

Wavefunctions of deformed nuclei in the collective space

DIVISION OF MATHEMATICAL PHYSICS - LUND UNIVERSITY

| | | | | | | | | | | | | | | | | | |
|---------------------------------|---------------------------------|--|---------------------------------|-------------------------------|---------------------------------|--|---------------------------------|--------------------------------|------------------------------------|-----------------------------------|-----------------------------------|---------------------------------|---------------------------------|---------------------------------|-----------------------------------|----------------------------------|---------------------------------|
| Li Lithium 6.941 | Be Beryllium 9.012 | | | | | | | | | | | B Boron 10.811 | C Carbon 12.011 | N Nitrogen 14.007 | O Oxygen 15.999 | F Fluorine 18.998 | |
| 11 Na Sodium 22.990 | 12 Mg Magnesium 24.305 | 3 IIIB 3B | 4 IVB 4B | 5 VB 5B | 6 VIB 6B | 7 VIIB 7B | 8 VIII 8 | 9 VIII 8 | 10 VIII 8 | 11 IB 1B | 12 IIB 2B | 13 Al Aluminum 26.982 | 14 Si Silicon 28.086 | 15 P Phosphorus 30.974 | 16 S Sulfur 32.066 | 17 Cl Chlorine 35.453 | 18 |
| 19 K Potassium 39.098 | 20 Ca Calcium 40.078 | 21 Sc Scandium 44.956 | 22 Ti Titanium 47.867 | 23 V Vanadium 50.942 | 24 Cr Chromium 51.996 | 25 Mn Manganese 54.938 | 26 Fe Iron 55.845 | 27 Co Cobalt 58.933 | 28 Ni Nickel 58.693 | 29 Cu Copper 63.546 | 30 Zn Zinc 65.38 | 31 Ga Gallium 69.723 | 32 Ge Germanium 72.631 | 33 As Arsenic 74.922 | 34 Se Selenium 78.971 | 35 Br Bromine 79.904 | 36 |
| 37 Rb Rubidium 85.468 | 38 Sr Strontium 87.62 | 39 Y Yttrium 88.906 | 40 Zr Zirconium 91.224 | 41 Nb Niobium 92.906 | 42 Mo Molybdenum 95.95 | 43 Tc Technetium 98.907 | 44 Ru Ruthenium 101.07 | 45 Rh Rhodium 102.906 | 46 Pd Palladium 106.42 | 47 Ag Silver 107.868 | 48 Cd Cadmium 112.414 | 49 In Indium 114.818 | 50 Sn Tin 118.711 | 51 Sb Antimony 121.760 | 52 Te Tellurium 127.6 | 53 I Iodine 126.904 | 54 |
| 55 Cs Cesium 132.905 | 56 Ba Barium 137.328 | 57-71 Lanthanide Series | 72 | 73 | 74 W Tungsten 183.84 | 75 | 76 | 77 | 78 Pt Platinum 195.085 | 79 Au Gold 196.967 | 80 Hg Mercury 200.592 | 81 Tl Thallium 204.383 | 82 Pb Lead 207.2 | 83 | 84 Bi Bismuth 208.980 | 85 | 86 |
| 87 Fr Francium 223.020 | 88 Ra Radium 226.025 | 10 Ne Neon 20.180 | | | | 24 Cr Chromium 51.996 | | | 110 Ds Darmstadtium [281] | 111 Rg Roentgenium [280] | 112 Cn Copernicium [285] | 113 Nh Nihonium [286] | 114 Fl Flerovium [289] | 115 Mc Moscovium [290] | 116 Lv Livermorium [293] | 117 Ts Tennessine [294] | 118 Og Oganesson [294] |
| | | Pr Praseodymium 140.908 | Ce Cerium 140.116 | Nd Neodymium 144.243 | Pm Promethium [145] | Sm Samarium 150.36 | Eu Europium 151.964 | Gd Gadolinium 157.25 | Tb Terbium 158.925 | Dy Dysprosium 162.500 | Ho Holmium 164.930 | Er Erbium 167.256 | Tm Thulium 168.930 | Yb Ytterbium 173.054 | Lu Lutetium 174.967 | | |
| | | Ac Actinium 227.028 | Th Thorium 232.038 | Pa Protactinium 231.036 | U Uranium 238.029 | Np Neptunium 237.048 | Pu Plutonium 244.064 | Am Americium 243.061 | Cm Curium 247.070 | Bk Berkelium 247.070 | Cf Californium 251.080 | Es Einsteinium [261] | Fm Fermium [267] | Mn Mendelevium [268] | Lr Lawrencium [260] | | |



