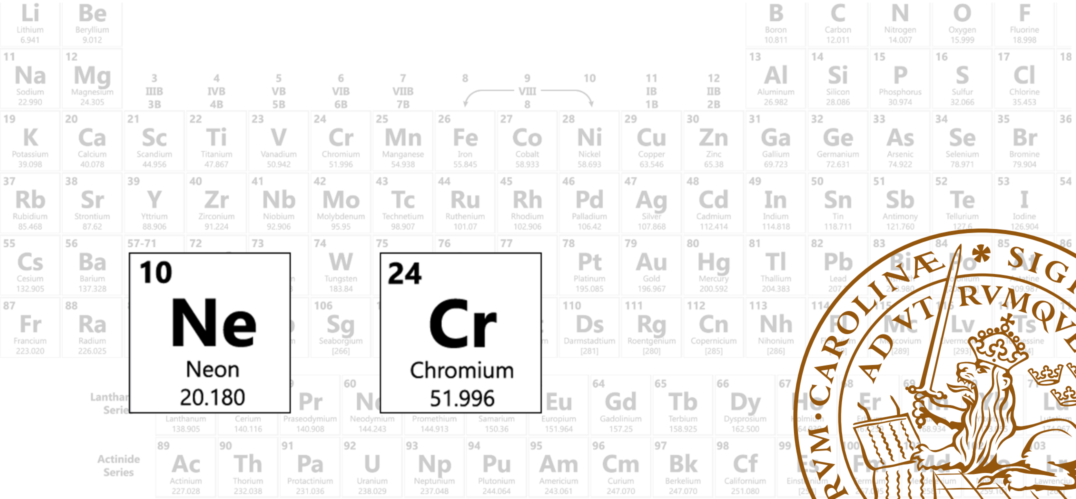
Wavefunctions of deformed nuclei in the collective space

DIVISION OF MATHEMATICAL PHYSICS - LUND UNIVERSITY



Wavefunctions of deformed nuclei in the collective space

- Filip Agert
- Erik Kronkvist

A project supervised by:

- Gillis Carlsson
- Andrea Idini

How do we motivate the investigation of deformed nuclei?

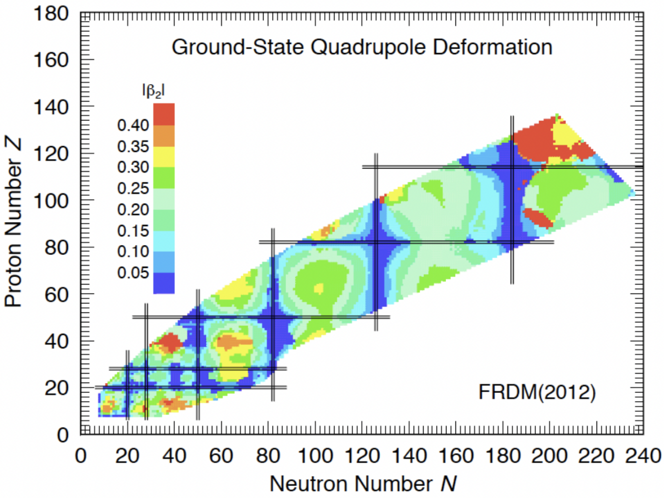


Figure: from P. Möller et.al (2012)

Problems of exploring deformed nuclei

Laboratory frame

- Loses information about the nuclear deformation.
- The nucleus is a superposition of intrinsic wave functions.
- Spherical symmetry
- $[\hat{L}^2, \hat{H}] = 0$

Intrinsic frame

- Well defined deformation.
- Can be found using approaches like the GCM.
- Symmetries are broken
- $[\hat{L}^2, \hat{H}] \neq 0$.

Quark Gluon Plasma at CERN

- Extreme temperatures.
- Protons and neutrons separates into quarks and gluons $\rightarrow$ quark gluon plasma.
- Can be achieved through high-energy ion collisions $\sim$ TeV.
- A way to probe the deformation of the nucleus.

