H(t) d t\right)|s t a r t\rangle$,
QAOA: use $\mathcal{T} \exp \left(\int_{0}^{T} H(t) d t\right) \approx$
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Free to choose $\mid$ start $\rangle$ and $H_{D}$ (which causes transitions)

## QAOA: Folding [Manuscript in preparation]

In collaboration with Leif Gellersen and Stefan Prestel

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Make a graph with spins as nodes and the quadratic terms in $E_{i} s$ as edges
Remember that $H_{D}$ causes transition. We choose an $H_{D}$ that only move between sets of spins with no edges in common.

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There is such a Hamiltonian called the MIS-mixer

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In collaboration with Leif Gellersen and Stefan Prestel

$$
\begin{aligned}
& E_{\mathrm{HP}}=-\sum_{\left|f-f^{\prime}\right|>1} C\left(h_{f}, h_{f^{\prime}}\right) \sum_{\left\langle s, s^{\prime}\right\rangle} \sigma_{s}^{f} \sigma_{s^{\prime}}^{f^{\prime}} \\
& E_{1}=\sum_{f}\left(\sum_{s} \sigma_{s}^{f}-1\right)^{2}, \quad E_{2}=\frac{1}{2} \sum_{f_{1} \neq f_{2}} \sum_{s} \sigma_{s}^{f_{1}} \sigma_{s}^{f_{2}} \\
& E_{3}=\sum_{1 \leq f<N} \sum_{s} \sigma_{s}^{f} \sum_{\left|s^{\prime}-s\right|>1} \sigma_{s^{\prime}}^{f+1}
\end{aligned}
$$

We choose an $H_{D}$ that only move between sets of spins with no edges in common.

There is such a Hamiltonian called the MIS-mixer
If we combine this with divide-and-conquer methods we can do large proteins with few qubits and gates

