Gauge theories in extra dimensions

ANCA PREDA

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PhD Days
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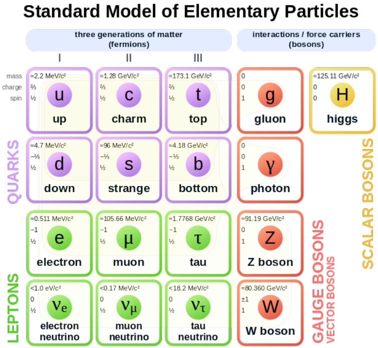
Outline

The Standard Model (SM)



source: Wikipedia

Tested to high precision but...

leaves some open questions:

hierarchy problem, charge quantization

neutrino mass...

Beyond the Standard Model (BSM) physics

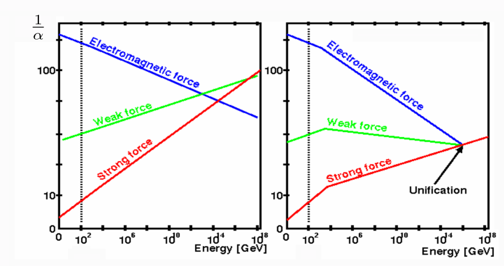
Beyond the Standard Model

There are many ways to include extensions

⇒ new particles, extra dimensions, grand unified theories (GUTs), supersymmetry...

⇒ our work: GUTs in higher dimensions $\equiv$ asymptotic GUTs[1]

Unification:

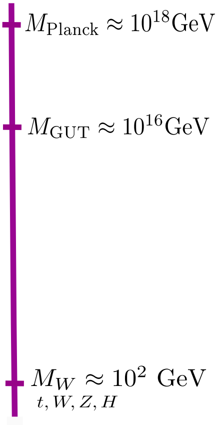


SM gauge couplings meet
at some high scale
$\updownarrow$
Physics described by a
unified gauge group
$\mathcal{G} \supset \mathcal{G}_{\rm SM}$

e.g. $SU(5)$, $SO(10)$, E_6

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)

Beyond the Standard Model



Why Grand Unification? [2]

⇒ can explain some of the puzzles (neutrino mass, dark matter...) and more fundamental issues, e.g. charge quantization

...but, GUT scale is very high, orders of magnitude away from hadron colliders

[2] H. Georgi and S. Glashow, Phys. Rev. Lett., 438 (1974)

Asymptotic Grand Unified Theories (aGUTs)

What we do: standard picture of unification but in higher dimensions

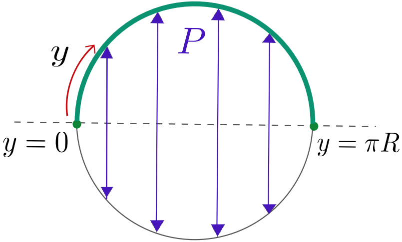
GUTs defined on $\mathbb{R}^4 \times K$, where $\mathbb{R}^4$ is the usual 4-dimensional Minkowski space and K defines δ *compact* extra dimensions .

Motivation:

- lower GUT scale
- less parameters/smaller representations
- solution to hierarchy problem

Example: 5D case

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:



Example: 5D case

Introduction

Asymptotic unification

Renormalization

Conclusions

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:

Example: 5D case

- One extra dimension ($\delta = 1$) compactified on $K = \mathbb{S}^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$:

- The inverse radius R^{-1} sets the scale of compactification.

Gauge-Higgs Unification[3] [4]

Assume a 5D gauge theory and A_M $(M = 1, \ldots, 5)$ a gauge field

⇓

$$\underbrace{A_\mu(\mu = 1 \ldots 4)}_{\text{4D}} \quad \text{and} \quad \underbrace{A_5}_{\text{extra dimension}}$$

⇓

A_5 behaves as a scalar field in 4D

$\equiv$ **Higgs field**

[3] Y. Hosotani, Phys. Lett. B 126 (1983)
[4] R. Contino,et al, Nucl. Phys. B 671 (2003)

Gauge-Higgs Unification

A_5 $\equiv$**gauge-scalar** embedded in the gauge fields

There will be a scalar potential for A_5!

...but gauge symmetry forbids the potential at tree level

one loop effective potential [5]

(dictates symmetry breaking, mass of the scalars etc.)

[5] I. Antoniadis, et al, New Journal of Physics 3 (2001)

Previously on PhD Days ’24...