Wavefunctions of deformed nuclei in the collective space

Introduction

Experiments

Method

Results

Chromium-50

Neon-20

Conclusion

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- Erik Kronkvist

A project supervised by:

- Gillis Carlsson
- Andrea Idini



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How do we motivate the investigation of deformed nuclei?

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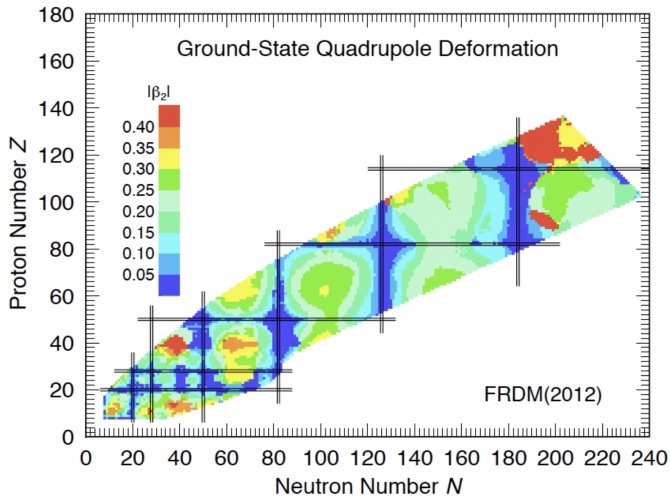


Figure: from P. Möller et.al (2012)

Problems of exploring deformed nuclei

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Laboratory frame

- Loses information about the nuclear deformation.
- The nucleus is a superposition of intrinsic wave functions.
- Spherical symmetry
- $[\hat{L}^2, \hat{H}] = 0$

Intrinsic frame

- Well defined deformation.
- Can be found using approaches like the GCM.
- Symmetries are broken
- $[\hat{L}^2, \hat{H}] \neq 0$.



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Quark Gluon Plasma at CERN

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- Extreme temperatures.
- Protons and neutrons separates into quarks and gluons \rightarrow quark gluon plasma.
- Can be achieved through high-energy ion collisions \sim TeV.
- A way to probe the deformation of the nucleus.



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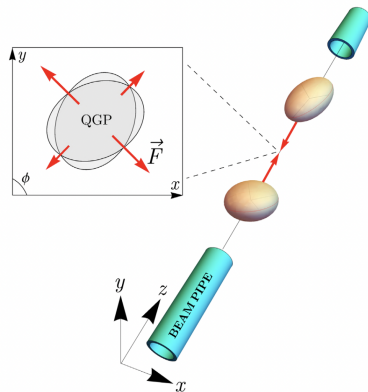


Figure: from B. Bally (2022)

Deformation

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The deformation of the nucleus can be shown intuitively in the (β_2, γ) -plane

- $\gamma = 0^\circ$: Prolate shape
- $\gamma = 60^\circ$: Oblate shape
- $0^\circ < \gamma < 60^\circ$: Triaxial shape



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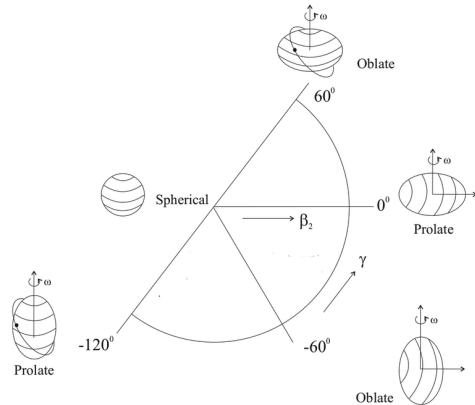


