Exercise II. Freeze-in Mechanism

Jose A. R. Cembranos *

Departamento de Física Teórica I, Universidad Complutense de Madrid, E-28040 Madrid, Spain.

II.a.- Estimate the dark matter abundance from the Freeze-in mechanism for a Dirac fermion which interact with the standard model through hypercharge interactions with hypercharge $Y = 1/2$. Assume that the maximum temperature is equal to the reheat temperature.

In such a case, the annihilation cross section into the total bosonic and fermionic content of the SM is s-wave dominated with:

$$c_0^Y \simeq \frac{\pi}{8} Y^2 (41 + 8Y^2)\, \alpha_Y^2 \,. \tag{1}$$

Estimate the reheat temperature for which the Freeze-out abundance is equal to the Freeze-in abundance.

FREEZ-IN RELIC ABUNDANCES

The thermalization does not take place if reheat or maximum temperatures are small with respect to the new particle masses within the inflationary framework. If we assume that the production is dominated by scattering processes in the thermal bath and not by direct inflaton decays, the abundances associated with these new states can be efficiently suppressed. In order to compute the number density n, of any of the new stable particles, we can use the Boltzmann equation:

$$\frac{d}{dt} n + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\rm EQ}^2 \right) \,, \tag{2}$$

where $\langle \sigma v \rangle$ is the thermal averaged annihilation cross section times velocity, H is the Hubble parameter, and $n_{\rm EQ}$ is the corresponding thermal equilibrium number density. Neglecting a possible back-reaction from the thermal bath, the Hubble parameter and the temperature of the universe T are determined by the inflaton energy density, so the dependence on the scale factor a, after the end of inflation is [1–5]:

$$\left(\frac{H}{H_R} \right)^2 = \left(\frac{T}{T_R} \right)^8 = \left(\frac{a}{a_R} \right)^{-3} \,, \tag{3}$$

where the subscript R means the value at the end of the reheating stage. By assuming that these particles have not thermalized at any time ($n \ll n_{\rm EQ}$), we can estimate their present abundance as: [6]

$$\Omega_0 h^2 \simeq \frac{s_0 g^2 x_R^{-7}}{36\pi^6 H_0^2 M_{\rm pl}} \left(\frac{90}{g_*} \right)^{\frac{3}{2}} \mathcal{F}(x_{\rm max}) \,, \tag{4}$$

where $M_{\rm pl} \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \cdot 10^{18}$ GeV is the reduced Planck scale, g the number of degrees of freedom associated with the stable particle (for instance, $g = 12$ for a quark), g_* the effective number of relativistic degrees of freedom produced by the reheating mechanism (for example, $g_* = 106.75$ accounts for all the SM particles, which are relativistic for $T \gtrsim 300\,{\rm GeV}$), "max" denotes the maximum temperature reached by the thermal bath, $x = M/T$ (where M denotes the mass of the stable particle), $H_0 = 100\,{\rm km/s\ Mpc^{-1}}$, $s_0 \simeq 2890\,{\rm cm^{-3}}$ the present entropy density, and

$$\mathcal{F}(y) = M^2 \int_y^\infty \langle \sigma v \rangle\, x^8 e^{-2x}\, dx \,. \tag{5}$$

Typically, the annihilation cross section is dominated by a particular wave channel characterized by an integer number j as:

$$\langle \sigma v \rangle \simeq M^{-2} c_j x^{-j} \,, \tag{6}$$

* Electronic address: cembra@fis.ucm.es

so $\mathcal{F}(y)$ can be written in terms of the incomplete Gamma function: [6]

$$\mathcal{F}(y) \simeq \frac{\Gamma(9-j, 2y)}{2^{9-j}} c_j \simeq \begin{cases} \frac{(8-j)!}{2^{9-j}} c_j \,, & y \ll 3\,; \\ \frac{y^{8-j}}{2e^{2y}} c_j \,, & y \gg 3\,. \end{cases} \tag{7}$$

Therefore, if $M \lesssim 3\,T_{\max}$, the abundance is suppressed by $(T_R/M)^7$ (by assuming $M \gg T_R$) due to an important entropy production before reheating. On the other hand, if $M \gg T_{\max}$, the abundance is exponentially suppressed by the Boltzmann statistical factor.

Concrete restrictions about the maximum temperature have not been established, but its value is bounded from below by the reheat temperature. The agreement of the observations of primordial abundances with the predictions of the Big Bang nucleosynthesis model implies $T_R \gtrsim 2\,\mathrm{MeV}$ [7, 8]. This constraint can be slightly improved by taking into account more cosmological data ($T_R \gtrsim 4\,\mathrm{MeV}$ [9]).

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