Orbifold stability

What we did [6] [7]**:**

- Study of the vacuum structure for different aGUT models

  $\Rightarrow$ Computed the effective potential for general $SU(N)$, $Sp(2N)$ and $SO(N)$ gauge theories

- Derive constraints $\leftrightarrow$ orbifold stability
- Impose constraints: list of viable models

Model	Breaking pattern	Stability	Gauge-scalar
SU(N)	$\mathrm{SU}(A) \times \mathrm{SU}(N-A) \times \mathrm{U}(1)$	$\forall A$	$(F, \bar{F})$ or none
	$\mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(s) \times \mathrm{U}(1)^2$	$p \geq N/2$	$(F, 1, \bar{F})$
Sp($2N$)	$\mathrm{Sp}(2A) \times \mathrm{Sp}(2(N-A))$	$\forall A$	(F, F) or none
	$\mathrm{Sp}(2p) \times \mathrm{Sp}(2q) \times \mathrm{Sp}(2s)$	$p \geq N/2$	$(F, 1, F)$
	$\mathrm{SU}(A) \times \mathrm{SU}(N-A) \times \mathrm{U}(1)^2$	$\forall A$	$(F, \bar{F})$
	$\mathrm{SU}(N) \times \mathrm{U}(1)$	always	none
SO($2N$)	$\mathrm{SO}(2A) \times \mathrm{SO}(2(N-A))$	$\forall A$	(F, F) or none
	$\mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$p \geq N/2$	$(F, 1, F)$
	$\mathrm{SU}(A) \times \mathrm{SU}(N-A) \times \mathrm{U}(1)^2$	$\forall A$	$(F, \bar{F})$
	$\mathrm{SU}(N) \times \mathrm{U}(1)$	always	none
SO($2N+1$)	$\mathrm{SU}(2A+1) \times \mathrm{SU}(2(N-A))$	$\forall A$	(F, F) or none
	$\mathrm{SO}(2p+1) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s)$	$2p+1 > N$	$(F, 1, F)$
	$\mathrm{SO}(2p) \times \mathrm{SO}(2q+1) \times \mathrm{SO}(2s)$	$2p > N$	$(F, 1, F)$
	$\mathrm{SO}(2p) \times \mathrm{SO}(2q) \times \mathrm{SO}(2s+1)$	$2p > N$	$(F, 1, F)$

[6] G. Cacciapaglia, [..], AP, Phys. Rev. D 111 (2025)

[7] G. Cacciapaglia, [..], AP, Eur. Phys. J. C 85 (2025)

Are these models theoretically consistent?

Renormalization of 5D gauge theories

$\Rightarrow$ 5D gauge theories (perturbatively) non-renormalizable: $[g] = -1/2$

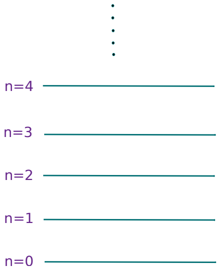
✱ can the behaviour in the UV be tamed?

- asymptotic safety scenario: UV fixed point
- nonpertubatively renormalizable [8]: fundamental and mathematically consistent

✱ if the fixed point exists, how do we renormalize these theories in the bulk and on the boundaries?

[8] H. Gies, Phys. Rev. D 68 (2003)

Renormalization of 5D gauge theories

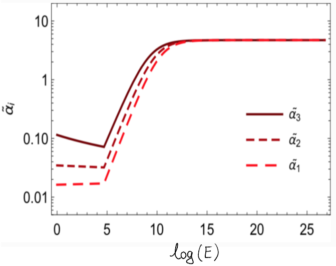


Each 5D field $\equiv$ **infinite tower** of 4D fields

RGEs for the couplings get modified:

logarithmic $\rightarrow$ power-law dependence

couplings will flow **asymptotically** towards a UV fixed point ?



Renormalization of 5D gauge theories

Existence of the fixed point depends on the power law running of the 5D coupling

- **gauge couplings**:

$$16\pi^2 \frac{dg}{dt} = b_{\rm SM} g^3 + (S(t) - 1) b_5 g^3 \,,$$

where

$$S(t) = \begin{cases} \mu R = M_Z R e^t, & \text{for } \mu \geq 1/R \,, \\ 1, & \text{otherwise} \,. \end{cases}$$

Fixed point exists if $b_5 < 0$.

- **Yukawa** and **scalar** couplings are more problematic (Landau poles) and impose more limitations on models.

Renormalization of 5D gauge theories

Existence of the fixed point depends on the power law running of the 5D coupling

- **gauge couplings**:

5D dynamics

$$16\pi^2 \frac{dg}{dt} = b_{\mathrm{SM}} g^3 + (S(t) - 1) b_5 g^3 \,,$$

where

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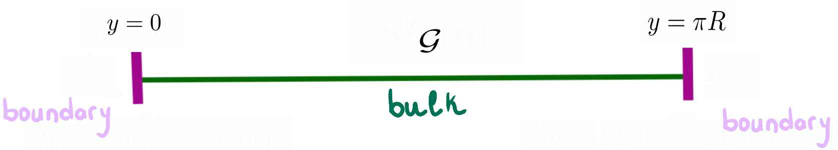
Fixed point exists if $b_5 < 0$.

