DARK MATTER

Problem Sheet 4: Baryonic effects

We studied in class a simple 'inflow-outflow' model for how baryons can heat dark matter. But gas need not necessarily flow in to dark matter halos adiabatically. Imagine instead in this question that it flows in in discrete lumps. These lumps lose orbital energy via frictional forces that can be 'dynamical friction' due to momentum exchange with background stars and dark matter particles, or could be friction against the surrounding smooth gas distribution. In this question, it does not really matter. We will simply assume that: (i) the gas lump moves on a circular orbit of radius r_i about a galaxy of mass M_i; (ii) the galaxy is approximated by a point mass; (iii) the specific orbital energy of the lump is lowered slowly so that the lump moves from one circular orbit to a more tightly bound circular orbit with radius r_f.

1. Show that the specific orbital energy of the lump lost as it moves from r_i to r_f is given by:

$$\Delta E = -\frac{1}{2}GM_i\left(\frac{1}{r_t} - \frac{1}{r_i}\right) \tag{1}$$

2. Assuming that the galaxy is in Virial equilibrium, show that the ratio of the energy (*not specific energy*) lost by the lump to the galactic binding energy $E_{\rm bind}$ is:

$$\frac{\Delta E}{E_{\rm bind}} \sim \frac{M_p}{M_i}\left(1 - \frac{r_t}{r_i}\right) \tag{2}$$

   Does this equation make sense? What happens in the limit $M_p \to 0$ and $r_t \to r_i$?

3. Assume that the lump falls all the way to the centre ($r_t = 0$) and that all of its lost orbital energy is absorbed locally by the dark matter halo. For what value of M_p is the effect significant?

4. What is the key difference of this 'lumpy' inflow model as compared to adiabatic, smooth, gas inflow?