DARK MATTER

Problem Sheet 5: Exam

This problem sheet brings together everything that you have learnt on the course.

1. **Classical evidence**

   (a) Give three pieces of evidence for *dark matter* in the Universe.

   (b) A spherical galaxy has enclosed mass:

   $$M(<r) = M_* + M_0 r^\alpha \tag{1}$$

   where M_* is the total visible mass in stars; M_0 and $\alpha \geq 0$ are constants; and r is the radius from the centre of the galaxy.

   (c) Balance the centripetal force and gravity to show that the velocity of a particle of mass m moving on a circular orbit in the galaxy at radius r is given by:

   $$v(r) = \sqrt{\frac{G\left(M_* + M_0 r^\alpha\right)}{r}} \tag{2}$$

   (d) At large distance $r \to \infty$ from the centre of the galaxy, the rotational velocity is observed to tend towards a constant value $v(r) = v_0$. Derive the value of α in this case and show that $v_0^2 = GM_0$.

   (e) Show that for the above value of α, the cumulative mass $M(<r)$ is given by:

   $$M(<r) = M_* + \frac{v_0^2}{G} r \tag{3}$$

   (f) What do you think the above equation for the cumulative mass is telling us? Explain your reasoning.

2. **Cosmological Probes**

   The Friedmann equation is given by:

   $$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4} - (\Omega_0 - 1) a^{-2}\right] \tag{4}$$

   where $\Omega_0 = \Omega_\Lambda + \Omega_m + \Omega_r$ and a is the dimensionless scale factor.

   (a) Hansen et. al (2004) estimate an age of 12.7 ± 0.7 Gyrs for the white dwarf stars in the star cluster M4[1]. Assuming $\Omega_\Lambda = 0$ and $H_0 = 70$ km/s/Mpc, can you construct a Universe of this age? Is this a reasonable model of the Universe? [Hint: you will need to include the curvature term; see the notes for the solution in this case.]

3. **Baryonic effects**

   Suppose I have a galaxy of mass M_i. We will assume that it is a point mass with dark matter orbiting around it on circular orbits of radius r_i.

   (a) Suppose I slowly add a mass of M_b in baryons to the above galaxy. Derive the resulting radii of the dark matter particle orbits r_t.

   (b) Imagine now that I instantaneously remove the mass M_b (due to e.g. a galactic wind). Prove that the galaxy becomes unbound if $M_b = M_i$.

[1] https://arxiv.org/abs/astro-ph/0205087.