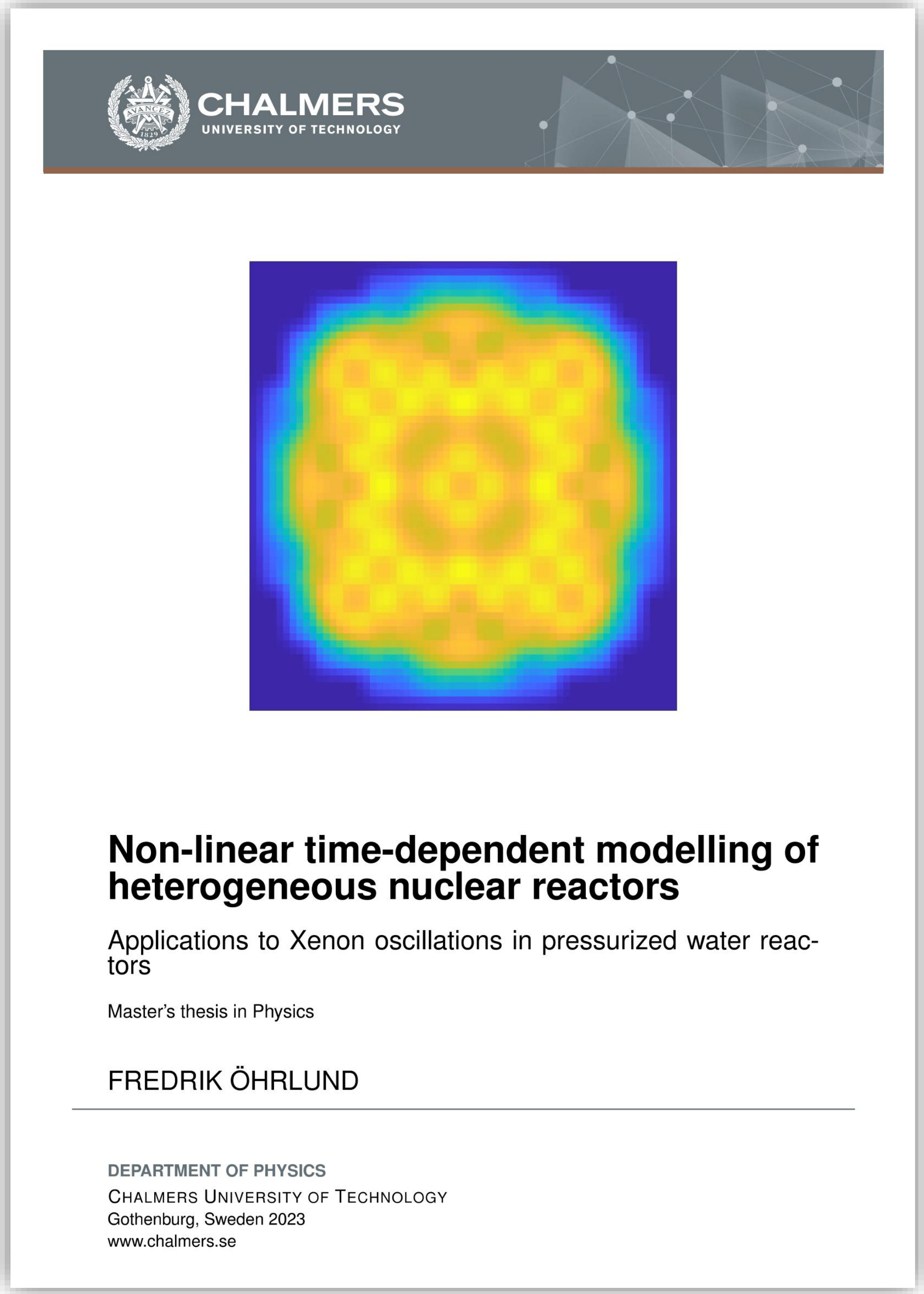

CHALMERS
UNIVERSITY OF TECHNOLOGY

Non-linear time-dependent modelling of heterogeneous nuclear reactors

Applications to Xenon oscillations in pressurized water reactors

Master's thesis in Physics

FREDRIK ÖHRLUND

DEPARTMENT OF PHYSICS
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2023
www.chalmers.se

Master's thesis presentation:

Non-linear time-dependent modelling of heterogeneous nuclear reactors

Applications to Xenon oscillations in pressurized water reactors

Awarded the 2024 SKC Sigvard Eklund price for best master's thesis

Fredrik Öhrlund, MSc

Outline of Presentation

• My Master Thesis	5 min
• Simulating a reactor	3 min
• Xenon-135	7 min

Thesis background

From
Page 1

- In thermal nuclear reactors, there is a fission product called Xenon-135
- Its well known in reactor physics that Xe-135 can cause ”xenon oscillations”
- Xenon oscillations are expected to be a more frequent problem in the future
- The DREAM group at Chalmers is currently developing methods for predicting xenon-oscillations
- ”This master thesis project aims at developing modelling capabilities that allow for the calculation of the three-dimensional time-dependence of the neutron flux in a nuclear reactor for the specific purpose of studying the xenon effect.”
- Unlike proprietary software, will have full access to the source code

Problem definition

$$\frac{1}{v_1}\frac{\partial}{\partial t}\phi_1(\vec{\mathbf{r}},t) = \left[\vec{\boldsymbol{\nabla}}\cdot D_1(\vec{\mathbf{r}},t)\vec{\boldsymbol{\nabla}} + \nu\Sigma'_{f,1}(\vec{\mathbf{r}},t) - \Sigma_{a,1}(\vec{\mathbf{r}},t) - \Sigma_r(\vec{\mathbf{r}},t)\right]\phi_1(\vec{\mathbf{r}},t)$$
$$+ \nu\Sigma'_{f,2}(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t)$$
$$+ \alpha_{1\to 1}(\vec{\mathbf{r}})(\phi_1(\vec{\mathbf{r}},t) - \phi_{eq,1}(\vec{\mathbf{r}})) + \alpha_{2\to 1}(\vec{\mathbf{r}})(\phi_2(\vec{\mathbf{r}},t) - \phi_{eq,2}(\vec{\mathbf{r}}))$$

From
Page 2

To summarize the remaining 5 pages of the project description:

”Here is a long list of contraints and requirements...
... please solve these equations, using a computer...
... then...
... somehow perturb the system to generate xenon oscillations”

Problem definition

$$\begin{bmatrix} \phi_1(\vec{\mathbf{r}},t) \\ \phi_2(\vec{\mathbf{r}},t) \\ I(\vec{\mathbf{r}},t) \\ X(\vec{\mathbf{r}},t) \end{bmatrix}$$

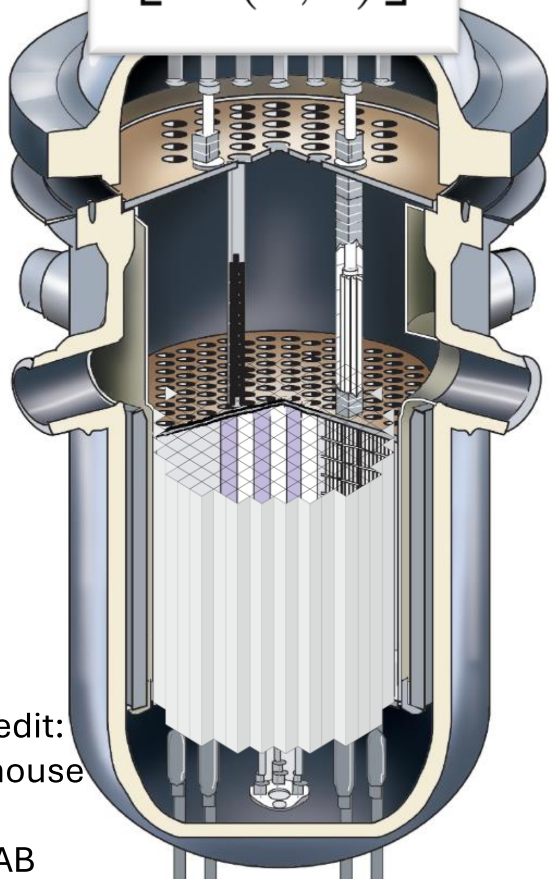


Figure credit: Westinghouse Electric Sweden AB

$$\left\{\begin{aligned}
\frac{1}{v_1}\frac{\partial}{\partial t}\phi_1(\vec{\mathbf{r}},t) &= \Big[\vec{\boldsymbol{\nabla}}\cdot D_1(\vec{\mathbf{r}},t)\vec{\boldsymbol{\nabla}} + \nu\Sigma'_{f,1}(\vec{\mathbf{r}},t) - \Sigma_{a,1}(\vec{\mathbf{r}},t) - \Sigma_r(\vec{\mathbf{r}},t)\Big]\phi_1(\vec{\mathbf{r}},t) \\
&\quad + \nu\Sigma'_{f,2}(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t) \\
&\quad + \alpha_{1\to1}(\vec{\mathbf{r}})(\phi_1(\vec{\mathbf{r}},t) - \phi_{eq,1}(\vec{\mathbf{r}})) + \alpha_{2\to1}(\vec{\mathbf{r}})(\phi_2(\vec{\mathbf{r}},t) - \phi_{eq,2}(\vec{\mathbf{r}})) \\
\frac{1}{v_2}\frac{\partial}{\partial t}\phi_2(\vec{\mathbf{r}},t) &= \Sigma_r(\vec{\mathbf{r}},t)\phi_1(\vec{\mathbf{r}},t) + \Big[\vec{\boldsymbol{\nabla}}\cdot D_2(\vec{\mathbf{r}},t)\vec{\boldsymbol{\nabla}} - \Sigma_{a,2,wox}(\vec{\mathbf{r}},t)\Big]\phi_2(\vec{\mathbf{r}},t) \\
&\quad + \alpha_{1\to2}(\vec{\mathbf{r}})(\phi_1(\vec{\mathbf{r}},t) - \phi_{eq,1}(\vec{\mathbf{r}})) + \alpha_{2\to2}(\vec{\mathbf{r}})(\phi_2(\vec{\mathbf{r}},t) - \phi_{eq,2}(\vec{\mathbf{r}})) \\
&\quad - \sigma_X X(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t) \\
\frac{\partial}{\partial t}I(\vec{\mathbf{r}},t) &= \gamma_I\Sigma'_{f,1}(\vec{\mathbf{r}},t)\phi_1(\vec{\mathbf{r}},t) + \gamma_I\Sigma'_{f,2}(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t) - \lambda_I I(\vec{\mathbf{r}},t) \\
\frac{\partial}{\partial t}X(\vec{\mathbf{r}},t) &= \gamma_X\Sigma'_{f,1}(\vec{\mathbf{r}},t)\phi_1(\vec{\mathbf{r}},t) + \gamma_X\Sigma'_{f,2}(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t) + \lambda_I I(\vec{\mathbf{r}},t) - \lambda_X X(\vec{\mathbf{r}},t) \\
&\quad - \sigma_X X(\vec{\mathbf{r}},t)\phi_2(\vec{\mathbf{r}},t).
\end{aligned}\right.$$

XS INPUT DATA

$D_1(\vec{\mathbf{r}})$ $D_2(\vec{\mathbf{r}})$

$\nu\Sigma_{f,1}(\vec{\mathbf{r}})$ $\nu\Sigma_{f,2}(\vec{\mathbf{r}})$

$\Sigma_{a,1}(\vec{\mathbf{r}})$ $\Sigma_{a,2}(\vec{\mathbf{r}})$

$\Sigma_r(\vec{\mathbf{r}})$

CONSTRAINTS

Minimum solver capability ~50h sims

Marshak Boundary Conditions

Mesh-centered Finite Differences

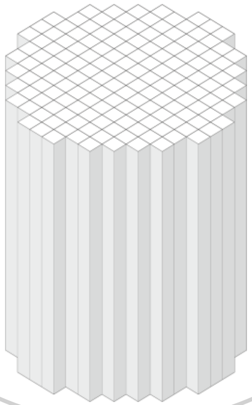
Compatibility with CORE SIM

Etc etc etc etc...