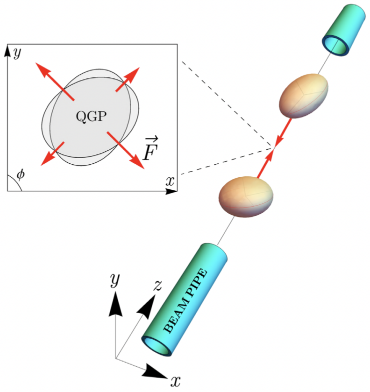


Figure: from B. Bally (2022)

Deformation

The deformation of the nucleus can be shown intuitively in the $(\beta_2, \gamma)-$plane

- $\gamma = 0^\circ$: Prolate shape
- $\gamma = 60^\circ$: Oblate shape
- $0^\circ < \gamma < 60^\circ$: Triaxial shape

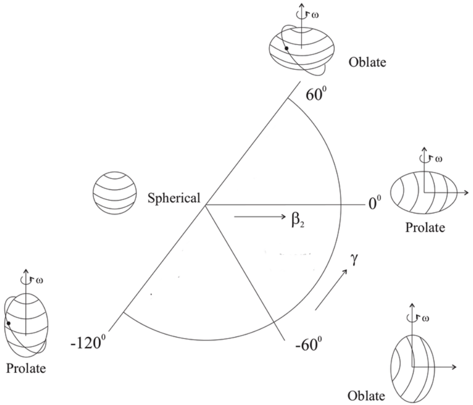


Figure: from M. Siciliano (2013)

GCM generating reference states

- Generate basis states (reference state) with constraints from generator coordinates.
  - Deformation of the basis states is well defined
  - This is the intrinsic frame wave function
  - Each reference state has a unique combination of generator coordinates

- Find eigenvector using these basis states
- These are the lab frame wave functions

More in depth discussion of GCM by Jennifer Boström tommorow

GCM generator coordinates

Generator coordinates: $[\beta_2, \gamma, \Delta_p, \Delta_n, j_x]$

- Chromium-50: 192 reference states.
- Neon-20: 191 reference states.

Chromium-50

Collective wavefunction of even spin yrast states I = 0 to I = 14

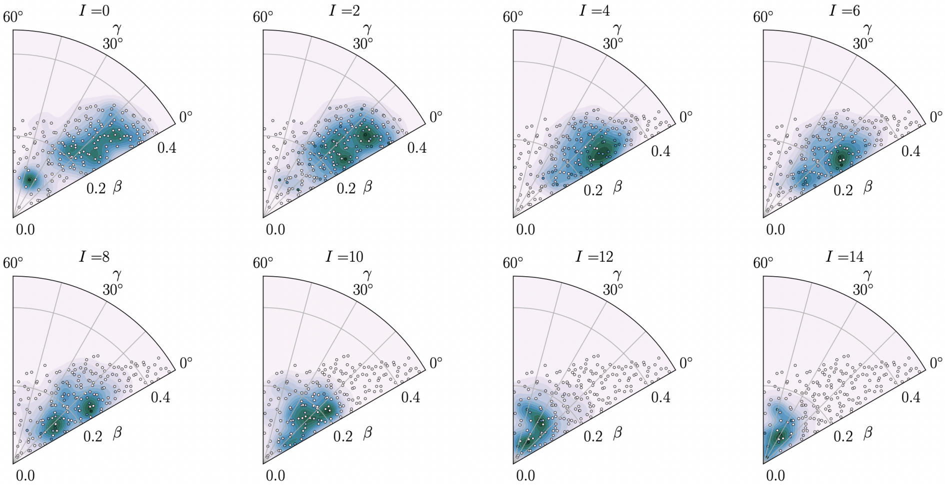
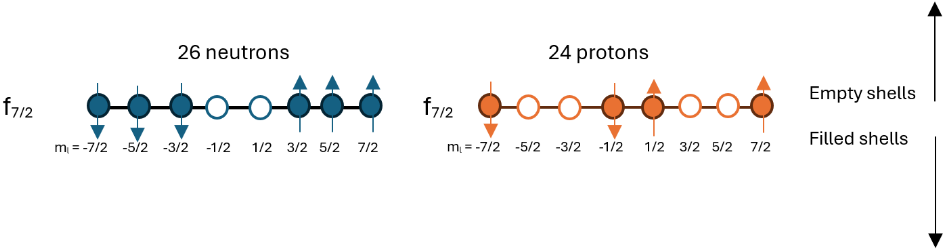