- **Yukawa** and **scalar** couplings are more problematic (Landau poles) and impose more limitations on models.

Renormalization of 5D gauge theories

Assume a theory with couplings that flow asymptotically toward a fixed point

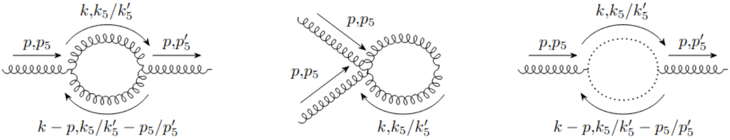
How do we renormalize this theory in the **bulk** and on the **boundary**?



Renormalization of 5D gauge theories

Pure Yang Mills theory

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$



Renormalization of 5D gauge theories

1. **Bulk**: structure of the divergencies same as in the 4D theory (one loop)

➔ scaling is different from 4D ($\sim \log \Lambda$) to 5D linear ($\sim \Lambda$)

➔ same renormalization procedure as that of a 4D theory

2. **Boundary**: renormalizability of a pure Yang Mills at the fixed points

➔ less straightforward than bulk computations (work in progress)

➔ study of Yukawa theory [9]: renormalizable on the boundary

[9] H. Georgi, et al, Phys. Let. B 506 (2001)

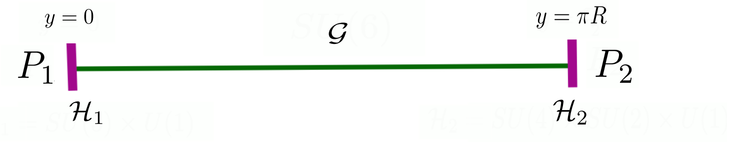
Conclusions

- aGUTs as an alternative to standard GUTs
- Viable models have to pass certain criteria $\Rightarrow$ **orbifold stability**
- Common lore: 5D theories are cut-off dependent
- Under certain conditions the **UV physics** is tamed by the existence of **fixed points**
- Pure YM **renormalizable** at one loop (finite number of counterterms cancel divergencies)

Back-up slides

Example: 5D case

- Each intrinsic $\mathbb{Z}_2$ transformation is specified by a parity matrix P acting on the fields [3]



- Each P_i will break $\mathcal{G} \to \mathcal{H}_i$ on one boundary, such that

$$\mathcal{G}_{\text{4D}} \equiv \mathcal{H}_i \cap \mathcal{H}_j$$

- Viable model must contain the Standard Model

$$\mathcal{G}_{\text{4D}} \supset \mathcal{G}_{\text{SM}}$$

[3] G. Cacciapaglia, arXiv:2309.10098 (2023)

Example: 5D case

For a given field $\Phi\left(x^\mu, y\right)$ we can do a **Kaluza-Klein decomposition**

Decomposition

$$\Phi\left(x^\mu, y\right) = \underbrace{\sum_{n=0}^{\infty} \phi_+^{(n)}\left(x^\mu\right) \cos\left(\frac{ny}{R}\right)}_{\text{parity-even}} + \underbrace{\sum_{n=1}^{\infty} \phi_-^{(n)}\left(x^\mu\right) \sin\left(\frac{ny}{R}\right)}_{\text{parity-odd}}$$

- The 4D fields $\phi_\pm^{(n)} \equiv$ Kaluza-Klein (KK) modes with mass of n/R.
- The Standard Model fields are the massless zero modes of ϕ_+.
- For $E \ll 1/R$, the heavy Kaluza-Klein towers are integrated out.

4D effective field theory

Orbifold stability: $SU(N)$ results

- All breaking patterns of the form

$$SU(N) \to SU(a) \times SU(N-a) \times U(1)$$

satisfy the constraint.

- Breaking patterns of the form

$$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2$$

satisfy the constraint only if $p \geq N/2$.

- For breaking patterns of the form

$$SU(N) \to SU(p) \times SU(q) \times SU(r) \times SU(s) \times U(1)^3$$

the constraint is never satisfied.

Orbifold stability: $SU(N)$ results

Examples:

$$
\begin{aligned}
&SU(5) \to SU(3) \times SU(2) \times U(1) \\
&SU(6) \to SU(3) \times SU(2) \times U(1)^2 \qquad \text{satisfy } p \geq N/2 \quad ✅ \\
&SU(8) \to SU(4) \times SU(2) \times SU(2)
\end{aligned}
$$

whereas

$$SU(7) \to SU(3) \times SU(3) \times U(1)^2 \qquad \text{has } p \leq N/2 \quad ❌$$

$\Rightarrow$ Analysis was extended to $Sp(2N)$ and $SO(N)$: more group theory needed, but results follow in a similar way

$\Rightarrow$ Next: derive similar constraints for exceptional groups E_6, E_8