Problem definition

$$\begin{bmatrix} \phi_1(\vec{\mathbf{r}}, t) \\ \phi_2(\vec{\mathbf{r}}, t) \\ I(\vec{\mathbf{r}}, t) \\ X(\vec{\mathbf{r}}, t) \end{bmatrix}$$

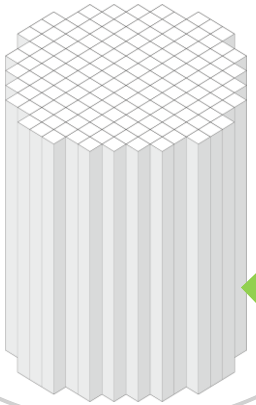
Xenon oscillations



My workflow

$$\begin{bmatrix} \phi_1(\vec{\mathbf{r}}, t) \\ \phi_2(\vec{\mathbf{r}}, t) \\ I(\vec{\mathbf{r}}, t) \\ X(\vec{\mathbf{r}}, t) \end{bmatrix}$$

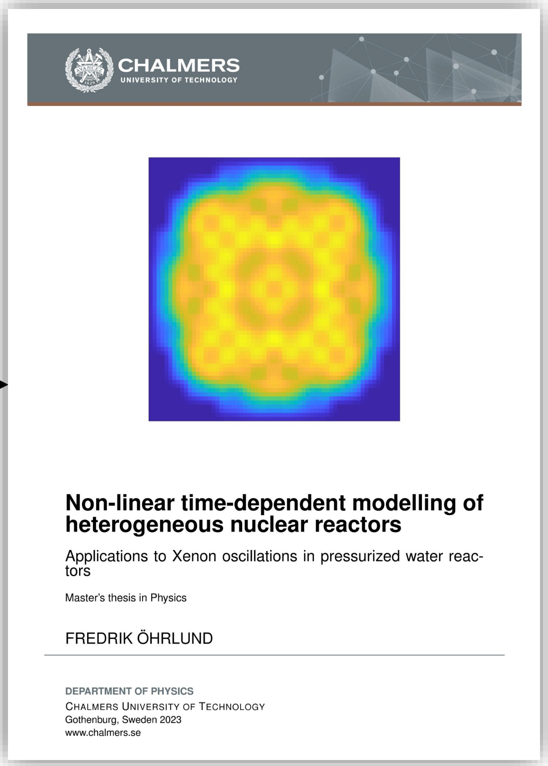
Xenon oscillations



My workflow

- Significant time studying/preparing
- Wrote a steady-state solver
- Wrote a time-integration solver
- Implemented CRB perturbations
- Generated some xenon oscillations

Lines of code
3328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Simulating a reactor

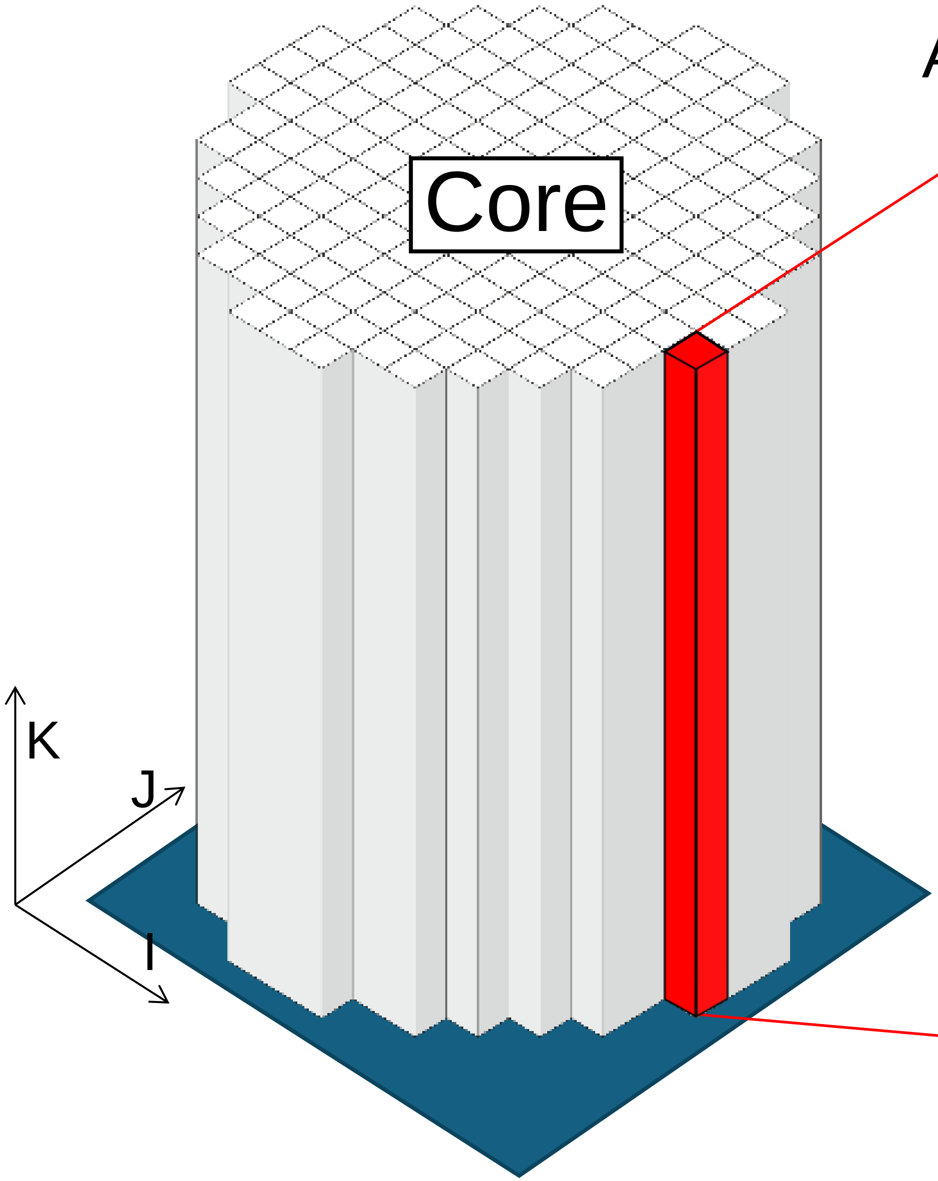

Figure Credit: Westinghouse Electric Sweden AB

Fuel Assemblies

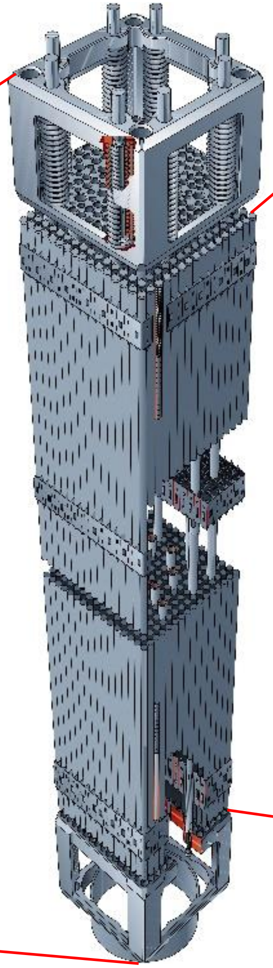

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Figure Credit: Swedish Academic Initiative in radiation and Nuclear Technology research and development

Fuel Pins

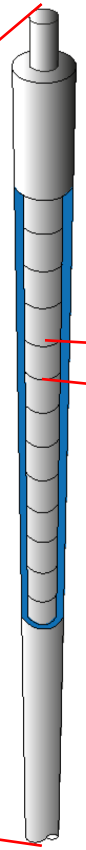

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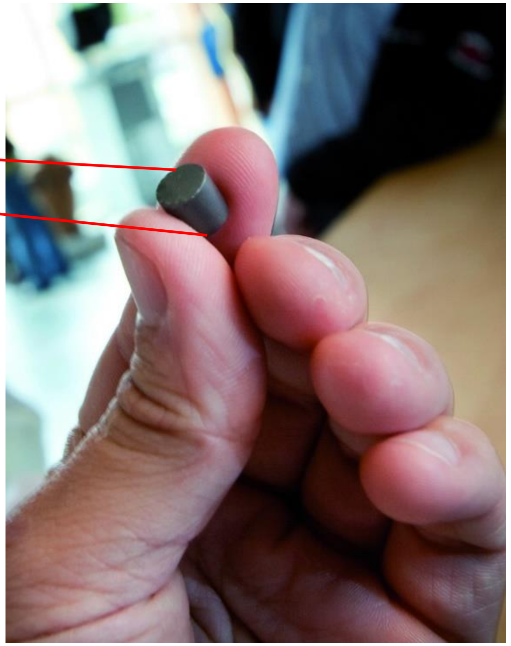
Figure Credit: Fredrik Ekenborg, Analysgruppen vid KSU