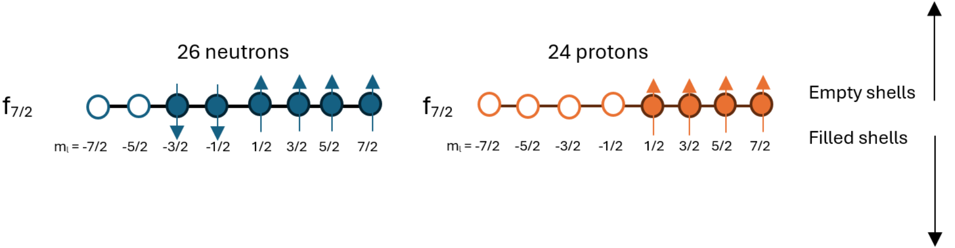
Figure: From A. Idini et.al (2024)

Chromium-50 nuclear shell model



$M = 0$

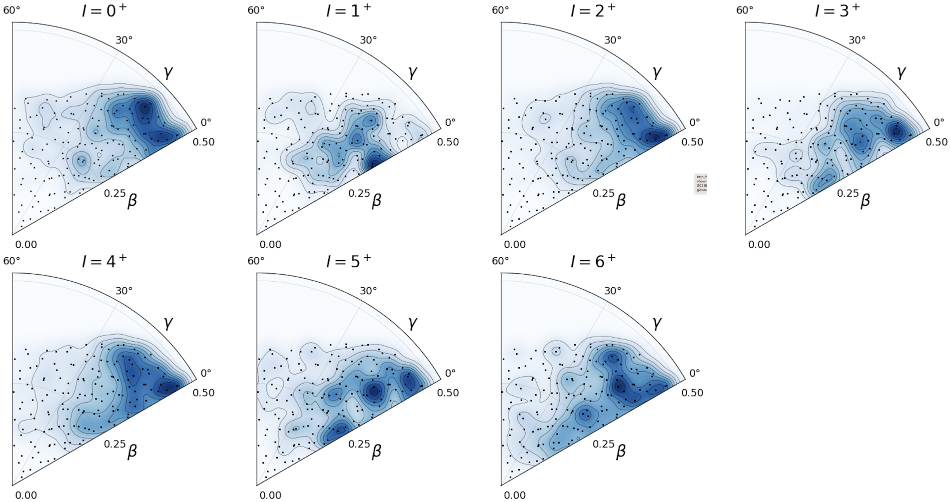
Chromium-50 nuclear shell model



$$M = 14$$

Neon-20

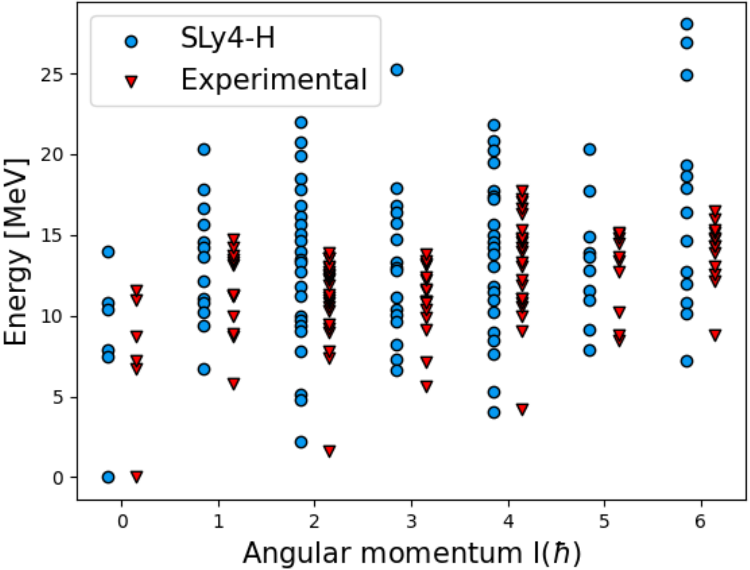
Collective wavefunction of Yrast states



Neon-20

Energy spectrum

Exp. data from Brookhaven National Laboratory database



Average values and smearing of the generator coordinates

Average value:

$$\overline{C} = \sum_j |g_j|^2 C_j$$

Standard deviation:

$$\Delta C = \left\{ \left[\sum_j |g_j|^2 C_j^2 \right] - \left[\,\overline{C}\, \right]^2 \right\}^{1/2}$$

C_j collective coordinate and g_j collective coefficient for reference state j.

