

# Grand unification in extra dimensions

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PhD Days  
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# Outline

Introduction

Asymptotic  
unification

Orbifold  
stability:  
results

Conclusions

## 1 Introduction

## 2 Asymptotic unification

## 3 Orbifold stability: results

## 4 Conclusions

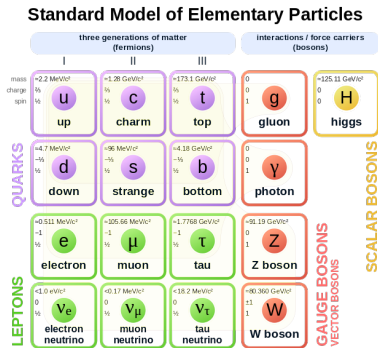
# The Standard Model (SM)

Introduction

Asymptotic unification

Orbifold stability: results

Conclusions



source: Wikipedia



Beyond the Standard Model (BSM) physics

Tested to high precision but...

leaves some open questions:

hierarchy problem, strong CP

neutrino mass, dark matter, baryon asymmetry...

# Beyond the Standard Model

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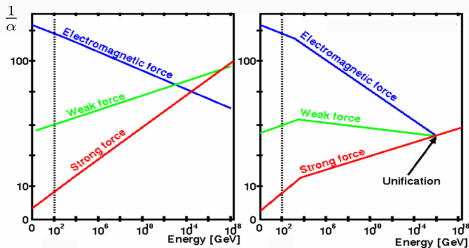
Conclusions

There are many ways to include extensions

⇒ new particles, extra dimensions, grand unified theories (GUTs), supersymmetry...

⇒ our work: GUTs in higher dimensions  $\equiv$  asymptotic GUTs<sup>1</sup>

**Unification:**



**SM gauge couplings meet  
at some high scale**



Physics described by a  
**unified gauge group**

$$\mathcal{G} \supset \mathcal{G}_{\text{SM}}$$

e.g.  $SU(5)$ ,  $SO(10)$ ,  $E_6$

<sup>1</sup>A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

# Beyond the Standard Model

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$$M_{\text{Planck}} \approx 10^{18} \text{ GeV}$$

$$M_{\text{GUT}} \approx 10^{16} \text{ GeV}$$

$$M_W \approx 10^2 \text{ GeV}$$

*t, W, Z, H*

## Why Grand Unification?<sup>2</sup>

⇒ can explain some of the **puzzles** (neutrino mass, dark matter...) and more **fundamental issues**, e.g. charge quantization

...but, GUT scale is very high, orders of magnitude away from hadron colliders

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<sup>2</sup>H. Georgi and S. Glashow, Phys. Rev. Lett., 438 (1974)

# Asymptotic Grand Unified Theories (aGUTs)

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**What we do:** standard picture of unification but in higher dimensions



GUTs defined on  $\mathbb{R}^4 \times K$ , where  $\mathbb{R}^4$  is the usual 4-dimensional Minkowski space and  $K$  defines  $\delta$  compact extra dimensions .

## Motivation:

- lower GUT scale
- less parameters/smaller representations
- solution to hierarchy problem

# Example: 5D case

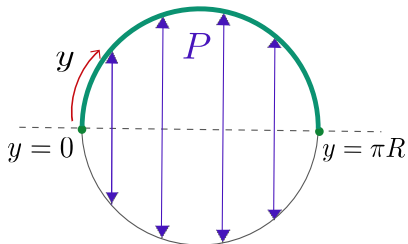
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- One extra dimension ( $\delta = 1$ ) compactified on  $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$ .



# Example: 5D case

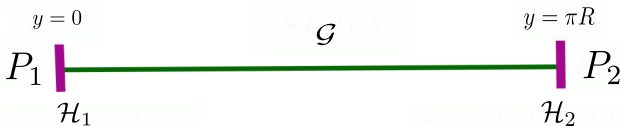
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- Each intrinsic  $\mathbb{Z}_2$  transformation is specified by a parity matrix  $P$  acting on the fields <sup>3</sup>



- Each  $P_i$  will break  $\mathcal{G} \rightarrow \mathcal{H}_i$  on one boundary, such that

$$\mathcal{G}_{4D} \equiv \mathcal{H}_i \cap \mathcal{H}_j$$

- Viable model must contain the Standard Model

$$\mathcal{G}_{4D} \supset \mathcal{G}_{SM}$$

<sup>3</sup>G. Cacciapaglia, arXiv:2309.10098 (2023)