Fuel Pellets

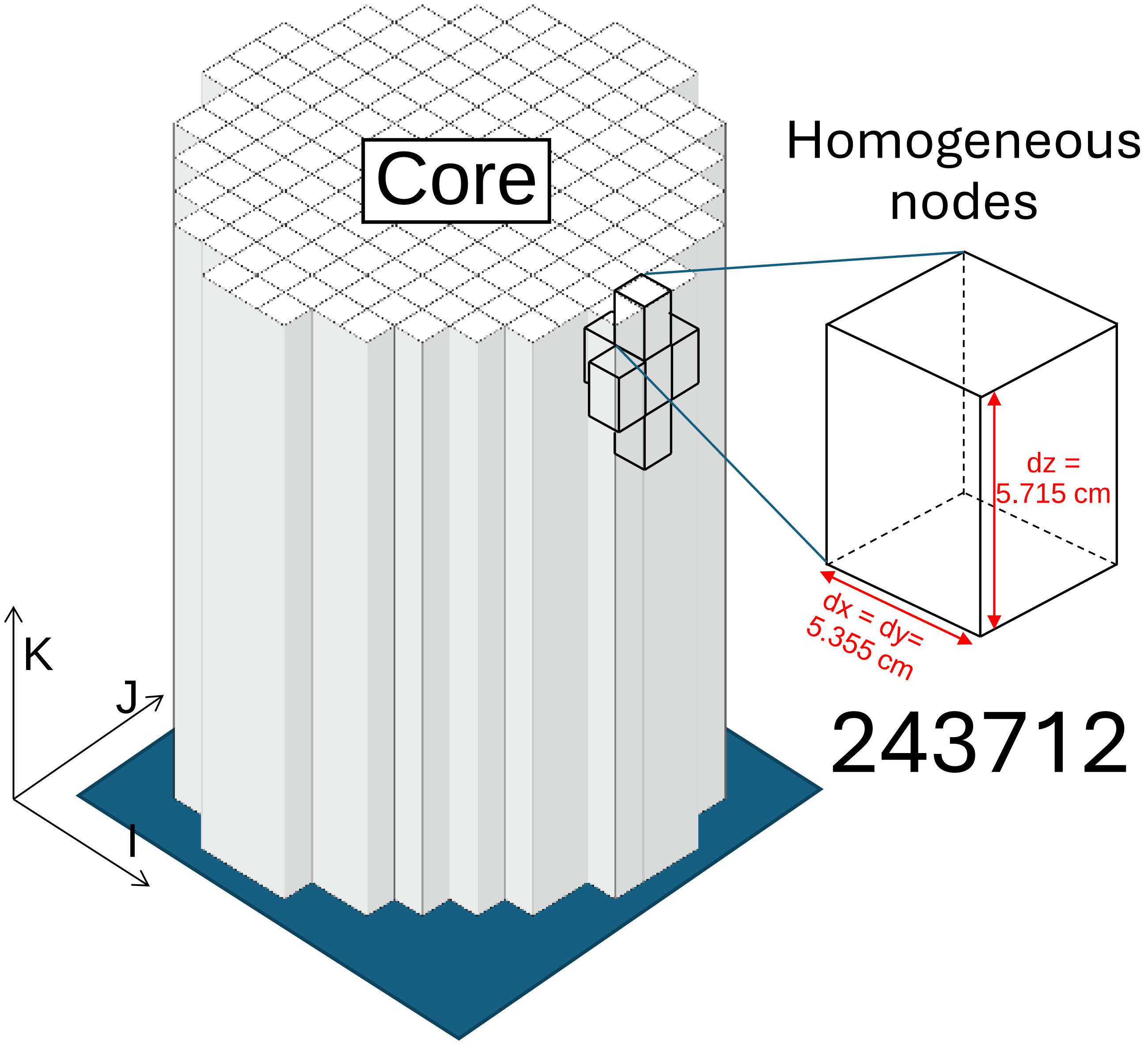


Serpent

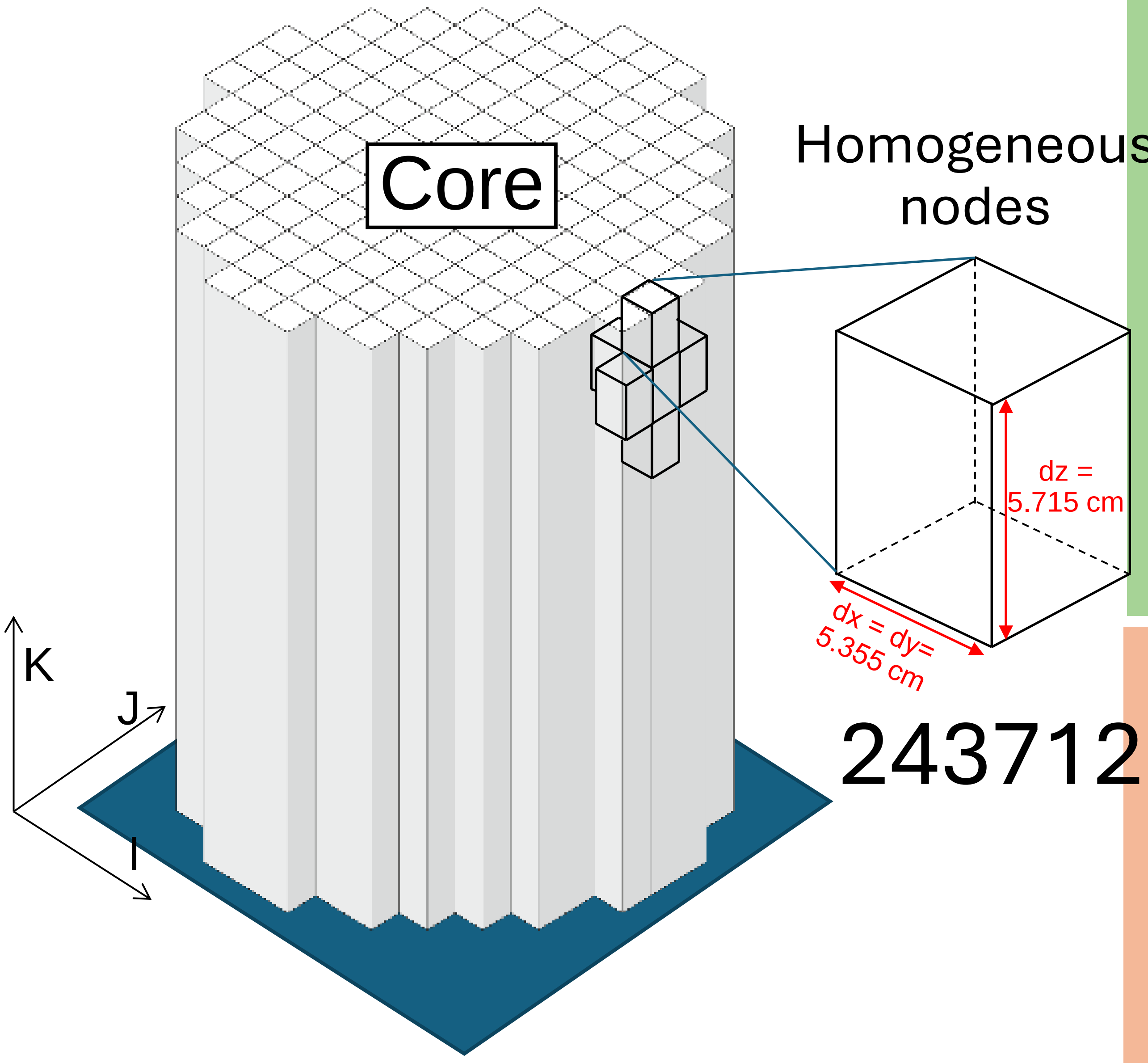
a Continuous-energy Monte Carlo neutron and photon transport code

~17 million

Simulating a reactor



Simulating a reactor



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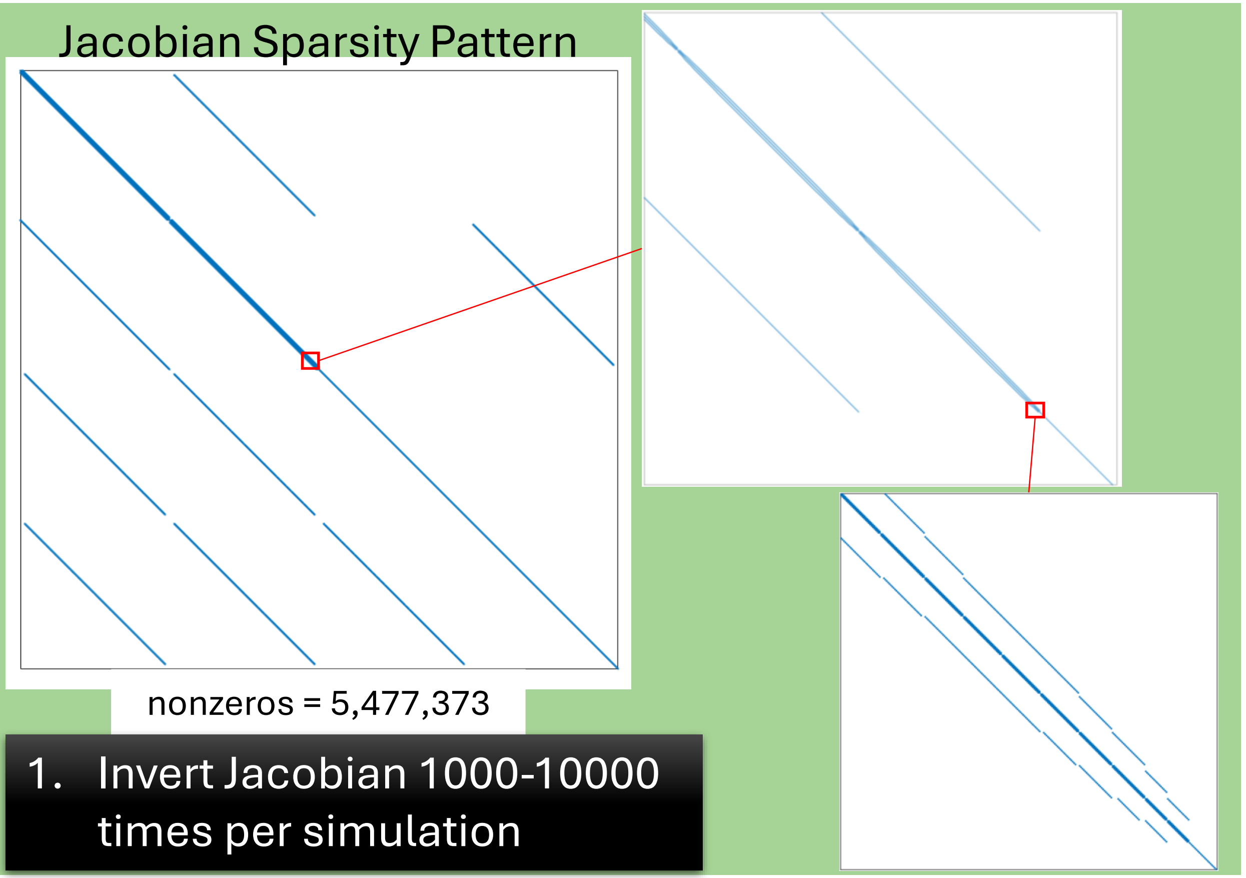