Figure: from M. Siciliano (2013)

GCM generating reference states

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- Generate basis states (reference state) with constraints from generator coordinates.
 - Deformation of the basis states is well defined
 - This is the intrinsic frame wave function
 - Each reference state has a unique combination of generator coordinates
- Find eigenvector using these basis states
- These are the lab frame wave functions

More in depth discussion of GCM by Jennifer Boström tomorrow



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GCM generator coordinates

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Generator coordinates: $[\beta_2, \gamma, \Delta_p, \Delta_n, j_x]$

- Chromium-50: 192 reference states.
- Neon-20: 191 reference states.



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Chromium-50

Collective wavefunction of even spin yrast states $I = 0$ to $I = 14$

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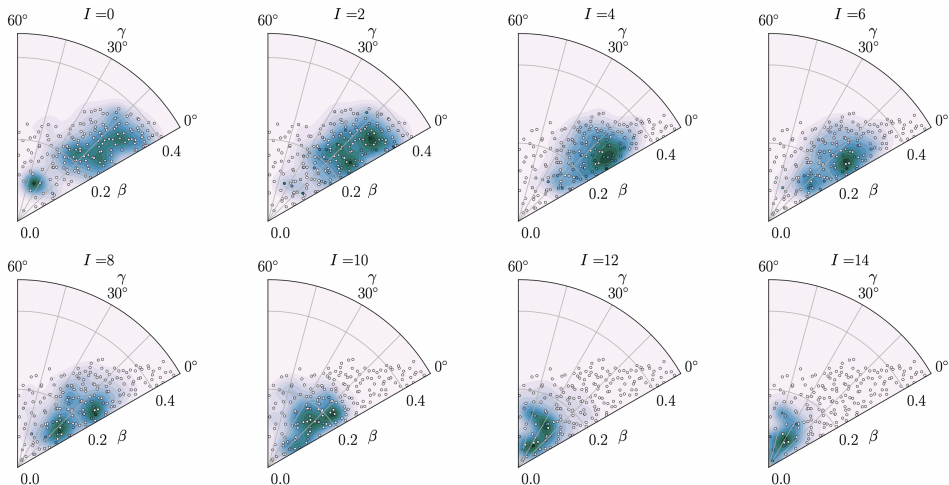


Figure: From A. Idini et.al (2024)



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Chromium-50 nuclear shell model

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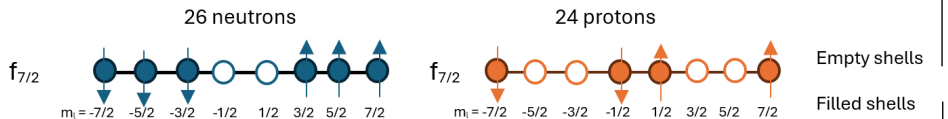
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$$M = 0$$



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Chromium-50 nuclear shell model

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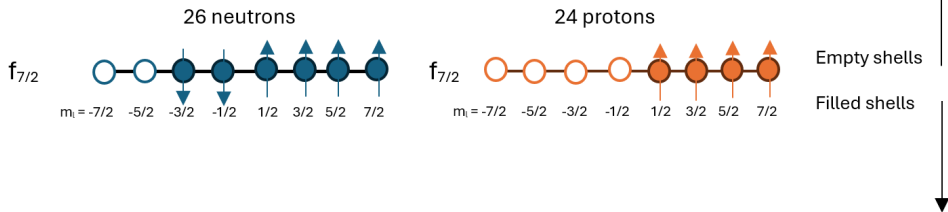
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$$M = 14$$



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Collective wavefunction of Yrast states

