DARK MATTER

Problem Sheet 2: The Homogeneous Universe

1. **The homogeneous Universe**

   (a) In the notes we proved that an isotropic Universe must be homogeneous. Is the converse true? If not, come up with a counter example.

2. **The Friedmann equation**

   The Friedmann equation can be written as:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4} - (\Omega_0 - 1)a^{-2}\right] \tag{1}$$

   where $H_0 = 70\,\mathrm{km/s/Mpc}$ is Hubble's constant; $\Omega_\Lambda = 0.7$, $\Omega_m = 0.3$, and $\Omega_r = 8 \times 10^{-5}$ are the dark energy, matter, and radiation contributions to the total energy density of the Universe at the current time; and $\Omega_0 = \Omega_\Lambda + \Omega_m + \Omega_r$.

   (a) Calculate the age of the Universe for a matter dominated flat Universe ($\Omega_m = \Omega_0 = 1; \Omega_\Lambda = \Omega_r = 0$).

   (b) Compare this age with the age of the oldest stars in the Milky Way. What do you think this means?

   (c) Derive $\ddot{a}$ from the Friedmann equation. Use this to derive the relationship between Ω_Λ and $a = a_0 =$ const. for an eternal static Universe.

   (d) Why can't I construct an eternal static Universe just using curvature with $\Omega_\Lambda = 0$?

   (e) Calculate the redshift at which radiation dominates over all other terms in the Friedmann equation.