1. Invert Jacobian 1000-10000 times per simulation

$$\begin{bmatrix} \phi_1(\vec{\mathbf{r}}, t) \\ \phi_2(\vec{\mathbf{r}}, t) \\ I(\vec{\mathbf{r}}, t) \\ X(\vec{\mathbf{r}}, t) \end{bmatrix}$$

Characteristic Time-scales:

~10 ns

~1 ms

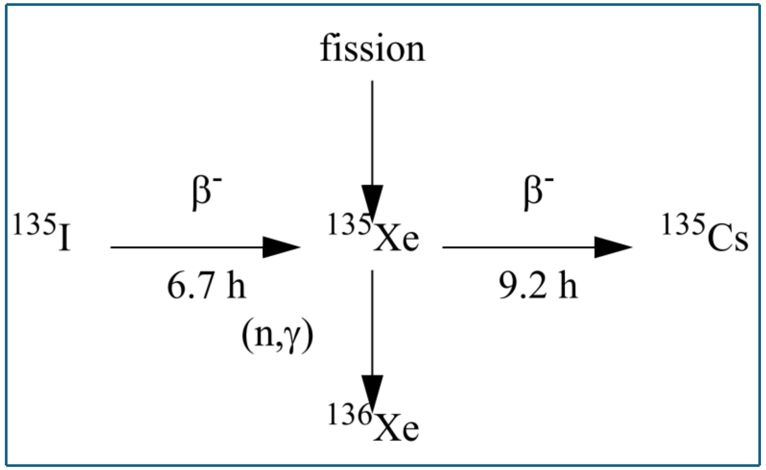
~1 h

Time-scales spanning $\sim 10^{11}$

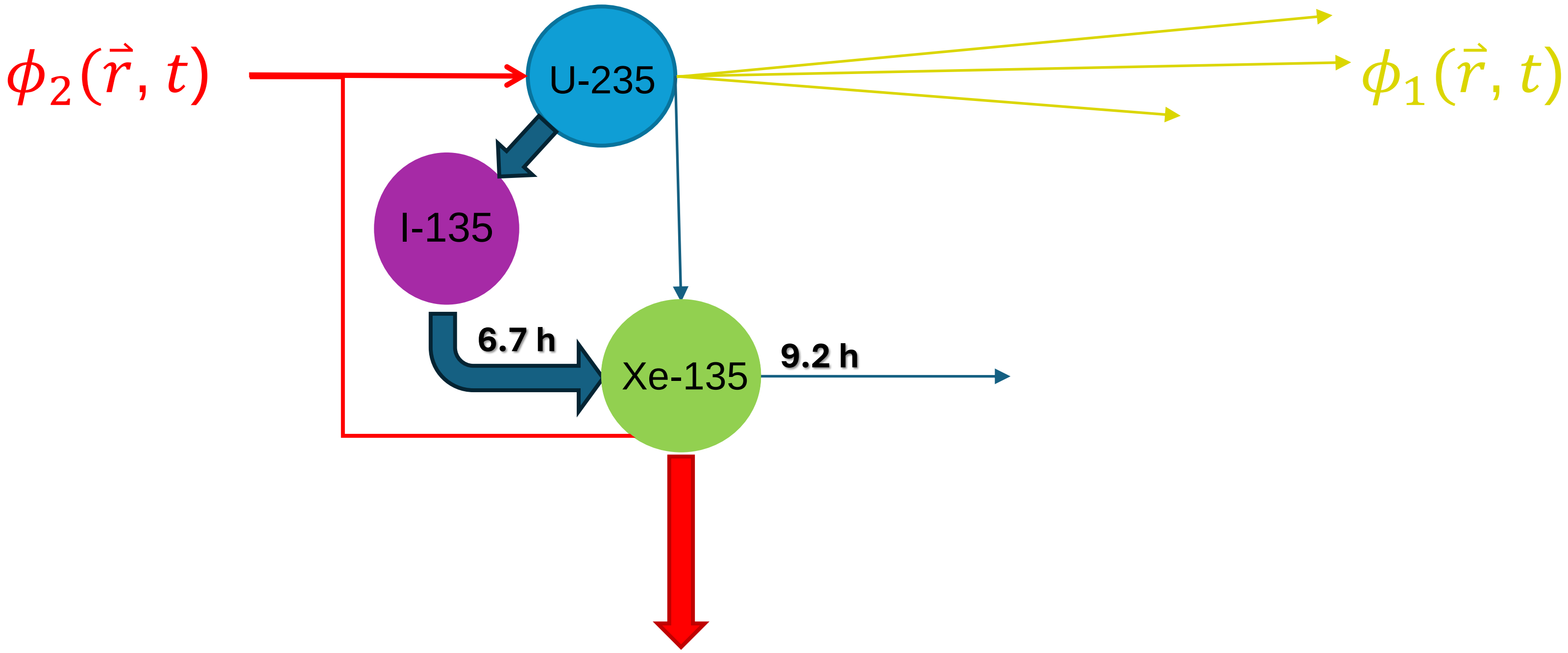
2. Extremely stiff dynamics -> -> Numerical oscillations

Xenon poisoning

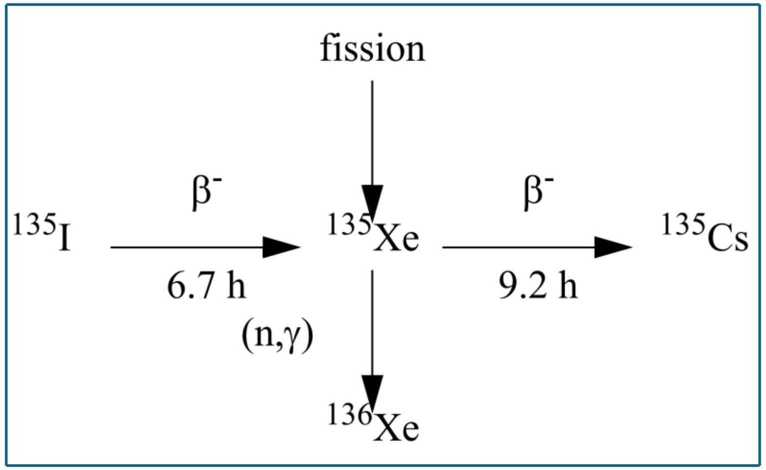
Evolution chain of Xenon-135



Xenon poisoning



Evolution chain of Xenon-135



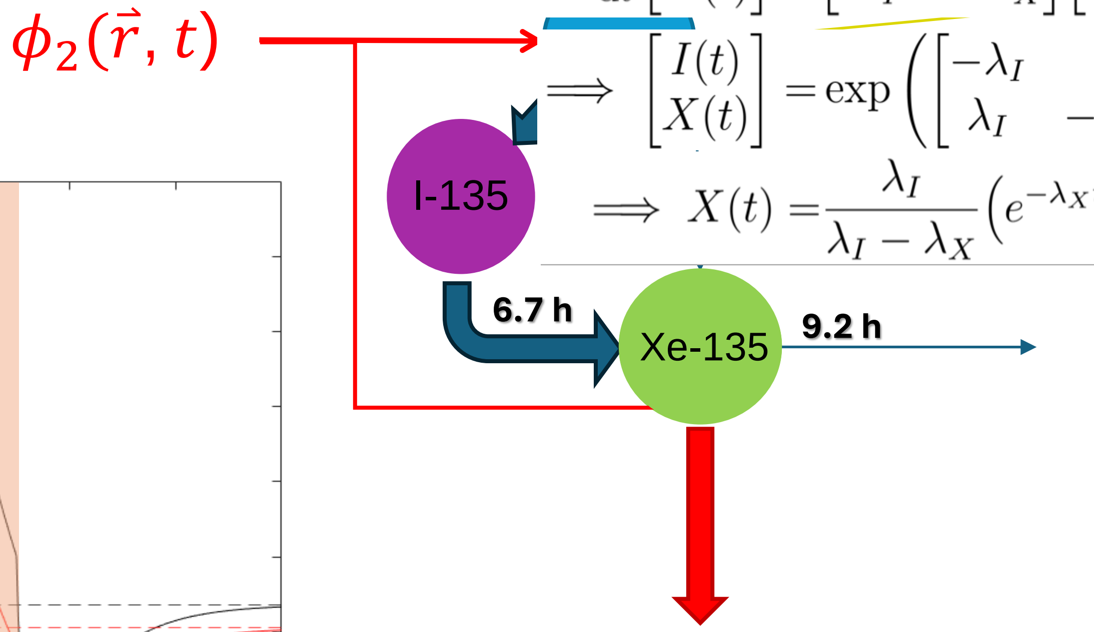
$time\ [h]$

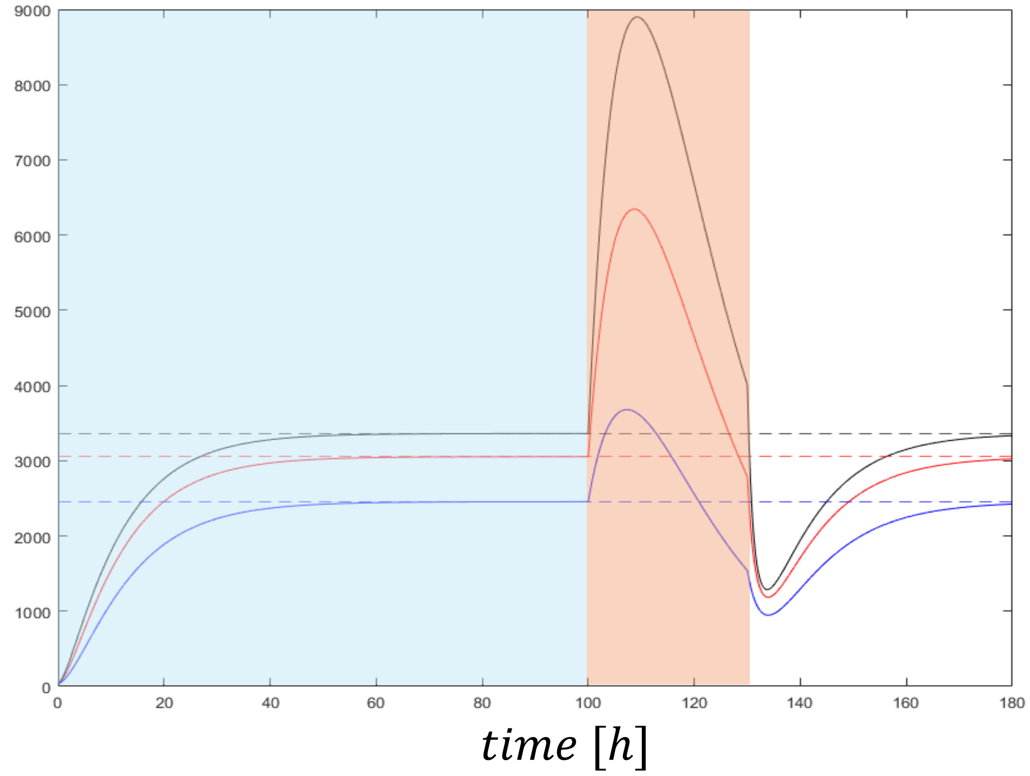
Xenon poisoning

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}I(t)\\X(t)\end{bmatrix}=\begin{bmatrix}-\lambda_I & \\ \lambda_I & -\lambda_X\end{bmatrix}\begin{bmatrix}I(t)\\X(t)\end{bmatrix}$$