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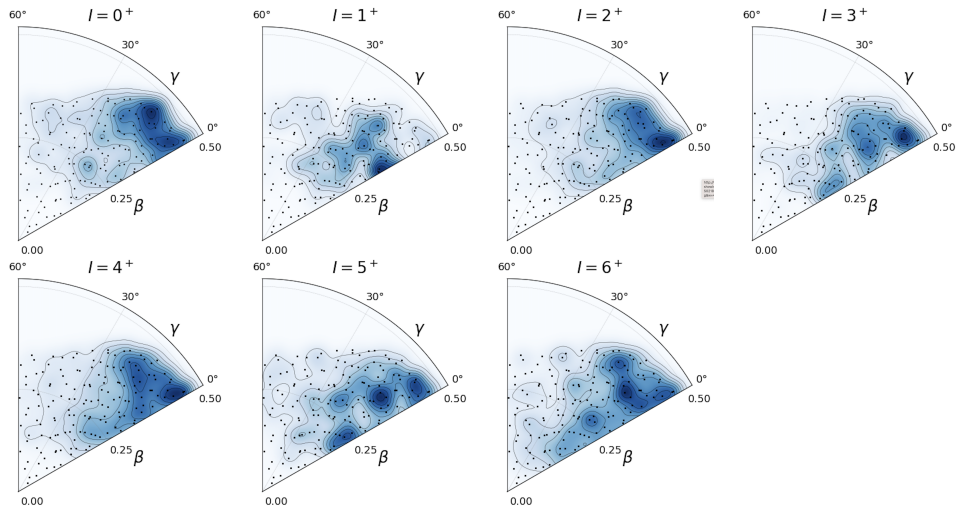
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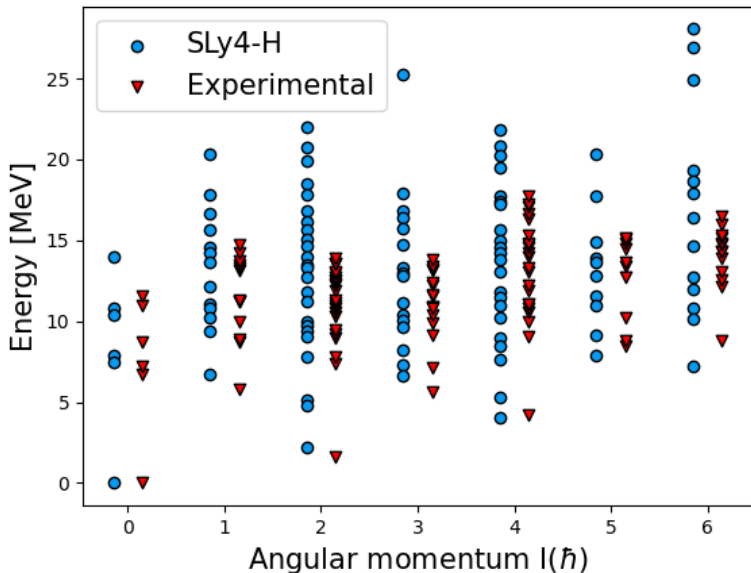
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Neon-20

Energy spectrum



Exp. data
from
Brookhaven
National
Laboratory
database



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Average values and smearing of the generator coordinates

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Average value:

$$\bar{C} = \sum_j |g_j|^2 C_j$$

Standard deviation:

$$\Delta C = \left\{ \left[\sum_j |g_j|^2 C_j^2 \right] - [\bar{C}]^2 \right\}^{1/2}$$

C_j collective coordinate and g_j collective coefficient for reference state j .



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Correlation energy

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$$\text{Correlation energy} = \bar{E} - \langle E \rangle$$

where \bar{E} is the average energy and $\langle E \rangle$ is the energy of the eigenstate to the Hamiltonian.



