DARK MATTER

Problem Sheet 3: The Inhomogeneous Universe

1. Show that the evolution equation for an overdensity at linear order for a non-relativistic fluid is given by:

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_0\delta + \frac{c_s^2}{a^2}\nabla_r^2\delta \tag{1}$$

2. Assuming a single plane wave form for $\delta(\mathbf{r}, t)$ and neglecting the $\dot{a}$ term, derive the 'Jeans length' for this fluid.

3. A relativistic fluid has a similar evolution equation (if you are feeling keen you can derive this one too!):

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = \frac{32\pi}{3}G\rho_0\delta + \frac{c_s^2}{a^2}\nabla_r^2\delta \tag{2}$$

4. How does the Jeans length for this relativistic fluid compare with the non-relativistic case? What do you think this means?