# Gauge-Higgs Unification<sup>4 5</sup>

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Assume a 5D gauge theory and  $A_M$  ( $M = 1, \dots, 5$ ) a gauge field



$$\underbrace{A_\mu (\mu = 1 \dots 4)}_{4\text{D}} \quad \text{and} \quad \underbrace{A_5}_{\text{extra dimension}}$$



$A_5$  behaves as a scalar field in 4D

$\equiv$  Higgs field

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<sup>4</sup>Y. Hosotani, Phys. Lett. B 126 (1983)

<sup>5</sup>R. Contino, et al, Nucl. Phys. B 671 (2003)

# Gauge-Higgs Unification

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$A_5 \equiv$  **gauge-scalar** embedded in the gauge fields

There will be a scalar potential for  $A_5$ !

...but gauge symmetry forbids the potential at tree level



**one loop effective potential**<sup>6</sup>

(dictates symmetry breaking, mass of the scalars etc.)

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<sup>6</sup> I. Antoniadis, et al, New Journal of Physics 3 (2001)

# One loop effective potential

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Total potential given by:

$$V_{\text{eff}}^{\text{total}} = V_{\text{eff}}^{\text{gauge}} + V_{\text{eff}}^{\text{scalar}} + V_{\text{eff}}^{\text{fermionic}}$$

Global minimum of  $V_{\text{eff}}^{\text{gauge}}$  must be at 0.



constraint on models in order to be phenomenologically relevant

# Orbifold stability

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G. Cacciapaglia, A. Deandrea, A. Cornell, W. Isnard, R. Pasechnik,  
A. Preda and Z. Wang [in preparation]

## What we did:

1. Computed the effective potential for general  $SU(N)$ ,  $Sp(2N)$  and  $SO(N)$  gauge theories
2. Imposed the global minimum constraint
3. Derived **orbifold stability** conditions based on this constraint

# Orbifold stability: $SU(N)$ results

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- All breaking patterns of the form

$$SU(N) \rightarrow SU(a) \times SU(N - a) \times U(1)$$

satisfy the constraint.

- Breaking patterns of the form

$$SU(N) \rightarrow SU(p) \times SU(q) \times SU(s) \times U(1)^2$$

satisfy the constraint only if  $p \geq N/2$ .

- For breaking patterns of the form

$$SU(N) \rightarrow SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3$$

the constraint is never satisfied.

# Orbifold stability: $SU(N)$ results

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## Examples:

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$


$$SU(6) \rightarrow SU(3) \times SU(2) \times U(1)^2$$

$$SU(8) \rightarrow SU(4) \times SU(2) \times SU(2)$$

satisfy  $p \geq N/2$  

whereas

$$SU(7) \rightarrow SU(3) \times SU(3) \times U(1)^2$$

has  $p \leq N/2$  

$\Rightarrow$  Analysis was extended to  $Sp(2N)$  and  $SO(N)$ : more group theory needed, but results follow in a similar way

$\Rightarrow$  Next: derive similar constraints for exceptional groups  $E_6, E_8$

# Conclusions

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- aGUTs as an alternative to standard GUTs
- Viable models have to pass certain criteria  $\Rightarrow$  **orbifold stability**
- For  $SU(N)$ : **two-blocks** and **three-blocks** with  $p \geq N/2$  are **stable**, while four-blocks are not
- The criteria of **orbifold stability** helps identify **potentially interesting models**
- **Systematic classification** that discards phenomenologically unrealistic scenarios



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# Back-up slides



## Example: 5D case

For a given field  $\Phi(x^\mu, y)$  we can do a **Kaluza-Klein decomposition**



### Decomposition

$$\Phi(x^\mu, y) = \underbrace{\sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right)}_{\text{parity-even}} + \underbrace{\sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right)}_{\text{parity-odd}}$$

- The 4D fields  $\phi_{\pm}^{(n)} \equiv$  Kaluza-Klein (KK) modes with mass of  $n/R$ .
- The Standard Model fields are the massless zero modes of  $\phi_+$ .
- For  $E \ll 1/R$ , the heavy Kaluza-Klein towers are integrated out.



**4D effective field theory**

# Example: 5D case

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⋮

n=4 \_\_\_\_\_

n=3 \_\_\_\_\_

n=2 \_\_\_\_\_

n=1 \_\_\_\_\_

n=0 \_\_\_\_\_

Each 5D field  $\equiv$  **infinite tower** of 4D fields

“ground state” (n=0) are the SM states

RGEs for the couplings get modified:

logarithmic  $\rightarrow$  power-law dependence



couplings will flow **asymptotically** towards a UV fixed point

