Exercise sheet III: Misalignment mechanism

Jose A. R. Cembranos *

Departamento de Física Teórica I, Universidad Complutense de Madrid, E-28040 Madrid, Spain.

III.a.- Estimate the abundance associated with the QCD axion from the misalignment mechanism taking into account the QCD axion mass (m) dependence with temperature:

$$m(T) \simeq 0.1\, m_0 \left[\frac{100\text{ MeV}}{T}\right]^{3.7}, \quad T > 100\,, \tag{1}$$

with $m_0 = m(T=0)$.

In order to reproduce the expressions that can be found in the literature, the misalignment of the axion field is usually written in terms of the misaligment angle:

$$\phi_1 \equiv \theta_1 f_a\,. \tag{2}$$

In the case of the QCD axion, the (zero temperature) mass and the Peccei-Quinn scale are related by the QCD scale ($\Lambda_{\rm QCD} \simeq 100$ MeV):

$$m_0 \simeq \Lambda_{\rm QCD}^2/f_a \simeq 0.6\times 10^{-4}\,\text{eV}\left(\frac{10^{11}\text{ GeV}}{f_a}\right). \tag{3}$$

ABUNDANCE FROM THE MISALIGNMENT MECHANISM

There is another abundance source for scalars or generic bosonic particles, such as axions [1]. These fields may have associated big abundances through the so called *misalignment mechanism*. There is no reason to expect that the initial value of the scalar field (ϕ_1) should coincide with the minimum of its potential ($\phi = 0$) if $H(T) \gg m_0$. Below the temperature T_1 for which $3H(T_1) \simeq m_0$, ϕ behaves as a standard scalar. It oscillates around the minimum. These oscillations correspond to a zero-momentum condensate, whose initial number density: $n_\phi \sim m_0\phi_1^2/2$ (where ϕ_1 is the maximum value of ϕ in the oscillations at T_1), will evolve as the typical one associated to standard non-relativistic matter.

Taking into account that the number density of scalar particles scales as the entropy density of radiation ($s = 2\pi^2 g_{s1}T_1^3/45$) in an adiabatic expansion, we can write:

$$\Omega_\phi h^2 \simeq \frac{(n_\phi/s)(s_0/\gamma_{s1})}{\rho_{crit}}\, m_0\,, \tag{4}$$

where $\rho_{crit} \equiv 1.0540\times 10^4\,\text{eV}\,\text{cm}^{-3}$ is the critical density, $s_0 = 2970\,\text{cm}^{-3}$ is the present entropy density of the radiation, and γ_{s1} is the factor that this entropy has increased in a comoving volume since the onset of scalar oscillations.

If we supposed a radiation dominated universe at T_1 ($3H_1 = \pi(g_{e\,1}/10)^{1/2}T_1^2/M_{\rm Pl}$), we can estimate T_1 by solving $m_0 = 3H_1(T_1)$:

$$T_1 \simeq 15.5\,\text{TeV}\left[\frac{m_0}{1\,\text{eV}}\right]^{\frac{1}{2}}\left[\frac{100}{g_{e\,1}}\right]^{\frac{1}{4}}, \tag{5}$$

and calculate the abundance as: [2]

$$\Omega_\phi h^2 \simeq 0.86\left[\frac{m_0}{1\,\text{eV}}\right]^{\frac{1}{2}}\left[\frac{\phi_1}{10^{12}\,\text{GeV}}\right]^2\left[\frac{100\, g_{e\,1}^3}{(\gamma_{s1}g_{s1})^4}\right]^{\frac{1}{4}}, \tag{6}$$

* Electronic address: cembra@fis.ucm.es

where $g_{e\,1}$ (g_{s1}) are the effective energy (entropy) number of relativistic degrees of freedom at T_1. We see that initial values for the scalar field of order of $\phi_1 \sim 10^{12}$ GeV can lead to the non-baryonic DM (NBDM) abundance depending on the rest of parameters and the early physics of the Universe.

[1] J. Preskill, M.B. Wise and F. Wilczek, *Phys. Lett.* **B120**, 127 (1983); J.A. Frieman and A.H. Jaffe, *Phys. Rev.* **D45**, 2674 (1992).
[2] J. A. R. Cembranos, Phys. Rev. Lett. **102**, 141301 (2009) [arXiv:0809.1653 [hep-ph]].