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Neon-20: Table of average values for Yrast states

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| I | $\langle E \rangle$ | Correlation energy | β_2 | γ |
|-------|---------------------|--------------------|--------------------|----------------------------------|
| 0^+ | -162.033299 | 9.473676 | 0.35 ± 0.09 | 17.6° $\pm 13.6^\circ$ |
| 1^+ | -155.316006 | 2.114562 | 0.32 ± 0.07 | 16.2° $\pm 12.8^\circ$ |
| 2^+ | -159.831035 | 7.097223 | 0.35 ± 0.09 | 15.9° $\pm 13.1^\circ$ |
| 3^+ | -155.375207 | 2.061610 | 0.33 ± 0.09 | 14.1° $\pm 11.7^\circ$ |
| 4^+ | -157.969636 | 5.117888 | 0.32 ± 0.09 | 15.9° $\pm 13.3^\circ$ |
| 5^+ | -154.189394 | 1.088491 | 0.30 ± 0.10 | 16.6° $\pm 13.6^\circ$ |
| 6^+ | -154.848582 | 2.494840 | 0.28 ± 0.11 | 16.1° $\pm 13.0^\circ$ |

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- Although nuclei with spin $I = 0$ are not deformed in the labframe, they can be considered so in the intrinsic frame.
- Chromium:
 - Deformation decreases with spin
- Neon:
 - β_2 is more static and well defined than γ which is more spread.
 - $\beta_2 \sim 0.35 \pm 0.15$
 - $\gamma \sim 16^\circ \pm 13^\circ$
- Behaviour of correlation energy interesting for further investigation.



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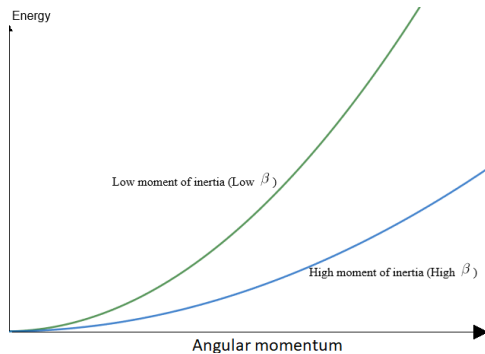
Neon-20

Conclusion

$$E_{rot} = \frac{\hbar^2 I(I+1)}{2\mathcal{I}}$$

Larger $\beta_2 \implies$ larger $\mathcal{I} \implies$
lower E_{rot}

Larger deformation (blue line)
Smaller deformation (green line)



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Deformation

- How can we couple the output of GCM to nuclear deformations?

$$\beta_x \propto \frac{\langle r^2 Y_2^0 \rangle}{\langle r^2 \rangle} \quad \beta_y \propto \frac{\langle r^2 Y_2^2 + r^2 Y_2^{-2} \rangle}{\langle r^2 \rangle}$$

$$\beta_2 = \sqrt{\beta_x^2 + \beta_y^2} \quad \gamma = \arctan \frac{\beta_y}{\beta_x}$$



(a) $Y_2^2 \propto \hat{Q}_{22}$



(b) $Y_2^0 \propto \hat{Q}_{20}$



(c) $Y_2^{-2} \propto \hat{Q}_{2-2}$



Table of average values

Grid 1

The values in this table correspond to the Yrast states (lowest energy) of the corresponding angular momentum in the first column.