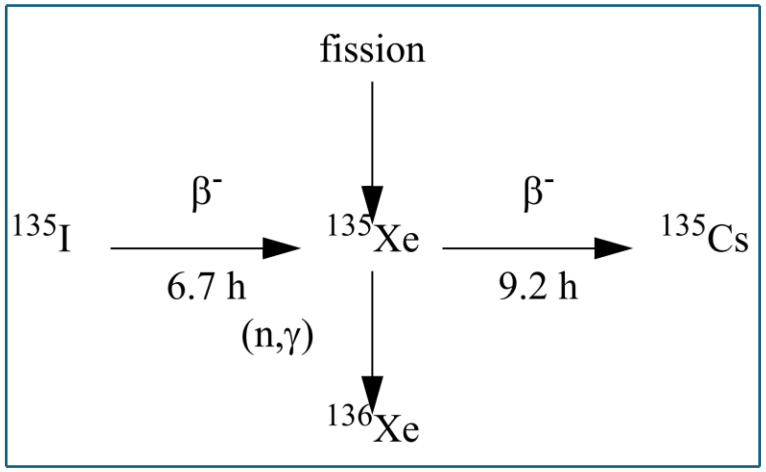
$$\implies \begin{bmatrix}I(t)\\X(t)\end{bmatrix}=\exp\left(\begin{bmatrix}-\lambda_I & \\ \lambda_I & -\lambda_X\end{bmatrix}t\right)\begin{bmatrix}I_0\\X_0\end{bmatrix}$$

$$\implies X(t)=\frac{\lambda_I}{\lambda_I-\lambda_X}\left(e^{-\lambda_X t}-e^{-\lambda_I t}\right)I_0+e^{-\lambda_X t}X_0$$

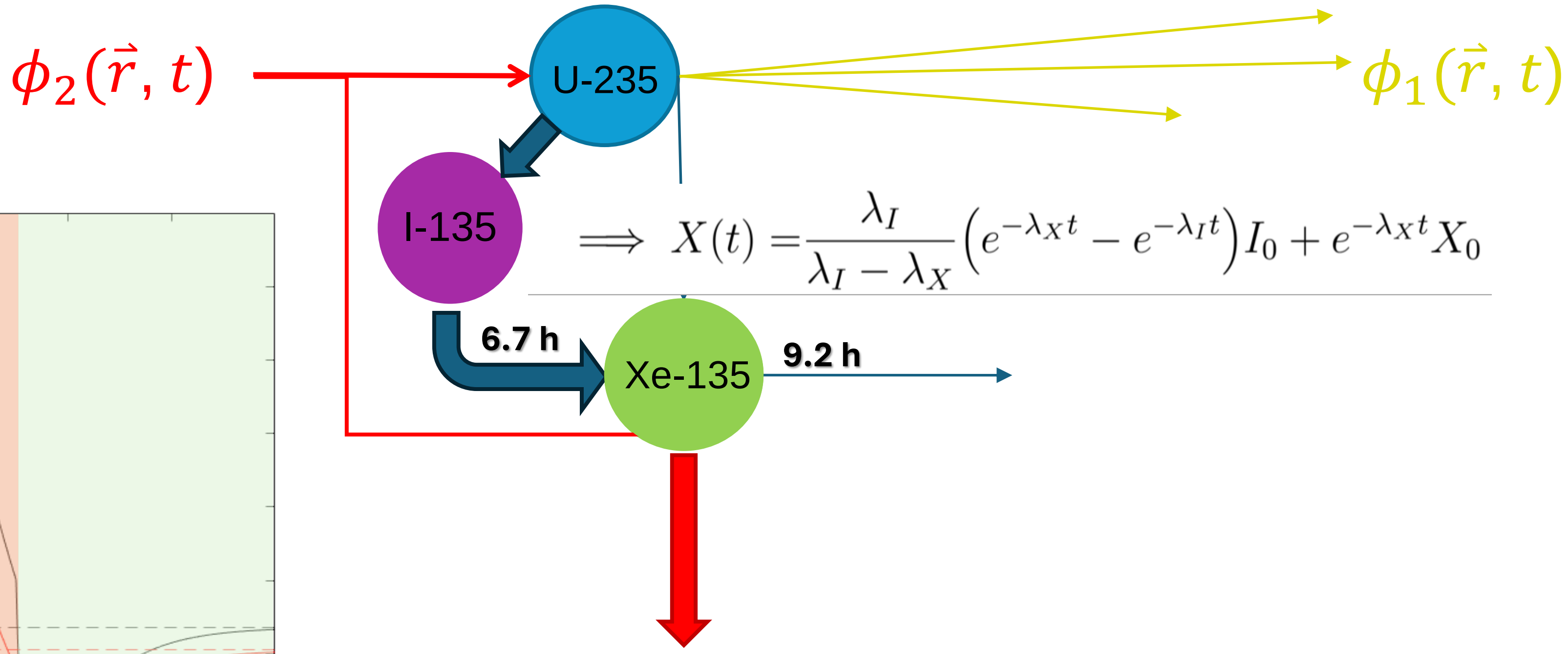




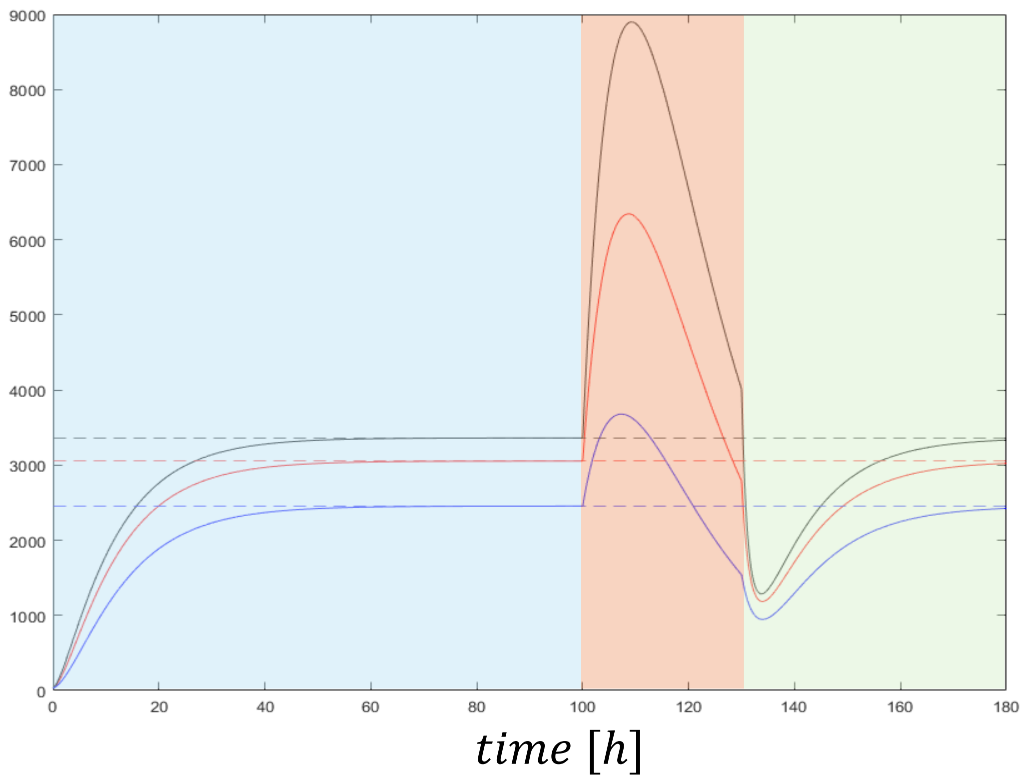
Evolution chain of Xenon-135



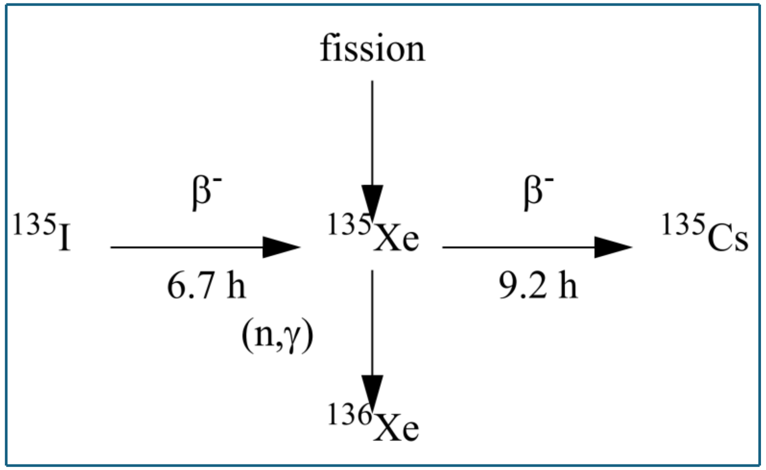
Xenon poisoning



$$\Longrightarrow \; X(t) = \frac{\lambda_I}{\lambda_I - \lambda_X}\left(e^{-\lambda_X t} - e^{-\lambda_I t}\right) I_0 + e^{-\lambda_X t} X_0$$