Correlation energy

$$\text{Correlation energy} = \overline{E} - \langle E \rangle$$

where $\overline{E}$ is the average energy and $\langle E \rangle$ is the energy of the eigenstate to the Hamiltonian.

Neon-20: Table of average values for Yrast states

I	$\langle E \rangle$	Correlation energy	β_2	γ
0^+	-162.033299	9.473676	0.35 ± 0.09	17.6° $\pm 13.6°$
1^+	-155.316006	2.114562	0.32 ± 0.07	16.2° $\pm 12.8°$
2^+	-159.831035	7.097223	0.35 ± 0.09	15.9° $\pm 13.1°$
3^+	-155.375207	2.061610	0.33 ± 0.09	14.1° $\pm 11.7°$
4^+	-157.969636	5.117888	0.32 ± 0.09	15.9° $\pm 13.3°$
5^+	-154.189394	1.088491	0.30 ± 0.10	16.6° $\pm 13.6°$
6^+	-154.848582	2.494840	0.28 ± 0.11	16.1° $\pm 13.0°$

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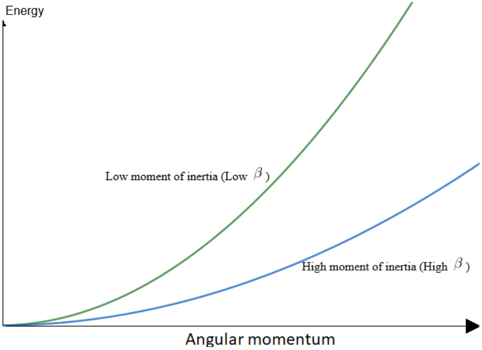
Conclusion

- Altough nuclei with spin $I = 0$ are not deformed in the labframe, they can be considered so in the intrinsic frame.
- Chromium:
  - Deformation decreases with spin
- Neon:
  - β_2 is more static and well defined than γ which is more spread.
  - $\beta_2 \sim 0.35 \pm 0.15$
  - $\gamma \sim 16^\circ \pm 13^\circ$
- Behaviour of correlation energy interesting for further investigation.

Neon-20

$$E_{rot} = \frac{\hbar^2 I(I+1)}{2\mathscr{I}}$$

Larger $\beta_2 \implies$ larger $\mathscr{I} \implies$ lower E_{rot}
Larger deformation (blue line)
Smaller deformation (green line)



Deformation

- How can we couple the output of GCM to nuclear deformations?

$$\beta_x \propto \frac{\langle r^2 Y_2^0 \rangle}{\langle r^2 \rangle} \quad \beta_y \propto \frac{\langle r^2 Y_2^2 + r^2 Y_2^{-2} \rangle}{\langle r^2 \rangle}$$

$$\beta_2 = \sqrt{\beta_x^2 + \beta_y^2} \quad \gamma = \arctan \frac{\beta_y}{\beta_x}$$

(a) $Y_2^2 \propto \hat{Q}_{22}$

(b) $Y_2^0 \propto \hat{Q}_{20}$

(c) $Y_2^{-2} \propto \hat{Q}_{2-2}$

Table of average values

Grid 1

The values in this table correspond to the Yrast states (lowest energy) of the corresponding angular momentum in the first column.