Table 1: Statistical average value and standard deviation results for grid C

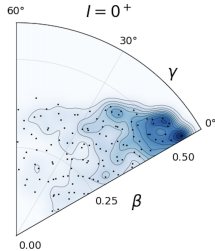
| I | E | Correlation E | ω | Δ | β_2 | γ | j_x | |
|----------------|--------------------------|----------------|----------|------------|------------|------------|------------------|------------|
| 0 ⁺ | $\overline{A_i}(\Gamma)$ | -152.559623 | 9.473676 | 0.46 | 1.02 | 0.35 | 17.6° | 1.06 |
| | $\Delta A_i(\Gamma)$ | ± 2.722149 | | ± 0.27 | ± 0.12 | ± 0.09 | $\pm 13.6^\circ$ | ± 1.12 |
| 1 ⁺ | $\overline{A_i}(\Gamma)$ | -153.201444 | 2.114562 | 0.46 | 1.03 | 0.32 | 16.2° | 2.18 |
| | $\Delta A_i(\Gamma)$ | ± 2.093619 | | ± 0.26 | ± 0.11 | ± 0.07 | $\pm 12.8^\circ$ | ± 1.59 |
| 2 ⁺ | $\overline{A_i}(\Gamma)$ | -152.733812 | 7.097223 | 0.50 | 1.02 | 0.35 | 15.9° | 1.22 |
| | $\Delta A_i(\Gamma)$ | ± 2.636391 | | ± 0.28 | ± 0.11 | ± 0.09 | $\pm 13.1^\circ$ | ± 1.23 |
| 3 ⁺ | $\overline{A_i}(\Gamma)$ | -153.313597 | 2.061610 | 0.50 | 1.01 | 0.33 | 14.1° | 1.48 |
| | $\Delta A_i(\Gamma)$ | ± 1.983384 | | ± 0.26 | ± 1.03 | ± 0.09 | $\pm 11.7^\circ$ | ± 1.39 |
| 4 ⁺ | $\overline{A_i}(\Gamma)$ | -152.851748 | 5.117888 | 0.54 | 1.02 | 0.32 | 15.9° | 1.86 |
| | $\Delta A_i(\Gamma)$ | ± 2.431483 | | ± 0.28 | ± 0.11 | ± 0.09 | $\pm 13.3^\circ$ | ± 1.79 |
| 5 ⁺ | $\overline{A_i}(\Gamma)$ | -153.100904 | 1.088491 | 0.52 | 1.00 | 0.30 | 16.6° | 2.04 |
| | $\Delta A_i(\Gamma)$ | ± 1.985209 | | ± 0.28 | ± 0.10 | ± 0.10 | $\pm 13.6^\circ$ | ± 1.81 |
| 6 ⁺ | $\overline{A_i}(\Gamma)$ | -152.353740 | 2.494840 | 0.66 | 1.03 | 0.28 | 16.1° | 3.24 |
| | $\Delta A_i(\Gamma)$ | ± 2.182303 | | ± 0.30 | ± 0.11 | ± 0.11 | $\pm 13.0^\circ$ | ± 2.30 |



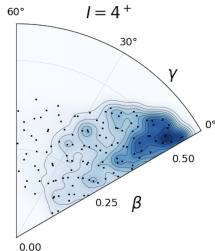
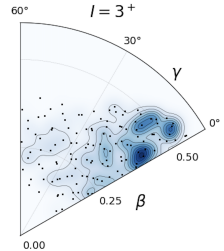
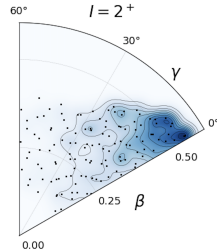
Neon-20

Collective wavefunction of Yrast states

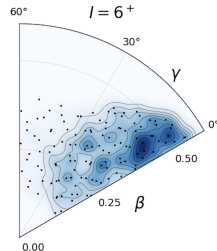
Grid without considering cranking.



No spin 1



No spin 5



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Ground state comparison of the two grids

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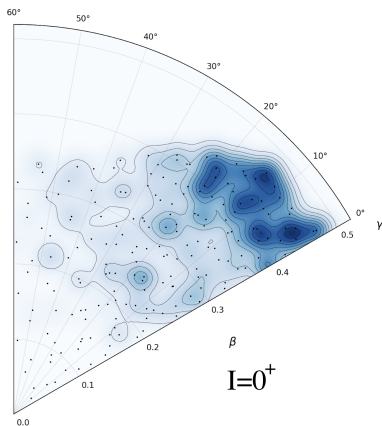
Chromium-50

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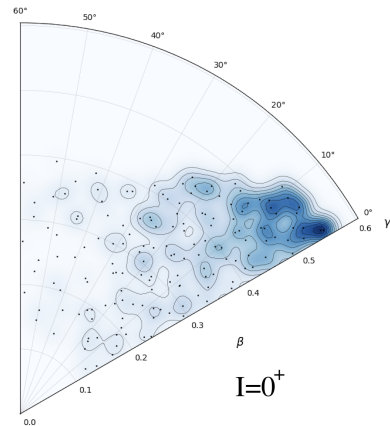
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(a) Grid with cranking



(b) Grid without cranking