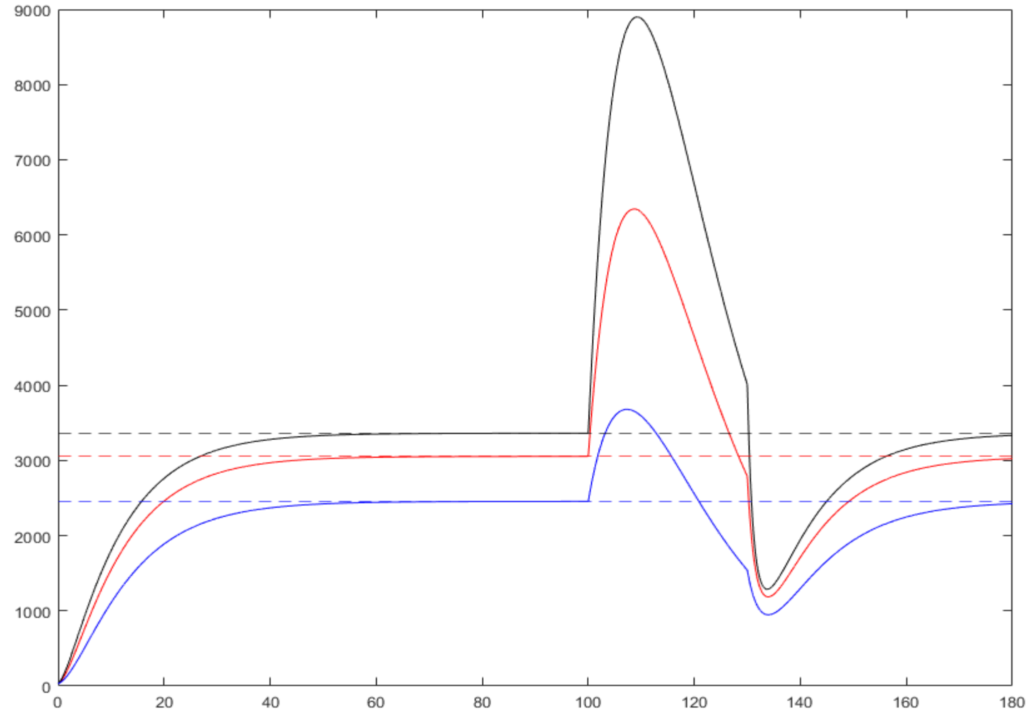


Evolution chain of Xenon-135



Xenon poisoning

Recreation using my time-evolver



$time\ [h]$

Figure Credit:
C. Demaziere, ”Physics of Nuclear Reactors”, p. 179

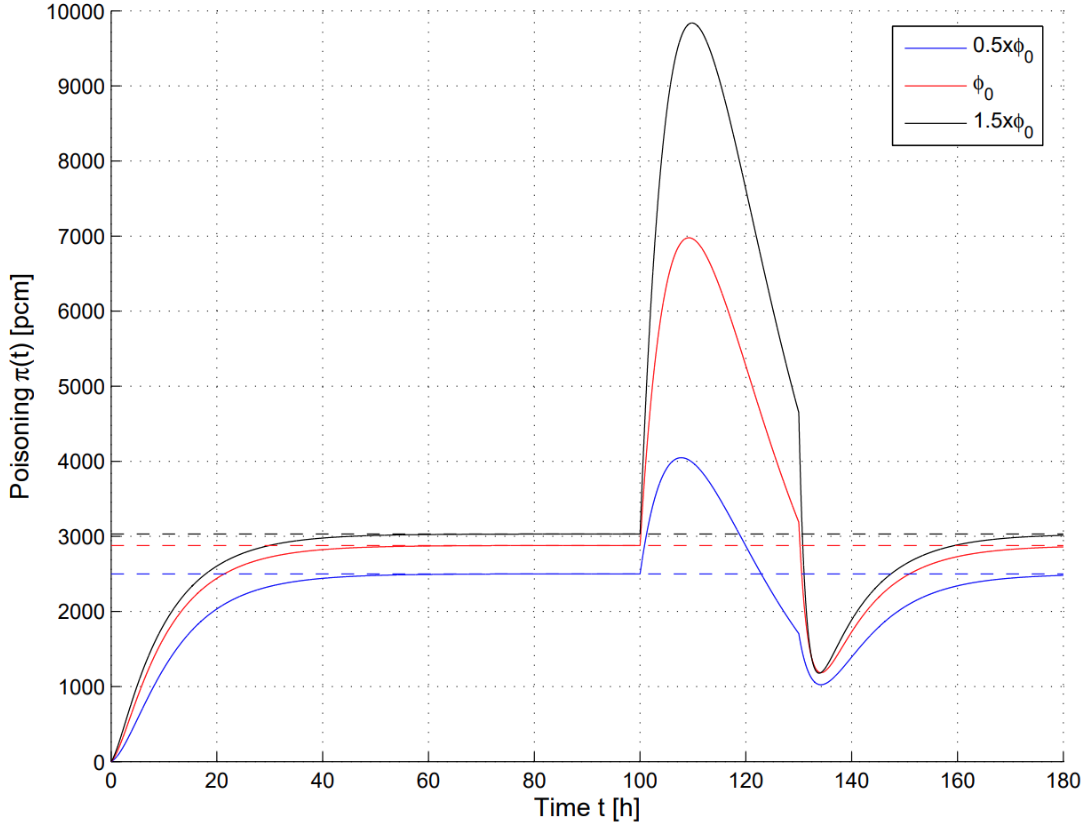
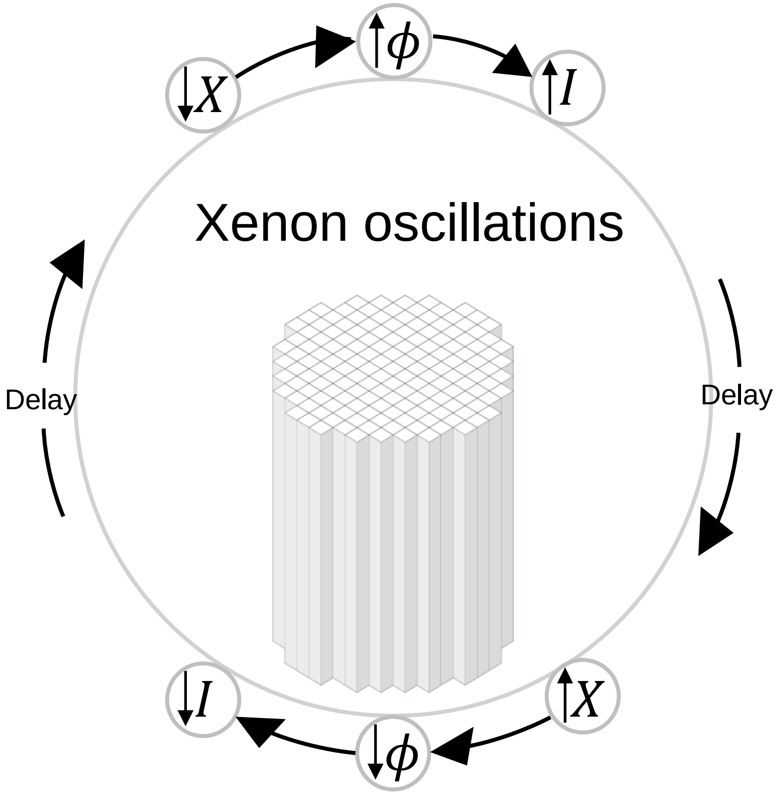
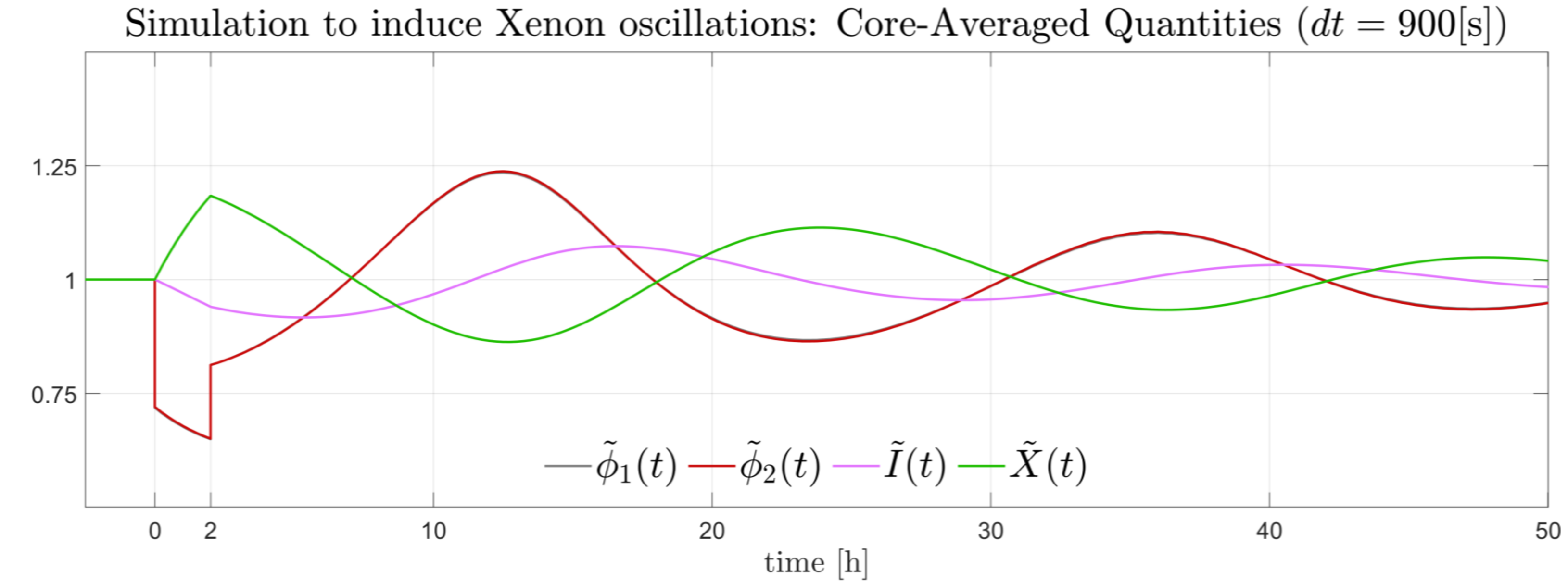