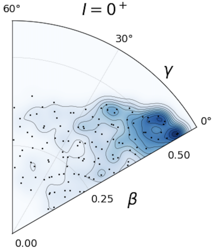
Table 1: Statistical average value and standard deviation results for grid C

I		E	Correlation E	ω	Δ	β_2	γ	j_x
0^+	$\overline{A_i}(\Gamma)$	−152.559623	9.473676	0.46	1.02	0.35	17.6°	1.06
	$\Delta A(\Gamma)$	±2.722149		±0.27	±0.12	±0.09	±13.6°	±1.12
1^+	$\overline{A_i}(\Gamma)$	−153.201444	2.114562	0.46	1.03	0.32	16.2°	2.18
	$\Delta A_i(\Gamma)$	±2.093619		±0.26	±0.11	±0.07	±12.8°	±1.59
2^+	$\overline{A_i}(\Gamma)$	−152.733812	7.097223	0.50	1.02	0.35	15.9°	1.22
	$\Delta A_i(\Gamma)$	±2.636391		±0.28	±0.11	±0.09	±13.1°	±1.23
3^+	$\overline{A_i}(\Gamma)$	−153.313597	2.061610	0.50	1.01	0.33	14.1°	1.48
	$\Delta A_i(\Gamma)$	±1.983384		±0.26	±1.03	±0.09	±11.7°	±1.39
4^+	$\overline{A_i}(\Gamma)$	−152.851748	5.117888	0.54	1.02	0.32	15.9°	1.86
	$\Delta A_i(\Gamma)$	±2.431483		±0.28	±0.11	±0.09	±13.3°	±1.79
5^+	$\overline{A_i}(\Gamma)$	−153.100904	1.088491	0.52	1.00	0.30	16.6°	2.04
	$\Delta A_i(\Gamma)$	±1.985209		±0.28	±0.10	±0.10	±13.6°	±1.81
6^+	$\overline{A_i}(\Gamma)$	−152.353740	2.494840	0.66	1.03	0.28	16.1°	3.24
	$\Delta A_i(\Gamma)$	±2.182303		±0.30	±0.11	±0.11	±13.0°	±2.30

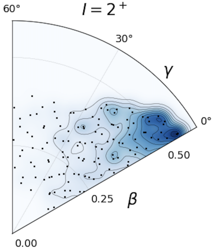
Neon-20

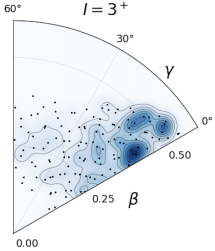
Collective wavefunction of Yrast states

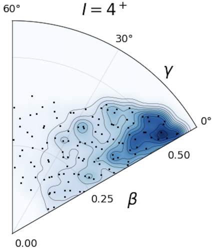
Grid without considering cranking.



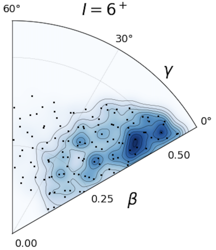
No spin 1





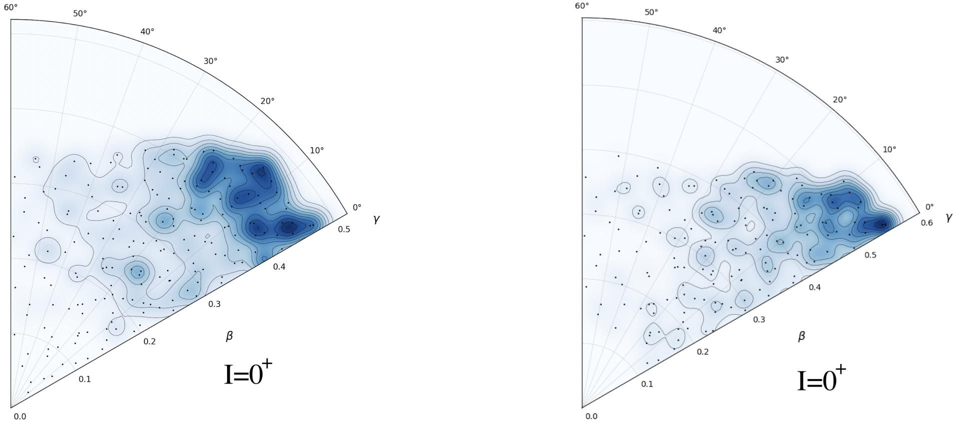


No spin 5



Neon-20

Ground state comparison of the two grids



(a) Grid with cranking

(b) Grid without cranking