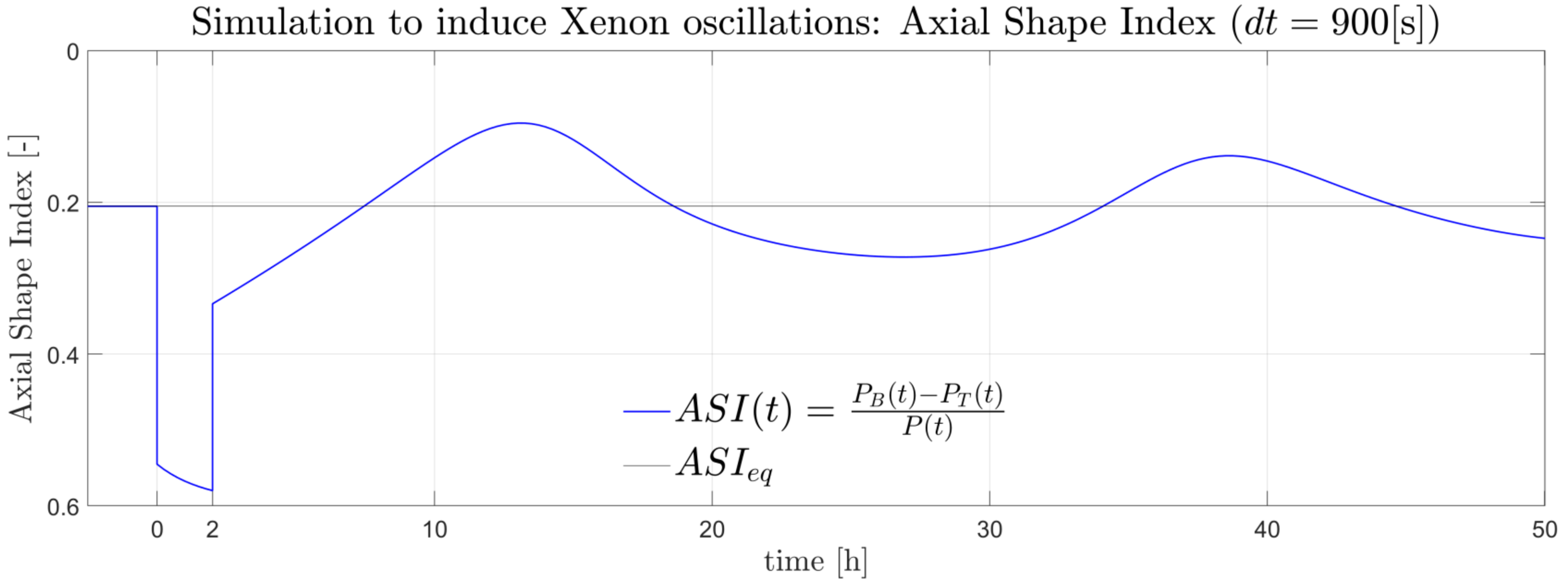
Fig. 4.10 Evolution of the poisoning due to ^{135}Xe after the start of the reactor (t=0), after a reactor scram (t=100 h), and after a reactor restart (t=130 h) (the horizontal dashed lines represent the equilibrium poisonings).

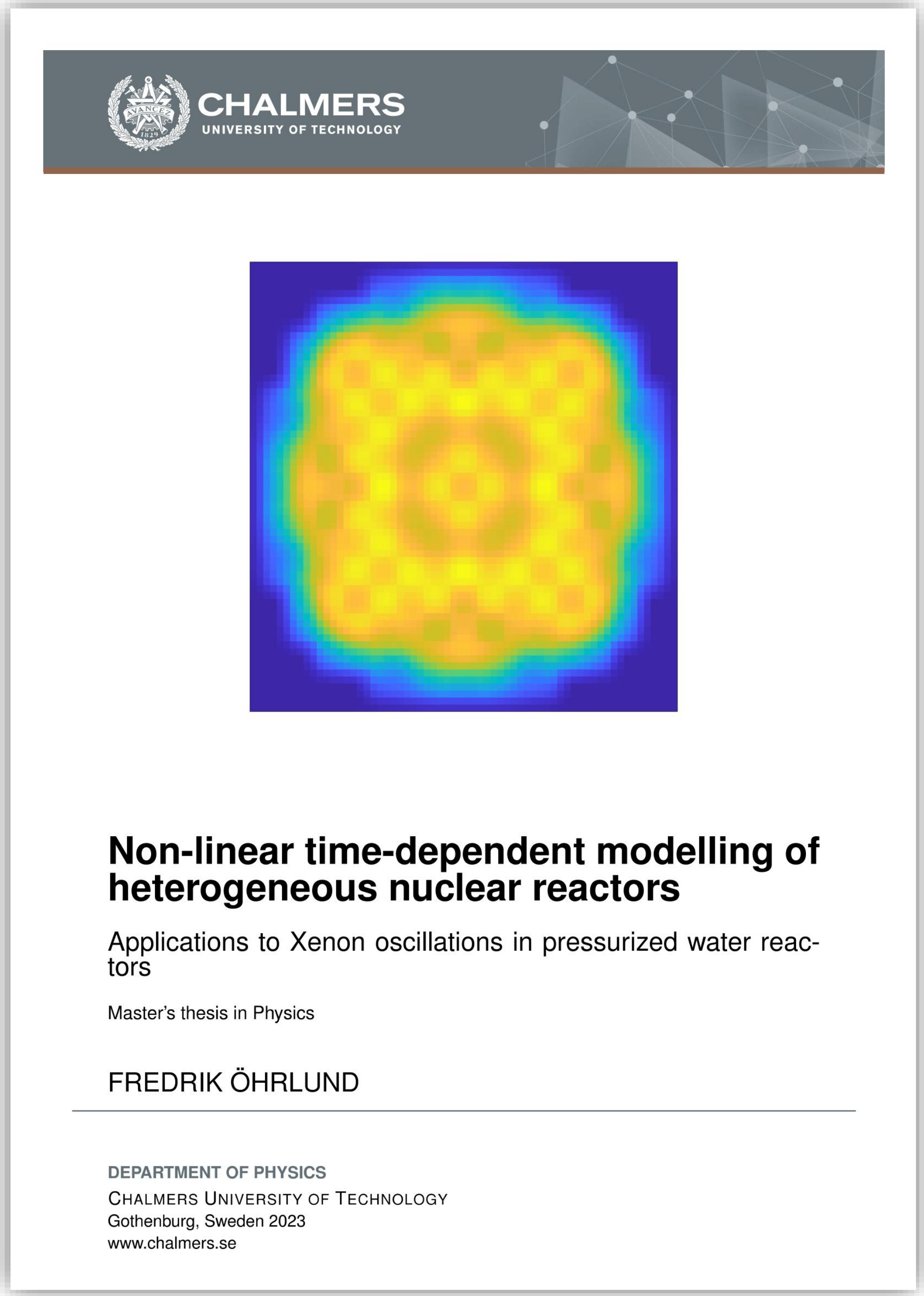
Xenon oscillations

$$\begin{bmatrix} \phi_1(\vec{\mathbf{r}}, t) \\ \phi_2(\vec{\mathbf{r}}, t) \\ I(\vec{\mathbf{r}}, t) \\ X(\vec{\mathbf{r}}, t) \end{bmatrix}$$





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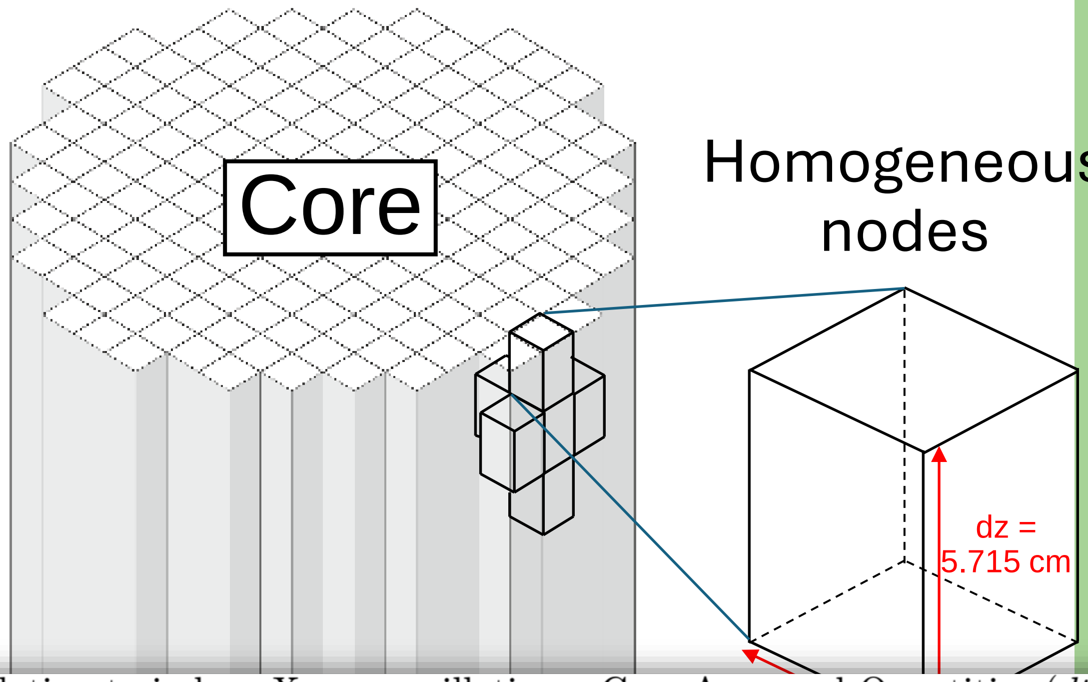
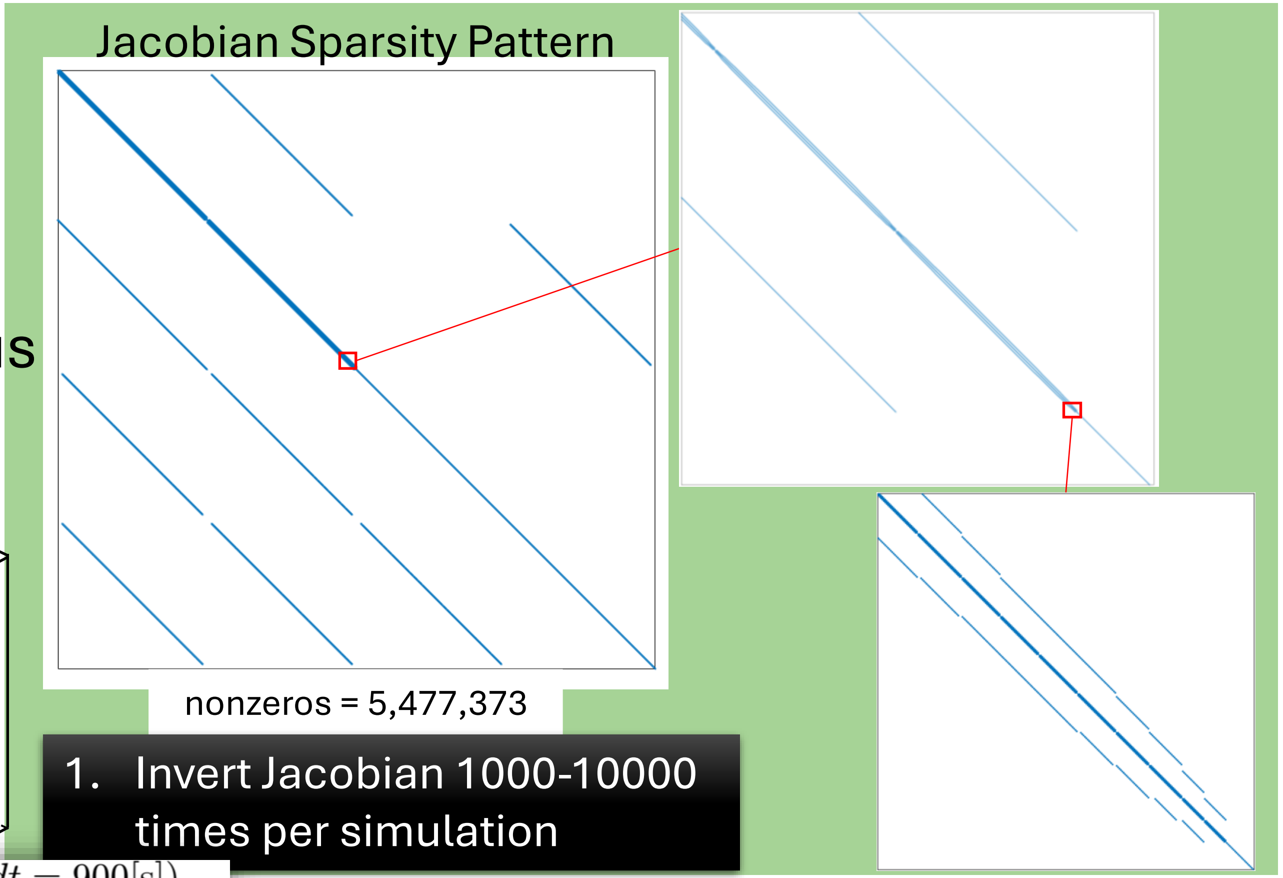
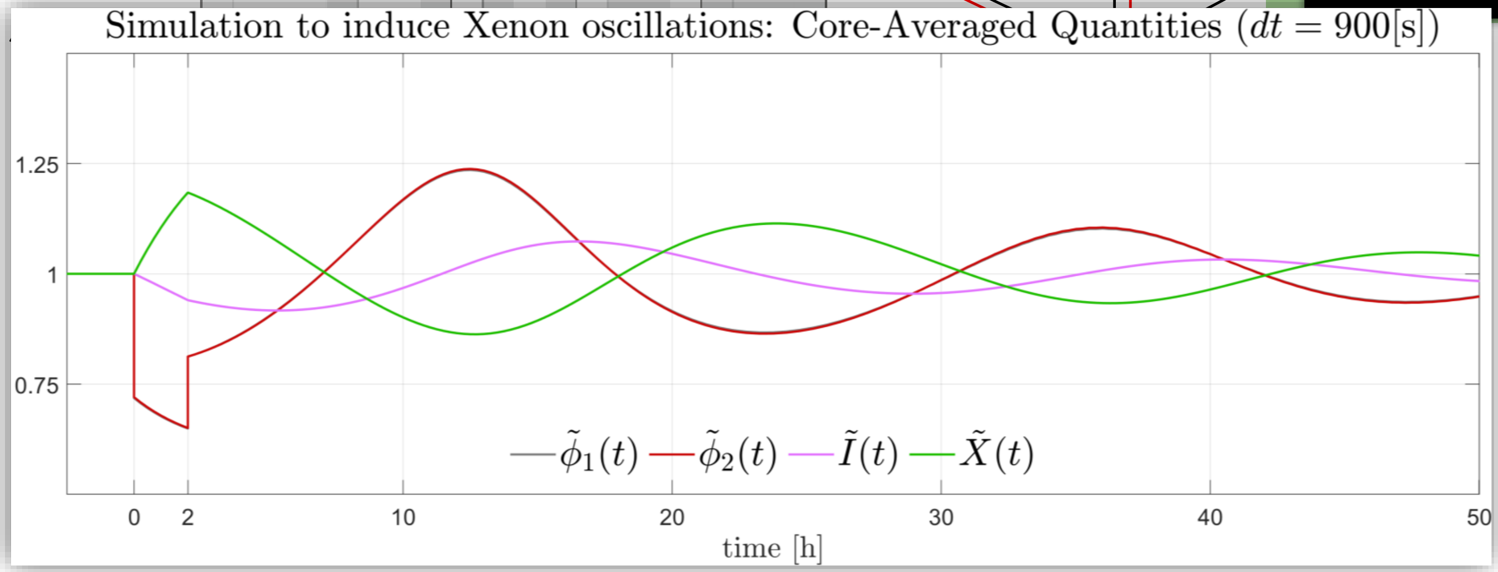
Simulating a reactor





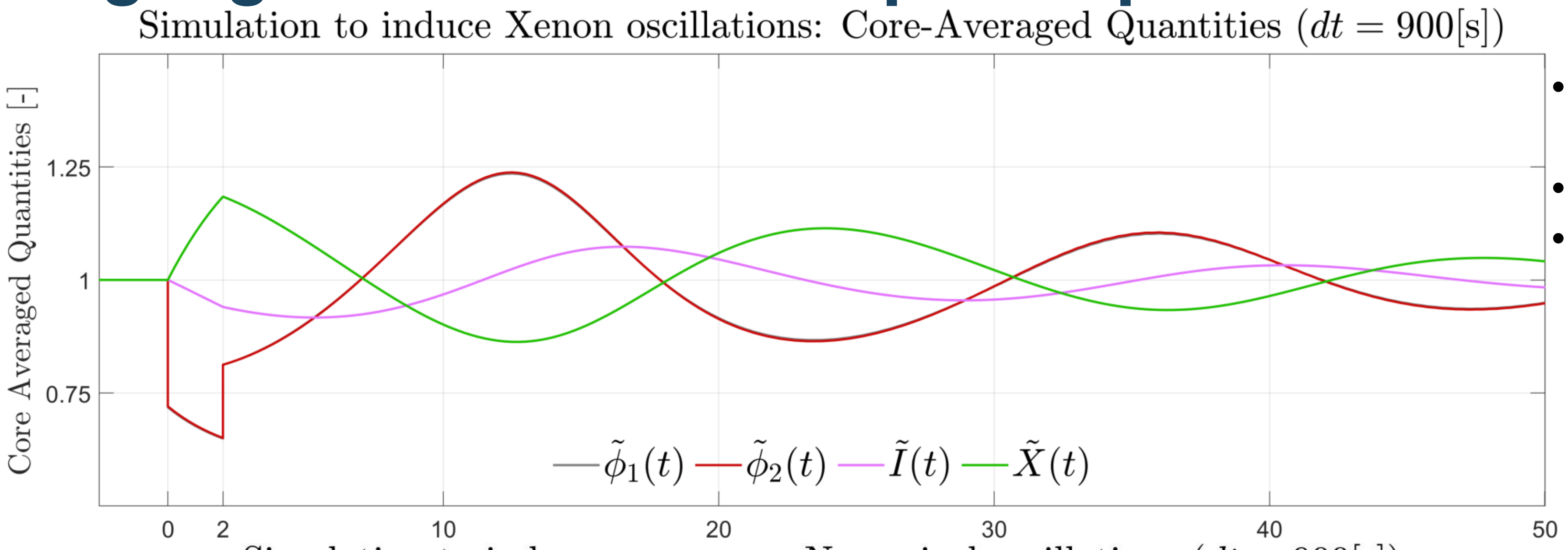
1. Invert Jacobian 1000-10000 times per simulation



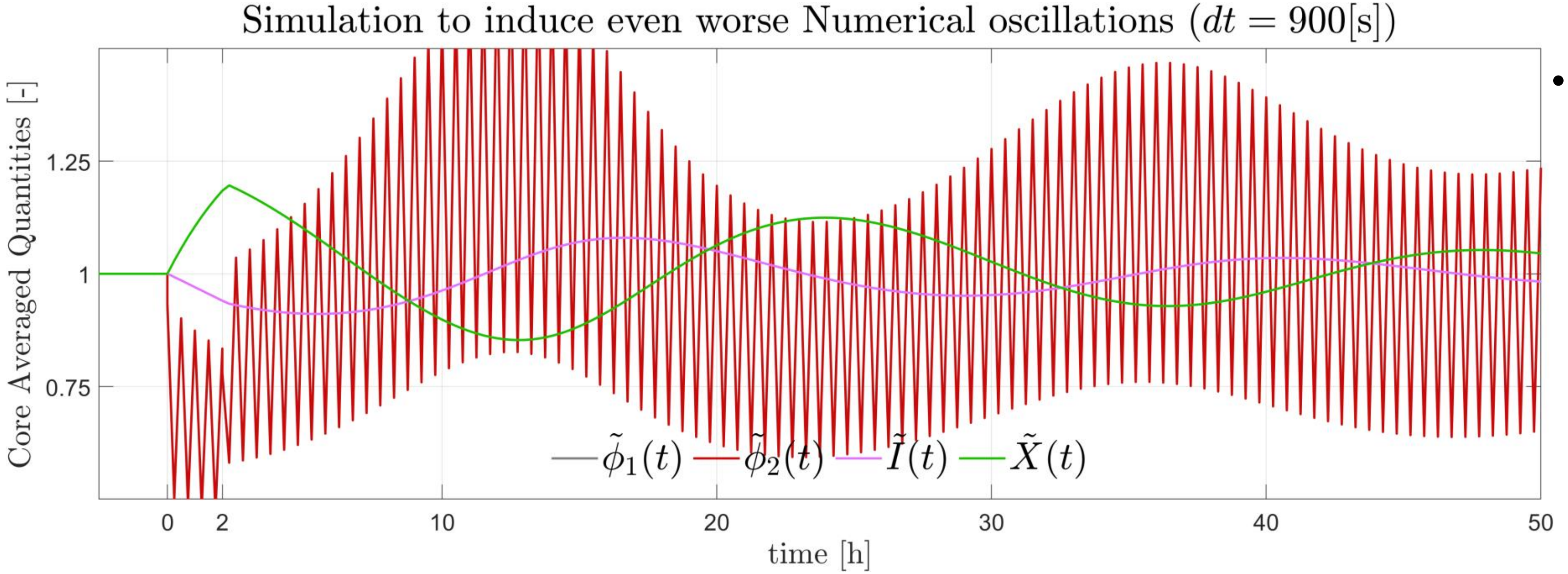
- Real example from my code: Solve Ax = b
- A\b (MatLab): 125s
- ILU(A) + GMRES(A,b) 0.11s
- 1136x speedup

- The total run-time of a simulation becomes
- 2 days 6 hours
- 180s

Highlight from the development process



- Time-evolution is done with the Crank-Nicolson method,
- Asymptotically stable (A-stable)
- This means that the method is numerically stable for arbitrarily large time-steps...



- From a certain perspective, this is not an error/mistake, but is in fact the correct solution according to Crank-Nicolson...

- When I handed in my thesis for grading and I had officially finished my work on the project, I still wasn't done with the problem.

1. Numerical oscillations fascinated me, wanted to find general solution.
2. Arbitrary XS-perturbation
3. Adaptive time-stepping

- So after I finished my thesis, I completely rewrote my entire code from scratch, and fixed/implemented points 1-3.

- It turned out that this required roughly as much time and effort as the entire original master's thesis project.
- Lines of code: 4485

- I would happily tell you everything about this!